MST129: Applied Calculus

==============================

Nancy Al Aswad -2180385

Arab Open University

Jordan

01/12/2021

Answer for Question (1):

================

First step: I solve the composition equation by using the rule as follow

$\underline{(\boldsymbol{f} \circ \boldsymbol{g})(\boldsymbol{x}) = f(g(x))} =$

$$= \left(\sqrt{9 - x^2}\right)^2 - 9 =$$

$$= 9 - x^2 - 9 =$$

$$\underline{\boldsymbol{-x^2}}$$

Second step: I solve the domain of $(\boldsymbol{f} \circ \boldsymbol{g})(\boldsymbol{x})$

1.) $First\ the\ Domain\ of f(g(x)) = \boldsymbol{R}$

2.) The $Domain\ of\ g(x)\ need\ to\ analyse\ as\ below$

$= \ 9 - x^2 = 0$

$= (3 - x)(3 + x) = 0$

So the $x = \mp 3$ and the $Domain\ of\ g(x)$: $[\boldsymbol{-3}, \boldsymbol{3}]$

So finally the $Domain\ of (f \circ g)(x). = R \cap [-3,3] = [\boldsymbol{-3}, \boldsymbol{3}]$

Answer for Question (2):

================

a) $\mathbf{2^{2x} - 2^{x+2} - 2^5 = 0}$

=============

$$2^{2x} - 4.2^x - 2^5 = \mathbf{0}$$

Here I need to analyse as $(2^x - 8)(2^x + 4) = 0$

Now take the ***first*** *part and equal it with zero as* $(2^x - 8) = \mathbf{0}$

that equal to $2^x = 8$

which can write as $2^x = 2^3$

So the result of $\boldsymbol{x = 3}$

And now take the ***Second*** *part and equal it with zero as* $(2^x + 4) = 0$

that equal to $2^x = -4$ *which is not acceptable*

So the final result is the set $\{\mathbf{3}\}$

b) $\boldsymbol{\frac{ln\,x}{2} + 5 = 3\,ln\,x.}$

==========

Answer:

=====

$\boldsymbol{First}$; $I\ need\ to\ \ multiply\ by\ (\ 2)\ so\ the\ equation\ became$

$$\ln x + 10 = 6\ln x = \ln x^6$$

$Here\ \ I\ dedcted\ the\ (\ln x) from\ each\ side\ , so\ the\ \ equation\ became$

$$\ln x^6 - \ln x = \mathbf{10}$$

$$\boldsymbol{ln\frac{x^6}{x} = 10}$$

$$x^5 = \boldsymbol{e^{10}}$$

$$\boldsymbol{x = (e^{10})^{\frac{1}{5}} = e^2}$$

$\boldsymbol{So\ the\ final\ \ result\ is\ the\ set\ \{e^2\}}$

Answer for Question (3):

================

First ; ***I need to get the rule***

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

Second ; ***I wok on compensation to get the rule***

$(x+h)^2 = x^2 + 2xh + h^2$

$$(x+h)^3 = (x+h)^2(x+h)$$

$$= (x+h)^2(x+h) = (x^2 + 2xh + h^2)(x+h)$$

$$(x+h)^3 = x^3 + 2x^2h + xh^2 + xh^2 + 2h^2x + h^3$$

$$= x^3 + 2x^2h + 2xh^2 + 2h^2x + h^3$$

$$\textbf{\textit{Now we know that}}\ \ g(x) = x^3 - 2x^2 + x - 1$$

$$g(x+h) = (x+h)^3 - 2(x+h)^2 + x + h - 1$$

$$= x^3 + 2x^2h + 2xh^2 + 2h^2x + h^3 - 2x^2 - 4xh - 2h^2 + x + h - 1$$

$g'(x)$
$$= \lim_{h\to 0} \frac{x^3 + 2x^2h + 2xh^2 + 2h^2x + h^3 - 2x^2 - 4xh - 2h^2 + x + h - 1 - x^3 + 2x^2 - x + 1}{h}$$

$$= \lim_{h\to 0} \frac{2x^2h + 2xh^2 + 2h^2x + h^3 - 4xh - 2h^2 + h}{h}$$

$$= \lim_{h\to 0} \frac{h\left(2x^2 + 2xh + 2hx + h^2 - 4x - 2h + 1\right)}{h}$$

$$\lim_{h\to 0} \frac{(2x^2 + 2xh + 2hx + h^2 - 4x - 2h + 1)}{} = \mathbf{2x^2 - 4x + 1}$$

$$\boldsymbol{So\ the\ final\ \ result\ is\ 2x^2 - 4x + 1}$$

Answer for Question (4):

================

$\boldsymbol{First}\ ;\ \boldsymbol{Depending\ on\ th\ question\ equation\ below\ \ I\ imply\ the\ rule}$

$$since\ f(x) = \sqrt{x^2 - 2x + 2} + \sqrt{x^2 - 6x + 13}$$

$$f'(x) = \frac{2x - 2}{2\sqrt{x^2 - 2x + 2}} + \frac{2x - 6}{2\sqrt{x^2 - 6x + 13}}$$

$$= \frac{2(x - 1)}{2\sqrt{x^2 - 2x + 2}} + \frac{2(x - 3)}{2\sqrt{x^2 - 6x + 13}}$$

After get the (2) from all sides I got: -

$$= \frac{(x - 1)}{\sqrt{x^2 - 2x + 2}} + \frac{(x - 3)}{\sqrt{x^2 - 6x + 13}}$$

Now make uniform for the denominator

$$= \frac{(x - 1)\sqrt{x^2 - 6x + 13} + (\boldsymbol{x} - \boldsymbol{3})\sqrt{x^2 - 2x + 2}}{\left(\sqrt{x^2 - 2x + 2}\right)\left(\sqrt{x^2 - 6x + 13}\right)}$$

And finally for working in the horizontal tangent line I use the formula

$$\boldsymbol{f'(x) = 0}$$

$$= (x-1)\sqrt{x^2-6x+13} + (\boldsymbol{x}-\mathbf{3})\sqrt{x^2-2x+2} = \mathbf{0}$$

$$= (x-1)\sqrt{x^2-6x+13} = (3-x)\sqrt{x^2-2x+2}$$

Multiply them $(x^2-2x+1)(x^2-6x+13) = (9-6x+x^2)(x^2-2x+2)$

The result we got are :

$$x^4-8x^3+26x^2-32x+13 = x^4-8x^3+23x^2-30x+18$$

Make aggregation to simplify the result as much as we can with below steps

$$26x^2-23x+13-23x^2+30x-18=0$$

$$3x^2-2x-5=0$$

$$(3x-5)(x+1)=0$$

$$(3x-5)=0 \to x=\frac{5}{3} \ \boldsymbol{which\ indicates\ that}\ f'\left(\frac{5}{3}\right)=0$$

$$(x+1)=0 \to x=-1 \ \ \boldsymbol{not\ acceptable\ for}\ \ f'(-1)=0$$

$$\boldsymbol{So\ at\ x} = \frac{5}{3} \to f\left(\frac{5}{3}\right) = \sqrt{(\frac{5}{3})^2-2(\frac{5}{3})+2} + \sqrt{(\frac{5}{3})^2-6(\frac{5}{3})+13} = \sqrt{\frac{\mathbf{65}}{\mathbf{9}}}$$

$$\boldsymbol{And\ the\ final\ \ result\ is}\ (\frac{\mathbf{5}}{\mathbf{3}}, \sqrt{\frac{\mathbf{65}}{\mathbf{9}}})$$

Answer for Question (5):

=================

let say $\boldsymbol{h} = f(x) = x^{-x} \quad , \boldsymbol{x > 0}$ **so ,** $\boldsymbol{\ln h = \ln x^{-x}}$

$$\textbf{And } \boldsymbol{ln\, h = -x\, ln\, x}$$

now I make differentiation *to each side*

$$\frac{h'}{h} = -\frac{x}{x} - \ln x$$

$$h' = h(-1 - lnx)$$

$$h' = x^{-x}(-1 - \ln x)$$

$$h' = 0 \rightarrow \ln x = -1 \ \ \boldsymbol{here\ we\ take\ (\ e\ )\ to\ each\ side}$$

$$\boldsymbol{x = \frac{1}{e}}$$

So the Intervals are:
=================

1. $(0, \frac{1}{e})$ where f is increasing.
2. $(\frac{1}{e}, \infty)$ where f is decreasing.

And At $x = \frac{1}{e}$ the local **maximum** is ≈ 1.44 and **minimum** not founded

Answer for Question (6):

================

I will represent the **(large volume)** with **(L) ,** And as I got square sheet of cardboard with each side I will use the below ***equation***

$$\mathrm{L}(x) = (a - 2x)(a - 2x)(x)$$

$$\mathrm{L}(x) = (a^2 - 4ax + 4x^2)(x)$$

$$\mathrm{L}(x) = a^2x - 4ax^2 + 4x^3$$

$$\mathrm{L}(x) = a^2 - 8ax + 12x^2$$

$$\boldsymbol{Now\ we\ take\ \ L'(x) = 0}$$

$$12x^2 - 8ax + a^2 = 0$$

$$(2x - a)(6x - a) = 0$$

$$(2x - a) = 0$$

$$\boldsymbol{So}\ 2x = a \quad , \qquad \boldsymbol{And\ x = \frac{a}{2}}$$

$\boldsymbol{And\ in\ the\ anothe\ part\ we\ solve\ as}$

$$(6x - a) = 0$$

$$6x = a \rightarrow \boldsymbol{x} = \boldsymbol{\frac{a}{6}}$$

$\boldsymbol{And\ for\ L'(x) = -8a + 24x}$

Now $L''\left(\frac{a}{2}\right) = -8a + 24\left(\frac{a}{2}\right) = 8a > 0$ $(\,which\ is\ the\ minmum\ value\ of\ x)$

$$L''\left(\frac{a}{6}\right) = -8a + 24\left(\frac{a}{6}\right) = -4a < 0\ (which\ is\ the\ maximum\ value\ of\ x)$$

$\boldsymbol{final}$ $\quad at\ x = \boldsymbol{\frac{a}{6}}$ $\quad (our\ large\ volum\ of\ box\ \ will\ be\ as\ possible)$

Answer for Question (7):

===============

Here I make differentiation *to each side*

$$\boldsymbol{x^3 + y^3 - 3xy = 0}$$

$The\ result\ is\ =\ 3x^2 + 3y^2y' - 3xy' - 3y = 0$

The next step is to divide the equation by 3 and simplify the result as in the below steps:

$$\boldsymbol{y^2y' - xy' = y - x^2} \quad \text{------------}\rightarrow \quad \boldsymbol{y'(y^2 - x) = y - x^2}$$

$Now\ \ we\ know\ that\, y' = \frac{y-x^2}{y^2-x}\ at\ point\ \left(\frac{3}{2},\frac{3}{2}\right)$ So we compensate it as ;

$$y' = \frac{\frac{3}{2} - \frac{9}{4}}{\frac{9}{4} - \frac{3}{2}} = \boldsymbol{-1}$$

To discover the equation for the tangent line is

$$\frac{y - \frac{3}{2}}{x - \frac{3}{2}} = -\boldsymbol{1}$$

$$y - \frac{3}{2} = \frac{3}{2} - x$$

final the equation for the tangent line is $\boldsymbol{y = 3 - x}$

Answer for Question (8):

===============

From the Question the function below;

$f(x) = \frac{x^5 e^x(4x+3)}{5^{\ln x}(3-x)^2}$ ***We have to*** **ind an equation of the line tangent** ***at*** $x = 1$

$$\boldsymbol{f(1) = \frac{7e}{4} = 4.756 \approx 5}$$

Now *suppose* $\boldsymbol{h} = f(x) = \frac{x^5 e^x(4x+3)}{5^{\ln x}(3-x)^2}$ *and get* (**ln**) *to each side*

$$\ln f(x) = \ln[x^5 e^x(4x+3)] - \ln[5^{\ln x}(3-x)^2]$$

$$\ln \boldsymbol{h} = \ln x^5 + \ln e^x + \ln(4x+3) - \ln 5^{\ln x} - \ln(3-x)^2$$

$$\ln \boldsymbol{h} = 5\ln x + x + \ln(4x+3) - \ln 5 \; \ln x - 2\ln(3-x)$$

$$\frac{1}{h}\frac{dh}{dx} = (\frac{5}{x} + 1 + \frac{4}{4x+3} - \frac{ln\,5}{x} + \frac{2}{3-x})$$

$$\frac{dh}{dx} = h\left(\frac{5}{x} + 1 + \frac{4}{4x+3} - \frac{ln\,5}{x} + \frac{2}{3-x}\right)$$

$$\frac{dh}{dx} = \frac{x^5 e^x(4x+3)}{5^{ln\,x}(3-x)^2} \quad \left(\frac{5}{x} + 1 + \frac{4}{4x+3} - \frac{ln\,5}{x} + \frac{2}{3-x}\right)$$

$$at\ x = 1 \ \rightarrow \frac{dh}{dx} = \frac{7e}{4}\left(5 + 1 + \frac{4}{7} - \ln 5 + 1\right) \approx 56.7 \approx 57$$

The equation of the line: $\frac{\boldsymbol{h-5}}{\boldsymbol{x-1}} = 57$

So we can see that $\boldsymbol{h - 5 = 57x - 57}$ which indicate that the **equation** of the line tangent to the graph of f at $x = 1$ is :

$$\underline{\boldsymbol{h = 57x - 52}}$$