

# **D. Interpolasi Newton Gregory Maju- Mundur**

*Interpolasi Newton Gregory Maju-Mundur hanya dapat digunakan jika "x" memiliki selisih/beda yang tetap.*

Contoh:

Diketahui data-data berikut.

$(-2, -6) ; (0, -4) ; (2, 4) ; (4, 20)$ .

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2

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Soal seperti di atas dapat dikerjakan dengan interpolasi  
Newton Gregory Maju-Mundur

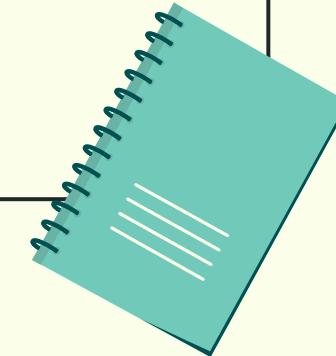
# 1. Interpolasi Newton Gregory Maju

## Rumus:

$$\begin{aligned}f(x) \approx P_n(x) = f(x_0) + \Delta f(x_0) \cdot s + \frac{\Delta^2 f(x_0)}{2!} \cdot s(s-1) + \frac{\Delta^3 f(x_0)}{3!} \cdot s(s-1)(s-2) \\+ \frac{\Delta^4 f(x_0)}{4!} \cdot s(s-1)(s-2)(s-3) + \dots \\+ \frac{\Delta^n f(x_0)}{n!} \cdot s(s-1) \dots (s-(n-1))\end{aligned}$$

dengan  $s = \frac{x - x_0}{h}$  dan  $h = x_{i+1} - x_i$

Jika diketahui 2 titik, maka  $f(x) \approx P_1(x) = f(x_0) + \Delta f(x_0) \cdot s$



## Contoh:

Diketahui data- data berikut:  $(-2, -6) ; (0, -4), (2, 4) ; (4, 20)$ .

Tentukan: (a)  $f(x)$  dan (b)  $f(1)$

Penyelesaian:

$x$	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	
-2	-6				$f(x_0)$
		2			$\Delta f(x_0)$
0	-4		6		$\Delta^2 f(x_0)$
		8		2	$\Delta^3 f(x_0)$
2	4		8		
		16			
4	20				

$$h = 2$$
$$s = \frac{x + 2}{2}$$


$$P_3(x) = f(x_0) + \Delta f(x_0) \cdot s + \frac{\Delta^2 f(x_0)}{2!} \cdot s(s-1) + \frac{\Delta^3 f(x_0)}{3!} \cdot s(s-1)(s-2)$$

$$= -6 + 2 \cdot \left(\frac{x+2}{2}\right) + \frac{6}{2} \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) + \frac{2}{6} \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) \left(\left(\frac{x+2}{2}\right) - 2\right)$$

$$= -6 + (x+2) + 3 \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) + \frac{1}{3} \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) \left(\left(\frac{x+2}{2}\right) - 2\right)$$


$$f(x) \approx P_3(x)$$

$$\approx -6 + (x+2) + 3 \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) + \frac{1}{3} \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) \left(\left(\frac{x+2}{2}\right) - 2\right)$$


$$f(x) \approx P_3(x)$$

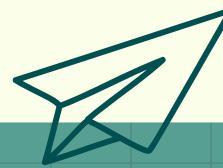
$$\approx -6 + (x+2) + 3 \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) + \frac{1}{3} \cdot \left(\frac{x+2}{2}\right) \left(\left(\frac{x+2}{2}\right) - 1\right) \left(\left(\frac{x+2}{2}\right) - 2\right)$$

$$f(1) \approx -6 + (1+2) + 3 \cdot \left(\frac{1+2}{2}\right) \left(\left(\frac{1+2}{2}\right) - 1\right) + \frac{1}{3} \cdot \left(\frac{1+2}{2}\right) \left(\left(\frac{1+2}{2}\right) - 1\right) \left(\left(\frac{1+2}{2}\right) - 2\right)$$

$$\approx -6 + 3 + 3 \cdot \left(\frac{3}{2}\right) \left(\frac{1}{2}\right) + \frac{1}{3} \cdot \left(\frac{3}{2}\right) \left(\frac{1}{2}\right) \left(-\frac{1}{2}\right)$$

$$\approx -3 + \frac{9}{4} - \frac{1}{8}$$

$$\approx -\frac{7}{8} = -0,875$$

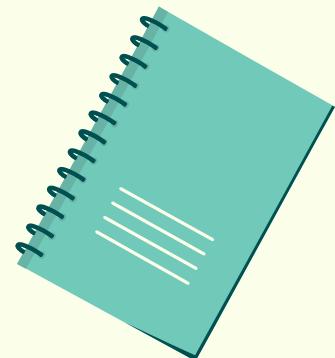


## 2. Interpolasi Newton Gregory Mundur

Rumus:

$$\begin{aligned}f(x) \approx P_n(x) = f(x_n) + \nabla f(x_n) \cdot s + \frac{\nabla^2 f(x_n)}{2!} \cdot s(s+1) + \frac{\nabla^3 f(x_n)}{3!} \cdot s(s+1)(s+2) \\+ \frac{\nabla^4 f(x_n)}{4!} \cdot s(s+1)(s+2)(s+3) + \dots \\+ \frac{\nabla^n f(x_n)}{n!} \cdot s(s+1) \dots (s+(n-1))\end{aligned}$$

dengan  $s = \frac{x-x_n}{h}$  dan  $h = x_{i+1} - x_i$





## contoh:



Diketahui data- data berikut: (-2, -6) ; (0, -4); (2, 4) ; (4, 20).

Tentukan: (a)  $f(x)$  dan (b)  $f(1)$

Penyelesaian:

$x$	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
-2	-6			
		2		
0	-4		6	
		8		
2	4		8	2
		16		
4	20			

$$h = 2$$
$$s = \frac{x - 4}{2}$$

$\rightarrow \nabla^3 f(x_3)$   
 $\rightarrow \nabla^2 f(x_3)$   
 $\rightarrow \nabla f(x_3)$   
 $\rightarrow f(x_3)$



$$P_3(x) = f(x_3) + \nabla f(x_3) \cdot s + \frac{\nabla^2 f(x_3)}{2!} \cdot s(s+1) + \frac{\nabla^3 f(x_3)}{3!} \cdot s(s+1)(s+2)$$

$$= 20 + 16 \cdot \left(\frac{x-4}{2}\right) + \frac{8}{2} \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) + \frac{2}{6} \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) \left(\left(\frac{x-4}{2}\right) + 2\right)$$

$$= 20 + 8(x-4) + 4 \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) + \frac{1}{3} \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) \left(\left(\frac{x-4}{2}\right) + 2\right)$$

$$f(x) \approx P_3(x)$$

$$\approx 20 + 8(x-4) + 4 \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) + \frac{1}{3} \cdot \left(\frac{x-4}{2}\right) \left(\left(\frac{x-4}{2}\right) + 1\right) \left(\left(\frac{x-4}{2}\right) + 2\right)$$

$$f(1) \approx 20 + 8(1-4) + 4 \cdot \left(\frac{1-4}{2}\right) \left(\left(\frac{1-4}{2}\right) + 1\right) + \frac{1}{3} \cdot \left(\frac{1-4}{2}\right) \left(\left(\frac{1-4}{2}\right) + 1\right) \left(\left(\frac{1-4}{2}\right) + 2\right)$$

# Latihan Soal

Diketahui data- data berikut:

$$(0, -6) ; (2, 4) ; (4, 10).$$

Tentukan perkiraan dari  $f(1)$



# TERIMA KASIH

