NATIONAL TEACHING COUNCIL

Numeracy

For

Ghana Teacher Licensure Examinations

ALASCA

0540770189

Chapter One

INVESTIGATION WITH NUMBERS AND NUMBER PATTERN

This chapter seeks to test teachers understanding on how to create, recognise, describe, extend and make generalizations about numeric patterns and how to practically use them in solving a typical classroom and educationally related problems. Patterns allow us to make predictions and in exploring of numbers in a variety of contexts.

A list of numbers which form a pattern is called a sequence. Each number in a sequence is called a term of the sequence. The first number is the first term of the sequence. For example, 14, 28, 20, 40, 32, 64 is a sequence/series because it forms a pattern. The first term is 14, second term is 28, third term is 20 and so on.

Numerical Pattern Rules

A numerical pattern is a sequence of numbers that has been created based on a formula or rule called a **pattern rule**. Pattern rules can use one or more mathematical operations to describe the relationship between consecutive numbers in the pattern.

There are two primary categories of numerical patterns.

1. When numbers in a pattern get larger as the sequence continues, they are in an **ascending pattern**. Ascending patterns often involve multiplication or addition.
   Example; 2, 5, 8, 11, 14, 17, 20, 23

2. When numbers in a pattern get smaller as the sequence continues, they are in a **descending pattern**. Descending patterns often involve division or subtraction.
   Example; 23, 20, 17, 14, 11, 8, 5, 2,

To figure out the pattern rule you must determine how the consecutive numbers are related.

Here are examples.

a. Find the pattern rule for the sequence: 243, 81, 27, 9.

Solution

i. First, take an overview of the numbers. The numbers get smaller in value as the sequence continues, so this is a descending pattern. This means the rule likely involves division or subtraction.

ii. Look at the smaller numbers at the end of the sequence. Think:"What could you do to 27 to get 9?" You could subtract 18. You could divide by 3. You could do a combination of two or more operations.

iii. Next, check if any of these potential pattern rules work with the rest of the sequence. Consider 81 and 27. If you subtract 18 from 81 you get 63, not 27. So the pattern rule is "not subtract 18." If you divide 81 by 3 you get 27. So the pattern rule "divide by 3" seems to work.

iv. Now, make sure "divide by 3" works throughout the whole sequence. "Divide by 3" works for the whole sequence. **The answer is that pattern rule is "divide by 3."**

b. Find the pattern rule for the sequence: 1, 3, 11, 43.

Solution

i. First, take an overview of the numbers. The numbers get larger in value as the sequence continues, so this is an ascending pattern. This means the rule likely involves multiplication or addition.

ii. Look at the smaller numbers at the beginning of the sequence. Think: "What could you do to 1 to get 3?" You could add 2. You could multiply by 3. You could do a combination of two or more operations.

iii. Next, check if any of these potential pattern rules work with the rest of the sequence. Consider 3 and 11. If you add 2 to 3 you get 5, not 11. So the pattern rule is not "add 2." If you multiply 3 by 3 you get 9, not 11. So the pattern rule is not "multiply by 3."

Since neither of those pattern rules work, the pattern rule must involve more than one operation. Notice how the jump between the numbers increases each time as you move through the sequence. This means multiplication must be involved, but addition or subtraction will be involved as well.

iv. Next, consider possible pattern rules that involve multiplication and addition or subtraction. Think: "What else can you do to 1 to get 3?" You could multiply by 2 and add 1. You could multiply by 4 and subtract 1. You could do some other combination of two or more operations. Now, look back at the rest of the sequence.

v. Again consider 3 and 11. If you multiply 3 by 2 and add 1 you get 7, not 11. So the pattern rule is not "multiply by 2 and add 1." If you multiply 3 by 4 and subtract 1 you get 11. So the pattern rule "multiply by 4 and subtract 1" seems to work.

vi. Now, make sure "multiply by 4 and subtract 1" works throughout the whole sequence. "Multiply by 4 and subtract 1" works for the whole sequence.

The answer is that the pattern rule is "multiply by 4 and then subtract 1."

Worked Examples:

Look at these series/sequence below and write down the next three numbers that should come next. Also explain in writing, in each case, how you figured out what the numbers should be.

1. 2, 5, 8, 11, 14, 17, 20, 23,
2. 2, 6, 18, 54, 162, 486,
3. =22, 21, 23, 22, 24, 23,
4. 7, 10, 8, 11, 9, 12,
5. 1, 5, 9, 13, 17, 21, 25, 29, 33,
6. 31, 29,24, 22, 17

Solution:

1. 2, 5, 8, 11, 14, 17, 20, 23,

A look at the first two numbers/terms (2, 5) in the sequence you will realize that you need to add 3 to go from 2 to 5. Again, a look at the next number/term (8) you will realize that you need to add 3 to go from 5 to 8. When you continue to test it works for all the next numbers.

RULE: This gives **a rule you can use to extend the sequence: add 3 to each number to find the next number in the pattern**.

You can equally find the pattern by working backwards and subtracting 3 each time. [14-3=11; 11-3=8; 8-3=5; 5-3=2]

Therefore, the next three numbers will be 26, 29, 32.

2. 2, 6, 18, 54, 162, 486,

Look at the first two terms (2, 6) in the sequence. Multiply the first number by 3, you will get the second number (2 × 3 = 6). Check to see if you can find the next number if you multiply 6 by 3 (6 × 3 = 18). Continue checking in this way: (18×3 = 54; 54×3 = 162; 54×3 = 162) and so on.

RULE: This gives you a rule you can use to extend the sequence **and the rule is: multiply each number by 3 to calculate the next number in the sequence.**

You can also find the pattern by working backwards and dividing by 3 each time:
[54/3=18; 18\3=6; 6\3=2]

Therefore, the next three numbers will be 1458, 4374, 13122.

3. 22, 21, 23, 22, 24, 23,

Look at the first two terms (22, 21) in the sequence. Subtract 1 from the first term you will get the second term (22-1=21). Check to see if you can find the next number (23) if you add 2 to 21 (2 + 21 = 23). Continue checking in this way: (22-1=21; 21+2=23; 23-1=22; 22+2=24; 24-1=23; 23+2 =25) and so on.

RULE: This gives you a rule you can use to extend the sequence **and the rule is: subtract 1 from the first term, add 2 to the next in that order to calculate the next number in the sequence. That's -1 +2**

Therefore, the next three numbers will be 25, 24, 26.

4. 7, 10, 8, 11, 9, 12,

To find the next series, add 3 to the first term (7+3) you will get the second term (10). Again, subtract 2 from the 10, (10-2) you will get the third term (8). Continue checking in this way: (8+3=11; 11-2=9; 9+3=12; 12-2=10; 10+3=13) and so on

RULE: This gives you a rule you can use to extend the sequence and the rule is +3 -2

Therefore, the next three numbers will be 10, 13, 11.

5. 1, 5, 9, 13, 17, 21, 25, 29, 33,

A look at the first two numbers/terms (1, 5) in the sequence you will realize that you need to add 4 to go from 1 to 5. Again, a look at the next number/term (9) you will realize that you need to add 4 to go from 5 to 9. When you continue to test this way (4+9=13; 13+4=17; 17+4=21; 21+4=25) and so on, you will get the next numbers.

RULE: This gives **a rule you can use to extend the sequence: add 4 to each number to find the next number in the pattern**.

Therefore, the next three numbers will be 37, 41, 45.

6. 31, 29,24, 22, 17

(31-2=29; 29-5=24; 24-2=22; 22-5=17; 17-2=15; 15-5=10)

RULE: -2 -5

Therefore, the next three numbers will be 15, 10, 8.

Review Questions

i. Look at these series/sequence below and write down the next three numbers that should come next. Also explain in writing, in each case, how you figured out what the numbers should be.

1. 1; 2; 4; 8; 16; 32; 64;
2. 3, 5, 7, 9, 11, 13, 15, 17, 19,
3. 4; 5; 7; 10; 14; 19; 25; 32; 40;
4. 4, 5, 8, 13, 20, 29, 40,
5. 2; 4; 8; 16; 32; 64;
6. A/2, B/4, C/6, D/8, ?, ?,

ii. Find the pattern rules for the following numerical patterns.

1. 95, 80, 65, 50
2. 3, 10, 17, 24
3. 3, 11, 43, 171
4. 81, 27, 9, 3
5. 4, 13, 40, 121
6. 3, 18, 108, 648
7. 100, 90, 80, 70
8. 45, 15, 5
9. 142, 70, 34, 16
10. 900, 300, 100

iii. Describe, in words, the rule for finding the next number in the sequence. Also write down the next five terms of the sequence if the pattern is continued.

1. 1; 10; 100; 1 000;
2. 16; 8; 4; 2;
3. 7; -21; 63; -189;
4. 3; 12, 48;

iv. Consider sequences A to H again and answer the questions that follow:

1. A: 2; 5; 8; 11; 14; 17; 20; 23; ...
2. B: 4; 5; 8; 13; 20; 29; 40;...
3. C: 1; 2; 4; 8; 16; 32; 64;...
4. D: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...
5. E: 4; 5; 7; 10; 14; 19; 25; 32; 40;...
6. F: 2; 6; 18; 54; 162; 486;...
7. G: 1; 5; 9; 13; 17; 21; 25; 29; 33;...
8. H: 2; 4; 8; 16; 32; 64;...

1. Which other sequence(s) is/are of the same kind as sequence B? Explain.

2. In what way are sequences B and E different from the other sequences?

v. Abrahim's school just won a contest! Ten lucky students will get to go to USA to participate in a science and math program. In order to determine who will get to go on the trip, the Headmaster, Mr Mensah assigns each of the 150 interested students a number between 1 and 150. Then, Mr Mensah starts listing off the numbers for the students who will be able to attend: 12, 27, 42, 57, ...

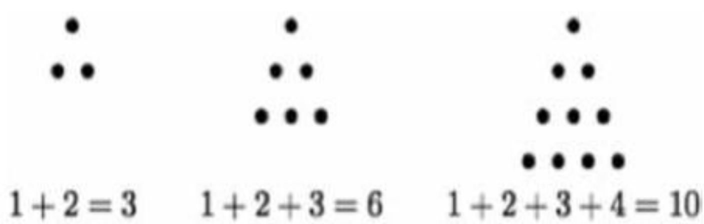
Abrahim realizes there is a pattern to the numbers being called. How is the Headmaster choosing which numbers to call? If Abrahim has number 82 will he get to go on the trip?

vi. Consider the sequence: 10; 17; 26; 37; 50; ...

1. Write down the next five numbers in the sequence.
2. Eric observed that he can calculate the next term in the sequence as follows: 10 + **7** = 17; 17 + **9** = 26; 26 + **11** = 37. Use Eric's method to check whether your numbers in question (a) above are correct.

Figurative Numbers

Triangular Number: is the number of dots needed to construct a triangle. Example, **T1=1; T2= 1+2=3; T3=1+2+3=6.** Therefore, Tn = n(n=1)/2



Triangular numbers

Example

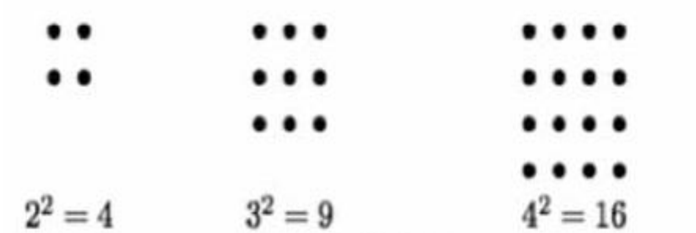
Find the 7th triangular numbers.

Solution:

From the formula, Tn = n(n+1)/2T7= 7(7+1)/2 = 7(8)/2 =28

Square Numbers:

Square Number: Is the number of dots needed to construct squares of varying sizes. Example, **S1=1; S2=2+2=4** Therefore, $Sn=n\times n=n^2$



Square numbers

Example

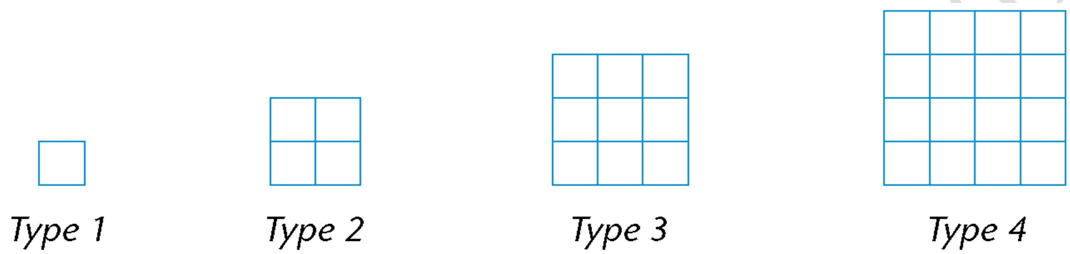
Find the 7th square numbers.

Solution:

From the formula, $S_n=n\times n=n^2$ $S_7=7\times 7=49$

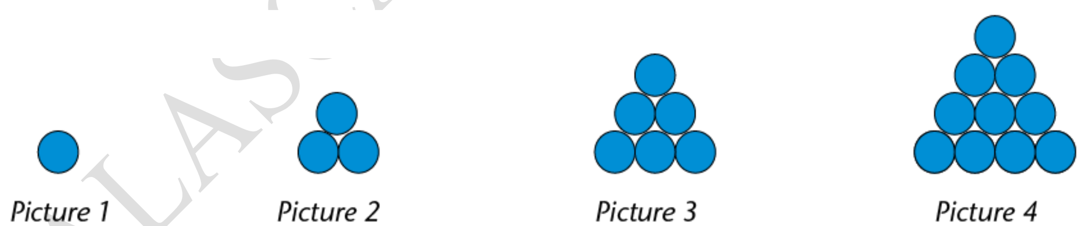
Review Question

1. A factory makes window frames. Type 1 has one windowpane, type 2 has four windowpanes, type 3 has nine windowpanes, and so on.



1. How many windowpanes will there be in type 5?
2. How many windowpanes will there be in type 6?
3. How many windowpanes will there be in type 7?
4. How many windowpanes will there be in type 12? Explain

2. Vincentia uses circles to form a pattern of triangular shapes:

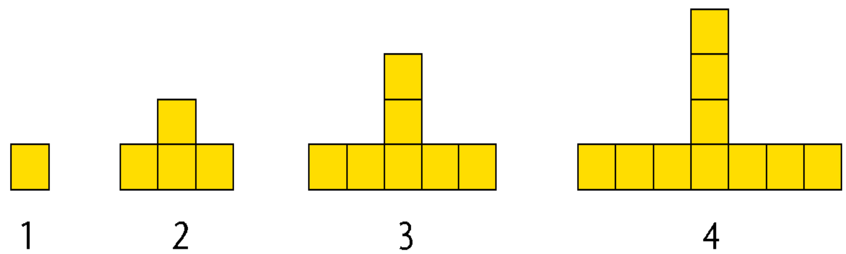


If the pattern is continued, how many circles must Vincentia have

1. in the bottom row of picture 5?
2. in the second row from the bottom of picture 5?
3. in the third row from the bottom of picture 5?
4. in the second row from the top of picture 5?

5. in the top row of picture 5?
6. in total in picture 5? Show your calculation.
7. How many circles does Therese need to form triangle picture 7? Show the calculation.
8. How many circles does Therese need to form triangle picture 8?

3. The pattern below is made from squares.



1. How many squares will there be in pattern 5?
2. How many squares will there be in pattern 15?

Chapter Two

FRACTIONS DECIMALS & PERCENTAGES

Let's say three of eight slices of a pizza have been eaten. What part of the pizza remains? If a sprinter runs a 40-yard dash in more than four seconds but less than five seconds, how can we represent the sprinter's exact time? A student answers 22 of 25 test questions correctly. How can we express the student's grade as a percent on the test?

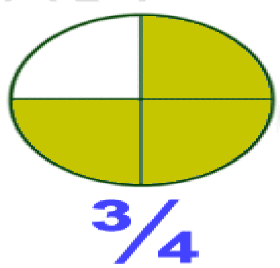
All of those questions can be answered using fractions, decimals, and percents. We need fractions, decimals, and percents to represent parts of a whole. Many real-life math problems involve whole numbers, but even more involve decimals.

This chapter intends to test teachers' knowledge on how to apply the basic concepts of fraction decimals and percentages to solve practical questions.

FRACTIONS

Fractions express parts of a whole. Fractions are given in the form of a numerator and a denominator. The denominator indicates what size part is being expressed and the numerator indicates how many of those parts are involved. For example, in $\frac{6}{7}$ the denominator (7) indicates that each part is 1 seventh of the whole. The numerator (6) indicates that there are 6 of these parts.

For example, consider the figure below with 3 of the 4 equal parts colored. We say that (three-fourth) of the figure is shaded.



When writing a fraction there are two main parts: the numerator and the denominator. The number above the fraction bar is called the numerator, and the number below is called the denominator. The numerator is how many parts you have. The denominator is how many parts the whole was divided into.

Types of Fractions

1. **Proper Fractions:** The numerator is less than the denominator. Examples: $\frac{1}{3}, \frac{3}{4}, \frac{2}{7}$
2. **Improper Fractions:** The numerator is greater than (or equal to) the denominator. Examples: $\frac{4}{3}, \frac{11}{4}, \frac{7}{7}$
3. **Mixed Fractions:** A whole number and proper fraction together. Examples: $1\frac{1}{3}, 2\frac{1}{4}, 16\frac{1}{5}$

Other Types of Fractions are:

4. **Like Fraction**: Fractions that have the same denominators are like fractions. For example, the fractions: $\frac{2}{7}, \frac{3}{7}$, and $\frac{6}{7}$ all have the same denominator – 7. Hence, these are like fractions.

   Simplification of like fractions is easy. For example, if you want to add the above three fractions, all you have to do is add the numerators. The denominator will remain the same. So, $\frac{2}{7}, + \frac{3}{7}, + \frac{6}{7} = \frac{11}{7}$

5. **Unlike Fraction**: Fractions that have different denominators are unlike fractions. For example, the fractions: $\frac{2}{3}$ and $\frac{1}{4}$ have different denominators. So, they are unlike fractions.

   Simplifications involving unlike fractions are not as straightforward as like fractions. You need to determine the LCM of the denominators. This LCM will be the denominator of both the fractions. For example, the above fraction is added as follows: $\frac{2}{3} + \frac{1}{4} = \frac{8+3}{12} = \frac{11}{12}$

6. **Equivalent Fraction:** Fractions that give the same value are called equivalent fractions upon simplification. Any fraction can be changed into an equivalent fraction by multiplying both the numerator and denominator by the same number Example, write $\frac{3}{5}$ as an equivalent fraction with a denominator of 35. To answer the question "What number times 5 equals 35?" we divide 35 by 5 to get 7. $\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$

7. **Unit Fractions:** Fraction whose numerator is one and the denominator is a positive integer is called a unit fraction. Examples of unit fractions are $\mathbf{\frac{1}{2}, \frac{1}{5}, \frac{2}{8}}$ etc.

Simplifying a Fraction (Reducing to its Lowest Terms)

It is usual to reduce a fraction until it can't be reduced any further. A simplified fraction has no common factors other than 1 which will divide into both numerator and denominator.

Example:

Simplify the following fractions:

1. $\frac{120}{200}$
2. $\frac{20}{25}$
3. $\frac{108}{144}$
4. $\frac{6}{8}$
5. $\frac{14}{21}$
6. $\frac{18}{80}$
7. Are the following fractions in simplest form?
   a. $\frac{12}{27}$ b. $\frac{5}{8}$

8. How will you help a Basic five pupil to simplify $\frac{90}{105}$?

Solution

1. $\frac{120}{200} = \frac{120 \div 40}{200 \div 40} = \frac{3}{5}$
2. $\frac{20}{25} = \frac{20 \div 5}{25 \div 5} = \frac{4}{5}$
3. $\frac{108}{144} = \frac{108 \div 36}{144 \div 36} = \frac{3}{4}$
4. $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$
5. $\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$
6. $\frac{18}{80} = \frac{18 \div 2}{80 \div 2} = \frac{9}{40}$
7. **a.** The factors of the numerator, 12, are: 1,2,3,4,6,12
   The factors of the denominator, 27, are: 1, 3,9,27
   Since the numerator and denominator have a common factor of 3, the fraction is $\frac{12}{27}$ not in simplest form.

   b. The factors of the numerator, 5, are: 1,5
   The factors of the denominator, 8, are: 1,2,4,8
   Since the only common factor of the numerator and denominator is 1, the fraction is $\frac{5}{8}$ in simplest form.

Adding and Subtracting Fractions

Rule: $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$

Examples

1. Evaluate the following fractions
   a. $\frac{2}{5} + \frac{4}{5}$
   b. $\frac{4}{2} + \frac{6}{10}$
   c. $\frac{2}{1} - \frac{4}{5}$
   d. $\frac{7}{10} + \frac{4}{5} - \frac{2}{3}$

Solution

a. $\frac{2}{5} + \frac{4}{5} = \frac{2+4}{5} = \frac{6}{5}$
b. $\frac{4}{2} + \frac{6}{10} = \frac{20+6}{10} = \frac{26}{10} = 2\frac{6}{10}$
c. $\frac{2}{1} - \frac{4}{5} = \frac{10-4}{5} = \frac{6}{5} = 1\frac{1}{5}$
d. $\frac{7}{10} + \frac{4}{5} - \frac{2}{3} = \frac{21+24-20}{30} = \frac{25}{30}$

The following are some application questions on addition and subtraction of fractions we will solve them in our online master class.

1. A pizza is covered with multiple toppings. $\frac{1}{8}$ of the pizza is covered in meat, $\frac{1}{5}$ of the pizza is covered with onions, and the rest of the pizza is covered with only cheese. What fraction of the pizza is covered with only cheese?

2. Ayisah baked 4 pizzas for a dinner party. She topped $2\frac{1}{9}$ of the pizzas with hot peppers and $\frac{5}{9}$ of the pizzas with mushrooms. She put cold pineapple slices on the rest of the pizza. How much of her pizza was topped with cold pineapple slices?

3. Alasca baked $2\frac{1}{7}$ pizzas and ordered another $\frac{5}{7}$ of a pizza from his local pizzeria. If he shares $\mathbf{1}\frac{2}{7}$ of his pizza with his friends, how much pizza does he have left over?

4. You give $\frac{1}{3}$ of a pan of brownies to Susan and 1/6 of the pan of brownies to Patrick. How much of the pan of brownies did you give away?

5. A school wants to make a new playground by cleaning up an abandoned lot that is shaped like a rectangle. They give the job of planning the playground to a group of students. The

students decide to use 1/4 of the playground for a basketball court and 3/8 of the playground for a soccer field. How much is left for the swings and play equipment?

Multiplication and Division of Fractions

Multiplication Rule: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$

Division Rule: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

Example:

Evaluate the following fractions

1. $\frac{5}{1} \times \frac{8}{10}$
2. $\frac{10}{20} \div \frac{6}{8}$

Solution:

1. $\frac{5}{1} \times \frac{8}{10} = \frac{5 \times 8}{1 \times 10} = \frac{40}{10}$
2. $\frac{10}{20} \div \frac{6}{8} = \frac{10}{20} \times \frac{8}{6} = \frac{10 \times 8}{20 \times 6} = \frac{80}{120}$

Here are some practical questions on multiplication and division of fractions we will solve together

1. Jackson’s mother buys 4 packs of cold fish. Each pack contains $\frac{4}{6}$ of a pound of meat. How much total meat did she buy?

2. Keisha is making potato casserole for a dinner party. She needs $\frac{1}{2}$ of a potato per guest. How many potatoes will she need for 9 guests?

3. Meandre has 8 pints of milk. If he drinks $\frac{1}{4}$ of a pint of milk each day, how long will the 8 pints of milk last him?

4. Mr. Simpson feeds his dog $\frac{1}{2}$ of a cup of dog food each day. How long will 4 cups of dog food last?

5. After a party, 1/3 of a pizza remains. Rachel eats 1/2 of the remaining pizza. What fraction of a pizza does she eat?

Converting a Decimal to a Fraction

A decimal such as 0.7 can be written as a fraction by remembering that the first decimal represents tenths, the second hundredths the third thousandths and so on.

Examples:

Converting the following decimal numbers into fractions

1. $0.7 = \frac{7}{10}$
2. $0.46 = \frac{46}{100}$
3. $0.234 = \frac{234}{1000}$

Application of Factions-Word Problem

1. One half of the students in a school are girls, $\frac{3}{5}$ of these girls are studying in lower classes. What fraction of girls are studying in lower classes?

2. In a music school there were 30 children. One-fifth of the children learnt piano, two-fifths of the children learnt guitar and the rest of the children learnt drums. What fraction of children learnt only drums?

3. Mike reads three-fifth of 75 pages of his lesson. How many more pages he needs to complete the lesson?

4. A herd of cows gives 4 litres of milk each day. But each cow gives one-third of total milk each day. They give 24 litres milk in six days. How many cows are there in the herd?

5. James went to a shop to buy a refrigerator. The cost of brand A refrigerator was GHC350.00. The cost of brand B refrigerator was one-fifth lesser than the price of brand A refrigerator. What is the price of brand B refrigerator?

Solution:

1. Fraction of girls studying in school = $\frac{1}{2}$

Fraction of girls studying in lower classes = $\frac{3}{5}$ of $\frac{1}{2}$

$$\frac{3}{5} \times \frac{1}{2} = \frac{3\times1}{5\times2} = \frac{3}{10}$$

2. Fraction of children learning piano= $\frac{1}{5}$

Fraction of children learning guitar= $\frac{2}{5}$

Fraction of children learning drums= $1 - \frac{1}{5} + \frac{2}{5}$

$1 - \left(\frac{1+2}{5}\right)$

$1 - \left(\frac{3}{5}\right)$

$\frac{5-3}{5} = \frac{2}{5}$

Therefore fraction of children learning drums=

$\frac{2}{5} \times 30 = 12$

3. Mike reads = $\frac{3}{5}$ of 75 = $\frac{3}{5} \times 75$ = 45 pages.

Mike has to read = 75 – 45 = 30 pages.

Therefore, Mike has to read 30 more pages.

4. A herd of cows gives 4 litres of milk each day.

Each cow gives one-third of total milk each day = 1/3 of 4

Therefore, each cow gives 4/3 of milk each day.

Total no. of cows = 4 ÷ 4/3

= 4 × ¾

= 3

Therefore there are 3 cows in the herd.

Revise Questions

1. $\frac{2}{9}$ of the people on a restaurant are adults. If there are 95 more children than adults, how many children are there in the restaurant?

2. Gary and Henry brought an equal amount of money for shopping. Gary spent GHC95 and Henry spent GHC350. After that Henry had $\frac{4}{7}$ of what Gary had left. How much money did Gary have left after shopping?

3. Jenny's mom says she has an hour before it's bedtime. Jenny spends $\frac{3}{5}$ of the hour texting a friend and $\frac{3}{3}$ of the remaining time brushing her teeth and putting on her pajamas. She spends the rest of the time reading her book. How long did Jenny read?

4. John had GHC5.00. His mom gave him GHC5.00 for washing dishes. John went to the store and bought a note book for GHC6.00. How much money does John have left?

5. Nancy wants to sell 54 oranges to her 6 friends for GHC5.00 each. How much will Nancy receive from selling her oranges? If each friend buys the same amount of orange, how many will each friend buy?

DECIMALS

Decimals are used widely. Our money is based on a decimal system. Common measurements such as the height of students in a class, the price of petrol or the amount of gas used for cooking, the time taken to perform an act are usually expressed using decimals.

Decimals can be used to represent both rational and irrational numbers. They are often used where measurements are taken and recorded to a given accuracy. Decimals lend themselves easily to approximations which can be given to any stated accuracy.

Comparing Decimals

One method for comparing decimals is to write the numbers one underneath the other, with the decimal points lined up.

For example, to compare 3.78 and 3.612, we first write the numbers one beneath the other, aligning the decimal points. As shown

$$\begin{array}{r} 3.780 \\ \underline{3.612} \\ \hline \end{array}$$

Next, we compare the whole number parts. Both are the same, so we proceed to the next digit. We compare the tenths: 7 tenths is larger than 6 tenths. So 3.78 is larger than 3.612.

Addition and Subtraction of Decimals

Adding Decimal Numbers

We can always relate the addition of decimal numbers to addition of fractions.

Example:

How will guide a Basic five pupil to add 4.2 and 5.09

$$4.2 + 5.09$$

$4\frac{2}{10} + 5\frac{9}{100}$ (chage the decimal into fraction)

$4\ \frac{20}{100} + 5\ \frac{9}{100}$ (make the denominators the same)

$9\ \frac{29}{100}$ (proceed to add the fractions)

9.29 (change the frsction to decimal)

Alternatively, we can use the decimal notation which gives us an easier way to calculate such sums.

$$\begin{array}{r} 4.20 \\ +5.09 \\ \hline 9.29 \\ \hline \end{array}$$

The decimal point in each number should be lined up one under the other. The vertical addition algorithm is shorthand for adding hundreds to hundreds, tens to tens, ones to ones, tenths to tenths, and so on. It is important to line up the place value columns when lining up the decimal point.

Subtracting Decimals Numbers

When subtracting one decimal from another, write the numbers one under the other as with whole number subtraction, making sure the decimal points are aligned.

Here are the two standard subtraction algorithms showing the methods used for subtracting 16.532 from 23.84.

$$\begin{array}{r} 23.840 \\ -\ 16.532 \\ \hline 7.308 \\ \hline \end{array}$$

Sometimes, the number of decimal places is different in each of the numbers. It is helpful when doing subtractions that deal with these ‘ragged’ decimals to place additional zeroes at the end of the top number (in this case 23.84). This does not change the number; it simply says that there are ‘no thousandths’.

Multiplication and Division Decimals

Multiplication of Decimals

A very common student error is to write $0.3 \times 0.2 = 0.6$. Clearly that is to be avoided and suggests that multiplying one decimal by another is best done by converting the decimals to fractions. The fractions can be multiplied in the usual way and converted back to decimals. For example,

$$
\begin{aligned}
&0.6 \times 0.4 \\
&= \tfrac{6}{10} \times \tfrac{4}{10} \text{(converted into fraction)} \\
&= \tfrac{6 \times 4}{10 \times 10} = \tfrac{24}{100} \text{(convert again to fraction)} \\
&= 0.24
\end{aligned}
$$

There is another commonly used method for multiplying decimals. Students seem to not carry out this method well. It was the pre-calculator method that was used widely.

The steps are as follows:

1. Ignore the decimal points and multiply the factors as if they were whole numbers.
2. Insert a decimal point in the product so that the total number of decimal places is the same on both sides of the equation.

For example:

$$
\begin{aligned}
&2.5 \times 0.06 \\
&\text{Step 1: } 25 \times 6 = 150 \\
&\text{Step 2: } 2.5 \times 0.06 = .150 = 0.15
\end{aligned}
$$

Division of Decimals

Division of decimals by whole numbers

Consider this statement:

If you had a piece of rope that was 4.8 metres long and wanted to cut it into 4 equal pieces you could cut it in two and then cut each piece in two again to get 4 pieces, each 1.2 metres in length. We can use the division algorithm to show this.

$$
\begin{array}{r}
1.2 \\
4\overline{)\,4.8}
\end{array}
$$

The procedure is the same as for whole numbers. The decimal point in the answer is aligned directly above the decimal point in the question. We can relate this to division by fractions (invert and multiply).

$$4.8 \div 4 = \frac{48}{10} \div \frac{4}{1} = \frac{\cancel{48}}{10} \times \frac{1}{\cancel{4}} = \frac{\cancel{12}}{\cancel{10}}\ \frac{6}{5} = 1.2$$

Dividing Decimals by Decimals

When the numbers are 'nice' it is possible to perform some divisions by decimal using common sense. For example

1.8 ÷ 0.3

By multiplying each number by the same multiple of ten the division is easier to deal with.

$$1.8 \div 0.3 = \frac{1.8\times10}{0.3\times10} = \frac{18}{3} = 6$$

This suggests that we multiply both numbers by the same power of 10 to make the divisor a whole number and then use the division algorithm.

1.355 ÷ 0.05 = 135.5 ÷ 5

Alternatively we can relate division of decimals to division of fractions by writing each decimal as a fraction and completing the division.

$$0.6 \div 0.4 = \frac{6}{10} \div \frac{4}{10} = \frac{6}{10} \times \frac{10}{4} = \frac{60}{40} = \frac{6}{4}$$

PERCENTAGES

A percent is a number out of 100. We use the symbol % to represent the word percent. The value 36% is read as "36 percent" and represents 36 out of 100.

Let's consider this statement

67% of Abena's marbles are red. What does this mean? 67% represents "67 out of 100." For every 100 marbles Abena has 67 of them are red.

Let's look at another example

28 of every 100 pizzas that Gifty makes are flour pizzas. What percent of Gifty's pizzas are flour pizzas?

Solution
28 out of 100 is 28%. If Gifty made 100 pizzas, 28 of them would be flour but what if Gifty made only 75 pizzas? Before we can answer this question, we must learn how to write percents as decimals.

Writing Percents as Decimals

We can rewrite a percent as a decimal by removing the percent symbol and dividing the number by 100. This causes the decimal to move two places to the left: example, 28% = 0.28.

Example
What is 43% as a decimal?

You know that 43% is 43 out of 100, or 43 hundredths. Moving the decimal point two places to the left (or dividing by 100), 43% becomes 0.43.

Finding the Percent of a Number

Example:
If 28% of Gifty's pizzas are flour pizzas, and Gifty makes 75 pizzas, how many flour pizzas does Gifty make?

Solution
To find the percent of a number, first, write the percent as a decimal. Then, multiply the decimal by the number. 28% = 0.28. $0.28 \times 75 = 21$. If Gifty makes 75 pizzas, 21 of them are pepperoni pizzas

Example:
Jason takes 60 photos. If 15% of them are black and white, how many black-and-white photos does he take?

This problem is asking us to find 15% of 60. Write 15% as a decimal: 15% = 0.15. Multiply the decimal by 60: $0.15 \times 60 = 9$. Jason took 9 black-and-white photos.

Convert a Percent to a Fraction

1. Write the percent as a ratio with the denominator 100.
2. Simplify the fraction if possible.

Example:
Convert 36% to a fraction.

Solution:

$$\frac{36 \div 4}{100 \div 4} = \frac{9}{25}$$

Example:

According to the local weather report, the probability of thunderstorms in Accra, Ghana on July 15 is 60%. Write this percent as a ratio in fraction form. Simplify the fraction.

Solution:

$$\frac{60 \div 10}{100 \div 10} = \frac{6}{10}$$

Convert a Percent to a Decimal

1. Drop the % sign.
2. Divide by 100. (Shortcut: Move the decimal point to the left 2 places.)

Let's consider the 6%. We begin by converting our percent to a fraction.

6% = 6/100. We get the fraction 6/100. To convert a fraction into a decimal we divide the numerator by the denominator. 6 ÷ 100 = 0.06. So 6% as a decimal is 0.06.

Convert a Decimal to a Percent

1. Multiply by 100. (Shortcut: Move the decimal point to the right 2 places.)
2. Add % sign.

Example:

Convert 0.41 to a percent.

Convert a Fraction to a Percent

1. Convert the fraction to a decimal.
2. Convert the decimal to a percent.

Example 1:

According to the Ghana Health Statistics, in 2012, 7/20 of Ghanaian adults were obese. Convert this fraction to a percent.

Example 2:

Convert 4/5 to a percent.

Example 3:

One-fourth of the total number of shoes in a shop were on discount sale. What percent of the shoes were there on normal price?

Revise Questions-Percent Problems

1. Carol went shopping for a cell phone. The price was listed as GHC400. She had a coupon for GHC50 off. What percent of the original price is the coupon savings amount?

2. Benedict is driving to Temale on a 10-hour road trip. So far, she has driven for 4 hours. What percent of the hours has Benedict already driven?

3. Daddy read 8 magazines this week, 60% more magazines than he read last week. How many magazines did he read last week?

4. A value is increased by 5% to 42. What was the original value?

5. A shirt at a clothing store normally costs $20, but is on sale for 10% off. What is the cost of the shirt?

6. A television at Electronics World costs GHC149. The store increases the price by 8%, but when sales are slow, they reduce that price by 10%. What is the price of the television now?

7. A value is decreased by 25%, and then increased by 7% to 54.57. What was the original value?

8. A painting is valued at GHC525 in 1990. Its value is 14% greater in 1995, but then drops by 6% five years later. What is the value of the painting in 2000?

9. Seventeen percent of the water in a tank is used to water crops. Then, the volume remaining in the tank is increased by 15% to 190.9 gallons. What was the original volume in the tank?
10. Alex scored 12 marks, while Ben scored 10 marks, in the first terminal examination. If in the second terminal examination (with same total number of marks) Alex scored 14 marks and Ben scored 12 marks, which student showed more improvement?

11. 30,000 students appeared in a contest. Of them 40% were girls and the remaining boys. If 10% boys and 12% girls won the contest with prizes, find the percentage of students who won prizes.
12. The Minister's salary was GHC49,500 last year. This year his salary was cut to GHC44,055 because of IMF conditions. Find the percent decrease.

RATIO AND PROPORTIONS

In our daily life, many a times we compare two quantities of the same type. Consider these two statements

1. Isha's weight is 25 kg and her father's weight is 75 kg. How many times Father's weight is of Isha's weight? It is three times.

2. Cost of a pen is GHC10 and cost of a pencil is GHC 2. How many times the cost of a pen that of a pencil? Obviously it is five times.

In the above examples, we compared the two quantities in terms of 'how many times'. This comparison is known as the Ratio. We denote ratio using symbol :

A ratio is therefore an ordered pair of numbers a and b, written a / b where b does not equal 0.

A proportion is an expression in which two ratios are set equal to each other. For example, if there is 1 boy and 3 girls you could write the ratio as: 1:3 (for every one boy there are 3 girls) 1 / 4 are boys and 3 / 4.

Note: Two quantities can be compared only if they are in the same unit

Consider the earlier examples again.

We can say, the ratio of father's weight to Isha's weigh = $\frac{75}{25} = \frac{3}{1}$ =3:1
The ratio of the cost of a pen to the cost of a pencil =$\frac{10}{2} = \frac{5}{1}$ =5:1

Let us look at this problem.

In a class, there are 20 boys and 40 girls. What is the ratio of

(a) Number of girls to the total number of students.
(b) Number of boys to the total number of students.

Solution:
Total number of students = 20+40=60
The ratio of number of girls to the total number of students = $\frac{40}{60} = \frac{2}{3}$ = 2:3

You can easily solve for the b

We will try these in our Master Class Lesson

1. Sarah takes 15 minutes to reach school from her house and Samuel takes one hour to reach school from his house. Find the ratio of the time taken by Sarah to the time taken by Samuel.

2. Cost of a toffee is GHC0.50 and cost of a chocolate is GHC10. Find the ratio of the cost of a toffee to the cost of a chocolate.

3. In a school, there were 73 holidays in one year. What is the ratio of the number of holidays to the number of days in one year?

4. In a class, there are 20 boys and 40 girls. What is the ratio of the number of boys to the number of girls?

5. Raph walks 6 km in an hour while Rama walks 4 km in an hour. What is the ratio of the distance covered by Raph to the distance covered by Rama?

6. Length of a pencil is 18 cm and its diameter is 8 mm. What is the ratio of the diameter of the pencil to that of its length?

7. Emma and Ruby started a business and invested money in the ratio 2:3. After one year the total profit was `GHC40,000. Emma said "we would divide it equally", Ruby said "I should get more as I have invested more". It was then decided that profit will be divided in the ratio of their investment. Find their share of the profit.

Example
Length and breadth of a rectangular field are 100 m and 20 m respectively. Find the ratio of the length to the breadth of the field.

Solution :
Length of the rectangular field = 100 m
Breadth of the rectangular field = 20 m
The ratio of the length to the breadth = 100 : 15 = 20:3

Example:
Find the ratio of 90 cm to 1.5 m.

Solution:
The two quantities are not in the same units. Therefore, we have to convert them into same units.

1.5 m = 1.5 × 100 cm = 150 cm.
Therefore, the required ratio is 90 : 150.
Required ratio is 3 : 5

Example:

Ratio of distance of the school from Mariam home to the distance of the school from Mohammed home is 10 : 5.
(a) Who lives nearer to the school?
(b) Complete the following table which shows some possible distances that Mariam and Mohammed could live from the school.

Distance from Mariam's home to school (in km.)	20		8		
Distance from Mohammed's home to school (in km.)	10	8		6	2

(c) If the ratio of distance of Mariam's home to the distance of Karim's home from school is 5 : 10, then who lives nearer to the school?

Solution :
(a) Mohammed lives nearer to the school (As the ratio is 10 : 5)

Distance from Maraim's home to school (in km.)	20	16	8	12	4
Distance from Mohammed's home to school (in km.)	10	8	4	6	2

(c) Since the ratio is 5 : 10, so Mariam lives nearer to the school.

Revise Questions

1. Abigail went to the market to purchase tomatoes. One shopkeeper tells her that the cost of tomatoes is GHC40 for 5 kg. Another shopkeeper gives the cost as 6 kg for GHC42. Now, what should Abigail do? Should she purchase tomatoes from the first shopkeeper or from the second? Will the comparison by taking the difference help her decide? No. Why not?

2. Bismarck has 28 mangos and Vincent has 180 oranges. They want to share these among themselves. Bismarck gave 14 mangos to Vincent and Vincent gave 90 oranges to Bismarck. But Vincent was not satisfied. He felt that he had given more oranges to Bismarck than the mangos given by Bismarck to him. What do you think? Is Vincent correct? To solve this problem Vincent and Bismarck approach you, how will you help them out?

3. Seyram purchased 3 pens for GHC15.00 and Effah purchased 10 pens for GHC50. Whose pens are more expensive?

4. Rahim sells 2 kg of apples for GHC180 and Karim sells 4 kg of apples for GHC360. Whose apples are more expensive?

5. A man travels 35 km in 2 hours. With the same speed would he be able to travel 70 km in 4 hours?

6. A scooter requires 2 litres of petrol to cover 80 km. How many litres of petrol is required to cover 1 km?

SETS OF NUMBERS

A set is a collection of distinct objects. This means that {1, 2, 3} is a set but {1, 1, 3} is not because 1 appears twice in the second collection. The second collection is called a multi set. Sets are often specified with curly brace notation.

Set Definitions and Notations

1. Empty Set: The empty set is a set containing no objects. It is written as a pair of curly braces with nothing inside {} or by using the symbol $\emptyset$.

2. The set membership symbol $\in$ is used to say that an object is a member of a set. It has a partner symbol $\in/$ which is used to say an object is not in a set.

3. We say two sets are equal if they have exactly the same members.

   Example, If A = {1, 2, 3} Then, $3 \in A$ and $4 \notin A$. The set membership symbol is often used in defining operations that manipulate sets. In another example, X = {2, 3, 1} is equal A, therefore, $X \in A$.\

4. The cardinality of a set is its size. For a finite set, the cardinality of a set is the number of members it contains. In symbolic notation the size of a set A is written |A|. Example, for the set A = {1, 2, 3} we show cardinality by writing |A| = 3

5. SUBSET: If A and B are two sets, and every element of set A is also an element of set B, then A is called a subset of B and we write it as $\mathbf{A \subseteq B}$ or $\mathbf{B \supseteq A}$ The symbol $\subset$ stands for 'is a subset of' or 'is contained in'

   • Every set is a subset of itself, i.e., $A \subset A$, $B \subset B$.
   • Empty set is a subset of every set.
   • Symbol '$\subseteq$' is used to denote 'is a subset of' or 'is contained in'.
   • $A \subseteq B$ means A is a subset of B or A is contained in B.
   • $B \subseteq A$ means B contains A.

Compliment

The compliment of a set A is the collection of objects in the universal set that are not in A. The compliment is written A' or A^c\In curly brace notation $A' = \{x : (x \in U) \wedge (x \in A)\}$ or more compactly as $A' = \{x : x \in A\}$

Example Set Compliments

U = {1, 2, 3, 4, 5}
A = {1, 2, 3}
B = {1, 3, 5}
$A^{'}$ = {4. 5}
$B^{'}$ = {2, 4}.

6. The intersection of two sets A and B is the collection of all objects that are in both sets. It is written $A \cap B$. Using curly brace notation $A \cap B = \{x : (x \in A) \wedge (x \in B)\}$ where $\wedge$ is a symbolic equivalent of 'and'.

Example of Intersections of sets

Suppose A = {1, 2, 3, 5}, B= {1, 3, 4, 5}, and U = {2, 3, 4, 5}. Then: $A \cap B = \{1, 3, 5\}$, $A \cap U = \{2, 3, 5\}$, and $B \cap U = \{3, 4, 5\}$

7. If A and B are sets and $A \cap B = \emptyset$ then we say that A and B are disjoint, or disjoint sets.

8. The union of two sets A and B is the collection of all objects that are in either set. It is written $A \cup B$. Using curly brace notion $A \cup B = \{x : (x \in A) \vee (x \in B)\}$ where $\vee$ is a symbolic equivalent of 'or'.
Example of Unions of sets
Suppose A = {1, 2, 3}, B = {1, 3, 5}, and U = {2, 3, 4, 5}.
Then:
$A \cup B = \{1, 2, 3, 5\}$,
$A \cup U = \{1, 2, 3, 4, 5\}$, and
$B \cup U = \{1, 2, 3, 4, 5\}$

The difference of two sets

The difference of two sets A and B is the collection of objects in A that are not in B.

Let A and B be two sets. The difference of A and B, written as A - B, is the set of all those elements of A which do not belongs to B.

Thus $A - B = \{x : x \in A \text{ and } x \notin B\}$ or $A - B = \{x \in A : x \notin B\}$. Clearly, $x \in A - B \Rightarrow x \in A$ and $x \notin B$

9. The universal set, at least for a given collection of set theoretic computations, is the set of all possible objects. When performing set theoretic computations, you should declare the domain in which you are working. In set theory this is done by declaring a universal set.

10. Disjoint sets: two sets are said to be disjoint sets if they have no element in common. Equivalently, two disjoint sets are sets whose intersection is the empty set.

For example, A= {1, 2, 3} and B ={4, 5, 6} are *disjoint sets,* while C= {1, 2, 3} and D= {3, 4, 5} are not disjoint. A collection of two or more sets is called disjoint if any two distinct sets of the collection are disjoint

Venn Diagrams

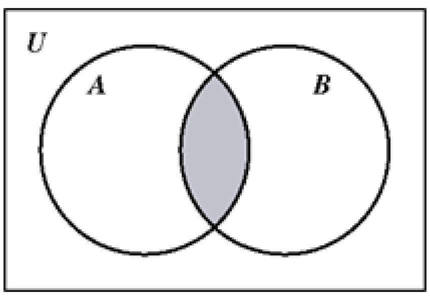
A Venn diagram is a way of depicting the relationship between sets. Each set is shown as a circle and circles over lap if the sets intersect.

• A rectangle is used to represent a universal set.
• Circles or ovals are used to represent other subsets of the universal set.

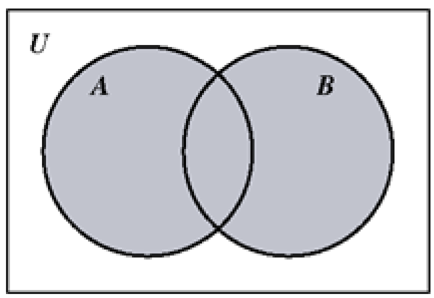
Venn diagrams in different situations

Intersection and union

Example: The following are Venn diagrams for the intersection and union of two sets. The shaded parts of the diagrams are the intersections and unions respectively.



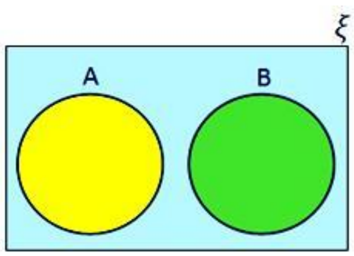
$A \cap B$



$A \cup B$

Disjoint Set

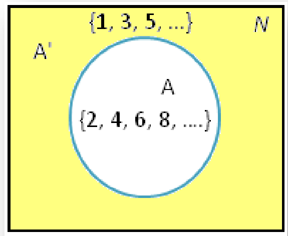
If set A and set B are disjoint, then they are represented by two non intersecting circles



In this diagrams, the universal set is represented by a rectangular region and its subsets by circles inside the rectangle. We represented disjoint set by disjoint circles and intersecting sets by intersecting circles.

Compliment

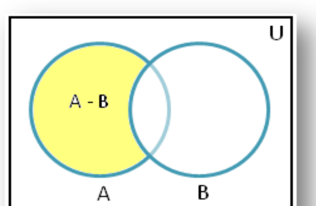
1. Let the set of natural numbers N = {1, 2, 3, …} be the universal set and let A = {2, 4, 6, 8, …}
Then A' = {1, 3, 5, …}



Complement of A or A'

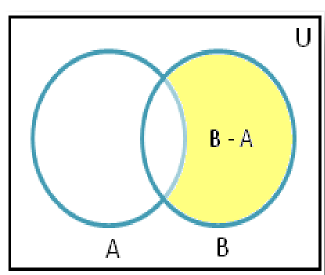
The difference of two sets

In the adjoining figure the shaded part represents A – B.



Difference of two sets
A - B

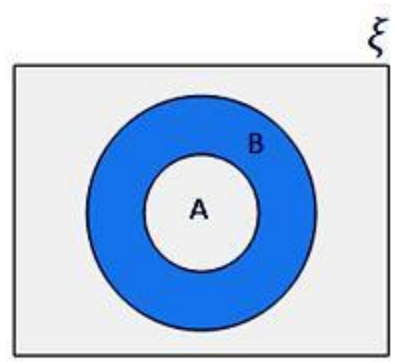
Similarly, the difference B – A is the set of all those elements of B that do not belongs to A.
Thus, B – A = {x : x ∈ A and x ∉ B} or A – B = {x ∈ B : x ∉ A}.



Difference of two sets
B - A

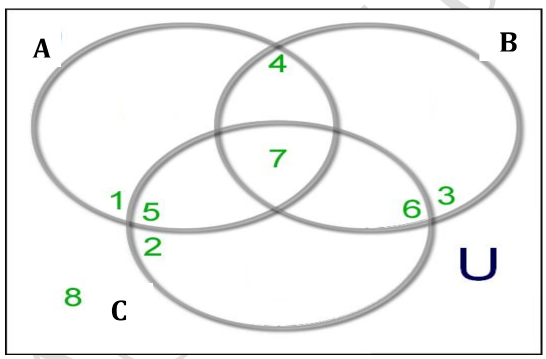
Subset

If a set A is a subset of set B, then the circle representing set A is drawn inside the circle representing set B.



Three Set Problem

Set and operations can be represented by Venn diagrams like the one shown below.



Observe from above, that Region (7) is in A, B and C. It is $A \cap B \cap C$, the intersection of A, B and C.

A look at Region (5), you observe that it is inside A and C; but outside B? It is $A \cap B^I \cap C$, the intersection of A; B^I and C. Similar; Region (6) is inside B and C; but outside A; i.e. $A^I \cap B \cap C$. Region (4) is inside A and B; but outside C; $A \cap B \cap C^I$.

Now look carefully at Region (1), it is clear that it is inside A, but outside B and C, so we write $A \cap B^I \cap C^I$

Similar, Region (3) is inside B, but outside A and C; i.e. $A^I \cap B \cap C^I$.

Region (2) is inside C, but outside A and B, i.e. $A^I \cap B^I \cap C$.

It should be clear to you that though Region (8) is inside U, it is outside A, B and C. This is expressed as $A^I \cap B^I \cap C^I$.

Revise Questions

1. Let A and B be two finite sets such that n(A) = 20, n(B) = 28 and n(A ∪ B) = 36, find n(A ∩ B).

2. In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

3. There are 35 students in art class and 57 students in dance class. Find the number of students who are either in art class or in dance class.
   i. When two classes meet at different hours and 12 students are enrolled in both activities.
   ii. When two classes meet at the same hour.

4. If n(A - B) = 18, n(A ∪ B) = 70 and n(A ∩ B) = 25, then find n(B).

5. In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

6. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories

7. Each student in a class of 40 plays at least one indoor game chess, carrom and scrabble. 18 play chess, 20 play scrabble and 27 play carrom. play chess and scrabble, 12 play scrabble and carrom and 4 play chess, carrom and scrabble. Find the number of students who play
   (i) chess and carrom.
   (ii) chess, carrom but not scrabble.

8. Suppose that we take the universal set U to be the integers. Let S be the even integers, let T be the integers that can be obtained by tripling any one integer and adding one to it, and let V be the set of numbers that are whole multiples of both two and three.
   (i) Write S, T , and V using symbolic notation.
   (ii) Compute S ∩ T , S ∩ V and T ∩ V and give symbolic representations that do not use the symbols S, T , or V on the right hand side of the equals sign.

9. Suppose that we have the set U = {n : 0 ≤ n < 100} of whole numbers as our universal set. Let P be the prime numbers in U, let E be the even numbers in U, and let F = {1, 2, 3, 5, 8, 13, 21, 34, 55, 89}. Describe the following sets either by listing them or with a careful English sentence.
   (i) E' (ii) P ∩ F (iii) P ∩ E (iv) $F \cap E \cup F \cap E'$ (v) $F \cup F'$

10. Suppose X = {1, 2, 3}, Y = {1, 3, 5}, and Z = {2, 3, 4, 5} Find,
   i. S ∪ T ii. S ∪ U iii. T ∪ U

Number System

Numbers: a number is an arithmetic value which is used to represent the quantity of an object.

Types of Numbers

Real Numbers

Any number such as positive integers, negative integers, fractional numbers or decimal numbers without imaginary numbers are called the real numbers. It is represented by the letter "R".

Examples: ¾, 0.333, √2, 0, -10, 20, etc.

Properties of Real Numbers:

- Real Numbers are commutative, associate, and distributive under addition and multiplication.
- Real numbers obey the inverse property.
- Additive and multiplicative identity elements of real numbers are 0 and 1, respectively.

1. Natural numbers includes only positive integers from 1 and that can be counted on our hands. Natural numbers are also known as counting numbers. For example, the numbers that we count on our hands are 1, 2, 3, 4, 5, 6, and so on. **Note:** zero, negative numbers, fractions, and decimals are **not** considered to be natural numbers.

Properties of Natural Numbers:

- Addition of natural numbers is closed, associative, and commutative.
- Natural Number multiplication is closed, associative, and commutative.
- The identity element of a natural number under addition is zero.
- The identity element of a natural number under Multiplication is one.

2. Whole numbers are those numbers that include positive integers along with 0. A few examples of whole numbers are 0, 15, 37, 97, 67, etc.

Properties of Whole Numbers:

- Whole numbers are closed under addition and multiplication.
- Zero is the additive identity element of the whole numbers.
- 1 is the multiplicative identity element.
- It obeys the commutative and associative property of addition and multiplication.
- It satisfies the distributive property of multiplication over addition and vice versa.

3. **Integers:** Integers are defined as the set of all whole numbers with a negative set of natural numbers. The integer set is represented by the symbol “Z”. The set of integers is defined as: **Z = {-3, -2, -1, 0, 1, 2, 3} Examples:** -52, 0, -1, 16, 82, etc.

Properties of Integers:

- Integers are closed under addition, subtraction, and multiplication.
- The commutative property is satisfied for addition and multiplication of integers.
- It obeys the associative property of addition and multiplication.
- It obeys the distributive property for addition and multiplication.
- Additive identity of integers is 0.
- Multiplicative identity of integers is 1.

4. **Rational numbers**: are numbers that can be expresses in the form $\frac{a}{b}$ examples: 1/2, -6/10, 0, -4, 6, 4/3 irrational numbers cannot be expressed in the form $\frac{a}{b}$

Properties of Rational Numbers:

- Rational numbers are closed under addition, subtraction, multiplication, and division.
- It satisfies commutative and associative property under addition and multiplication.
- It obeys distributive property for addition and subtraction.

Irrational Numbers

The number that cannot be expressed in the form of p/q. It means a number that cannot be written as the ratio of one over another is known as irrational numbers. It is represented by the letter ”P”. **Examples:** √2, π, Euler’s constant, etc

Properties of Irrational Numb**ers:**

- Irrational numbers do not satisfy the closure property.
- It obeys commutative and associative property under addition and multiplication.
- Irrational Numbers are distributive under addition and subtraction.

Prime number and Composite number

A prime number is a number that has exactly two factors, 1 and the number itself. For example, 2, 5, 7, 11, and so on are prime numbers. It can be said that any whole number greater than 1 that has exactly two factors, 1and itself is defined to be a prime number.

A composite number is a number that has more than two factors, which means it can be divided by the number 1 and itself, and at least one more integer. It can also be said that any number greater than 1 that is not a prime number, is defined to be a composite number. Composite numbers always have more than 2 factors. For example, 6, 8, 9, 12, and so on are composite numbers because these numbers have more than 2 factors.

Even Numbers and Odd Numbers

Even numbers are those numbers that can be divided into two equal groups or pairs and are exactly divisible by 2. For example, 2, 4, 6, 8, 10, and so on.

Odd numbers are those numbers that are not completely divisible by 2. Examples, 1, 3, 5, 7 ,9,…

Perfect Numbers

Perfect numbers are the positive integers that are equal to the sum of its factors except for the number itself. In other words, perfect numbers are the positive integers that are the sum of their proper divisors. The smallest perfect number is 6, which is the sum of its proper divisors: 1, 2 and 3

Factors and Multiples

Factors and multiples are the two key concepts that are studied together. Factors are the numbers that divide the given number completely without leaving any remainder, whereas the multiples are the numbers that are multiplied by the other number to get specific numbers.

Factors of a given number are numbers that can perfectly divide that given number.

Examples:

- Factors of 6: 1, 2, 3, 6
- Factors of 8: 1, 2, 4, 8

Applications of G C F

1. Kiara baked 30 oatmeal cookies and 48 chocolate chip cookies to package in plastic containers for her friends at school. She wants to divide the cookies into identical containers so that each container has the **same number of each kind of cookie**. If she wants each container to have the **greatest number of cookies possible**, how many plastic containers does she need?

Answer:

30 – 1, 2, 3, 5, **6**, 10, 15, 30

48 – 1, 2, 3, 4, **6**, 8, 12, 16, 24, 48

Kiara needs **6 plastic containers** for her cookies.

2. The choir teacher plans to arrange the students in equal rows. Only girls or boys will be in each row. What is the **greatest number of students** that could be in each row?

Answer:

48 – 1, 2, 3, 4, 6, 8, 12, **16**, 24, 48

64 – 1, 2, 4, 8, **16**, 32, 64

A multiple of a number is a number obtained by multiplying the given number by another whole number. The multiples of a number are infinite. **Examples:**

- Multiples of 3: 3, 6, 9, 12, 15, ...
- Multiples of 5: 5, 10, 15, 20, 25, ...
- Multiples of 10: 10, 20, 30, 40, 50,....

Least Common Multiple (L.C.M.)

The least common multiple of 2 or more numbers is the smallest multiple of the common multiples. Example: Find the L.C.M. of 5 and 8.

Multiples of 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, …

Multiples of 8 = 8, 16, 24, 32, 40, 48, …

The L.C.M. is 40.

Applications of H.C.F

Find the greatest number which divides 304 to leave a remainder of 4 and which also divides 298 to leave a remainder of 4.

As we will have a remainder of 4 in each case, we subtract the 4 from the given numbers:

$304 - 4 = 300$

$298 - 4 = 294$

We express 300 and 294 as a product of their prime factors:

$300 = 2 \times 2 \times 5^2 \times 3$

$294 = 2 \times 3 \times 7^2$

H.C.F. of 300 and 294 = 3 x 2 = 6

The greatest number which will divide 304 and 298 so as to leave remainder of 4 in each case = 6.

We can verify that $\frac{304}{6} = 50 \text{ R } 4$ and $\frac{298}{6} = 49 \text{ R } 4$

Mira has two pieces of ribbon of lengths 18 cm and 24 cm respectively. She wants to cut both pieces into smaller pieces of equal length that are as long as possible. What would be the length of each smaller piece?

We express 18 and 24 as the product of their prime factors:

$$18 = 2 \times 3 \times 3$$
$$24 = 2 \times 3 \times 2 \times 2$$

H.C.F of 18 and 24 = 6

Length of each smaller piece = 6 cm

Applications of L.C.M.

A cinema runs its movies in two different halls 24/7. One movie runs for 80 minutes and the second one runs for 120 minutes. Both movies start at 1.00 p.m. When will the movies begin again at the same time?

We express 80 and 120 as a product of their prime factors:

$$80 = 2^4 \times 5$$
$$120 = 2^3 \times 5 \times 3$$

L.C.M. of 80 and 120 = $2^4 \times 3 \times 5 = 240$

So, the two movies will start again at the same time after 240 minutes, that is, in 4 hours.

The movies will start together again at 5.00 p.m.

Equation and inequalities

Linear equation

A linear equation is made up of two expressions that are equal to each other. A linear equation may have one or two variables in it, where each variable is raised to the power of 1. No variable in a linear equation can have a power greater than 1.

Linear equation: $2\square\square = 3\square\square + 1$ (each variable in the equation is raised to the power of 1)

Not a linear equation: $\square^2 = 3\square\square + 1$ (y is raised to the power of 2, therefore this is not linear)

The solution to an equation is the value, or values, that make the equation true. Given a solution, we plug the value(s) into the respective variable(s) and then simplify both sides. The equation is true if both sides of the equation equal each other.

Solve for the variable in each of the following equations

a) $\square + 7 = 18$

$3(4\square\square - 2) = 5(\square\square + 3)$

$\square\square - 5 = 4\square + 7$

$4 - (2\square - 1) = 2(5\square\square + 9) + \square\square$

$\frac{2}{3}(\square + 4) = 5\left(\frac{5}{6}x - \frac{7}{15}\right)$

$-\frac{\square}{\square} - \frac{\square}{\square}x = -\frac{4}{3}\square - \frac{2}{3}\left(-\frac{13}{4}x + 1\right)$

$\frac{\square}{\square x - 2} = \frac{\square}{\square x\ 1}$

0.12t - 2.1 = 0.07t - 0.2

Linear Inequalities

An algebraic inequality is a mathematical sentence connecting an expression to a value, variable, or another expression with an inequality sign. Below is a table of inequalities we will be using

Symbol	In Words	Examples
$<$	Less than	$1 < 2$ "1 is less than 2"
$>$	greater than	$4 > 3$ "4 is greater than 3"
$\leq$	less than or equal to	$0 \leq 5$ "0 is less than 5"

≥	greater than or equal to	$-1 \geq -1$ "−1 is equal to −1"
≠	not equal	$3 \neq 4$ "3 is not equal to 4"

Solve the inequality, check your answer, and graph the solution on a number line. Give the solution in interval notation.

a) 3□□ > □□ + 6

b) 3 − 5□□ ≤ 2(□□ + 5)

c) −5(□□ + 2) ≥ −3(□□ + 4)

Problem Solving

The ability to use the tools of algebra to solve problems requires that we be able to translate the English language into the language of algebra. More specifically, at this time we need to translate English sentences into algebraic equations so that we can use our equation-solving skills.

You Try: Write an algebraic expression for each verbal expression

1. eight less than a number

3. the quotient of m and n

5. nine times a number

7. seven more than the cube of a number

2. a number increased by seven

4. a number squared

6. a number decreased by three

8. one-half the product of x and y

9. the product of twice a and b

10. twice the product of a and b

11. two less than five times a number

12. twice a number increased by 3 times that number

Examples:

1. If 2 is subtracted from five times a certain number, the result is 28. Find the number.

Solution

Let n represent the number to be found. The sentence If 2 is subtracted from five times a certain number, the result is 28 translates into the equation $5n - 2 = 28$.

Solving this equation, we obtain

$5n - 2 = 28$

$5n = 28 + 2$

$5n = 30$

$n = 6$

The number to be found is 6

2. Find three consecutive integers whose sum is 245.

Solution

Let n represent the smallest integer; then n + 1 is the next integer and n + 2 is the largest of the three integers. Because the sum of the three consecutive integers is to be -45, we have the following equation.

$n + (n + 1) + (n + 2) = -45$

$3n + 3 = -45$

$3n = -45 - 3$

$3n = -48$

$n = -16$

If n = -16, then n + 1 is -15 and n + 2 is -14. Thus the three consecutive integers are -16, -15, and -14.

3. Tina is paid time-and-a-half for each hour worked over 40 hours in a week. Last week she worked 45 hours and earned $380. What is her normal hourly rate?

Solution

$2\left[40\square + 5\frac{3}{2}\square\right] = 2(380)$

$2(40r) + 2\left[5(\frac{3}{2}\square)\right] = 760$

80r + 15r =760

95r = 760

r = 8

Her normal hourly rate is thus $8 per hour.

Questions

Establish a variable, write an inequality to represent the scenario, and solve. Write a complete sentence to describe your solution.

1. The sum of two numbers is 84, and one of them is 12 more than the other. What are the two numbers?

2. The cost of leasing a new Ford mustang is $2,311 for a down payment and processing fee plus $276 per month. For how many months can you lease this car with $10,000?

3. There are 51 students in a certain class. The number of females is 5 less than three times the number of males. Find the number of females and the number of males in the class.

4. Keith has $500 in a savings account at the beginning of the summer. He wants to have at least $200 at the end of the summer. He withdraws $25 per week for food, clothing, and movie tickets. How many weeks can Keith withdraw money from his account?

5. A taxi charges a flat rate of $1.75, plus an additional $0.65 per mile. If Erica has at most $10 spend on the cab ride, how far could she travel?

6. Fixed fair of hiring a taxi is $12, fare for every additional mile is $5 per mile. Set up an inequality to show how much a person can travel with $50 or less. Represent the additional mile by m.

7. Lauren goes shopping and has £50 to spend. She bought a T-shirt and 3 pairs of leggings. The T-shirt cost £23. Each pair of leggings cost £x
(a) Form an inequality in terms of x. (b) Solve the inequality to find the possible price of the leggings

Capacity, mass and time

Capacity refers to the amount a container can hold and is usually associated with liquid. Common capacity measurements are millilitres and litres.

1,000 millilitres = 1 litre
1,000 ml = l

Example:

1.Convert these amounts to litres:

3,452 ml

10,000 ml

12,674 m

235 ml

56,780 ml

2. Convert these amounts to millilitres:

1.78 l

7.305 l

3.999 l

0.35 l

20.4564 l

3. The capacity of a standard size bathtub is 100 gallons. Taking a 15-minute shower uses 30 gallons of water. How much water can be saved from taking a 15-minute shower instead of bath every day for a week?

Answer:

100 – 30 = 70

70 gallons of water can be saved each day.

In a week 7 x 70 = 490 gallons of water can be saved.

4. Capacity of a small swimming pool is 860 gallons. To clean the pool, it must be drained so that only 360 gallons of water are left. If it takes an hour for each 100 gallons to drain, how long does it take to drain the pool to the level required?

Answer:

860 – 360 = 500

500 gallons of water must be drained from the pool.

500 ÷ 100 = 5

It will take 5 hours to drain the pool to the level required.

5. A sink holds 10 L of water. The tap can fill up the sink with 1,500 ml of water every minute. The drain of the sink can drain away 1,300 ml of water every minute. a. Will the sink overflow if the tap is on for a long time and the sink is unplugged? b. how long before the sink is full?

Answer:

1,500 > 1,300

a. The tap fills the sink up quicker than it can drain, so the sink will overflow if the tap is on for a long time.
b. Each minute, the tap adds 1,500 ml and the drain drains 1,300 ml.

1,500 – 1,300 = 200 ml

So, each minute an additional 200 ml of water are added.

The sink holds 10 L = 10,000 ml of water.

10,000 ÷ 200 = 50

So, it takes 50 minutes to fill the sink when the drain is open.

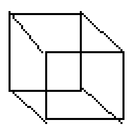
We will solve these word problems in our online classes. They all involve conversion.

c. Omar was filling up a 3 l container with cordial. He only had a small 300 ml jug. How many times did hehave to fill the jug to totally fill the container?
d. I poured 375 ml out of a 2 l milk container. How much was left? I then poured out another 375 ml. How much is left now?
e. How many 315 ml glasses can be filled from a 1.7 l jug? How much is left over?
f. Paula is making a punch for her party. She uses 1.5 l of orange juice, 750 ml pineapple juice, 1.25 l of lemonade and 1.25 l of ginger ale. How much punch does she have altogether? How many 250 ml cups will she be able to fill?

Volume

Volume refers to the amount of space occupied by an object or substance. Commonly used volume measurements are the cubic centimetre and the cubic metre.

One cubic centimetre is 1 cm long, 1 cm wide and 1 cm high. The symbol we use for cubic cm is cm3. 1 cm × 1 cm × 1 cm = 1 cm3



One cubic metre is 1 m long, 1 m wide and 1 m high. The symbol we use is m3. 1 m × 1 m × 1 m = 1 m3

We will learn how to find volume of regular objects in our plane geometry lesson.

Mass

Mass measures how much matter is in an object. We usually measure this by finding out what the object weighs. Mass and weight are slightly different but we often use weight terms when we are talking about day to day mass measurements. Common measurements are grams (g), kilograms (kg) and tonnes (t).
There are 1,000 g in each kilogram and 1,000 kg in a tonne.

Write each mass in kilograms

1,000 g

350 g

160g

Write each mass in grams

2 kg

5 kg

20kg

We will solve these word problems in our online classes.

1. There are 28 students in Mr Brown's class. Being the dedicated and hardworking teacher that he is, he lugs their books home to mark each week.
2. Each maths book has a mass of 550 g. He puts them all in a tray which has a mass of 345 g. What is the total mass he will carry to his car?
3. Last week he took home the spelling books in the same tray. The total mass was 9.445 kg. What was the mass of each spelling book?
4. Next week, the football starts again. There goes the marking. Mr Brown will now be sitting in the grandstand munching crisps and cheering on the Mighty Blues. If he consumes four 375 g bags of crisps in a particularly tense game, how much does he eat?

Time

In order to calculate answers to questions involving time, you must know which units are being used. In questions referring to a week, please note that it is a school week of 5 days.

Note: When working out children's reading ages it is necessary to consider time given in years and months. A child may have an actual age of 8 years and 4 months, but a reading age of 8 years and 10 months. Another child of the same age might have a reading age of 7 years and 9 months. Reading ages are presented in several forms and 8 years 6 months, 8.6 or 8-6 are all common. If the form 8.6 is used, it has to be remembered that here the point is not a decimal point, just a means of separating months and years.

Example 1
A junior school is putting on entertainment. If each of the 4 classes is allowed 25 minutes and the entertainment starts at 2.15pm, when is it expected to finish?

If each class were allowed half an hour, the entertainment would finish 2 hours after the start, therefore 4.15pm.
However, each class has 25 minutes, which is 5 minutes less than half an hour. So 4 classes would take 4 x 5 minutes less, ie 20 minutes less.
The entertainment should therefore finish 20 minutes before 4.15pm.

15 minutes before 4.15pm is 4.00pm,
20 minutes before 4.15pm is 3.55pm.

An alternative method would be to calculate:
25 minutes x 4 = 100 minutes = 1 hour 40 minutes
So the 4 classes take 1 hour 40 minutes.1 hour 40 minutes after 2.15pm is 3.55pm.

Example 2
A parent comes in to a school for 1 and a half hours to hear a group of children reading.
If there are 10 children in the class, can she give them all 10 minutes each?

One and a half hours is 60 + 30 = 90 minutes.
To give 10 children 10 minutes each would take 100 minutes. So she cannot give each of them 10 minutes.

Revise Questions

1. The shop is open for 5 days each week. It is open for 8 hours each day. How many hours is the shop open each week? Show how you work it out. (Ans 40 hours)

2. Class 3 needs to get to the museum at 11.30am. It is a 17 minute walk from school to the museum. At which time must Class 3 leave school if they are to arrive at the museum at exactly 11.30am? (Ans 11.13am)

3. Tracy started to watch TV at 4 o'clock. She watched cartoons for 1 hour and BBC for half an hour. Then she switched off. What time did she switch off? (Ans 5.30)

4. The shop is open for 5 days each week. It is open for 8 hours each day. How many hours is the shop open each week? Show how you work it out.(Ans.40 hours}

5. We have playtime at 10.30am. We usually have a quarter of an hour but today we are having double that! What time will we go back into class? (Ans. 11.00am)

6. Passengers usually start to board their flights 1 hour before the scheduled take off time. The gate will be closed 15 minutes before the takeoff time. Jack's plane is scheduled to take off at 12:05 a.m. If he arrives at the gate at 11:45 p.m., can he board the flight? (Ans The boarding time starts at 11:05 p.m. and the gate closes at 11:50 p.m. So, Jack can board his flight)

Conversions

Conversion involves changing information from one unit of measurement to another. For example, converting cedis to dollar, distances in kilometres to miles, or weights from pounds to kilograms.

Physical quantity

A physical quantity is an attribute or property of a substance that can be expressed in a mathematical equation. A quantity, for example the amount of mass of a substance, is made up of a value and a unit. If a person has a mass of 72kg: the quantity being measured is Mass, the value of the measurement is 72 and the unit of measure is kilograms (kg). Another quantity is length (distance), for example the length of a piece of timber is 3.15m: the quantity being measured is length, the value of the measurement is 3.15 and the unit of measure is metres (m).

Unit of measurement

A unit of measurement refers to a particular physical quantity. A metre describes length, a kilogram describes mass, a second describes time etc. A unit is defined and adopted by convention, in other words, everyone agrees that the unit will be a particular quantity.

International System of Units

Quantity	Name	Symbol
Length	Metre	m
Mass	Kilogram	kg
Time	Second	s
Area	square metre	m^2
Volume	cubic metre	m^3
Speed	metre per second	m/s
Capacity	L (or l)	dm^3

An important feature of the metric system is the use of prefixes to express larger and smaller values of a quantity. For example, a large number of grams can be expressed in kilograms, and a fraction of a gram could be expressed in milligrams.

Commonly used prefixes are listed in the table below.

Example 1

On a school trip, the entrance fee to a museum for a group of pupils is 64 cedis. If the exchange rate is GHC1 = $12, what is the group entrance fee in Ghana cedis?

Example 2

A teacher led a group of pupils on a camping trip to France. The teacher wanted to arrange a cycle ride of approximately 8 miles for the pupils to a local village and back. The sign post at their campsite showed the distance to the following villages: Pet 7km, Fo 6km, Tra 5km. Taking 5 miles to be equivalent to 8 kilometres, which of the three villages should the teacher choose as the destination for the cycle ride?

Solution

The return distance from the campsite to Pet is 2 x 7km = 14km
Convert this to miles by dividing by 8 and multiplying by 5:14 ÷ 8 x 5 = 8.75 miles
The return distance from the campsite to Fo is 2 x 6 = 12km
Convert this to miles by dividing by 8 and multiplying by 5:12 ÷ 8 x 5 = 7.5 miles
The return distance from the campsite to Tra is 2 x 5km = 10km
Convert this to miles by dividing by 8 and multiplying by 5:10 ÷ 8 x 5 = 6.25 miles

The distance to Tra is less than the distance to Fo in kilometres. Given that the return distance from the camp to Fo is 7.5 miles, it is clear that Tra is not the right answer.
You can see that 7.5 miles is closer to 8 miles than 8.75 miles, so Fo is the village to choose as the destination for the cycle ride.

Revise Question

1. a. What are the SI units for length, mass and time?

b. What is difference between the prefix m and M?

c. What is the difference between volume and capacity?

2. Using the traditional method of unit conversions, perform the following:

(a) 495mm to m (b) 1.395kg to g (c) 58g to kg

(d) 0.06km to mm (e) 25 000m2 to ha (f) 3.5m3 to L

Choose a unit that would be suitable to measure

(a) The length of the Bruxner Highway

(b) The floor area of a house

(c) The mass of a newly born chicken

(d) The volume of water in a water storage dam supplying a city.

(e) The length of wood-screws

2. Change the following length measurements to the units shown in brackets

(a) 3.6m (cm) (b) 4500m (km)

(c) 55m (km) (d) 0.325km (mm)

(e) 4 550 000 mm (km) (f) 5.2 cm (km)

3. Change the following mass measurements to the units shown in brackets

(a) 8550 kg (t) (b) 0.52g (mg)

(c) 9.1mg (mcg or µg) (d) 1.25 g (kg)

(e) 2 905 mg (kg) (f) 35mg (g)

4. Change the following capacity measurements to the units shown in brackets

(a) 8500mL (L) (b) 0.451kL (L)

(c) 85.9L (kL) (d) 1.6 ML (L)

(e) 75L (kL) (f) 0.000 6kL (L)

Time Conversion

Time units cause problems because conversions are not based on powers of tens, or in other words, time is not a decimal system.

Units of time include secs, min, hours, days, weeks, etc. Stop watches will work in smaller units, usually mins, secs and hundredths of seconds (or centiseconds). A stopwatch reading of 20:31:90 means 20 minutes, 31seconds and 90 hundredths of a second. Notice that a colon (:) is used to separate the different units to avoid confusion with decimal points.

The unit conversions for time are:

60 seconds = 1 minute

60 minutes = 1 hour

24 hours = 1 day

7 days = 1 week

365 days = 1 year

52 weeks = 1 year

4 weeks = 1 month

12 months = 1year

Unit conversions involving time

Example:

Change 180 minutes to hours.

The conversion to be used is 60 minutes = 1 hour.

In the traditional method, the conversion is the right way round so to go from mins to hours, dividing by 60 must occur.

180mins ÷ 60 = 3 hours

In the dimensional analysis method,

180 mins x $\frac{\square\ \square\square\square\square}{\square\square\ \square\square\square\square}$

$\frac{\square\square\square}{\square\square}\ \square\square\square\square\square$

3 hours

Change 252 minutes to hours (and hours and mins)

The conversion to be used is 60 minutes = 1 hour.

In the traditional method, the conversion is the right way round so to go from mins to hours, dividing by 60 must occur.

252 mins ÷ 60 = 4.2 hours

To change this to hours and minutes, the decimal part of the hour is multiplied by 60. 0.2 x 60 = 12 252 minutes = 4.2 hours = 4 hours 12 minutes

Change 2 mins 41 seconds to seconds.

The conversion to be used is 60 seconds = 1 minute. Note: 2 mins 41 seconds cannot be written as 2.41 mins. As part of the question already contains seconds, only the 2 minutes needs changing to seconds. The best method is to convert 2 mins to seconds and then add on the 41 seconds. 2 mins is 2 x 60 seconds + 41 seconds gives 161 seconds.

Change 2.45 hours into minutes.

Because this time is just hours, the normal conversion strategies can be used. The conversion to be used is 60 minutes = 1 hour which is changed around to be 1 hour = 60 minutes.

In the traditional method, to go from mins to hours, multiplying by 60 must occur. 2.45 hours x 60 = 147 minutes

In the dimensional analysis method,

2.45 hours 60 mins/1 hour

2.4 x 60 mins

147 mins

24 hour time

Note: Twenty four hour time is commonly used around the world in situations where confusion could arise due to omitting am or pm from a time. Some countries have adopted 24 hour time as the standard way to express time. The time using 24 hour time is the elapsed time from the beginning of the day, that is, midnight. At 7.30am, the elapsed time from the beginning of the day is 7 hours 30 mins, so in 24 hour time the time is written as 0730. It is conventional to write 24 hour time using 4 digits.

The 24 hour time at midnight is 0000 as no time has elapsed since the beginning of the day.

The 24 hour time at midday is 1200 as 12 hours has elapsed since the beginning of the day.

The 24 hour time at 3:21pm is 1521 as 15hrs and 21 minutes has elapsed since the beginning of the day.

Review Questions

1. Example: If a car is moving at a speed of 60km/hr, how long will it take (in hours and mins) to cover 75km?
2. John travelled for 3hrs 41 mins before lunch and another 2 hours 27 mins after lunch, how long did he travel for?
3. A nurse commenced an IV at 7:58pm. It should take 4 hrs 20 mins for the medication to be infused. At what time will it be finished?
4. A teacher takes lessons of 2 hour duration. There are 17 students in the group. How much time (on average) does the teacher spend with each student?
5. A car travelling at an average speed of 85 km/hr takes how long to cover 400km?
7. Students at a local school attend six, fifty minute lessons each day. How long have they spent in class over a 5 day school week.
8. A family needs to travel 575 km to reach their holiday destination. If they leave at 6.45am and travel at an average speed of 85 km/hr, what time will they arrive at their destination?

PROBABILITY

Probability measures the likelihood of an event occurring on a scale from 0 (i.e. impossible) to 1 (i.e. certain). We write this as P(name of event), and its value can be expressed as a fraction, decimal or percentage. The greater the probability, the more likely the event is to occur.

Probability theory provides us with a precise understanding of uncertainty. This understanding can help us make predictions, make better decisions, assess risk, and even make money.

Experiments, events and outcomes

The result of an experiment is called an outcome or elementary event, and a combination of these is known simply as an event.

Example, rolling an ordinary fair die is an experiment that has six possible outcomes: 1, 2, 3, 4, 5 or 6. Obtaining an odd number with the die is an event that has three favourable outcomes: 1, 3 or 5.

Random selection and equiprobable events

The purpose of selecting objects at random is to ensure that each has the same chance of being selected. This method of selection is called fair or unbiased, and the selection of any particular object is said to be equally likely or equiprobable.

Note: When one object is randomly selected from n objects, P(selecting any particular object)=$\frac{1}{\square}$
The probability that an event occurs is equal to the proportion of equally likely outcomes that are favourable to the event.

P(event) =

Example Random selection and equal probable events

A student is randomly selected from a group of 19, where 11 are boys and eight are girls.

6. How many possible outcomes are there?
7. What is the probability of selecting any particular boy?
8. What is the probability of selecting any particular girl?

9. What is the probability of selecting any particular student?
10. What is the probability of selecting a boy?
11. What is the probability of selecting a girl?

Solution

1. There are 19 possible outcomes: 11 are favourable to the event selecting a boy and eight are favourable to the event selecting a girl.

   P(event) = $\frac{Number\ of\ events}{sample\ space}$

   P(selecting any particular boy)= $\frac{1}{19}$
2. P(selecting any particular girl)= $\frac{1}{19}$
3. P(selecting any particular student)= $\frac{1}{19}$
4. P(selecting a boy)= $\frac{11}{19}$
5. P(selecting a girl)= $\frac{8}{19}$

   Note: The word particular specifies one object. It does not matter whether that object is a boy, a girl or a student; each has a $\frac{1}{19}$ chance of being selected.

Exhaustive events

A set of events that contains all the possible outcomes of an experiment is said to be exhaustive. In the special case of event A and its complement, not A, the sum of their probabilities is 1 because one of them is certain to occur.

Formula: P(A)+P(not A)=1 or P(A)+P(A$^{'}$)=1

Examples of complementary exhaustive events

Experiment:	**Exhaustive events**		**Probabilities**
	A	Not A	
Toss a fair coin	heads	tails	$\frac{1}{2}+\frac{1}{2}=1$
Roll a fair die	less than 2	2 or more	$\frac{1}{6}+\frac{5}{6}=1$
Play a game of chess	win	not win	P(win) P(not+win)

Trials and expectation

Each repeat of an experiment is called a trial. The proportion of trials in which an event occurs is its relative frequency, and we can use this as an estimate of the probability that the event occurs. If we know the probability of an event occurring, we can estimate the number of times it is likely to occur in a series of trials. This is a statement of our expectation.

Formula: In n trials, event A is expected to occur n×P(A)times.

Example Trials and expectation

The probability of rain on any particular day in a mountain village is 0.2. On how many days is rain not expected in a year of 365 days?

Answer

n = 365 and P(does not rain)=1– 0.2 = 0.8
365 ×0.8=292 days

Mutually exclusive events and the addition law

To find the probability that event A or event B occurs, we can simply add the probabilities of the two events together, but only if A and B are mutually exclusive. Mutually exclusive events have no common favourable outcomes, which means that it is not possible for both events to occur, so P(A and B)= 0.

Example:

When we roll an ordinary die, the events ‘even number = {2,4,6}’ and ‘factor of 5 = {1, 5}’ are mutually exclusive because they have no common favourable outcomes. It is not possible to roll a number that is even and a factor of 5. We say that the intersection of these two sets is empty.
Therefore: P(even or factor of 5) = P(even) + P(factor of 5)

Events are not mutually exclusive if they have at least one common favourable outcome, which means that it is possible for both events to occur, so P(A and B) ≠ 0. For example, when we roll an ordinary die, the events ‘odd number = {1,3,5}’ and ‘factor of 5 = {1, 5}’ are not mutually exclusive because they do have common favourable outcomes. It is possible to roll a number that is odd and a factor of 5. We say that the intersection of these two sets is not empty.
Therefore: P(odd or factor of 5) ≠ P(odd) + P(factor of 5)

The addition law for mutually exclusive events is P(A or B) = P(A)+ P(B) This can be extended for any number of mutually exclusive events: P(A or B or C or …) = P(A)+P(B)+P(C)+…

Independent events and the multiplication law

Two events are said to be independent if either can occur without being affected by the occurrence of the other. Examples of this are making selections with replacement and performing separate actions, such as rolling two dice.

The multiplication law for independent events is $P(A \text{ and } B) = P(A\cap B)=P(A)\times P(B)$ This can be extended for any number of independent events:
$P(A \text{ and } B \text{ and } C \text{ and } \ldots) = P=(A\cap B\cap C\cap\ldots) = P(A)\times P(B)\times P(C)\times\ldots$

Example:

Consider the following bag, which contains two blue balls (B) and five white balls (W). We will select one ball at random, replace it and then select another ball.

For the first selection: $P(B)= \frac{2}{7}$ = and $P(W) = \frac{5}{7}$

For the second selection: P(B))= $\frac{2}{7}$ and P(W)= $\frac{5}{7}$

Note that the first and second selections are made from the same seven balls, so probabilities are identical and independent.

Conditional probability

The word conditional is used to describe a probability that is dependent on some additional information given about an outcome or event.

Example:

For example, if your friend randomly selects a letter from the word ACE, then P(selects E) = $\frac{1}{3}$

However, if we are told that she selects a vowel, we now have a conditional probability that is not the same as P(selects E).

This conditional probability is P(selects E, given that she selects a vowel) = $\frac{1}{2}$

Note: Conditional probabilities are usually written using the symbol | to mean given that. We read P(A|B) as ‘the probability that A occurs, given that B occurs.

Independence and conditional probability

In general, we can use the multiplication law given in Key point 4.8 as the definition of ‘independent’. However, a more formal definition can now be given.

Events X and Y are said to be independent if each is unaffected by the occurrence of the other. If this is the case then the probability that X occurs is the same in two complementary situations: (i) when Y occurs, and (ii) when Y does not occur.

From these, we can now say that X and Y are independent if and only if $P(X|Y)=P(X|Y')$

First note that 1 is odd and square, so X and Y are not mutually exclusive; but are the independent?

When Y occurs, the die shows 1, 3 or 5, so $P(X|Y) = \frac{1}{3}$

When Y does not occur, the die shows 2, 4 or 6, so $P(X|Y') = \frac{1}{3}$

$P(X|Y) = P(X|Y')$ means that P(X) is unaffected by the occurrence of Y. Events X and Y are not mutually exclusive, but they are independent.

Dependent events and conditional probability

Two events are mutually dependent when neither can occur without being affected by the occurrence of the other. An example of this is when we make selections without replacement; that is, when probabilities for the second selection depend on the outcome of the first selection. The multiplication law for independent events (see Key point 4.8) is a special case of the multiplication law of probability. The multiplication law of probability is used to find the probability that 'this and that' occurs when the events involved might not be independent.

$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B|A)$

$P(B \text{ and } A) = P(B \cap A) = P(B) \times (A|B)$

$P(A) \times P(B|A) \equiv P(B) \times P(A|B)$.

Example

Two children are randomly selected from 11 boys (B) and 14 girls (G). Find the probability that the selection consists of: a. two boys b. a boy and a girl, in any order.

Answer

a $P(2\text{boys}) = P(B \text{ and } B) = P(B1) \times P(B_2|B_1)$

$$\frac{11}{25} \times \frac{10}{24}$$

$$\frac{11}{60}$$

b. $P(\text{a boy and a girl}) = P(B \text{ and } G) + P(G \text{ and } B)$

$P(B_1) \times P(G_2|B_1) + P(G_1) \times P(B_2|G_1)$

$$\left(\frac{11}{25} \times \frac{14}{24}\right) + \left(\frac{14}{25} \times \frac{11}{24}\right) = \frac{77}{150}$$

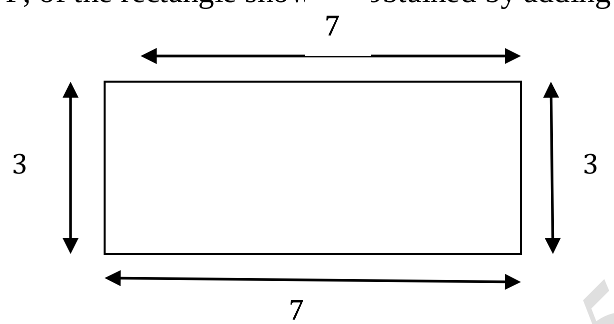
Plane Shapes-Area And Perimeter

Perimeter of Plane Shapes

The perimeter is the distance or length all around a two dimensional shape or figure

For example, if you measure the distance all around the below shape, you will obtain its perimeter.

The perimeter, P, of the rectangle shown is obtained by adding the length of the four sides. That is,

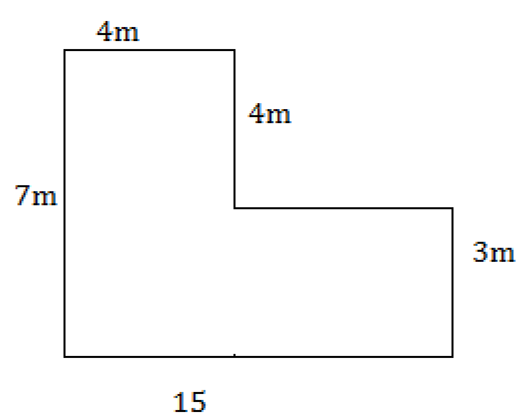


Perimeter = 3+7+3+7 = 20 units Add the lengths of ALL the sides

Note: a regular polygon has equal sides. For example, the formula for finding the perimeter, P of a square is P= 4L where L is the length of its sides. Likewise that of triangle of sides L, is P(Perimeter) = 3L, Regular pentagon of size, L, P =5L. For irregular polygon, to find the perimeter, just add up the lengths of all the sides.

Question

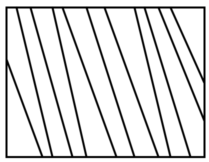
1. A man wants to fence his rectangular garden which is 15m long and 8 m wide. Find the minimum length of fencing he needs to buy.
2. Given that I =60cm and w=30cm, verify that the perimeter of the rectangle is 180cm.
3. Find the perimeter of the below shape



Area of Plane Shapes

The term 'area' refers to the amount of space inside the boundaries of a figure. We frequently use area in our daily life. For instance, if you want to paint the walls of your room, you have to know the area to be painted so as to buy the adequate amount of paint. Calculating area of a room is important to determine the number of tiles needed or the size of a new carpet to be fitted into the room. The area of a classed figure is the measure of the space in the interior of the figure.

Example, the area of the below spade in shaded.



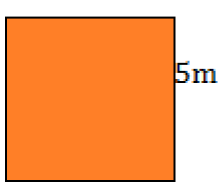
The formula for finding the area of a figure are shown below

Square:	$A = x^2$	where x is the length of the sides
Rectangle	A = I × w	where I is the length and w is the width
Parallelogram	A = a × b	where a is the height and b is the base
Triangle	A = $\frac{1}{2}$ × □ × □	where a is the height and b is the base
Trapezium	$\frac{1}{2}$ × □ × (□ + □)	where (□ + □) is the sum of the length of the two parallel sides

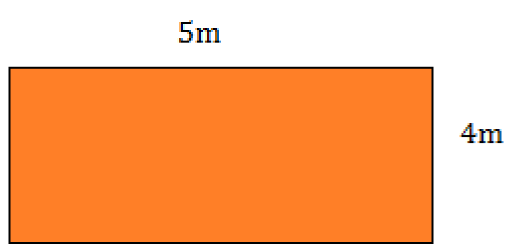
Squares and Rectangles

To find the area of a square or a rectangle, we use the formula: Area=length x width

As a square has all sides equal, its area is often calculated as Area of a square =length x length= $(\text{length})^2$

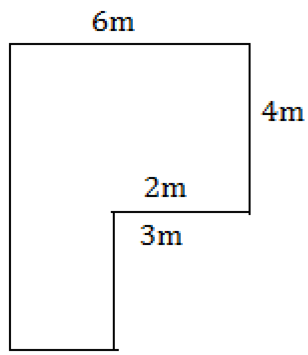


Area = 5 cm x 5 cm
= 25 cm2

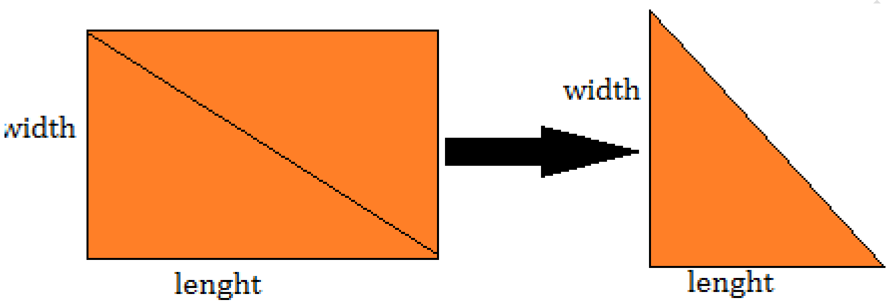


Area = 5cm x 4 cm
= 20 cm2

Calculate the area of the following figure:

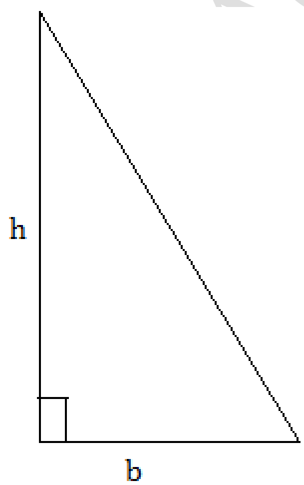


Area of triangles

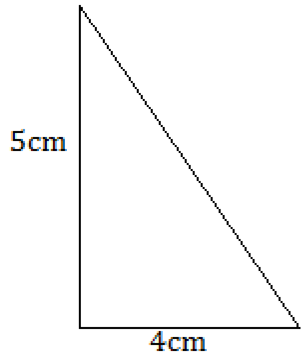


Cut the rectangle into two equal parts as shown on the diagram and consider the triangle obtained. The area is half that of the rectangle, So area of the triangle is ½ X length X width. However for triangle we take the length as the base and the width as the perpendicular height

Formula for area of triangles



Area= 1/2 x Base x height



Find the area

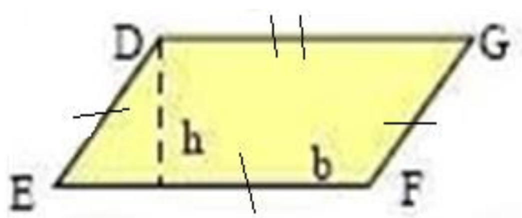
Area= $\frac{□}{□}$ × □□□□ × □□□□□□

$\frac{□}{□}$ × □□□ × □□□

10cm^2

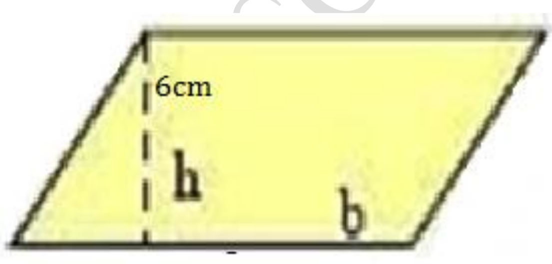
Parallelograms

A Parallelogram is a flat shape with opposite sides parallel and equal in length.



Area= Base(b) x height(h)

Example:

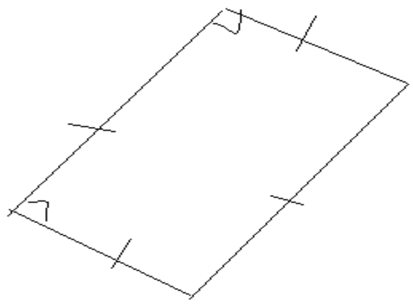


Area = base X perpendicular height

= 6 X 8 = 48 cm^2

Rhombus

A rhombus is a parallelogram with all sides equal

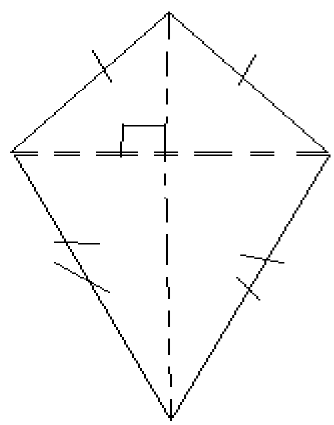


Now since a rhombus is a parallelogram, its area is also is base x height

Area of rhombus = base x height

Kites

A kite has 2 pairs of equal adjacent sides and its diagonals meet each other at right angle



Since we have two pairs of equal sides in a kite, the vertical diagonal is a line of symmetry. So we have two equal triangles on both sides of the vertical diagonal. Now the area of triangle is: The base of the triangles is the vertical diagonal, while the height of the triangle is the horizontal diagonal.
Hence the area of one of the triangle

$\frac{1}{2} \times \frac{1}{2}$ x horizontal diagonal x vertical diagonal

$\frac{1}{4}$ x horizontal diagonal x vertical diagonal

Therefore the area of the kite = 2x area of the triangle

= 2X ¼ horizontal diagonal X vertical diagonal

= ½ x product of the diagonals

Trapeziums

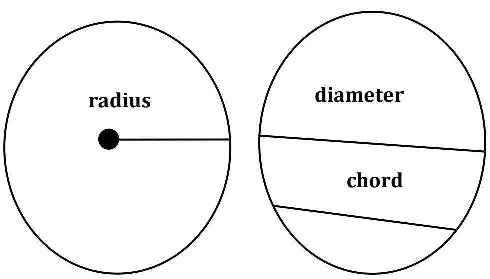
A trapezium is a quadrilateral having one pair of parallel sides.

Circle:

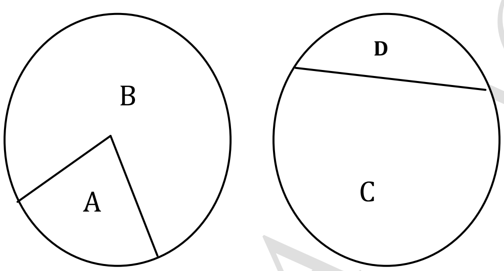
A circle is a set of points which are equidistant from a given point. The given point is known as the centre of that circle.

1. In equal circles (or in the same circle), if two arcs are equal, the chords associated with the arcs are also equal and vice-versa.
2. In equal circles (or in the same circle), if two arcs subtend equal angles at the centre, then they are equal.

Parts of A Circle



- The fixed point is called the **centre of the circle.**
- The distance of any point on the circle from the centre is called **radius** of the circle
- A **chord** is a line segment joining two points on the circle. It is impotent to know the basic difference between the **diameter** and the chord. The diameter **must pass through** the centre of the circle whiles the chord **must not**. Noted also that any of them can be horizontal, vertical or at any angle to the horizontal.



- **A** is called the **minor sector** and is an area enclosed by the radii (two radius) and the shorter of the two areas.
- **B** is called the **major sector** and is an area enclosed by the radii and the larger arc.
- A chord divides the area of a circle into two **segments**. The **minor segment (D)** is the area enclosed by the short arc and the chord and the **major segment(C)** is the area enclosed by the longer arc and the chord.

Circumference of A Circle

The perimeter of a circle is the length of the circle and is called its *circumstance.* It can be shown that for all circles, the ratio of the circumstance, C, to the length of the diameter, *d*, is the same number, □. That is, $\frac{□}{□} = □$ where □ is constant thus $\frac{22}{7}$ □□ *3.142*. Therefore, the formula for computing the circumference of a circle is □ = □□ □□ □ = 2□

Area of A Circle

It can also be shown that for all circles, the ratio of the area, A, to the square of the radius, r, is the same number, π. That is, $\frac{A}{r^2} = \pi$. Therefore, the formula for computing the area of a circle is $A = \pi r^2$

Angles in Circle:

1. The angle which is subtended at the centre by an arc of a circle is double the angle which is subtended at any point on the remaining part of the circle.
2. Angles in the same segment of a circle are equal.
3. The angle in a semi-circle is a right angle.

Triangle

In a ΔABC, there are three vertices A, B and C. There are three angles ∠A, ∠B and ∠C. There are three sides AB, BC and AC. The sum of all the angles of a triangle is 180°. If one side of a triangle is produced, then exterior angle so formed is equal to the sum of two interior opposite angles. If the three sides of a triangle be produced in order, then the sum of all the exterior angles so formed is 360°.

Types of Triangles

1. Equilateral triangle: A triangle having all sides equal is called an equilateral triangle.
2. Scalene triangle: A triangle having all sides of different length is called a scalene triangle.
3. Isosceles triangle: A triangle having two sides equal is called an isosceles triangle.
4. Right angled triangle: A triangle one of whose angles measures 90° is called a right angled triangle.
   Obtuse angled triangle: A triangle one of whose angles lies between 90° and 180° is called an obtuse angled triangle.
5. Acute angled triangle: A triangle each of whose angle is less than 90° is called an acute angled triangle.

Line and Angle

Line

A line is asset of points. Any two points A and B, and all the point between them form a line segment. A line is defined by its length but has no breadth. A line contains infinite points. Three or more points are said to be collinear, if there is a line which contains all of them.

Parallel Lines: Two lines in the same place are said to be parallel, if they never meet, however any far they are extended in either direction. They remain at same distance for the whole length. The sign of parallel is '||'. A line which cuts a pair of parallel lines is called a transversal.

Angle

An angle is formed when two rays originate from a common point called vertex of the angle. When two or more lines meet, an angle is formed.

Polygon
A polygon is a closed plane figure bounded by straight lines.

Convex polygon: A polygon in which none of its interior angles is more than 180° is called convex polygon.

Concave polygon: A polygon in which at least one angle is more than 180° is called concave

Regular polygon: A regular polygon has all its sides and angles equal.

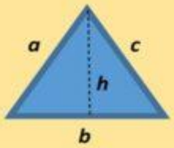
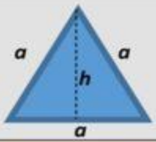
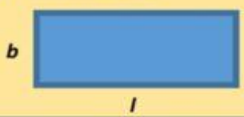
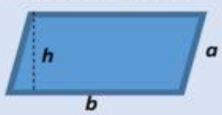
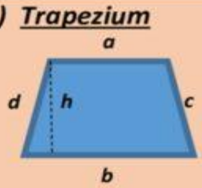
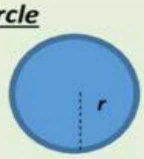
1. Each exterior angle of a regular polygon = 360° / Number of sides
2. Each interior angle = 180° – Exterior angle.

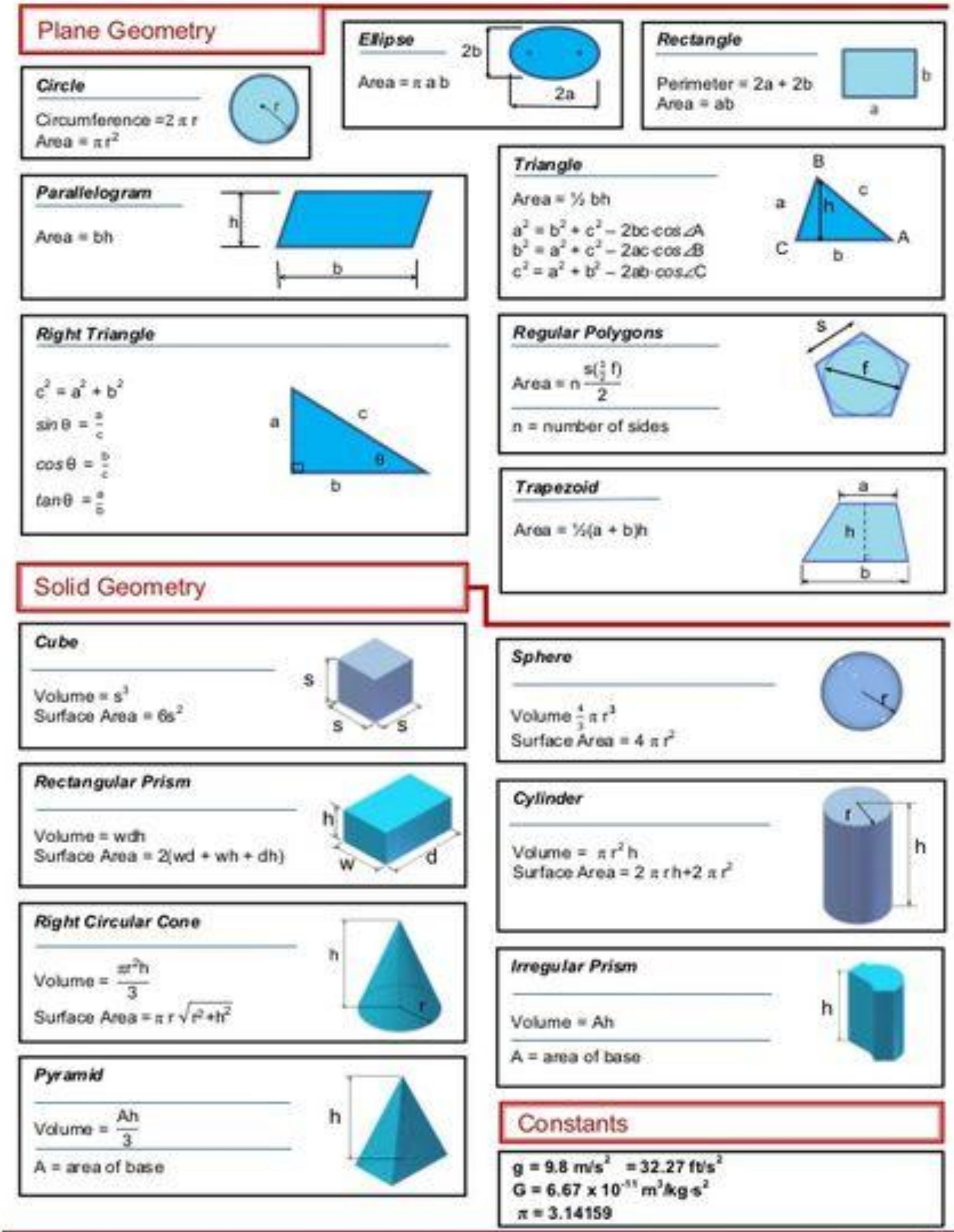
In a convex polygon of n sides, we have

1. Sum of all interior angles = (2n – 4) × 90°
2. Sum of all exterior angles = 360°
3. Number of diagonals of polygon on n sides = [n(n – 3) / 2]

A polygon is called a triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon and decagon according as it contains 3, 4, 5, 6, 7, 8, 9 and 10 sides, respectively.

Area and Perimeter of 2-D Figures

Shape	Terms	Perimeter	Area
1) Triangle 	b = base of triangle h = height of triangle	Perimeter = a + b + c and (Semi perimeter)s = $\frac{a+b+c}{2}$	Area = $\frac{1}{2}$ x base x height Or Area = $\sqrt{s(s-a)(s-b)(s-c)}$
2) Equilateral Triangle 	a = length of sides of equilateral triangle	Perimeter = 3a	Area = $\frac{\sqrt{3}}{4}a^2$
3) Square 	a = length of side	Perimeter = 4a	Area = a^2
4) Rectangle 	l = length b = breadth	Perimeter = 2(l + b)	Area= l x b
5) Parallelogram 	b = base of parallelogram h = height of parallelogram	Perimeter = 2(a + b)	Area = b x h
6) Trapezium 	a , b = length of parallel sides h = distance between the parallel sides	Perimeter = a + b + c + d	Area = $\frac{1}{2}(a+b)$ x h
7) Circle 	r = radius d = diameter	Perimeter or circumference = 2πr	Area = πr^2



Plane Geometry

Circle

Circumference $= 2\pi r$
Area $= \pi r^2$

Ellipse

Area $= \pi a b$

Rectangle

Perimeter $= 2a + 2b$
Area $= ab$

Parallelogram

Area $= bh$

Triangle

Area $= \frac{1}{2} bh$

$a^2 = b^2 + c^2 - 2bc \cdot \cos\angle A$
$b^2 = a^2 + c^2 - 2ac \cdot \cos\angle B$
$c^2 = a^2 + b^2 - 2ab \cdot \cos\angle C$

Right Triangle

$c^2 = a^2 + b^2$
$\sin\theta = \frac{a}{c}$
$\cos\theta = \frac{b}{c}$
$\tan\theta = \frac{a}{b}$

Regular Polygons

Area $= n\frac{s(\frac{1}{2}f)}{2}$

n = number of sides

Trapezoid

Area $= \frac{1}{2}(a + b)h$

Solid Geometry

Cube

Volume $= s^3$
Surface Area $= 6s^2$

Sphere

Volume $\frac{4}{3}\pi r^3$
Surface Area $= 4\pi r^2$

Rectangular Prism

Volume $= wdh$
Surface Area $= 2(wd + wh + dh)$

Cylinder

Volume $= \pi r^2 h$
Surface Area $= 2\pi r h + 2\pi r^2$

Right Circular Cone

Volume $= \frac{\pi r^2 h}{3}$
Surface Area $= \pi r \sqrt{r^2 + h^2}$

Irregular Prism

Volume $= Ah$

A = area of base

Pyramid

Volume $= \frac{Ah}{3}$

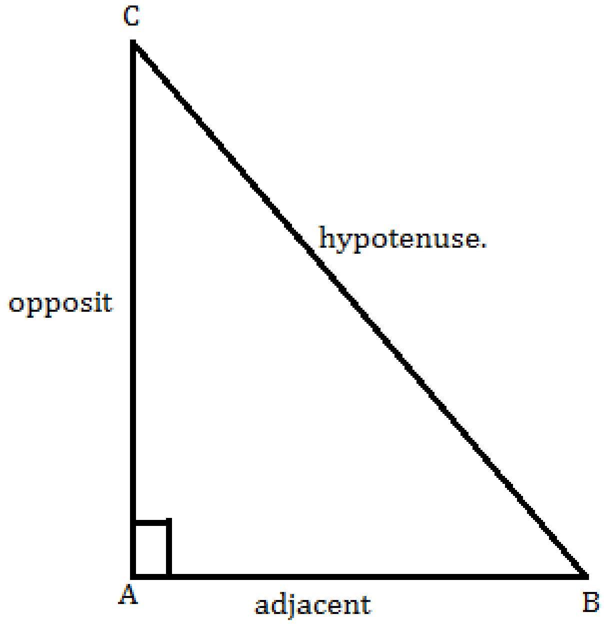
A = area of base

Constants

$g = 9.8\ m/s^2 = 32.27\ ft/s^2$
$G = 6.67 \times 10^{-11}\ m^3/kg \cdot s^2$
$\pi = 3.14159$

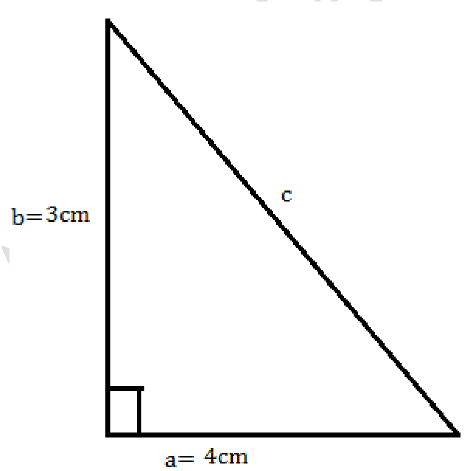
Pythagoras Theorem

The theorem makes reference to a right-angled triangle such as that shown below. The side opposite the right-angle is the longest side and is called the hypotenuse.



What the theorem says is that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the two shorter sides (opposite and adjacent). The above shows squares drawn on the hypotenuse and on the two shorter sides. The theorem tells us that
$|BC|^2 = |AB|^2 + |AC|^2$

Example:



$c^2 = a^2 + b^2$
$c^2 = 4^2 + 3^2$
$c^2 = 16+9$

$c^2 = 25$
$c = 5$

So 5 is the length of the hypotenuse, the longest side of the triangle.

Questions

1. One end of a ten foot ladder is four feet from the base of a wall. How high on the wall does the top of the ladder touch?
2. Could 3, 6 and 8 represent the lengths of the sides of a right triangle? Explain.
3. A 6 foot ladder is one and a half feet from a wall. How high on the wall does the ladder touch?
4. An airplane is flying from Accra to Temale. The airplane flies 50 miles East and then 180 miles South. How far is Temale from Accra directly?
5. A wooden flagpole is 25 foot tall. In a storm, the flagpole is broken and its top touches the ground 5 foot from the base. Find the lengths of the segments of the flagpole.

AVERAGES/ MEASUREMENT OF CENTRAL TENDENCIES

This topic seeks to test teachers understanding on how to transform raw data and scores of students into information for making decision.

An average is a single number which is used to represent a group of values collected for a particular purpose.

We use an average to summarise the values in a set of data. As a representative value, it should be fairly central to, and typical of, the values that it represents.

Three types of average

There are three measures of central tendency that are commonly used to describe the average value of a set of data. These are the mode, the mean and the median.

1. The mode is the most commonly occurring value.

2. The mean is calculated by dividing the sum of the values by the number of values.

3. The median is the value in the middle of an ordered set of data.

Mean

The mean is the most commonly used average. The mean is usually used when the data involved is fairly evenly spread, and there are no exceptional cases that are much higher or lower than the rest. If a few exceptional results exist, the mean may give a misleading impression, because it takes account of all the data given. It is found by adding together all the data values in a set of data and then dividing this total by the number of values in the set.

Example: In a test, a group of 11 pupils scored the following marks: 5, 10, 3, 4, 4, 8, 4, 3, 11, 9, 5. Find the mean mark.

The mean mark is found by adding together all the marks and dividing the total by the number of pupils.

(5+10+3+4+4+8+4+3+11+9+5) = 66
66 ÷ 11= 6. Therefore the mean mark is 6.

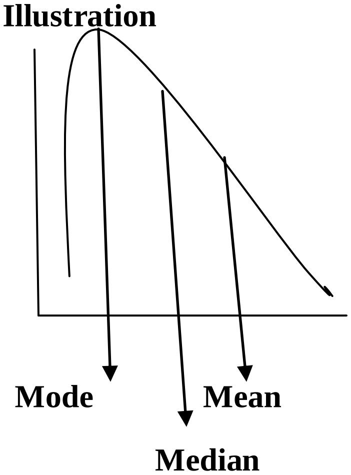
Median
The median is the middle value of a set of data when placed in order. It can be found with little or no calculation. The median is particularly useful when the data has a wide spread, as the middle value is not affected by exceptional cases. For example, to find the median age of a group of 11 pupils it is only necessary to arrange their ages in order, and the age of the middle child (the sixth child) is the median. This means that for the median there are as many values in the dataset above the median as there are below it. The median for any data can also be found by drawing a cumulative frequency graph.

Example: The median mark is the middle value in the group of marks when arranged in order of size. In order of size the marks are: 3, 3, 4, 4, 4, 5, 5, 8, 9, 10, 11. There are 11 numbers in this set of data. The sixth number is the middle value or median. In this example the median is 5. ***Note*****:** When there is an even number of values, the median is found half way between the two middle values. For example, if the test results for a particular pupil are: 12, 13, 16, 17, 21 and 25, then 16 and 17 are the middle values. The median is the mean of the 2 values, or half way between the 2 values. That is (16 + 17)/2. The median in this example is 16.5.

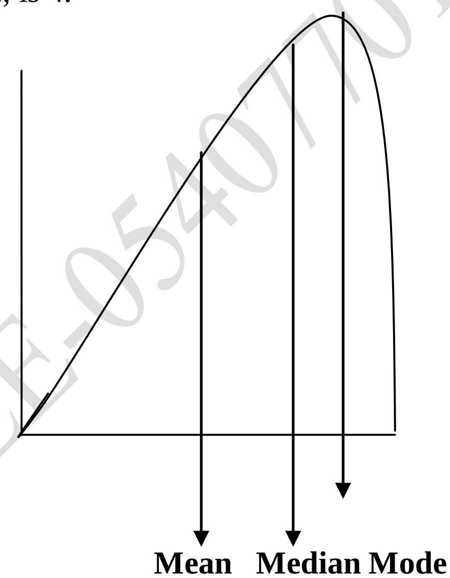
Mode

The mode is the most frequently occurring value in a set of data. For example, shoe manufacturers are not interested in the mean or median value of shoe sizes but they may want to know the size most frequently sold. Example, the mode is the value which occurs most often in the set of marks5, 10, 3, 4, 4, 8, 4, 3, 11, 9, 5. The score which occurs most often is 4 with 3 pupils scoring 4 marks, so the modal mark, or mode, is 4.

Illustration



Positive Skewness



Negative Skewness

Where Mean> Median, the distribution is skewed to the right (positive skewness) showing that performance tends to be low.

Where Mean<Median, the distribution is skewed to the left (negative skewness) showing that performance tends to be high.

Examples: Given the following scores, 15, 12, 10, 10, 9, 20, 14, 11, 13, 16, find the mean.

X = 15+12+10+10+19+20+14+11+13+16 = $\frac{130}{10}$ = 13

Formula used for ungrouped data to find the mean is X = $\frac{\square\square}{\square}$

Finding Mean From Grouped Data

Scores	Midpoint(X)	Freq(F)	Fx
46-50	48	4	192
41-45	43	6	258
36-40	38	10	380
31-35	33	12	380
26-30	28	8	224
21-25	23	7	161
16-20	18	3	54
Total		**50**	**1665**

Mean(X) $= \frac{\sum fx}{n} = \frac{1665}{50} = \mathbf{33.3}$

Finding The Median

From ungrouped data:

1. Arrange all observations in order of size from smallest to largest.
2. If the number of observations, n, is odd, the median is the number at the center or the number at the $\frac{(n+1)}{2}th$ position
3. If the number of observations, n, is even, the median is the mean of the two center numbers.

Example:

For old set of numbers:

Given the observation as: 8 11 26 7 12 9 6 20 1, find the median.

a. Rearrange the scores in a sequential order: 6 7 8 9 11 12 14 20 26
b. Find $\frac{(n+1)}{2}th$ position thus $\frac{(9+1)}{2}th = \frac{10}{2}th = 5th$ position
c. The median is at the 5^{th} position which is 11.

For even set of numbers:

Given the observation as: 45 61 76 44 89 31 14 8 98 12, find the median.

a. Rearrange the scores in a sequential order: 8 12 14 31 44 45 61 76 89 98
b. Find $\frac{(n+1)}{2}th$ position thus $\frac{(10+1)}{2}th = \frac{11}{2}th = 5.5th$ position. This means that the median lies half way between the 5^{th} and 6^{th} positions

c. The score at the 5th position is 44 and at the 6th position is 45. Half way between 44 and 45 is $\frac{(44+45)}{2}=\frac{(89)}{2}$=44.5=45. The median is therefore 45.

d. The median is at the 5th position which is 45.

From Grouped Data:

Scores	Midpoint(X)	Freq(F)	Cum. Freq
46-50	48	4	46+4=50
41-45	43	6	40+6=46
36-40	38	10	30+10=40
31-35	33	12	18+12=30
26-30	28	8	19+8=18
21-25	23	7	3+7=10
16-20	18	3	3
Total		**50**	

Using the formula Mdn=$\text{L1}\frac{□\frac{□}{2}-□□}{□□□□}$

Representation of data

Types of data

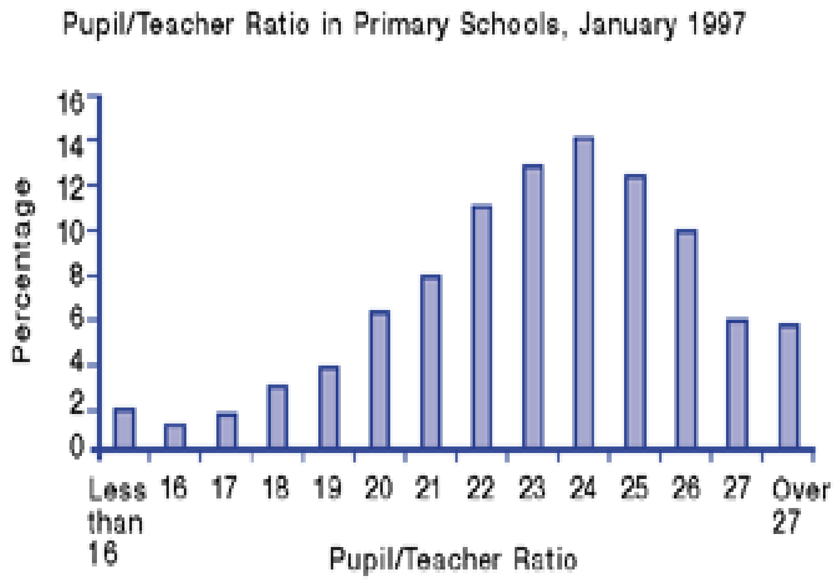
There are two types of data: qualitative (or categorical) data are described by words and are non-numerical, such as blood types or colours. Quantitative data take numerical values and are either discrete or continuous. As a general rule, discrete data are counted and cannot be made more precise, whereas continuous data are measurements that are given to a chosen degree of accuracy

Bar Chart/Graph

A bar chart shows items represented as vertical or horizontal bars. The length of each bar shows the number of times the item occurs.

Example 1

A survey of primary schools was carried out in January 1997. The number of pupils and all teachers, including the head teacher, was recorded for each primary school surveyed. The pupil to teacher ratio (PTR) was calculated for each school. The data was recorded in the table shown below.

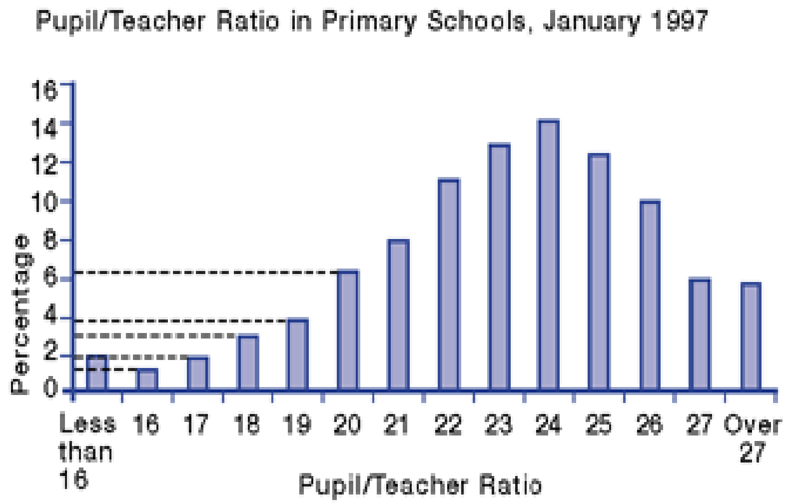


Pupil Teacher Ratio(PTR)	<16	16	17	18	19	20	21	22	23	24	25	26	27	>27	
% of schools with this PTR	2.0	1.2	1.9	2.9	4.0	6.5	8.0	11.2	13	14.3	12.8	10.3	6	5.9	

Interpretation of the Data:

The bar chart shows the data in a visual form. For each PTR (Pupil Teacher Ratio value), a vertical bar has been drawn. The top of the bar represents the corresponding percentage value from the table. For example, the bar for a PTR less than16 has a height of 2 on the vertical axis, representing 2%, and the bar for a PTR of 23 is represented by 13% on the vertical axis. It is often easier to see the pattern in data when it is displayed as a graph rather than in a table. This bar chart shows that most schools have a PTR between 22 and 26 and the most common PTR is 24. Bar charts are often used in documents containing data, rather than tables. Information must be obtained by using the charts.

Example 2
Use the bar chart to find what percentage of schools have a PTR (Pupil Teacher Ratio) of 20 or less.



This question can be solved by locating the bars for PTRs of 20 or less, (ie the bars for less than 16, up to and including 20). As the values must be read from the bar chart, some values are likely to be approximate because they cannot be read accurately.

2 + 1.2 + 1.9 + 3 + 4 + 6.5
= 18.6%
So, approximately 18.6%of schools have a PTR of 20 or less.

Performance of Inter –House Athletics at MPASS

House	Total Points
One	120
Two	100
Three	150
Four	60
Five	170

Prepare a Bar graph from the data above.

Box And Whisker Diagram

A box and whisker diagram illustrates the spread of a set of data. It also displays the upper quartile, lower quartile and inter-quartile range of the data set. A quartile is any one of the values which divide the data set into 4 equal parts, so each part represents a quarter of the sample. The upper quartile represents the highest 25% of the data. It can be considered as the median of the upper half of the values in the set.

The lower quartile represents the lowest 25%of the data. It can be considered as the median of the lower half of all the values in the set. The inter-quartile range is the difference in value between the upper quartile and the lower quartile values. The median is the middle value, half of the data set is below and half is above.

Example

You can draw a box and whisker diagram for the results below which were obtained by a year 6 class in an English test marked out of 20.

Pupil A - 14	Pupil B – 13	Pupil C - 3	Pupil D - 7
Pupil E - 9	Pupil F – 12	Pupil G - 17	Pupil H - 4
Pupil I –9	Pupil J - 10	Pupil K – 18	Pupil L - 16

There are 12 scores. You must place these in order as follows 3, 4, 7, 9, 9, 10, 12, 13, 14, 16, 17, 18.

The range is 18 - 3 = 15.13

There is an even number of values so the median is the midway between 10 and 12. The median has the value 11.

The upper quartile is the median value for 12, 13, 14, 16, 17, 18 and so is midway between 14 and 16. The upper quartile is 15. So 25% of all the pupils score above 15 and 75%score less than 15.

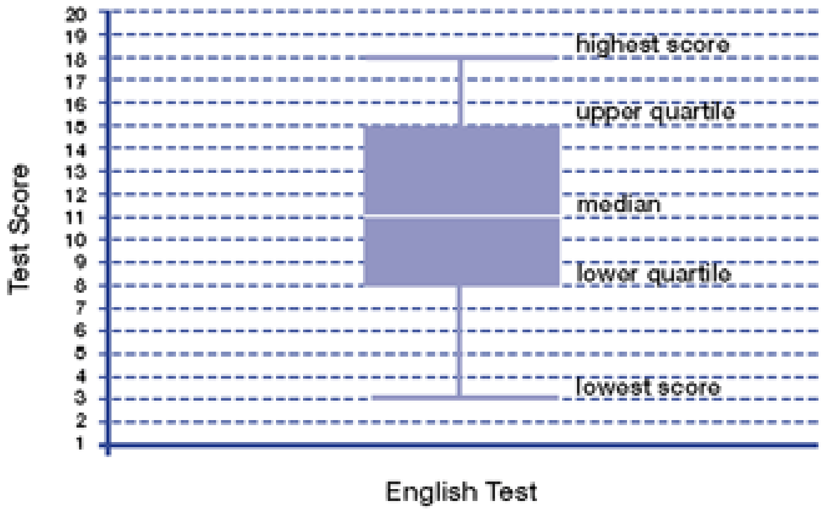
The lower quartile is the median value for 3, 4, 7, 9, 9, 10 and so is midway between 7 and 9. The lower quartile is 8. So 25%of all the pupils score less than 8 and 75% score more than 8.

Had there been an odd number of values, the median would be the middle value, but in determining the lower and upper quartiles the median value is then ignored. For example, if the values are: 2, 3, 5, 7, 9, 13, 15, 18, 19, 20, 22, 23. The median value is 14.

A box and whisker diagram can be used to display this information. The table below shows the results of an English test, constructed for presentation as a box and whisker diagram. English test scores out of 20

Lowest test score	3

Highest test score	18



Lower quartile	8
Upper quartile	15

The data in the table are represented by the box and whisker diagram as follows

Cumulative Frequency
A cumulative frequency graph shows the cumulative totals of a set of values up to each of the points on the graph.

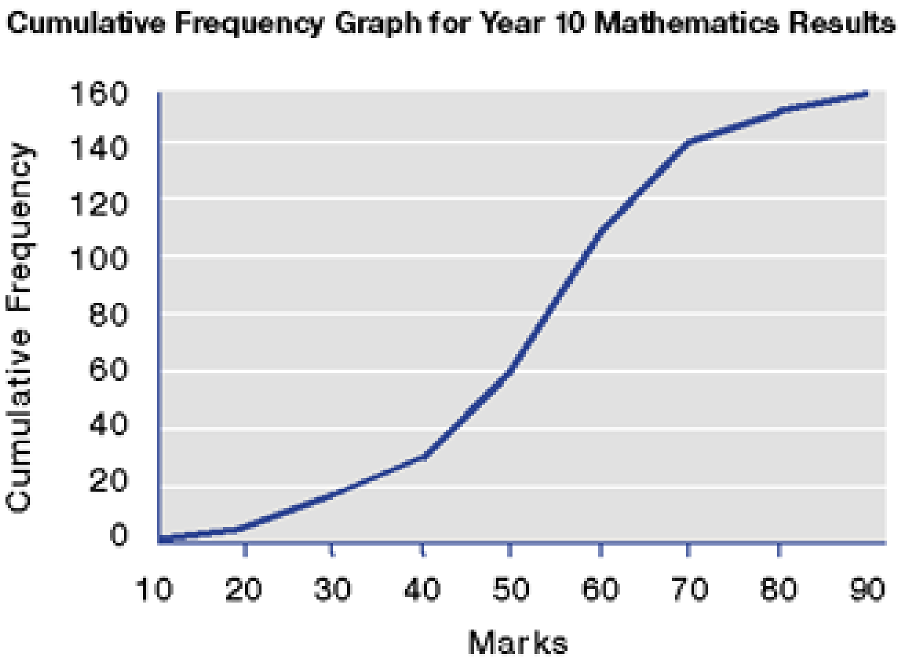
Example
A teacher arranged the marks gained by all year 10 pupils in a mathematics test in a table as shown below:

Marks	Frequency of pupils	Cumulative Percentage
11-20	2	2
21-30	11	2+11=13
31-40	19	13+19=32
41-50	36	33+36=68
51-60	42	68+42=110
61-70	31	110+31=141

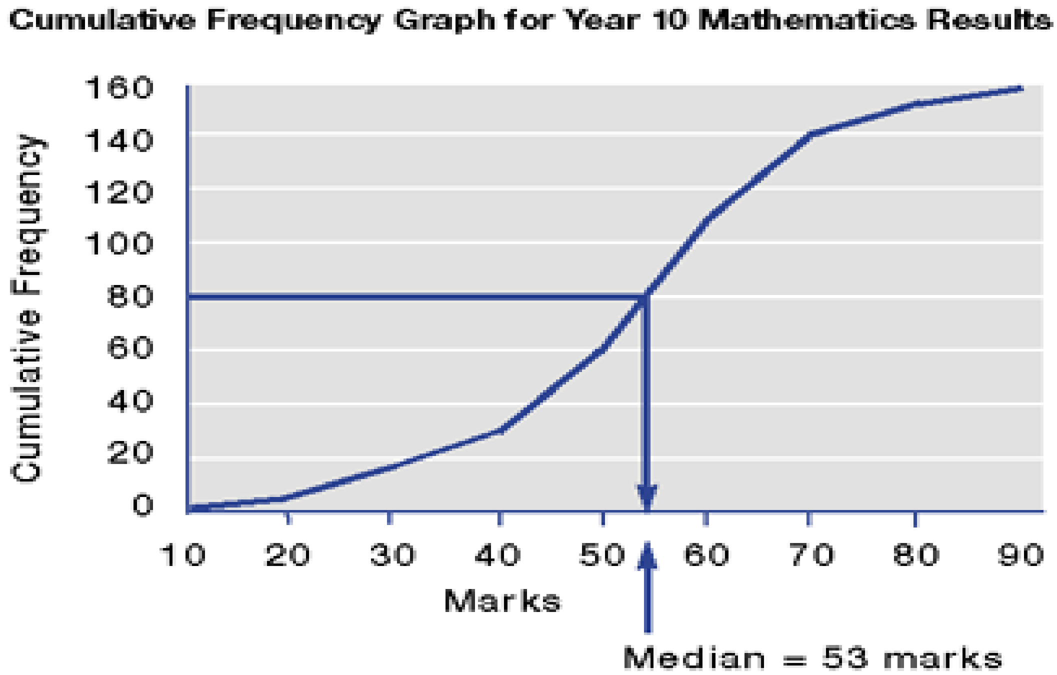
71-80	13	141+13=154
81-90	6	154+6=160

This table shows the number of pupils (called the frequency) who gained marks in the various mark bands (eg 31 to 40). For example, the number of pupils who scored between 21 and 30 marks was 11. No pupil scored fewer than 11 marks or more than 90 marks.

The cumulative frequency column makes it easy to see at a glance that 68 pupils scored 50 marks or fewer, and that 32 pupils scored 40 marks or fewer.



A cumulative frequency graph is a way of presenting information visually, which allows other information to be deduced.
For example, from the graph we can obtain the median (or middle) mark. The median is the mark which half of all pupils exceed and half do not reach. As there are 160 pupils, we need to find 80 pupils, and then draw a line across until it meets the graph. Drawing a vertical line down from this point and reading the number of marks at that point shows that the median is 53 marks.



A cumulative frequency graph is a way of presenting information visually, which allows other information to be deduced. For example, from the graph we can obtain the median (or middle) mark. The median is the mark which half of all pupils exceed and half do not reach. As there are160 pupils, we need to find 80 pupils, and then draw a line across until it meets the graph. Drawing a vertical line down from this point and reading the number of marks at that point shows that the median is 53 marks.
It is also possible to find the upper and lower quartile marks from the graph. In this example, the lower quartile is the mark which 1 quarter of all pupils' scores does not reach.

To find the lower quartile, find the point 1 quarter of the way up the vertical axis, which is40 on the cumulative frequency axis. Draw a line across from the 40 mark until it meets the graph. Draw a vertical line down from this point. The lower quartile is 43 marks.
Therefore a quarter of the marks lie below 43 marks.

To find the upper quartile, we need to find the point that is three quarters of the way up the vertical axis which is 120 on the cumulative frequency axis. Draw a line across from the 120 mark until it meets the graph. Draw a vertical line down from this point. An estimate of the upper quartile is 63. Therefore three-quarters of the marks lie below 63 marks.

The inter-quartile range is often used to give an idea of how widely the items of data are spread out. The inter-quartile range is found by calculating the difference in value between the upper quartile and the lower quartile, that is: upper quartile value - lower quartile value.

In this example, the estimate of the inter-quartile range is 63 - 43 = 20.The marks of the middle 50 per cent of pupils lie roughly between the lower quartile mark, 43, and the upper quartile mark, 63.If this information was shown using a box and whisker diagram, the box would be drawn with the left-hand edge at 43 and the right-hand edge at 63.

Line graphs

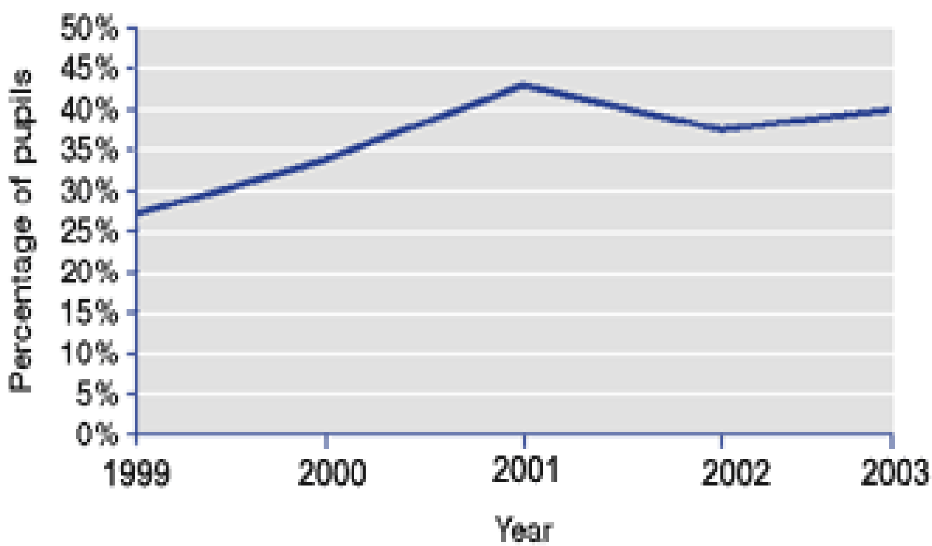
A line graph is a visual representation of two sets of related data. It is the name given to a graph where the individual points are joined by a line or lines.

Example

The percentage of pupils in a school achieving level 5 in the end of Stage 2 assessments was recorded in the table below.

Percentage of pupils achieving level 5 in the end of Stage 2 assessments

Year	1999	2000	2001	2002	2003
% of pupils achieving level 5	27%	34%	43%	37%	40%



The table and the line graph represent the same data. The graph provides a visual representation of the changes in the school's performance over a period of time, but the points relating to the years would not normally be joined up; they have been here to help show the trend.

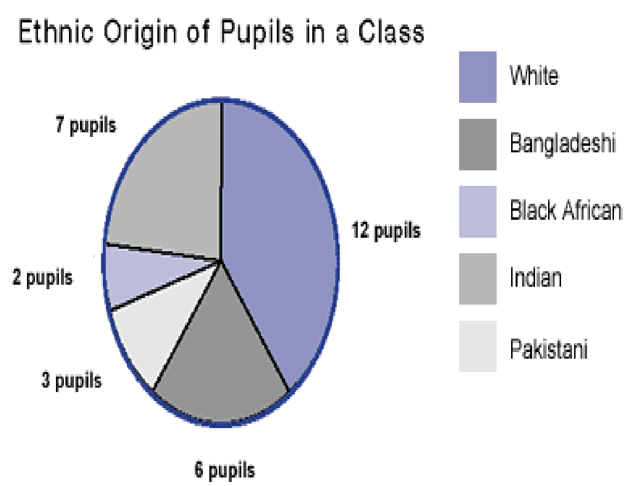
Pie Chart

A pie chart is a way of illustrating information by using sectors of a circle to represent parts of the whole.

Example: A newly qualified teacher (NQT) was given the following information about the ethnic origins of the pupils in a class.

Ethnic origin	No. of pupils
White	12
Indian	7
Black African	2
Pakistani	3
Bangladeshi	6
Total	30

The 30 pupils in the class are classified into 5 different ethnic origins. The whole pie chart represents the class of 30(360°) pupils, and the 5 sectors represent 12,(12/30×360°) 7,(7/30×360°), 2(2/30×360°), 3,(3/30×360°) and 6,(6/30×360°) pupils, as shown in the table.



Range

Range is a measure of the spread of data. It is the difference between the largest and the smallest values. To find the range of a set of data, take the smallest value from the largest value.

Example 1

In a set of tests, 2 pupils scored the following marks out of 10:
John: 5, 6, 4, 5, 5, 6, 5, 7
Sally: 3, 2, 3, 6, 5, 8, 6, 9
The lowest mark John scored is 4 and the highest mark he scored is 7, so the range of John's marks is 3 (from 4 to 7).
Sally's marks are between 2 and 9 making the range for Sally's marks 7.
From this we can see that Sally's marks are more widely spread than John's marks.
John's marks are fairly consistent, whereas Sally obtains some high and some low marks.

Example 2

The marks scored by a group of pupils in an end of term test marked out of 80 are as follows.
37, 45, 53, 61, 70, 50, 48, 29, 52, 59
The teacher was interested in the spread of marks. What is the range of the set of marks?
The lowest mark is 29
The highest mark is 70
The difference: 70 - 29 = 41

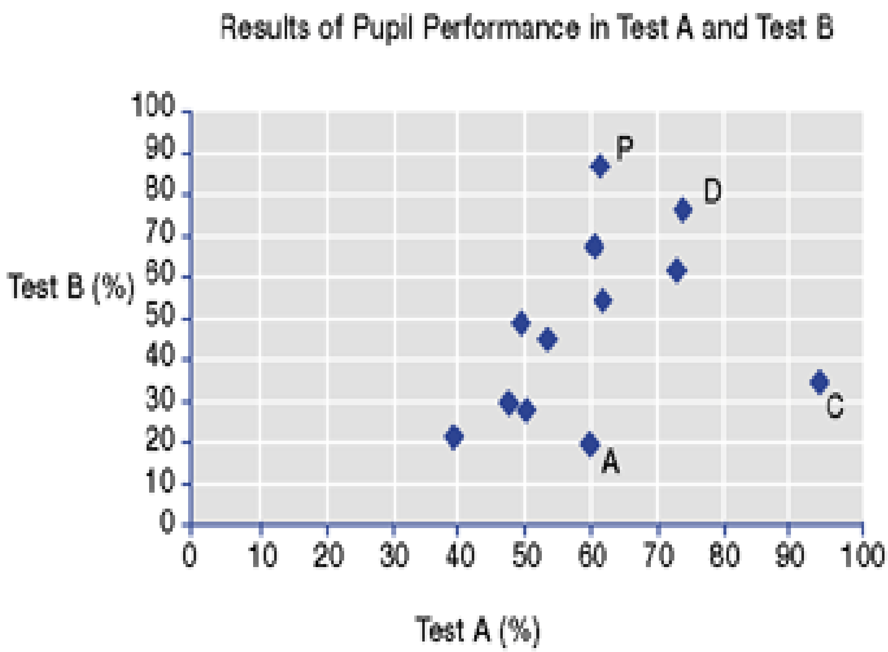
The range of marks is 41. This result shows wide variation in the group's performance on this test.

Scatter Graphs

A scatter graph is a statistical diagram drawn to compare two sets of data. It can be used to look for connections or a correlation between the two sets of data.

Example

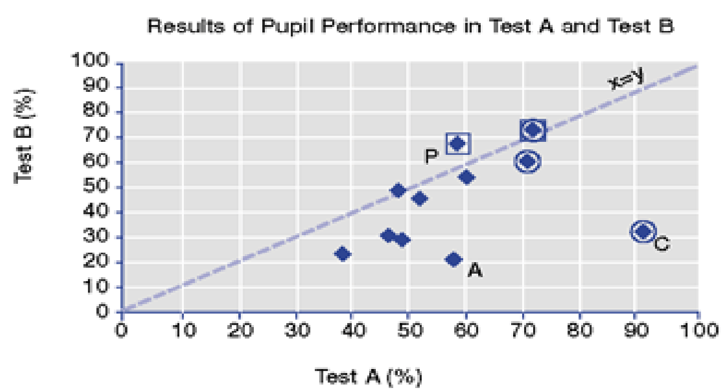
A class took tests. Test A was given early in the course and test B towards the end.
The comparative results of these tests are given in the scatter graph below



Each symbol on the graph shows the scores achieved in both tests by each of the pupils.
So, for example, pupil A scored 60% on test A but only 20% on test B.
Pupil C achieved the top mark in test A.
Pupil P achieved the top mark in test B.
Pupil marked D came second in both tests.

Making Comparisons

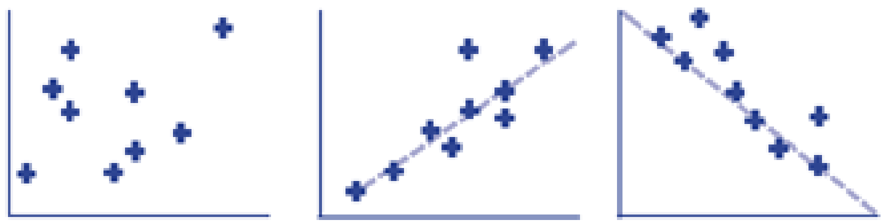
To find out whether the pupils generally did better in one test or the other, use a ruler or straight edge to draw a line joining marks that are the same on both tests, for example 0 for both, 50 for both, 70 for both, etc.



The two students above the line achieved higher marks on test B than in test A.
The eight students below the line achieved higher marks on test A than on test B.

An important use of scatter graphs is to show how one set of results relates to another. If the points on a scatter graph appear to be randomly scattered (see figure (i) below) there is unlikely

to be any correlation between the two sets of data being measured.
If they form a more regular pattern (as in figure (ii) or figure (iii)), there is likely to be a correlation between them.



The scatter graph for the results from test A and test B, discussed earlier, is similar to figure (ii) and shows a correlation.
This suggests that test A and test B have a correlation because the pupils performed similarly in both tests. So the tests are likely to be based on the same subject or related subjects and set at a similar level.
The scatter graph enables you to identify particular pupils for whom action might be needed. For example, the reason why C achieved a high mark in test A but a low mark in test B might be investigated.

RIGID MOTION

A rigid motion is a transformation (of the plane) that “preserves distance”. In other words, if A is sent/mapped/transformed to A′ and B is sent to B′, then the distance between A and B (the length of segment AB) is the same as the distance between A′ and B′ (the length of segment A′B′).

Types of Rigid Motion

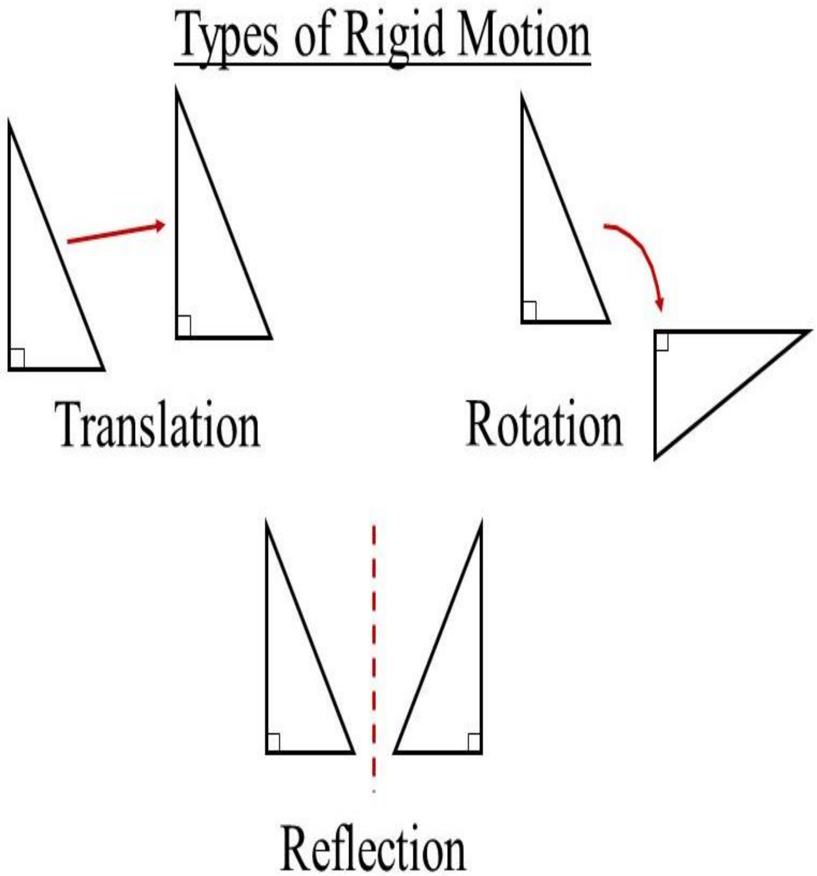
There are three types of rigid motions that we will consider: translation, rotation and reflection,

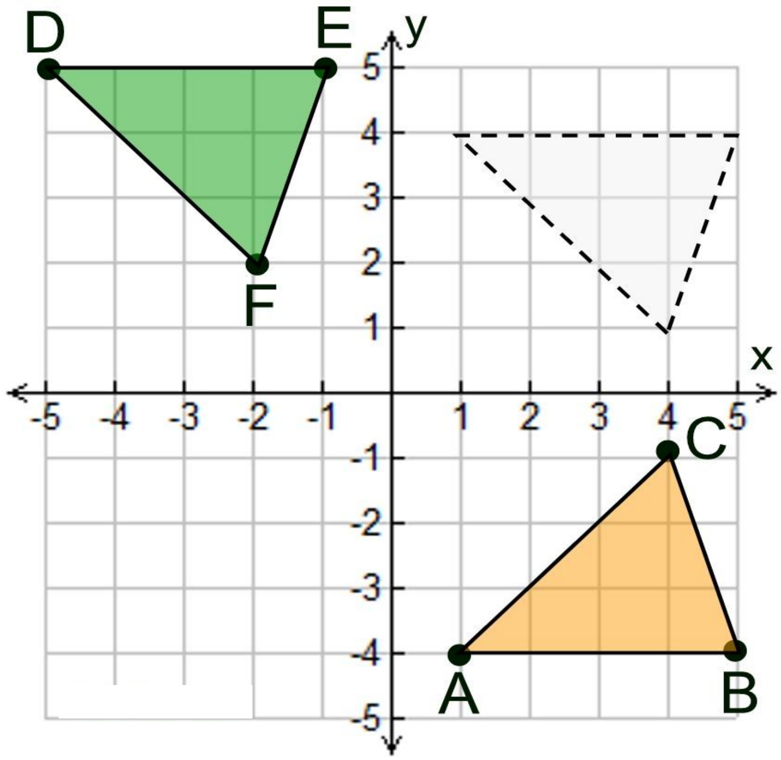
1. Translation: In a translation, everything is moved by the same amount and in the same direction. Every translation has a direction and a distance
2. Rotation: A rotation fixes one point (the rotocenter) and everything rotates by the same amunt around that point. Every rotation has a rotocenter and an angle.
3. Reflection: A reflection fixes a mirror line in the plane and exchanges points from one side of the line with points on the other side of the mirror. Every reflection has a mirror line. A reflection is called a rigid transformation or isometry because the image is the same size and shape as the pre-image.

Rules

1. Reflection in x-axis. (x, y) --> (x,-y)
2. Reflection in y-axis. (x, y) --> (-x, y)

3. Reflection in line y=x. (x, y) --> (y, x)
4. Translation in coordinate plane. (x, y) --> (x+ a, y+ b)
5. 90 degree rotation. (x, y) --> (-y, x)
6. 180 degree rotation. (x, y) --> (-x,-y)




D
E
F
y
x
5
4
3
2
1
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
1
2
3
4
5
C
A
B