

# Burning Graphs

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# Introduction to Graph Burning

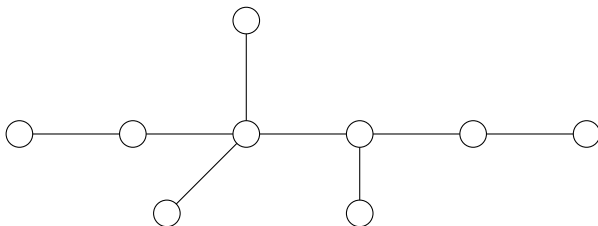
## Question

How fast can a meme spread through a network?

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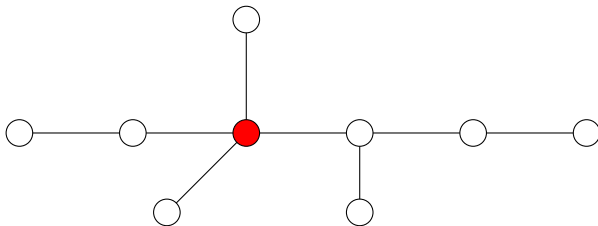
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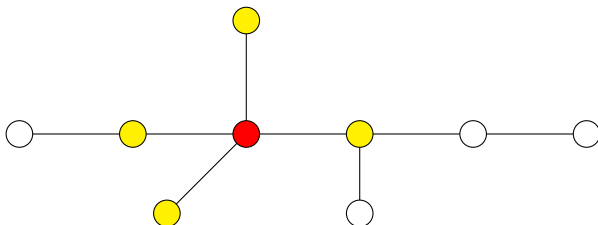
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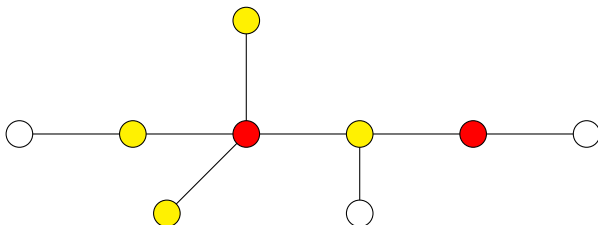
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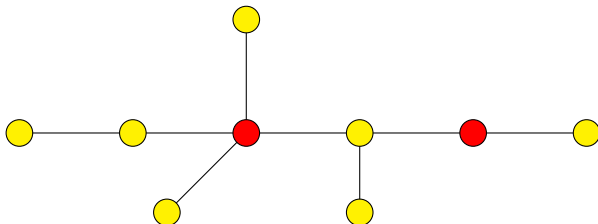
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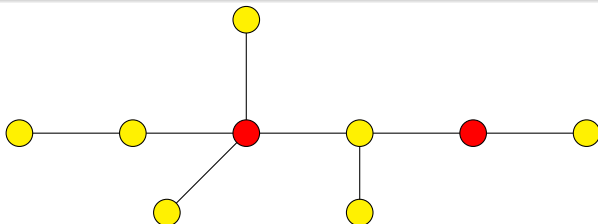
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## Burning Sequence

A burning sequence is the sources that are chosen over time.

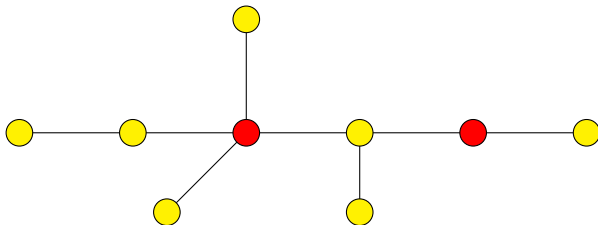




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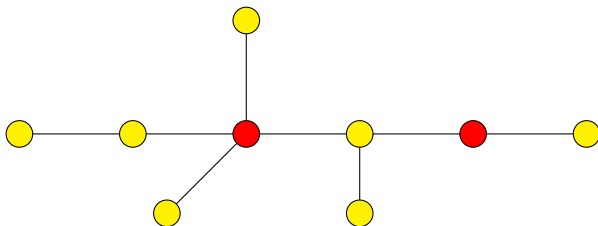
## Burning Number

Burning number of a graph  $G$ ,  $b(G)$ , is the minimum number of rounds needed for graph to get entirely burned.

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$$b(G) = 3$$

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# Introduction to the Burning Number Conjecture

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$$b(G) \leq \left\lceil \frac{\sqrt{12n + 64} + 8}{3} \right\rceil = \left\lceil \sqrt{\frac{4n}{3}} \right\rceil + O(1)$$

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Conjecture is true for:

- Paths
- Graphs with minimum degree 23

# Proof Ideas: Minimal Counterexample

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Pendant Path



Subgraph



# Future Plans

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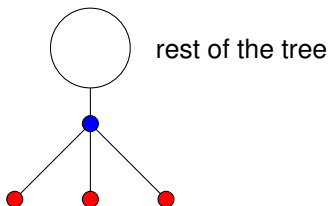
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• Example:



Thank you!