Minimal Counter Example - Leave Lower Bond

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February 2022

0.1 Conjecture

Let b(G) be the burning number of a graph G. Let n be the number of vertices that G has. For all simple connected graph G, b(G) $\leq \sqrt{n}$.

0.2 Definition

Let T be a tree with n vertices. Then T is a minimal counterexample of our conjecture if

1. $b(T) > \sqrt{n}$, and
2. for any $T*$ with $n*$ vertices($n* < n$), $b(T*) \leq \lceil \sqrt{n*} \rceil$

0.3 Proposition

Let T be the tree with $n = b^2$ number of the vertices and have less than $2\sqrt{n}$ leaves. Prove of disprove that it is a minimal counterexample.

0.4 Prove

We will disprove by contradiction. Assume that T* is T without its leaves. Thus once T* is fully burnt, it only takes one more round for the $2\sqrt{n}$ leaves to be on fire, or, for T to be fully burnt. Thus we know that:

$$b(T) \leq b(T*) + 1$$

or

$$b(T*) \geq b(T) - 1$$

we also know that

$$n^* \leq n - 2\sqrt{n}$$

$$n^* \leq n - 2\sqrt{n} + 1$$

$$n^* \leq b^2 - 2b + 1$$

$$n^* \leq (b-1)^2$$

Assume T is a minimum counterexample. Then we know that $b(T*) \leq \lceil\sqrt{n*}\rceil$ is true. Then:

$$b(T) \leq b(T*) + 1$$

$$b(T) \leq \lceil\sqrt{n*}\rceil + 1$$

$$b(T) \leq \lceil\sqrt{(b-1)^2}\rceil + 1$$

$$b(T) \leq b - 1 + 1$$

$$b(T) \leq b$$

$$b(T) \leq \sqrt{n}$$

This contradicts with our definition for minimum counterexamples, thus, tree with less than $2\sqrt{n}$ leaves can not be a minimum counterexample. Q.E.D.