



$$T(x) = \begin{bmatrix} \cosh(x) & \sinh(x) \\ \sinh(x) & \cosh(x) \end{bmatrix}$$

$$R(x) = \begin{pmatrix} -1 & 0 \\ \frac{2}{d(x)} & -1 \end{pmatrix}$$

$$T^{-1}(x) = \begin{bmatrix} \cosh(x) & -\sinh(x) \\ -\sinh(x) & \cosh(x) \end{bmatrix}$$

$$R^{-1}(x) = \begin{pmatrix} -1 & 0 \\ -\frac{2}{d(x)} & -1 \end{pmatrix}$$

$$T(x)R(x)T(z)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = R^{-1}(y)T^{-1}(y)R^{-1}(z)\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (?)$$

$$\sinh(x+z) - \sinh(y) = \frac{2\sinh(x)\sinh(z)}{d(x_0)} \quad (1)$$

$$\left( d(x_0) = \frac{\sinh \phi_0}{k(s_0)} \right) \quad (3)$$

$$\cosh(y) = \cosh(x)\cosh(z) - \sinh(x)\sinh(z)\cos(\gamma)$$

$$\lim \phi_0 = \lim \left( \frac{\pi - \gamma}{2} \right) = \cos \left( \frac{\pi}{2} \right) \quad (4)$$

$$\text{using } \cosh(x+z) = \cosh(x)\cosh(z) + \sinh(x)\sinh(z)$$

$$\cosh(y) = \cosh(x+z) - \sinh(x)\sinh(z) - \sinh(x)\sinh(z)\cos(\gamma)$$

$$\begin{aligned} \cosh(x+z) - \cosh(y) &= [1 + \cos(\gamma)] \sinh(x)\sinh(z) \\ &= 2\cos^2\left(\frac{\gamma}{2}\right) \sinh(x)\sinh(z) \quad (2) \end{aligned}$$

$$\frac{\cosh(x+z) - \cosh(y)}{2\cos^2\left(\frac{\gamma}{2}\right)} = \sinh(x)\sinh(z) = \frac{\sinh(x+z) - \sinh(y) \cdot \cos\left(\frac{\gamma}{2}\right)}{k(s_0)} \quad \text{from (1), (2), (3), (4)}$$

$$\therefore k(s_0) = \frac{\sinh(x+z) - \sinh(y)}{\cosh(x+z) - \cosh(y)} \cdot \cos^3\left(\frac{\gamma}{2}\right)$$

$$\text{using } \left. \begin{aligned} \sinh a + \sinh b &= 2 \cosh\left(\frac{a+b}{2}\right) \sinh\left(\frac{a-b}{2}\right) \\ \cosh a - \cosh b &= 2 \sinh\left(\frac{a+b}{2}\right) \sinh\left(\frac{a-b}{2}\right) \\ \text{and } x+y+z=L &\Rightarrow x+z=L-y \end{aligned} \right\}$$

$$k(s_0) = \frac{\sinh(L-y) - \sinh(y)}{\cosh(L-y) - \cosh(y)} \cos^3\left(\frac{\gamma}{2}\right)$$

$$= \frac{2 \cosh\left(\frac{L}{2}\right) \sinh\left(\frac{L-2y}{2}\right)}{2 \sinh\left(\frac{L}{2}\right) \sinh\left(\frac{L-2y}{2}\right)} \cos^3\left(\frac{\gamma}{2}\right) = \text{constant} \cdot \cos^3\left(\frac{\gamma}{2}\right)$$