



# Università degli Studi di Ferrara

Università degli Studi di Ferrara  
CORSO DI LAUREA IN FISICA

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*Statistical learning and  
simulating the paths of walking  
pedestrians*

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## Abstract

[ENG] The dynamics of pedestrian changes considerably depending on the surrounding space, not just for the intrinsic chaotic movements that people does walking but also due to the reciprocal collisions and environment condition. We have considered some scenarios to implement models and a tools that can gives us simulations of the movements of a single pedestrian. In order to properly simulate a pedestrians' dynamic, is to have information about the probability to change direction after every step, in every positions of the trajectory. This approach is linked to the path integral in the way that: given a trajectory, it's possible to say with certain probability where the next step is. This mathematical approach is computationally expensive, even more with the big amount of data we are using. So we started implementing a discrete system and a easy model and than we moved to more complex model. In total we get four types of models: two time dependent and tow independent. From now on we'll call those: D2Q9 and D2Q9Q9 the firsts two; TD2Q9 and TD2Q9Q9 the others two.

[ITA] L'obiettivo scientifico è stato creare un metodo, ispirato al Lattice-Boltzman, con cui apprendere, a partire da dati reali, la dinamica pedonale e quantificarla in termini di matrici di transizione su reticolo. L'obiettivo fondamentale è riuscire a quantificare il campo di probabilità, trovato utilizzando diversi modelli. Questo campo ci permette di studiare la dinamica e creare simulazioni di pedoni e traiettorie le cui statistiche sono indistinguibili per costruzione dalle statistiche delle traiettorie reali

# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
1.1	Assimilating pedestrian dynamics . . . . .	2
1.2	Challenges . . . . .	2
1.3	Relevance . . . . .	2
<b>2</b>	<b>Pedestrian dynamics measurements</b>	<b>3</b>
2.1	Idea of the recording technique . . . . .	3
2.1.1	Kinect: depth map based pedestrian tracking . . . . .	3
2.1.2	ProRail cameras . . . . .	3
2.2	Presenting datasets . . . . .	3
2.2.1	Glow experiment . . . . .	3
2.2.2	Utrecht Centraal (Floorefield 10) . . . . .	3
2.3	Technique of measure . . . . .	4
2.3.1	Kinect sensor . . . . .	4
2.3.2	ProRail data . . . . .	4
<b>3</b>	<b>Background</b>	<b>5</b>
3.1	Markov Chain . . . . .	5
3.2	Cellular-Automata . . . . .	5
3.3	Lattice-Boltzman . . . . .	5
<b>4</b>	<b>Propose data assimilation technique</b>	<b>6</b>
4.1	Approximation of path integrals: 3D histogram . . . . .	6
4.2	Learning transition matrices from data . . . . .	6
4.2.1	Model D2Q9 . . . . .	7
4.2.2	Model D2Q9Q9 . . . . .	9
4.2.3	Model TD2Q9 . . . . .	9
4.2.4	Model TD2Q9Q9 . . . . .	9
4.3	Simulations . . . . .	9
4.3.1	Trajectories simulation . . . . .	9
4.3.2	Distribution simulation . . . . .	9
<b>5</b>	<b>Results</b>	<b>10</b>
5.1	Comparison of assimilation methods . . . . .	10
5.1.1	Real data - D2Q9 . . . . .	10
5.1.2	Real data - D2Q9Q9 . . . . .	10
5.1.3	Real data - TD2Q9 . . . . .	10
5.1.4	Real data - TD2Q9Q9 . . . . .	10
5.2	Simulation with generative models . . . . .	10
5.2.1	Real data - D2Q9 . . . . .	10
5.2.2	Real data - D2Q9Q9 . . . . .	10
5.2.3	Real data - TD2Q9 . . . . .	10
5.2.4	Real data - TD2Q9Q9 . . . . .	10
<b>6</b>	<b>Discussion</b>	<b>11</b>
<b>7</b>	<b>Appendix</b>	<b>12</b>
7.1	Package pathintegralanalytics - code description . . . . .	12
7.1.1	Explanation of the library . . . . .	12
7.1.2	UML diagram . . . . .	12

# Chapter 1

## Introduction

The aim of this work is to clarify the possibility to analyze real life datasets and simulate the pedestrian crowd starting from the Lattice model.

### 1.1 Assimilating pedestrian dynamics

### 1.2 Challenges

**Starting from real data how can we define a good model to simulate a pedestrian in the crowd flow?** In this type of system there is a multitude of *forces* that determinate the path of a single pedestrian. So let's take into account a single pedestrian  $P$  that walks in a certain space. The first type of interaction is the structure where  $P$  can or cannot walk thought, that is defined as the whole domain  $\Omega$ . The second interaction is between  $P$  and the other pedestrians. Every pedestrian needs a personal space all around, that is variable due the circumstance and it is not easy to be analytically determinate. A third type of interaction are random events along the  $P$ 's path, real world events. ...

**How to visualise the pedestrian's path using a multi-dimensional histogram?** It is possible to plot every single trajectory, but this lead to a chaotic data representation and not so functional nor readable. It is also easily possible to plot the *heatmap* of a dataset to analyse the most "walked" areas. Even if this second plot choice can takes into account more trajectories than the first and still be readable, it has a problem. This second lead to a representation where the time dependency is completely lost. ...

### 1.3 Relevance

## Chapter 2

# Pedestrian dynamics measurements

### 2.1 Idea of the recording technique

The recording techniques employed are two: Kinect based technology and ProRail security cameras. In both cases the field of view is covered using multiple cameras working together. Starting from the raw images, each object is tracked down along its entire path. This is possible using imaging recognition software. The software give as output a data-frame with coordinates and time for each pedestrian.

#### 2.1.1 Kinect: depth map based pedestrian tracking

#### 2.1.2 ProRail cameras

### 2.2 Presenting datasets

#### 2.2.1 Glow experiment

The experiment was realized during the Glow Expo in 2017. The idea is that people that enter from a single entrance have to chose between two possible exits, because of the obstacle in the middle. In the (Figure 2.1) is represented a digital reconstruction of the domain.

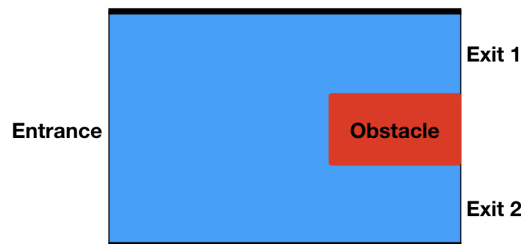


Figure 2.1: Reconstruction of the *glow* experiment, point of view from above.

#### 2.2.2 Utrecht Centraal (Floorefield 10)

In collaboration with ProRail - the company responsible of the train's stations in Netherlands - we had the possibility to use data from the Utrecht's train station. The (Figure 2.2) represents the camera's point of view of the analyzed domain in the station. This is an interesting spot given that this square has three free sides where people could walk through. It's also a huge corridor

and an highly crossed spot, that increase the statistic. With this domain we could study if the simulations we're doing are correct also in cases where people with different directions cross the same coordinates in the map. In fact - as described in the following paragraphs - the representation with the 3-dimensional histogram shows us a sort of *cross X*.



Figure 2.2: Utrecht Centraal, cameras point of view (Floorfield 10)

## 2.3 Technique of measure

Technique of measure documentation intro here

### 2.3.1 Kinect sensor

Technique of measure with Kinect sensor documentation intro here

### 2.3.2 ProRail data

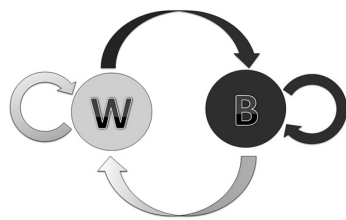
Technique of measure with ProRail data documentation intro here

## Chapter 3

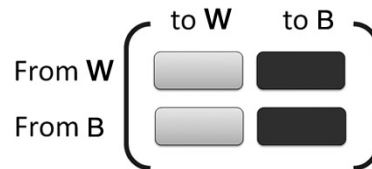
# Background

### 3.1 Markov Chain

A *Markov Chain* is a stochastic model. It describes the future outcome state based on the present state. In other words, the present state determinate the probabilities for every possible future outcome. The model's representation is a *stochastic matrix*, from now on called  $P$  matrix. The matrix's entries  $P_{ij}$  has as row-index  $i$  the starting state and as column-index  $j$  the ending state of the system.



(a) The diagram of a two-state Markov Chain



(b) The transition matrix, also named the Markov matrix

Figure 3.1: From the Markov diagram to the Markov matrix of a tow-state system

A two-state Markov chain is the most basic model which can be used for the illustration of the Markov process. The diagram in (Figure 3.1a) represents the possibility that the system has to change from both states. For instance, from the state  $W$  the system can move to the  $B$  state with the big black arrow or can remain in the  $W$  state with the small white arrow. The entries in the Markov Matrix in (Figure 3.1b) are positives numbers that represent the probability of changing state.

### 3.2 Cellular-Automata

Cellular-Automata documentation here

### 3.3 Lattice-Boltzman

Lattice-Boltzman documentation here

## Chapter 4

# Propose data assimilation technique

### 4.1 Approximation of path integrals: 3D histogram

Approximation of path integrals: 3D histogram documentation [here](#)

### 4.2 Learning transition matrices from data

In this study there is a total of four models:  $D2Q9$ ,  $D2Q9Q9$ ,  $TD2Q9$ ,  $TD2Q9Q9$ . The firsts two are only dependent by the position in space, also called *time-independents*. The others two are dependent by the position and time, also called *time-dependents*. Whereas there is also a distinction between the  $D2Q9$ s and the  $D2Q9Q9$ s. For the  $D2Q9$ s what it's doing is considering the velocity from a cell to another, so just the change in position. For the  $D2Q9Q9$ s it's also considering the acceleration, so the change in velocity. The starting point of each one is the dataset, collected from a real life situation. Since each of them are entirely based on real world pedestrian's path in a crowd, those models simulates an *effective potential* (EP). This potential considers the imposed limit due to the presence of others pedestrians, such as pedestrians tend to not collide each others. It also considers the boundary condition given by the structural environment. The strong point of this EP is that is generated by the real world observation and not built by hand. With the aim of reproducing realistic pedestrians movements, synthetic paths are created from the models. Every model generate one trajectory that simulate just one pedestrian in a statistical crowd. When simulating more paths it consider pedestrian that walks alone in the crowd. This model doesn't consider the interaction made by the others simulated pedestrians.

**Notation** Lets assume  $\gamma = \gamma(\vec{x}_c)$  a pedestrian's path, where  $\vec{x}_c = (x_c, y_c)$  has a bi-dimensional spacial dependency. Given a field  $\Omega_c$ , the continuous space where pedestrians are tracked, the path  $\gamma$  in that space has a start position  $A$  and an end position  $B$ . The field  $\Omega$  is then divided into *rectangular* cells, dividing the real space along  $x$ , with maximum extension indicated as  $L_x$ , in a certain number of cells  $D_x$ ; as well for the  $y$ -direction, with obvious notation:  $L_y$  and  $D_y$ . After this discretization is obtained a *grid space*  $\Omega_g$ . Where every path  $\gamma$  is converted from continuous  $\gamma = \gamma(x_c, y_c)$  to discrete coordinate  $\gamma = \gamma(x_g, y_g)$ , referred to the *grid*. To lighten up the notation when speaking of *grid space* it is simply used  $(x, y)$  in reference to the discrete grid position.

**The standard D2Q9 configuration** In reference to the (Figure 4.1). This *map* is set for each position  $(x_0, y_0)$  in the grid space and it represents the eight neighbors and the central position where a pedestrian could go. Each direction will be associated to a certain transition probability.

When a trajectory change position, in the grid space (Figure 4.2), from  $P_0 = (x_0, y_0)$  to  $P_1 = (x_1, y_1)$  is associated a transition. The transition is identified by a number  $k = 0, 1, \dots, 8$  such that is unique. It is derived from the series of coordinates for each trajectory and each step in time. When the calculation is made for each step, for every position in time is also associated a transition number, that represents where is going to go in the next step.

If this transition is associated to the change in position it identify a certain velocity, as vector, with a certain direction. Iterating this procedure to the entire pedestrian's trajectory it is possible to get something like what's illustrated in the (Figure 4.3). In that figure it is possible to distinguish



6	2	5
3	0	1
7	4	8

Figure 4.1: Transitions associated to possible movements from the center cell. The figure identify the  $k$  nine values for each cell.

$(-1, +1)$	$(+0, +1)$	$(+1, +1)$
$(+1, +0)$	$(+0, +0)$	$(+1, +0)$
$(-1, -1)$	$(+0, -1)$	$(+1, -1)$

Figure 4.2: Given the initial position at the center square, this is a representation of the change in coordinates to the next cell.

the path in the continuous space and the discrete path in the grid space. It also shows the direction of the next movement for each position with arrows that are consistent with the velocity arrows in each position. The numbers are the value of the  $k$ -index in each position, it is solid with the maps above. This lead the discussion directly to the first model *D2Q9* in the next paragraph.

#### 4.2.1 Model D2Q9

The simplest model considered here is called *D2Q9 – model*. This model is a time-independent and it consider the velocity of the pedestrian. Given a starting position  $(x_0, y_0)$  in the field  $\Omega$ . It uses the nine closest possible positions where a pedestrian could go from that point. With the *D2Q9 – model* is than possible to know, for each position  $(x_0, y_0)$ , the probability to go up, down, left, right or a combination of those movements.

**Transitions** from the initial position  $P_0 = (x_0, y_0)$  to the next closest cell in the grid  $P_k$  are defined by the index  $k$ . So that the index  $k$  gives the direction of the transition. For instance a transition from  $P_0$  to  $P_1$  and a transition from  $P_0$  to  $P_2$  are defined by:

$$\begin{array}{ll} P_0 \rightarrow P_1 & P_0 \rightarrow P_2 \\ (x_0, y_0) \rightarrow (x_0 + 1, y_0) & (x_0, y_0) \rightarrow (x_0, y_0 + 1) \end{array}$$

Considering the (Figure 4.4) all the transitions are associated to a specific  $k$ . This is a particular Markov Chain (see 3.1 ) where there are a total of *nine* states. Between these states the transitions always and only start from the  $P_0$  state to go to the others  $P_k$  states or itself. The same concept is graphically represented with the diagram in (Figure 4.5). To every transition is associated a certain probability to happen. Formally this probability is given by the initial and the final states:  $p_{if}$ . Since there always is the same starting state, it is possible to omit it. So that the probability of the transition from  $P_0$  to  $P_k$  is expressed by  $p_k$ , where the index  $k$  points to the ending state. It means that for each position in  $\Omega$  it's possible to say how likely is to "step forward" or "turn right" and so on. Then, once in the new position, it's again possible to say the most probable direction that the pedestrian will choose. The same prediction is applicable to the whole space, mapped by the real datas.

With this structure it is then possible to create a tensor  $A$  with three indices. Taking into account the simplest model, as above, the relative *tensor* is  $A_{xyk}$ . Where every entries is the probability  $p$  to move along the  $k$  direction from the location  $(x, y)$ . Since the aim of every models is to simulate a pedestrian in the crowd, this tensor is the key to get to the result.

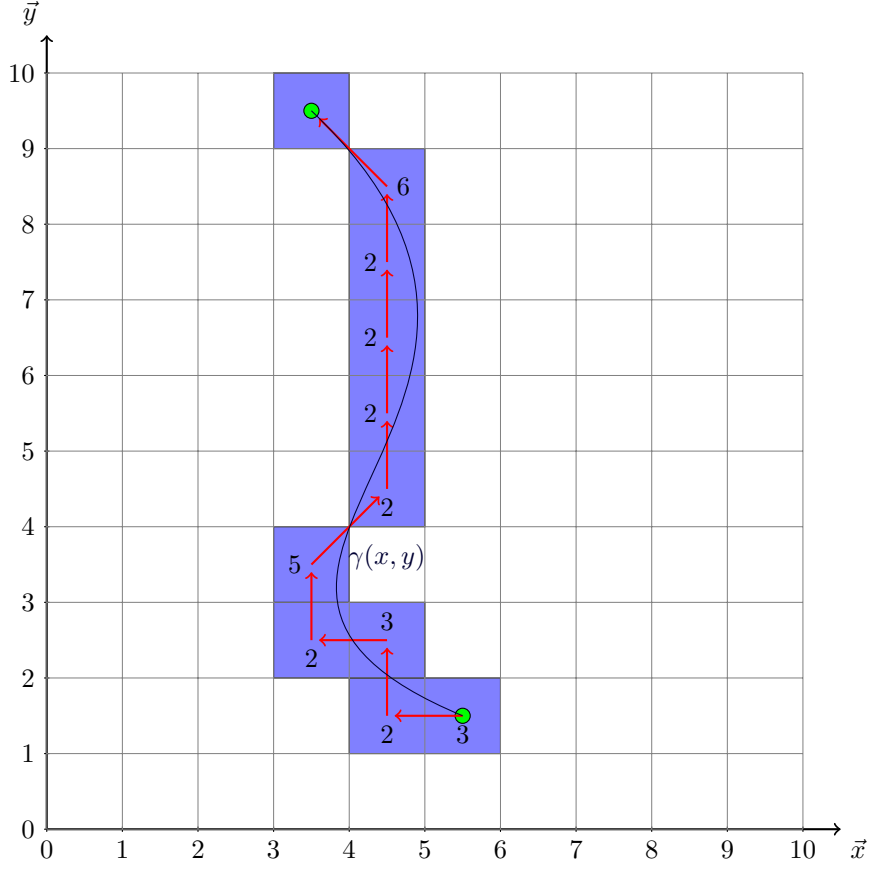


Figure 4.3: This illustration represents a trajectory in the *continuous space* as the blue line  $\gamma$ . That path  $\gamma$  is discretized in the *grid space*, represented by the blue cells. The red arrows represents the change from a cell to the next. The numbers are the associated to the D2Q9 indexes to those moves, also called *k-directions*

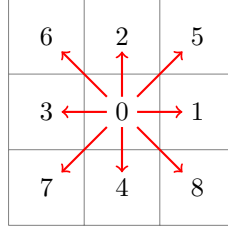


Figure 4.4: The possible transitions, represented as vectors. For the D2Q9 model those vector are also representing the velocity vectors.

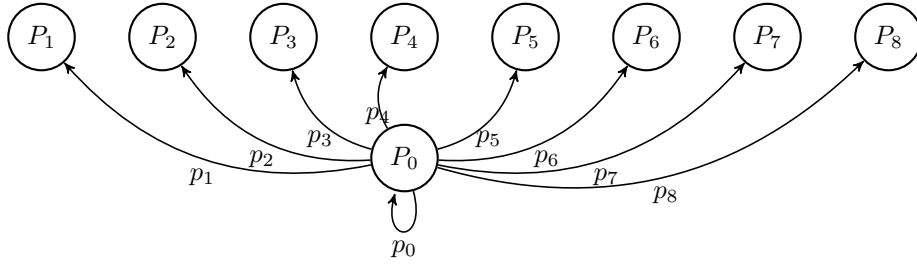


Figure 4.5: The Markov Chain diagram of the system. The states are indicated with circles and labeled with  $P_k$ . The transitions are indicated with arrows and labeled with their probability  $p_k$

### 4.2.2 Model D2Q9Q9

The conceptual step forward of the study is to not only consider the next position, but also the previous. Hence given a trajectory  $\gamma$  in the grid space of a pedestrian that make a transition for each time step. For each point  $P_0$  of  $\gamma$  it is possible to determinate where it was before at  $P_{-1}$  and where is going to be after at  $P_{+1}$ . The index that represents the *next* position is  $k$ , meanwhile the index that represents the *previous* position is  $h$ . For instance it is given the table of the coordinates and the two indexes related to the (Figure 4.5) in the (Table 4.1). The *tensor* associated to this

Time step	$x_g$	$y_g$	$k$ -index	$h$ -index
1	5	1	3	0
2	4	1	2	1
3	4	2	3	4
4	3	2	2	1
5	3	3	5	4
6	4	4	2	7
7	4	5	2	4
8	4	6	2	4
9	4	7	2	4
10	4	8	6	4
11	3	9	0	8

Table 4.1: This is tabulated the trajectory of the same illustrative pedestrian as above in (Figure 4.5). Here is expressed the position time to time, the index of the following move and the index of the previous move.

model is characterized by a total of four indexes as  $A_{xykh}$ . Every element of this tensor is now representing a certain probability to move away from a certain state to another. Taking into account the example in the (Table 4.1), and considering this the only one possible trajectory. It is easy to see that the probability at  $A_{xykh} = A_{3,3,5,4} = 1$  is maximum in the position (3, 3). The probability for every other  $k, h$  in the same position is zero,  $A_{3,3,k,h} = 0 \quad \forall k \neq 5, h \neq 4$ .

### 4.2.3 Model TD2Q9

### 4.2.4 Model TD2Q9Q9

## 4.3 Simulations

Simulations documentation here

### 4.3.1 Trajectories simulation

### 4.3.2 Distribution simulation

# Chapter 5

## Results

### 5.1 Comparison of assimilation methods

Comparison of assimilation methods documentation here

#### 5.1.1 Real data - D2Q9

#### 5.1.2 Real data - D2Q9Q9

#### 5.1.3 Real data - TD2Q9

#### 5.1.4 Real data - TD2Q9Q9

### 5.2 Simulation with generative models

Simulation with generative models documentation here

#### 5.2.1 Real data - D2Q9

#### 5.2.2 Real data - D2Q9Q9

#### 5.2.3 Real data - TD2Q9

#### 5.2.4 Real data - TD2Q9Q9

## Chapter 6

# Discussion

Discussion documentation here

## Chapter 7

# Appendix

### **7.1 Package pathintegralanalytics - code description**

Package pathIntegralAnalytics documentation here

#### **7.1.1 Explanation of the library**

Explanation of the library documentation here

#### **7.1.2 UML diagram**

UML pathIntegralAnalytics documentation code here