# Homework_5

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2025-10-13

**Setup**

First, we load in our data. This is the NOAA.new dataset from Canvas.

```
NOAA.new <- read.csv("/Users/nancyz/STAT486/Data/NewNOAA.csv") #replace with your path
x <- NOAA.new[,3] #temperature rise
y <- NOAA.new[,2] #rate of billion-dollar disasters
```

**Edited smoother.pck** This section contains the edited smoother.pck and its functions, with PRESS statistic in each smoother. The function chunks are fully commented, as this data was taken from Canvas/slides.

```
smoother.pck <- #received error that version of this package not available for this
  #version of R
c("smoother.pck", "bin.mean", "gauss.mean", "gauss.reg", "gauss.mean.trunc",
"gauss.reg.trunc", "my.hat.w")
bin.mean <-
function(x,y,nbin,xcol=2,do.plot=F)
{
  #order x and y
  o1<-order(x)
  x1<-x[o1]
  y1<-y[o1]
  #find min and max x
  r1<-range(x)
  #width of each bin = (max - min) / number of bins
  inc<-(r1[2]-r1[1])/nbin
  yvec<-NULL
  smat<-matrix(0,length(x1),length(x1))
  #for each bin:
  for(i in 1:nbin){
        #find min and max values for the bin
        bin.low<-r1[1]+(i-1)*inc
        bin.high<-r1[1]+i*inc
        #I1: true if x1 is within the current bin or a later bin

        I1<-x1>=bin.low
        #I2: true if x1 is within the current bin or an earlier bin

if(i<nbin){
        I2<-x1<bin.high
}else{
        I2<-x1<=(bin.high+200)
}       #I3: true if both I1 and I2 are true, meaning x1
```

```r
        #is within the current bin
        I3<-as.logical(I1*I2)
        #find mean of the bucket
        yval<-mean(y1[I3])
        n1<-sum(I3)
        if(n1>0){
        #convert values in each bucket to the mean and add to matrix
        smat[I3, I3]<-1/n1
        yvec<-c(yvec,rep(yval,n1))
        }
  }
n99<-length(x1)
#calculate degrees of freedom, delta1, delta2, and R based on number of data points
dferror<-length(x1)-sum(diag(2*smat-smat%*%(t(smat))))
delta1<-sum(diag(t(diag(n99)-smat)%*%(diag(n99)-smat)))
R<-t(diag(n99)-smat)%*%(diag(n99)-smat)
delta2<-2*sum(diag(R%*%R))
#plot if enabled
if(isTRUE(do.plot)) lines(x1,as.numeric(smat%*%y1),col=xcol)
#calculate residuals and use PRESS function on the sum of squares
ypred<-y
ypred[o1]<-smat%*%y1
resid<-y1-(smat%*%y1)
hii<-diag(smat); den<-1-hii; den[den<=1e-8]<-1e-8;
loo<-as.numeric(resid)/den;
loo[!is.finite(loo)]<-0
PRESS<-sum(loo^2)
#return
list(smat=smat,df=sum(diag(smat)),dferror=dferror,delta1=delta1,delta2=delta2,
     resid=resid,pred=ypred,
x=x,press=PRESS)

}

gauss.mean <-
function(x,y,lambda,xcol=3,do.plot=T)
{
#order data
o1<-order(x)
x1<-x[o1]
y1<-y[o1]
r1<-range(x)
smat<-NULL
#for each data point
n1<-length(x1)
for(i in 1:n1){
        #take a sample with the data point as the mean and lambda as the standard deviation
        v1<-dnorm(x1,x1[i],lambda)
        #normalize vector by dividing by mean and add to matrix
        v1<-v1/sum(v1)
        smat<-rbind(smat,v1)
}
yhat<-smat%*%y1
```

```r
#plot if enabled
if(do.plot){
lines(x1,yhat,col=xcol)
}
n99<-length(x1)
#calculate degrees of freedom, delta1, delta2, and R based on number of data points
dferror<-length(x1)-sum(diag(2*smat-smat%*%(t(smat))))
delta1<-sum(diag(t(diag(n99)-smat)%*%(diag(n99)-smat)))
R<-t(diag(n99)-smat)%*%(diag(n99)-smat)
delta2<-2*sum(diag(R%*%R))
#calculate residuals and use PRESS function on the sum of squares
resid<-y1-smat%*%y1
ypred<-y
ypred[o1]<-smat%*%y1
hii<-diag(smat);
den<-1-hii;
den[den<=1e-8]<-1e-8; loo<-as.numeric(resid)/den; loo[!is.finite(loo)]<-0
PRESS<-sum(loo^2)
#return
list(smat=smat,df=sum(diag(smat)),dferror=dferror,delta1=delta1,delta2=delta2,resid=resid,pred=ypred,
press=PRESS)

}

gauss.reg <-
function(x,y,lambda,xcol=4,do.plot=T)
{
#order data
o1<-order(x)
x1<-x[o1]
y1<-y[o1]
r1<-range(x)
smat<-NULL
n1<-length(x1)
#for each data point
for(i in 1:n1){
        #take a sample with the data point as the mean and lambda as the standard deviation
        v1<-dnorm(x1,x1[i],lambda)
        #normalize vector by dividing by mean
        v1<-v1/sum(v1)
        #generate hat matrix
        H1<-my.hat.w(x1,v1)
        smat<-rbind(smat,H1[i,])
}
yhat<-smat%*%y1
#plot of enabled
if(do.plot){
lines(x1,yhat,col=xcol)
}
n99<-length(x1)
#calculate degrees of freedom, delta1, delta2, and R based on number of data points
dferror<-length(x1)-sum(diag(2*smat-smat%*%(t(smat))))
delta1<-sum(diag(t(diag(n99)-smat)%*%(diag(n99)-smat)))
```

```r
R<-t(diag(n99)-smat)%*%(diag(n99)-smat)
delta2<-2*sum(diag(R%*%R))
#calculate residuals
resid<-y1-smat%*%y1
ypred<-y
ypred[o1]<-smat%*%y1
hii<-diag(smat);
den<-1-hii;
den[den<=1e-8]<-1e-8; loo<-as.numeric(resid)/den; loo[!is.finite(loo)]<-0
PRESS<-sum(loo^2)  #added PRESS statistic calculation
#return
list(smat=smat,df=sum(diag(smat)),dferror=dferror,delta1=delta1,delta2=delta2,resid=resid,pred=ypred,
press=PRESS)
}

gauss.mean.trunc <-
function(x,y,lambda,nnn,xcol=5,do.plot=T)
{
#order data
o1<-order(x)
x1<-x[o1]
y1<-y[o1]
r1<-range(x)
smat<-NULL
n1<-length(x1)
trunc.val<-n1-nnn
#for each data point
for(i in 1:n1){
  #generate sample with sd lambda and mean x1[i]
        v1<-dnorm(x1,x1[i],lambda)
        #order vector
        o2<-order(v1)
        #remove values less than the threshold for truncating
        thresh<-v1[o2[trunc.val]]
        v1<-v1*(v1>thresh)
        #normalize and add to matrix
        v1<-v1/sum(v1)
        smat<-rbind(smat,v1)
}
yhat<-smat%*%y1
#graph if enabled
if(do.plot){
lines(x1,yhat,col=xcol)
}
n99<-length(x1)
#calculate degrees of freedom, delta1, delta2, and R based on number of data points
dferror<-length(x1)-sum(diag(2*smat-smat%*%(t(smat))))
delta1<-sum(diag(t(diag(n99)-smat)%*%(diag(n99)-smat)))
R<-t(diag(n99)-smat)%*%(diag(n99)-smat)
delta2<-2*sum(diag(R%*%R))
#predict y values and calculate residuals
resid<-y1-smat%*%y1
ypred<-y
```

```
ypred[o1]<-smat%*%y1
hii<-diag(smat);
den<-1-hii;
den[den<=1e-8]<-1e-8; loo<-as.numeric(resid)/den; loo[!is.finite(loo)]<-0
PRESS<-sum(loo^2)  #added PRESS statistic calculation
#return
list(smat=smat,df=sum(diag(smat)),dferror=dferror,delta1=delta1,delta2=delta2,resid=resid,pred=ypred,
press=PRESS)

}

gauss.reg.trunc <-
function(x,y,lambda,nnn,xcol=6,do.plot=T)
{
#order data
o1<-order(x)
x1<-x[o1]
y1<-y[o1]
r1<-range(x)
smat<-NULL
n1<-length(x1)
trunc.val<-n1-nnn
#for all data
for(i in 1:n1){
        #generate sample with sd lambda and mean x1[i]
        v1<-dnorm(x1,x1[i],lambda)
        #order vector
        o2<-order(v1)
        #remove values less than threshold for truncating
        thresh<-v1[o2[trunc.val]]
        v1<-v1*(v1>thresh)
        #normalize vector
        v1<-v1/sum(v1)
        #generate hat matrix
        H1<-my.hat.w(x1,v1)
        smat<-rbind(smat,H1[i,])
}
yhat<-smat%*%y1
if(do.plot){ #plot if enabled
lines(x1,yhat,col=xcol)
}
n99<-length(x1)
#calculate degrees of freedom, delta1, delta2, and R based on number of data points
dferror<-length(x1)-sum(diag(2*smat-smat%*%(t(smat))))
delta1<-sum(diag(t(diag(n99)-smat)%*%(diag(n99)-smat)))
R<-t(diag(n99)-smat)%*%(diag(n99)-smat)
delta2<-2*sum(diag(R%*%R))
#predict y values and calculate residuals
resid<-y1-smat%*%y1
ypred<-y
ypred[o1]<-smat%*%y1
hii<-diag(smat); den<-1-hii;
den[den<=1e-8]<-1e-8;
```

```
loo<-as.numeric(resid)/den; loo[!is.finite(loo)]<-0
PRESS<-sum(loo^2)  #added PRESS statistic calculation
#return
list(smat=smat,df=sum(diag(smat)),dferror=dferror,
     delta1=delta1,delta2=delta2,resid=resid,pred=ypred,
press=PRESS)
}


my.hat.w <-
function(x,wt){
x1<-cbind(1,x)
x1%*%solve(t(x1)%*%diag(wt)%*%x1)%*%t(x1)%*%(diag(wt))
}
```

**Comparing Smoothers with PRESS and Plots** This section is to apply each smoother to the NOAA.new data using the parameters given in slide 17. The reference from the slides is provided first, then the updated code with more readability is below.

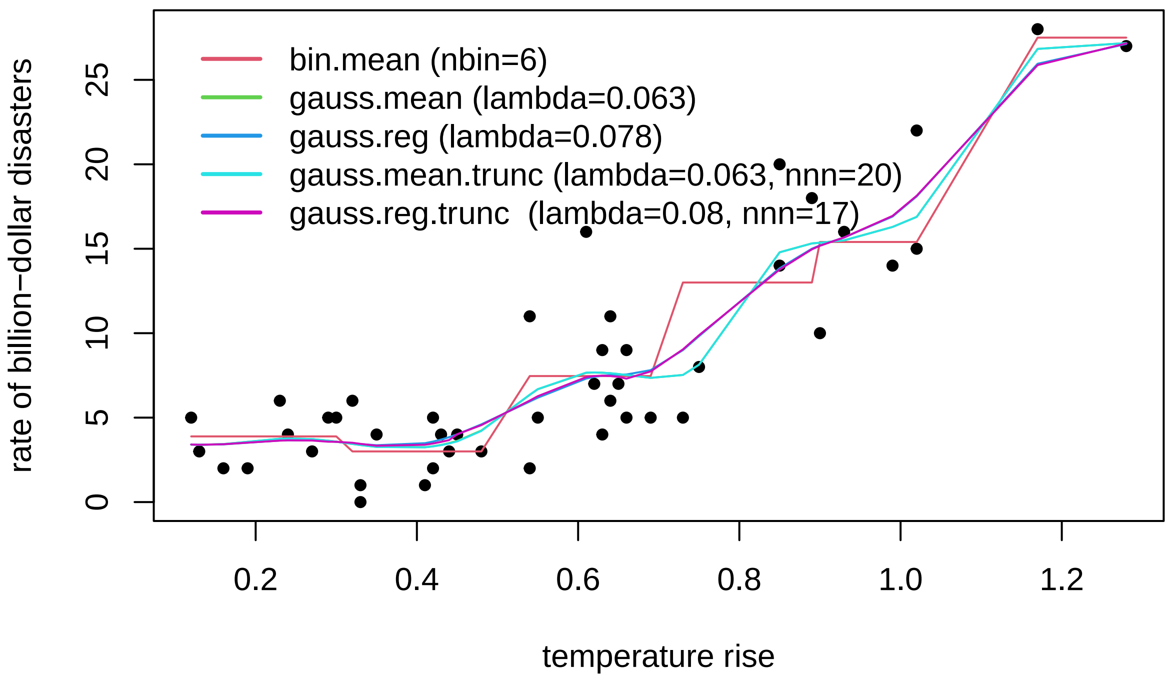
```
#plot(NOAA1[,3],NOAA1[,2],xlab="temperature rise",ylab="rate of billion dollar
#weather disasters")
#dum<-bin.mean(NOAA1[,3],NOAA1[,2],6)
#dum<-gauss.mean(NOAA1[,3],NOAA1[,2],.063)
#gauss.reg(NOAA1[,3],NOAA1[,2],.078,do.plot=T)
#gauss.mean.trunc(NOAA1[,3],NOAA1[,2],.063,20,do.plot=T)
#gauss.reg.trunc(NOAA1[,3],NOAA1[,2],.08,17,do.plot=T)
#lines(lowess(NOAA1[,3],NOAA1[,2]),col=7)
#lines(smooth.spline(NOAA1[,3],NOAA1[,2]),col=8)
```

**Plot** We have a nice plot to show the differences between all our smoothers.

```
#base plot where we'll have all our smoothers on
plot(x,y,xlab = "temperature rise",ylab = "rate of billion-dollar disasters",pch = 19,cex = 0.7)

#refit with plotting turned on for ALL
fit_bin<-bin.mean(x,y,nbin=6,xcol=2,do.plot=TRUE)  #bin.mean function
fit_gm<-gauss.mean(x,y,lambda=0.063,xcol=3,do.plot=TRUE) #gauss.mean function
fit_gr<-gauss.reg(x,y,lambda=0.078,xcol=4,do.plot=TRUE) #gauss.reg function
#gauss mean truncated function
fit_gm_trunc<-gauss.mean.trunc(x,y,lambda=0.063,nnn=20,xcol=5,do.plot = TRUE)
#gauss reg truncated function
fit_gr_trunc<-gauss.reg.trunc(x,y,lambda=0.08,nnn=17,xcol=6,do.plot =TRUE)

#legend for the lines
legend("topleft",inset = 0.02,bty = "n",lwd=2,lty=rep(1, 5),col=c(2,3,4,5,6),
       legend = c("bin.mean (nbin=6)",
                  "gauss.mean (lambda=0.063)",
                  "gauss.reg (lambda=0.078)",
                  "gauss.mean.trunc (lambda=0.063, nnn=20)",
                  "gauss.reg.trunc  (lambda=0.08, nnn=17)"))
```



```
#comparing PRESS in a table:
press_table<-data.frame(
  smoother=c("bin.mean (nbin=6)",
               "gauss.mean (lambda=0.063)",
               "gauss.reg (lambda=0.078)",
               "gauss.mean.trunc (lambda=0.063, nnn=20)",
               "gauss.reg.trunc (lambda=0.08,  nnn=17)"),
  PRESS=c(fit_bin$press,
          fit_gm$press,
          fit_gr$press,
          fit_gm_trunc$press,
          fit_gr_trunc$press))
print(press_table)
```

```
##                                  smoother    PRESS
## 1                       bin.mean (nbin=6) 632.1817
## 2               gauss.mean (lambda=0.063) 461.9978
## 3                gauss.reg (lambda=0.078) 560.7915
## 4 gauss.mean.trunc (lambda=0.063, nnn=20) 461.8456
## 5  gauss.reg.trunc (lambda=0.08,  nnn=17) 561.7968
```

**Greedy Random Search Function** This function references the pseudo-code from slide 19 of Class 9. Its purpose is to find better values of the adjustable parameters, based on minimizing PRESS.

```
greedy.search<-function(fit.eval,init.gen,perturb.gen,in.bounds,max.no.improve=300,target=Inf){
#parameters: fit.eval - function to use for fit, init,gen -
#function to generate parameters for fit,
#perturb.gen - function to randomly perturb parameters, in.bounds -
#function to check whether value is within the given bounds,
#max.no.improve - max number of trials, target - criterion for fit
#do a random fit, choose parameters = pi0
pi0<-init.gen()
crit0<-fit.eval(pi0) #check and save fit
```

```r
best.par<-pi0
best.crit<-crit0
noimp<-0
#while fit not as good as criterion-> we use patience OR target
while(noimp<max.no.improve&& best.crit>target)
  {#randomly perturb parameters, pi0 + small increments
    cand<-perturb.gen(best.par)
     #check if parameters meet any constraints
     if(in.bounds(cand)){
       #check fit
        ccrit<-fit.eval(cand)
        #if better save new parameters as pi0
        if(ccrit+1e-12<best.crit)
          {best.par<-cand
           best.crit<-ccrit
           noimp<-0
          }
        else
          {noimp<-noimp+1
          }
        }
     else{noimp<-noimp+1
         }
 }
#output pi0 and final fit criterion
list(par=best.par,press=best.crit,tries=noimp)}
```

**Performance of the Greedy Random Search Function** We'll now see how well the greedy random search function minimizes PRESS for each smoother. Using the implementations highlighted above, we use a few minimal wrappers for each of the different smoothers.

```r
#final greedy search chunk to to minimize PRESS for each smoother
#we use wrappers with similar deisng and structure
#bin.mean has discrete int nbin, gauss.* continuous lambda
#ranges and step sizes
dx<-diff(range(x))
n<-length(x)
lam.range<- pmax(1e-6,c(0.02, 0.30)*dx)  #width of search window
nbin.range<-c(2L,max(6L,min(50L,floor(n/3))))  #keep reasonable bins
nnn.range <-c(5L,max(10L,min(floor(0.6*n), n-1L)))  #kept neighbors

#helper methods
clamp<-function(v,lo,hi) pmin(hi,pmax(lo,v))
rand_int<-function(lo,hi) sample(seq.int(lo,hi), 1)

#optimize nbin for bin.mean with greedy search
fit.eval.bin<-function(p) {
  #clamp and round to keep nbin valid
  nbin<-as.integer(clamp(round(p$nbin),nbin.range[1],nbin.range[2]))
  #return PRESS for these params
  bin.mean(x,y,nbin = nbin,do.plot = FALSE)$press
}
#random valid start for nbin
init.gen.bin<-function()list(nbin=rand_int(nbin.range[1],nbin.range[2]))
```

```r
#try +/- 1 bin each step
perturb.gen.bin<-function(p)list(nbin=as.integer(p$nbin+sample(c(-1L,1L),1)))
#keep only finite, in-range nbin
in.bounds.bin<- function(p)is.finite(p$nbin)&&p$nbin>=nbin.range[1] && p$nbin<=nbin.range[2]

set.seed(486)
#run greedy for bin.mean
best_bin<-greedy.search(
fit.eval=fit.eval.bin,
init.gen=init.gen.bin,
perturb.gen=perturb.gen.bin,
in.bounds=in.bounds.bin,
max.no.improve=400)

#optimize lambda for gauss.mean
fit.eval.gm<-function(p) {
#keep lambda in allowed window
lam<-clamp(as.numeric(p$lambda),lam.range[1],lam.range[2])
#PRESS for gauss.mean
gauss.mean(x,y,lambda=lam,do.plot=FALSE)$press
}
#random lambda start
init.gen.gm<-function() list(lambda=runif(1,lam.range[1],lam.range[2]))
#lognormal step to nudge lambda up/down
perturb.gen.gm<-function(p) list(lambda=p$lambda*exp(rnorm(1,0,0.15)))
#lambda must be finite and positive
in.bounds.gm<-function(p) is.finite(p$lambda)&&p$lambda>0

#run greedy for gauss.mean
best_gm<-greedy.search(
  fit.eval=fit.eval.gm,
  init.gen=init.gen.gm,
  perturb.gen=perturb.gen.gm,
  in.bounds=in.bounds.gm,
  max.no.improve=400
)

#optimize lambda for gauss.reg
fit.eval.gr<-function(p){
  #clamp lambda to search range
  lam<-clamp(as.numeric(p$lambda),lam.range[1],lam.range[2])
  #PRESS for gauss.reg
  gauss.reg(x,y,lambda=lam,do.plot=FALSE)$press
}
#random start for lambda
init.gen.gr<-function() list(lambda=runif(1,lam.range[1],lam.range[2]))
#gentle multiplicative jitter on lambda
perturb.gen.gr<-function(p) list(lambda=p$lambda*exp(rnorm(1,0,0.15)))
#must be finite and > 0
in.bounds.gr<-function(p) is.finite(p$lambda)&&p$lambda>0

#run greedy for gauss.reg
best_gr<-greedy.search(
  fit.eval=fit.eval.gr,
  init.gen=init.gen.gr,
  perturb.gen=perturb.gen.gr,
  in.bounds=in.bounds.gr,
  max.no.improve=400
)

#optimize lambda and nnn for gauss.mean.trunc
fit.eval.gmt<-function(p){
  #clamp both params
  lam<-clamp(as.numeric(p$lambda),lam.range[1],lam.range[2])
  nnn<-as.integer(clamp(round(p$nnn),nnn.range[1],nnn.range[2]))
  #PRESS for truncated mean smoother
  gauss.mean.trunc(x,y,lambda=lam,nnn=nnn,do.plot=FALSE)$press
}
#random start for both params
init.gen.gmt<-function() list(
  lambda=runif(1,lam.range[1],lam.range[2]),
  nnn=rand_int(nnn.range[1],nnn.range[2])
)
#alternate stepping lambda or nnn
perturb.gen.gmt<-function(p){
  if(runif(1)<0.5){
    #jitter lambda, keep nnn
    list(lambda=p$lambda*exp(rnorm(1,0,0.15)),nnn=p$nnn)
  }else{
    #move nnn by a small integer step
    list(lambda=p$lambda,nnn=as.integer(p$nnn+sample(c(-2L,-1L,1L,2L),1)))
  }
}
#basic validity checks for both params
in.bounds.gmt<-function(p){
  ok1<-is.finite(p$lambda)&&p$lambda>0
  ok2<-is.finite(p$nnn)&&round(p$nnn)>=nnn.range[1]&&round(p$nnn)<=nnn.range[2]
  ok1&&ok2
}
#run greedy for gauss.mean.trunc
best_gmt<-greedy.search(
  fit.eval=fit.eval.gmt,
  init.gen=init.gen.gmt,
  perturb.gen=perturb.gen.gmt,
  in.bounds=in.bounds.gmt,
  max.no.improve=500
)
#optimize lambda and nnn for gauss.reg.trunc
fit.eval.grt<-function(p){
  #clamp both params
  lam<-clamp(as.numeric(p$lambda),lam.range[1],lam.range[2])
  nnn<-as.integer(clamp(round(p$nnn),nnn.range[1],nnn.range[2]))
  #PRESS for truncated local regression
  gauss.reg.trunc(x,y,lambda=lam,nnn=nnn,do.plot=FALSE)$press
}
#random start for both params
```

```r
init.gen.grt<-function() list(
  lambda=runif(1,lam.range[1],lam.range[2]),
  nnn=rand_int(nnn.range[1],nnn.range[2])
)
#alternate between nudging lambda or nnn
perturb.gen.grt<-function(p){
  if(runif(1)<0.5){
    #multiplicative jitter for lambda
    list(lambda=p$lambda*exp(rnorm(1,0,0.15)),nnn=p$nnn)
  }else{
    #small integer step for nnn
    list(lambda=p$lambda,nnn=as.integer(p$nnn+sample(c(-2L,-1L,1L,2L),1)))
  }
}
#validity checks for both params
in.bounds.grt<-function(p){
  ok1<-is.finite(p$lambda)&&p$lambda>0
  ok2<-is.finite(p$nnn)&&round(p$nnn)>=nnn.range[1]&&round(p$nnn)<=nnn.range[2]
  ok1&&ok2
}
#run greedy for gauss.reg.trunc
best_grt<-greedy.search(
  fit.eval=fit.eval.grt,
  init.gen=init.gen.grt,
  perturb.gen=perturb.gen.grt,
  in.bounds=in.bounds.grt,
  max.no.improve=500
)
#collect and print results
best_tbl <- data.frame(
  smoother = c("bin.mean", "gauss.mean", "gauss.reg", "gauss.mean.trunc", "gauss.reg.trunc"),
  PRESS    = c(best_bin$press, best_gm$press, best_gr$press, best_gmt$press, best_grt$press),
  nbin     = c(as.integer(best_bin$par$nbin), NA, NA, NA, NA),
  lambda   = c(NA,
               as.numeric(best_gm$par$lambda),
               as.numeric(best_gr$par$lambda),
               as.numeric(best_gmt$par$lambda),
               as.numeric(best_grt$par$lambda)),
  nnn      = c(NA, NA, NA,
               as.integer(best_gmt$par$nnn),
               as.integer(best_grt$par$nnn))
)

best_tbl <- best_tbl[order(best_tbl$PRESS), ]
rownames(best_tbl) <- NULL
print(best_tbl)
```

```
##           smoother    PRESS nbin     lambda nnn
## 1       gauss.mean 460.6817   NA 0.05713687  NA
## 2         bin.mean 488.8604    8         NA  NA
## 3        gauss.reg 527.3064   NA 0.15981955  NA
## 4 gauss.mean.trunc 544.3961   NA 0.14635581  15
## 5  gauss.reg.trunc 557.9847   NA 0.10810784  25
```

**Conclusion** So the hard-coded parameters referenced from slide 17, are defaults that may not be tuned to dataset. Our implemented greedy search function attempts to minimize PRESS on these data, so it naturally picks different bandwidths/neighbor counts when the x-range, point density, or noise level differs from the slide example. Also, since the Gaussian bandwidth lambda is in the units of delta.temp, extending the time span to 2024 and changing the scale/density of x shifts the optimal settings—so a difference between defaults and greedy picks is expected and desirable.

We set a seed for reproducability, and gauss.reg likely performed best because it cancels first-order bias and therefore reduces boundary bias. This can be a big deal when there are fewer points at the coolest/warmest delta temp values. The greedy run also selected a relatively wide bandwidth, which smooths over year-to-year spikes while tracking the smooth, mostly monotone trend in disasters, lowering the PRESS statistic. Truncated variants of the smoothers that were provided from class add an extra nnn that helps when x is extremely uneven, but the design here isn't that irregular, so we believe that the simpler non-truncated local-linear achieved the best bias–variance trade-off.