MT131 (M131): Discrete Mathematics

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Q–1:

a) $P \wedge Q \Longrightarrow R$ and $(\neg R \wedge P) \Longrightarrow \neg Q$.

They are equivalent because

- $\boldsymbol{P \wedge Q \Longrightarrow R \equiv \neg(P \wedge Q) \vee R \equiv \neg P \vee \neg Q \vee R}$

- $\boldsymbol{(\neg R \wedge P) \Longrightarrow \neg Q \equiv \neg(\neg R \wedge P) \vee \neg Q \equiv R \vee \neg P \vee \neg Q}$

  $\boldsymbol{\equiv \neg P \vee \neg Q \vee R}$

b) $P \Longrightarrow (Q \wedge R)$ and $(\neg Q \Longrightarrow \neg P) \wedge (P \Longrightarrow R)$.

They are equivalent because

- $\boldsymbol{P \Longrightarrow (Q \wedge R) \equiv \neg P \vee (Q \wedge R) \equiv (\neg P \vee Q) \wedge (\neg P \vee R)}$

- $\boldsymbol{(\neg Q \Longrightarrow \neg P) \wedge (P \Longrightarrow R) \equiv (\neg(\neg Q) \vee \neg P) \wedge (\neg P \vee R)}$

  $\boldsymbol{\equiv (Q \vee \neg P) \wedge (\neg P \vee R) \equiv (\neg P \vee Q) \wedge (\neg P \vee R)}$

c) $\neg(p \Longrightarrow Q)$ and $(\neg P \Longrightarrow \neg Q)$.

They are not equivalent because

- $\boldsymbol{\neg(p \Longrightarrow Q) \equiv \neg(\neg P \vee Q) \equiv P \wedge \neg Q}$

- $\boldsymbol{(\neg P \Longrightarrow \neg Q) \equiv \neg(\neg P) \vee \neg Q \equiv P \vee \neg Q}$

Q–2:

a) $(\exists x \in \boldsymbol{Z})(x + x \leq x)$.

Which is **true** because the statement express that **For some** x values ∈ integers , So we may check one value at least to be true example

$x = 0 \qquad \in Z$

$(0 + 0 <= 0)$

b) For all positive integers n; $n^2 + n + 41$ is a prime.

Because the statement express that **For all** x values ∈ integers , So we may check one values to find one False value for example

n=41 (positive integer)

$41^2+41+41=1763$ (**NOT PRIME)** $1763/41 = 43$

So it is **False Statement**

c) For integers a, b, c; if a divides bc, then either a divides b or a divides c.

Which is **False** as example

$6|36 = 6|4*9$

6 doesn't divide neither 4 nor 9 **T → F ∨ *F***

T → F

d) $(\forall x \in \boldsymbol{R})(-|x| \leq x \leq |x|)$.

Which is True because All values **are true because of equal signs in the both sided**

<u>for negative x</u>

$-|-5| \leq -5 \leq |-5|$

$-5 \leq -5 \leq 5$ **True**

<u>for positive x</u>

$-|5| \leq 5 \leq |5|$

$-5 \leq 5 \leq 5$ **True**

Q–3:

a) $\boldsymbol{f}: \boldsymbol{Q} \rightarrow \boldsymbol{Q}$ **where** $\boldsymbol{f}\left(\frac{p}{q}\right) = \boldsymbol{q}$**.**

Suppose $\mathrm{p} = 1$ and $\mathrm{q} = 2$

$\boldsymbol{f}\,(1/2) = 2$

$\boldsymbol{f}\,(4/8) = 8 = \boldsymbol{f}\,(1/2)$

$So\ f\left(\frac{1}{2}\right)$ Has **multiple value** in co-domain Which mean that **$\boldsymbol{f}$(x) is not function**

b) $\boldsymbol{g}: \boldsymbol{Z} \rightarrow \boldsymbol{Z}$ **where** $\boldsymbol{g}(\boldsymbol{x}) = \begin{cases} \boldsymbol{x^2} & \textbf{if } \boldsymbol{x} \leq \boldsymbol{2} \\ \boldsymbol{x - 1} & \textbf{if } \boldsymbol{x} \geq \boldsymbol{4} \end{cases}$.

g(x) is not function because we find **no** image in co-domain for **x=3** at the domain (set of integers Z) , According to intervals define $\boldsymbol{g}(\boldsymbol{x})$

$\boldsymbol{g}(\boldsymbol{3}) =$ **?!**

c) $\boldsymbol{h}: \boldsymbol{R} \rightarrow \boldsymbol{R}$ **where** $\boldsymbol{h}(\boldsymbol{x}) = \frac{\mathbf{1}}{\boldsymbol{x}+\mathbf{5}}$.

h (x) is not function because we find **no** image in co-domain for the domain $\boldsymbol{R}$ at **x= -5**

x= (-5)

$$\boldsymbol{h}(-\mathbf{5}) = \frac{\mathbf{1}}{-\mathbf{5}+\mathbf{5}} = \frac{\mathbf{1}}{\mathbf{0}} = \frac{\mathbf{1}}{\mathbf{0}}\ \boldsymbol{undefined}$$

d) $\boldsymbol{k}: \boldsymbol{N} \rightarrow \boldsymbol{N}$ **where** $\boldsymbol{k}(\boldsymbol{n}) =$ **any integer** $> \boldsymbol{n}$**.**

$\boldsymbol{k}(\boldsymbol{n})$ **is not function** because we find multiple value (more than one image) in co-domain for an element **x=3** at the domain (N)

k(n) = any integer > n
n=3
k (3) =3
k (3) =4
k (3) =5

Q–4:

a) How many words begin with R **and** end with T?

= 1*1*26^{5} **= 11,881,376**

R	26 possible letter	26	26	26	26	T

b) How many words begin with A **or** end with B?

$$P(A\cup B) = P(A) + P(B) - P(A \cap B) =$$
$$= 26^6 + 26^6 - 26^5 = 605,950,176$$

c) How many words begin with A or B and end with A or B?

$$= 2*26^5*2 = 4 * 26^5 = 47,525,504$$

d) How many words begin with A or B or end with A or B?

$$=2*26^6+26^6*2 - 4*26^5 = 1,188,137,600$$

e) How many words begin with a vowel **and** end with a vowel?

Possible vowel letters = 5

$$= 5*5*26^5 = 25 * 26^5 = 297,034,400$$

5 possible vowels	26	26	26	26	26	5

f) How many words begin with a vowel or end with a vowel?

$= 5*26^6+5*26^6 - 25 \ *26^5 =$ 2,792,123,360

g) How many words begin with AAB in some order?

A	A	B	26	26	26	26

$= 3*26^4 =$ 1,370,928

h) How many words have exactly one vowel?

$= 5*7*21^6 =$ 3,001,814,235

Q–5:

a) p (sum of the two numbers picked is < 4).

{(1,1), (1,2), (2,1)}

$$p(x*y<4) = \frac{3}{8*8} = \frac{3}{64}$$

1,1	2,1	3,1	4,1	5,1	6,1	7,1	8,1
1,2	2,2	3,2	4,2	5,2	6,2	7,2	8,2
1,3	2,3	3,3	4,3	5,3	6,3	7,3	8,3
1,4	2,4	3,4	4,4	5,4	6,4	7,4	8,4
1,5	2,5	3,5	4,5	5,5	6,5	7,5	8,5
1,6	2,6	3,6	4,6	5,6	6,6	7,6	8,6
1,7	2,7	3,7	4,7	5,7	6,7	7,7	8,7
1,8	2,8	3,8	4,8	5,8	6,8	7,8	8,8

b) p (both numbers match).

{(1,1), (2,2), (3,3),(4,4),(5,5),(6,6),(7,7),(8,8)}

p(x=y) $= \frac{8}{8*8} = \frac{8}{64} = \frac{1}{8}$

1,1	2,1	3,1	4,1	5,1	6,1	7,1	8,1
1,2	2,2	3,2	4,2	5,2	6,2	7,2	8,2
1,3	2,3	3,3	4,3	5,3	6,3	7,3	8,3
1,4	2,4	3,4	4,4	5,4	6,4	7,4	8,4
1,5	2,5	3,5	4,5	5,5	6,5	7,5	8,5
1,6	2,6	3,6	4,6	5,6	6,6	7,6	8,6
1,7	2,7	3,7	4,7	5,7	6,7	7,7	8,7
1,8	2,8	3,8	4,8	5,8	6,8	7,8	8,8

c) p (the sum of the two numbers is a prime).

Possible primes by sums = 1+2+4+6+6+4+2=23

P(x+y) = prime= $\frac{23}{64}$

A prime number is 2,3,5,7,11,13

{(1,1), (1,2), (2,1), (2,3), (3,2), (1,4), (4,1), (3,4), (4,3),....... (8,5)}

1,1	2,1	3,1	4,1	5,1	6,1	7,1	8,1
1,2	2,2	3,2	4,2	5,2	6,2	7,2	8,2
1,3	2,3	3,3	4,3	5,3	6,3	7,3	8,3
1,4	2,4	3,4	4,4	5,4	6,4	7,4	8,4
1,5	2,5	3,5	4,5	5,5	6,5	7,5	8,5
1,6	2,6	3,6	4,6	5,6	6,6	7,6	8,6
1,7	2,7	3,7	4,7	5,7	6,7	7,7	8,7
1,8	2,8	3,8	4,8	5,8	6,8	7,8	8,8

d)P (your number is greater than your friend's number).

Ways of being greater =

1(number below) +2 (numbers below) +3+4+5+6+7= 28

P(x>y)= $\frac{28}{64}$

1,1	2,1	3,1	4,1	5,1	6,1	7,1	8,1
1,2	**2,2**	**3,2**	**4,2**	**5,2**	**6,2**	**7,2**	**8,2**
1,3	**2,3**	**3,3**	**4,3**	**5,3**	**6,3**	**7,3**	**8,3**
1,4	**2,4**	**3,4**	**4,4**	**5,4**	**6,4**	**7,4**	**8,4**
1,5	**2,5**	**3,5**	**4,5**	**5,5**	**6,5**	**7,5**	**8,5**
1,6	**2,6**	**3,6**	**4,6**	**5,6**	**6,6**	**7,6**	**8,6**
1,7	**2,7**	**3,7**	**4,7**	**5,7**	**6,7**	**7,7**	**8,7**
1,8	**2,8**	**3,8**	**4,8**	**5,8**	**6,8**	**7,8**	**8,8**