IGL Brochure - Exploring Entropy and Complexity of Curves on a Surface

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Each closed curve γ with a base point p on a surface S determines a natural homeomorphism $\mathcal{P}(\gamma)$ of the surface $S \setminus \{p\}$ via **point-pushing**:

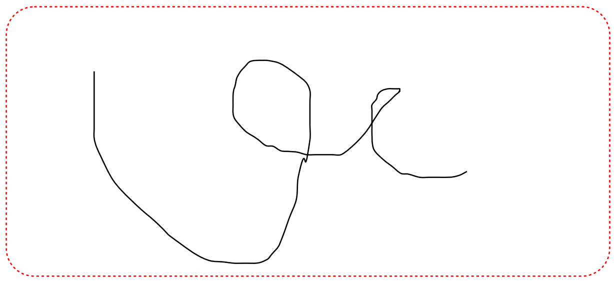

Figure 1: **Point-push** around γ by dragging basepoint p once around γ.

The goal of our project is to study point-pushing homeomorphisms on the genus-g surface, and to relate their topological entropy to their combinatorial complexity. To this end, we developed a computer program that randomly generates closed curves γ on the surface, and then calculates the self-intersection number $\iota(\gamma)$ of the curve (measuring the combinatorial complexity) and the dilatation number (measuring the topological entropy) of the resulting point-pushing homeomorphism $\mathcal{P}(\gamma)$. We then statistically analyzed the results by calculating the distributions of the dilatation and self-intersection number data and looking for correlations between these numbers.