- Discrete mathematics is a core subject of theoretical computer science. It is not a directly application-oriented subject, but it provides tolls and mathematical models, which are applied to different areas in computer science.

- **GATE (7-9 MARKS)**

- Has a good weightage in general all objective and subjective examination.

- Will be asked in Interview for M.Tech, PhD or other government jobs. Not that important in software industry.

Section 1: Engineering Mathematics

Discrete Mathematics: Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.

Linear Algebra: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Probability and Statistics: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

What is a SET

- Set are the fundamental discrete structures on which all the discrete structures are built. Sets are used to group objects together, formally speaking

- “An *unordered ,well-defined*, collection of *distinct* objects (Called elements or members of a set) of same type”. Here the type is defined by the one who is defining the set. For e.g.

- A = {0,2,4,6, ---}

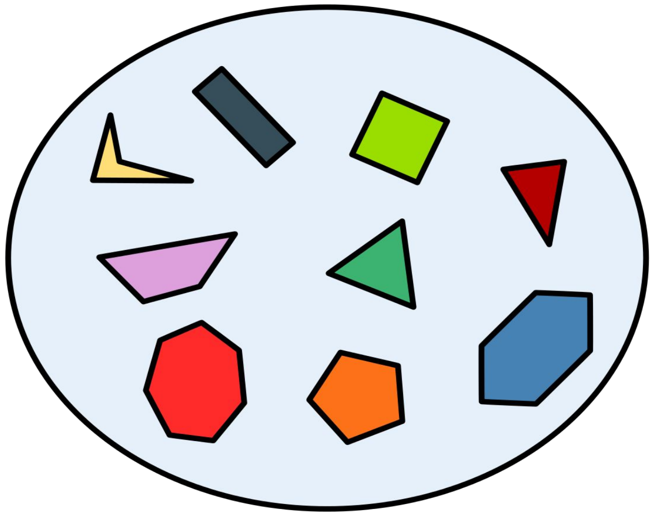
- B = {1,3,5, ---}

- C= {x| x ∈ Natural number}

- A Set is generally denoted usually by capital letter. The objects of a set called the elements, or members of the set.

- A set is said to contain its elements. Lower case letters are generally used to denote the elements of the set.

- x ϵ A, means element x is a member of A, x ∉ A means x is not a member of A



Representation of set

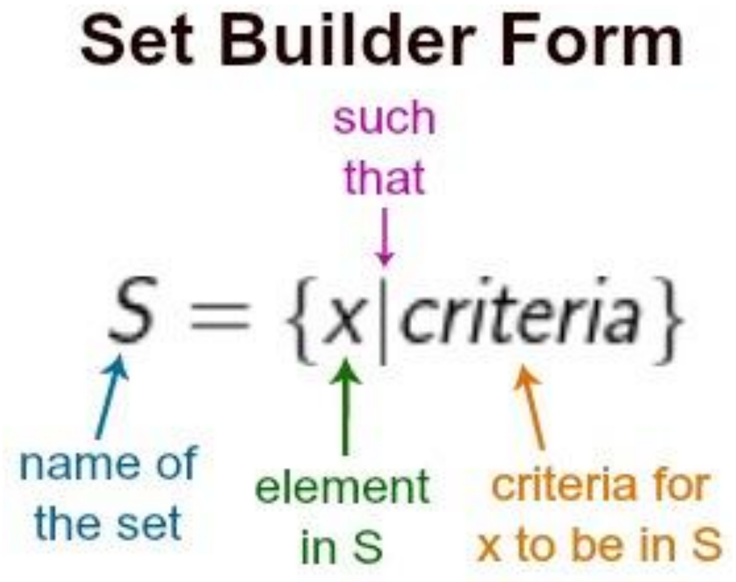
- **Tabular/Roster representation of set** - here a set is defined by actually listing its members. E.g.

- A = {a, e, i, o, u}

- B = {1, 2, 3, 4}

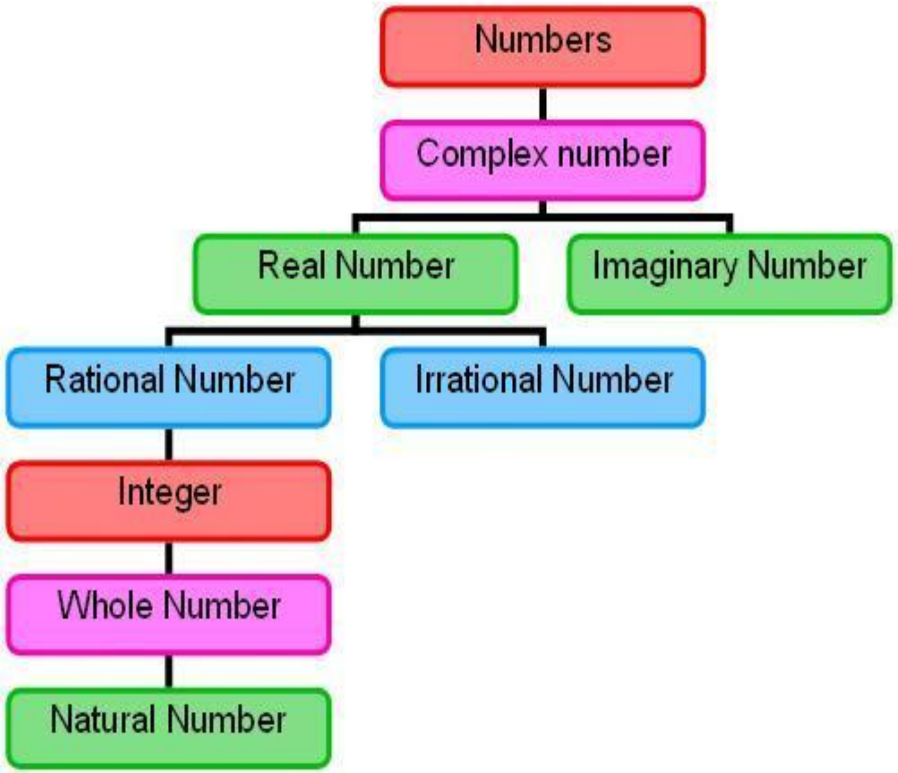
- C = {---- -4, -2, 0, 2, 4, -----}.

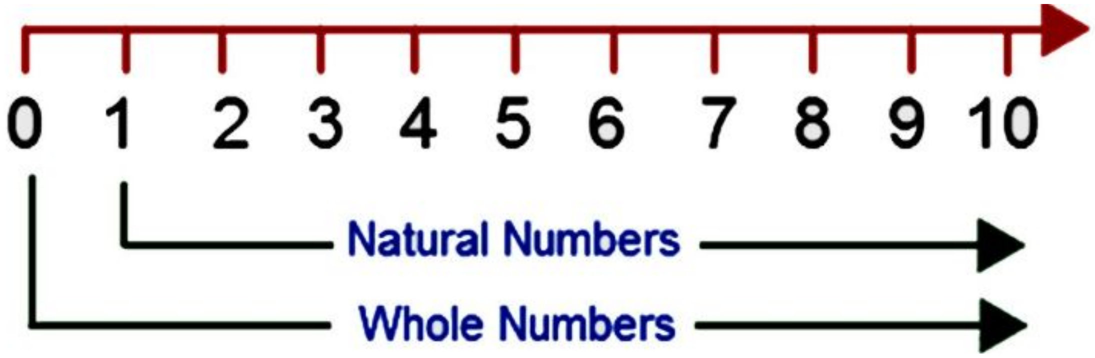
- **<u>Set Builder representations of set</u>**- here we specify the property which the elements of the set must satisfy. E.g.



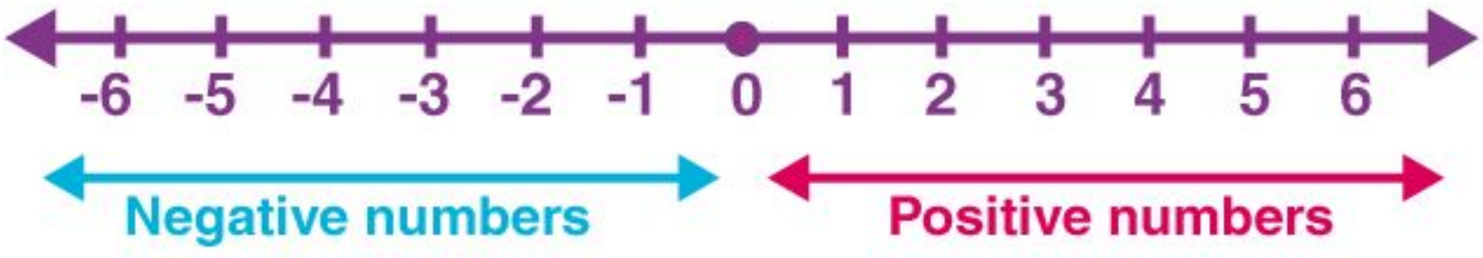
- A = {x| x is an odd positive number less than 10},
- A = {x | x ϵ English alphabet && x is vowel}
- B = {x | x ϵ N && x <5}
- C = {x | x ϵ Z && x%2 = 0 }

- **Set of all-Natural number(N) -** A natural number is a number that occurs commonly and obviously in nature. The set of natural numbers, can be defined as N = {1,2,3, 4.... ∞}
- **Set of all Whole number(W) -** A whole number is a science expanded natural number. Set of natural number and zero

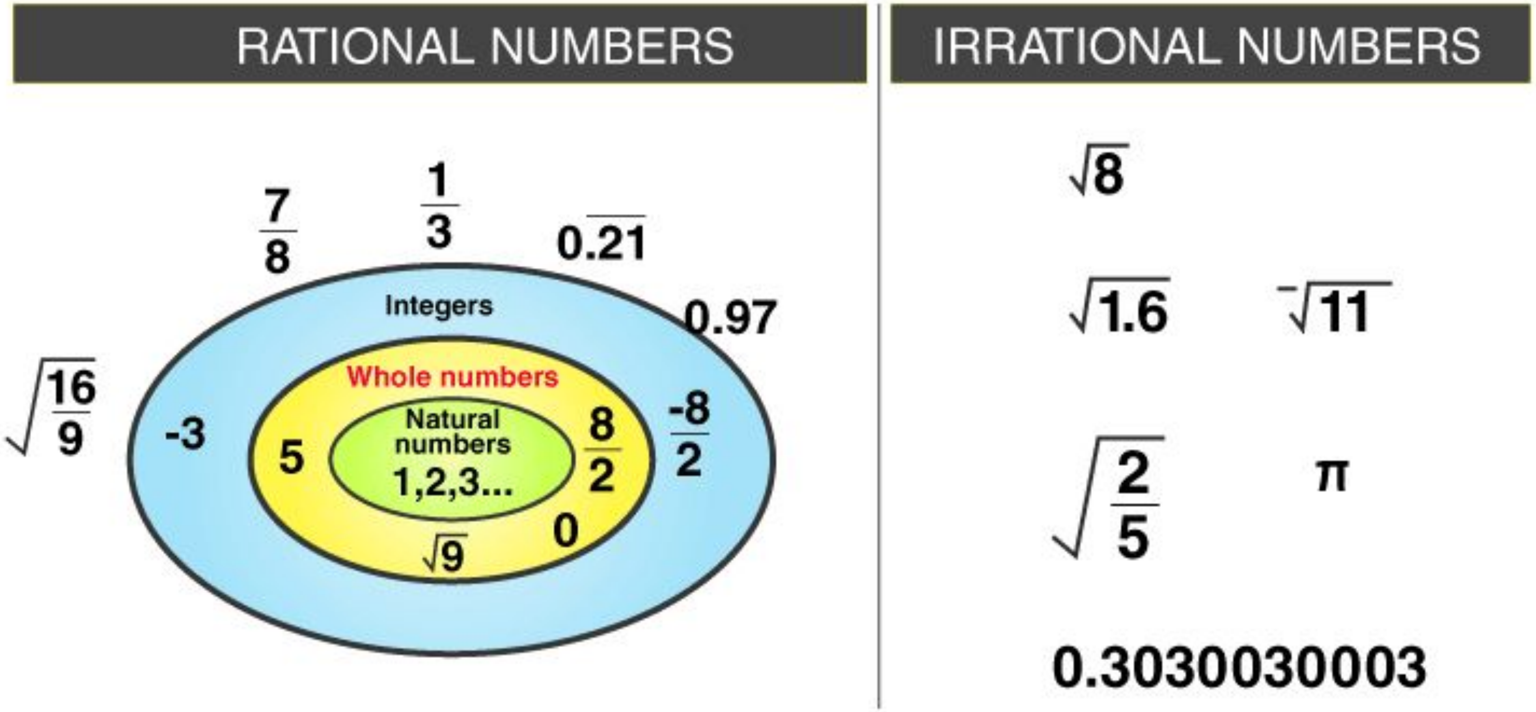




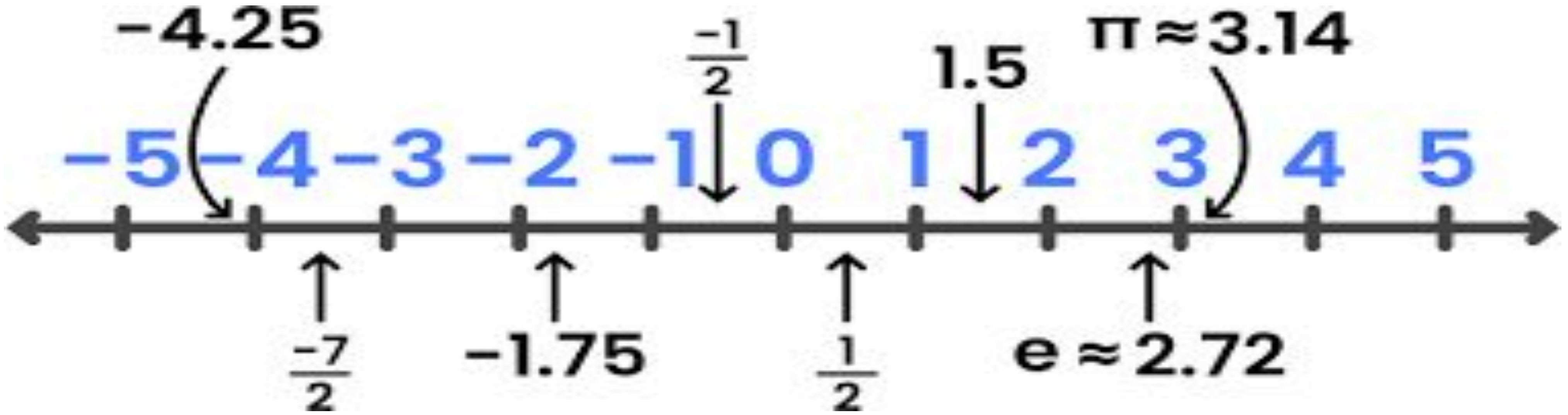
- **<u>Set of all Integer(Z)</u> -** An integer is a number that can be written without a fractional component.



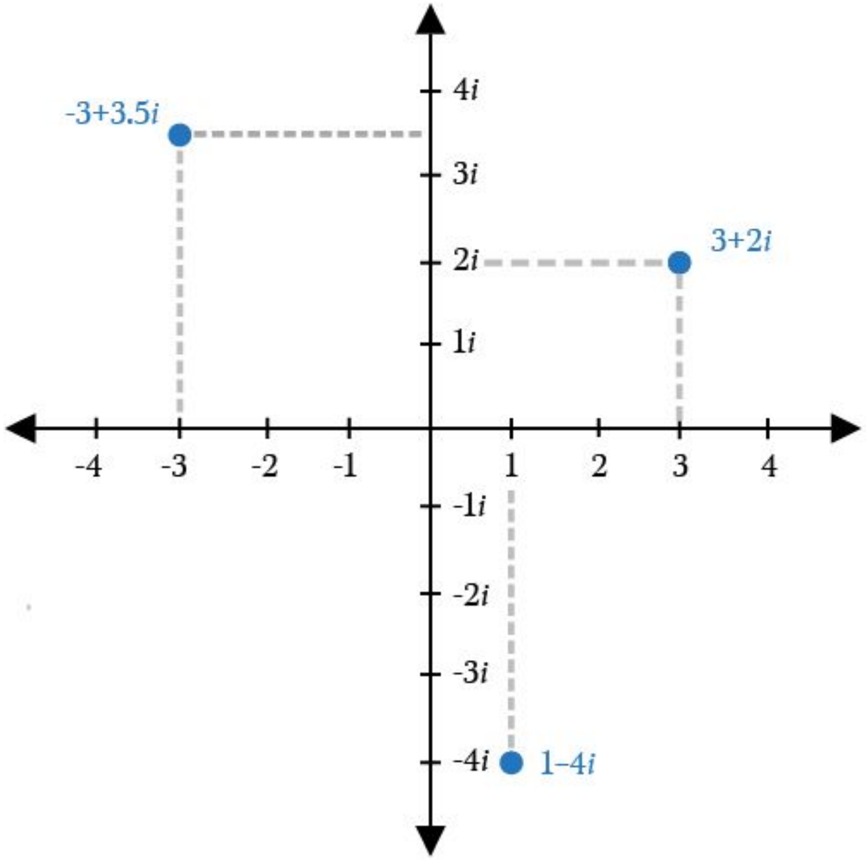
- **<u>Set of all Rational number (Q)</u> -** A rational number is any number that can be expressed as a fraction P/Q of two integers, a numerator P and a non-zero denominator Q.
- **<u>Set of all Irrational number (R-Q or R/Q or P)</u> -** An irrational number is a real number that cannot be expressed as a fraction i.e. as a ratio of integers. Therefore, irrational numbers, when written as decimal numbers, do not terminate, nor do they repeat. E.g. root2.



- **<u>Set of all Real number(R) </u>-** A real number is a value that represents a quantity along a continuous line, containing all of the rational numbers and all of the irrational numbers.



- **<u>Set of all Complex number(C)</u> -** A complex number is a number that can be expressed in the form ‘a + bi’, where ‘a’ and ‘b’ are real numbers and ‘i’ is the imaginary unit, that satisfies the equation $i^2 = -1$. In this expression, ‘a’ is the real part and ‘b’ is the imaginary part of the complex number.



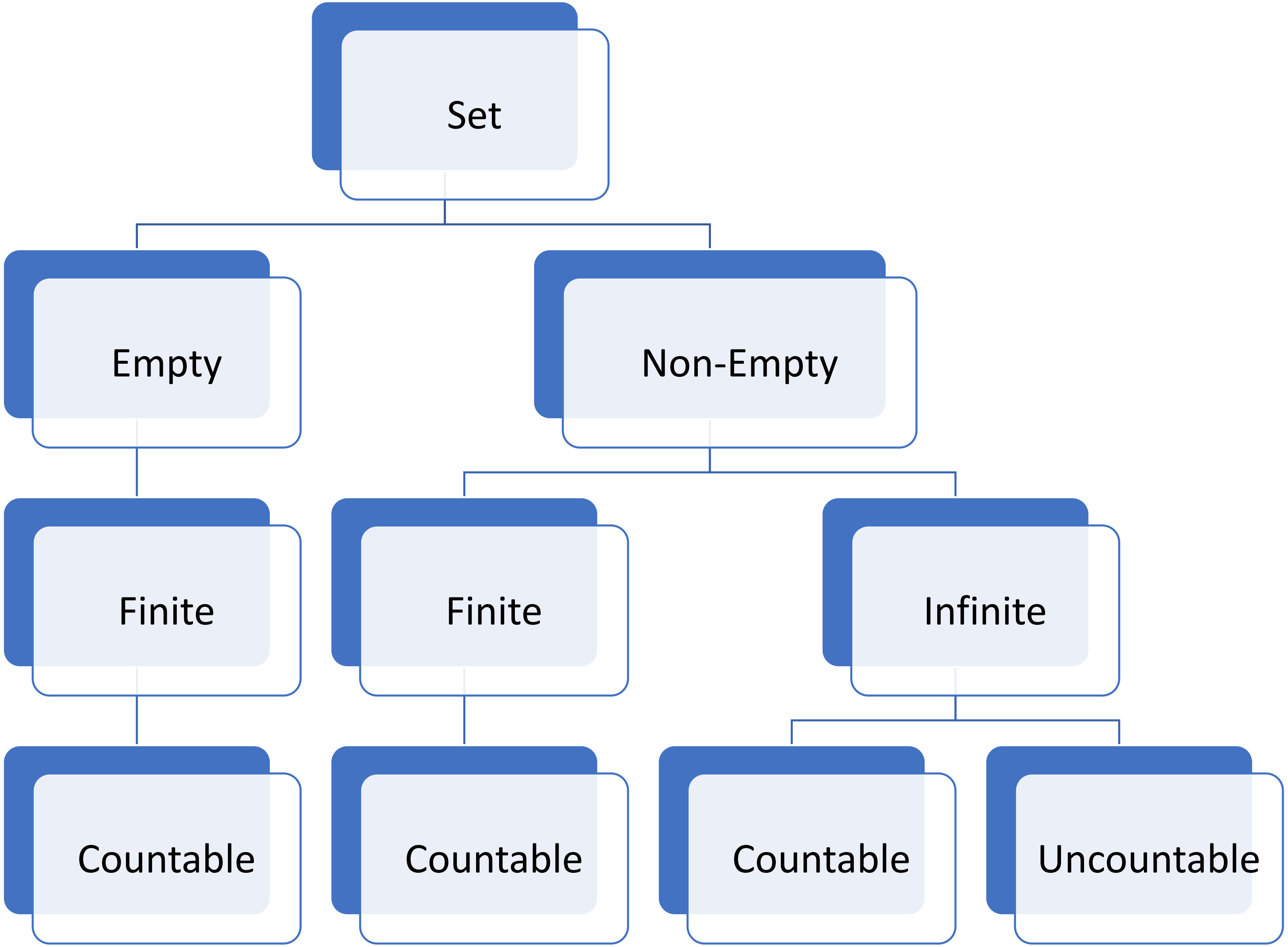
- **Finite set** - If there are exactly '*n*' elements in *S* where '*n*' is a nonnegative integer, we say that *S* is a *finite set.*

- *i.e. if a* set contain specific or finite number of elements is called is called finite set. For e.g. A = {1,2,3,4}

- **Cardinality of a set** — It is the number of elements present in a finite Set, denoted like $|A|$.

- For e.g. $A = \{0,2,4,6\}$, $|A| = 4$

- **Infinite set** – A set contain infinite number of elements is called infinite set, if the counting of different elements of the set does not come to an end. For e.g. a set of natural numbers.

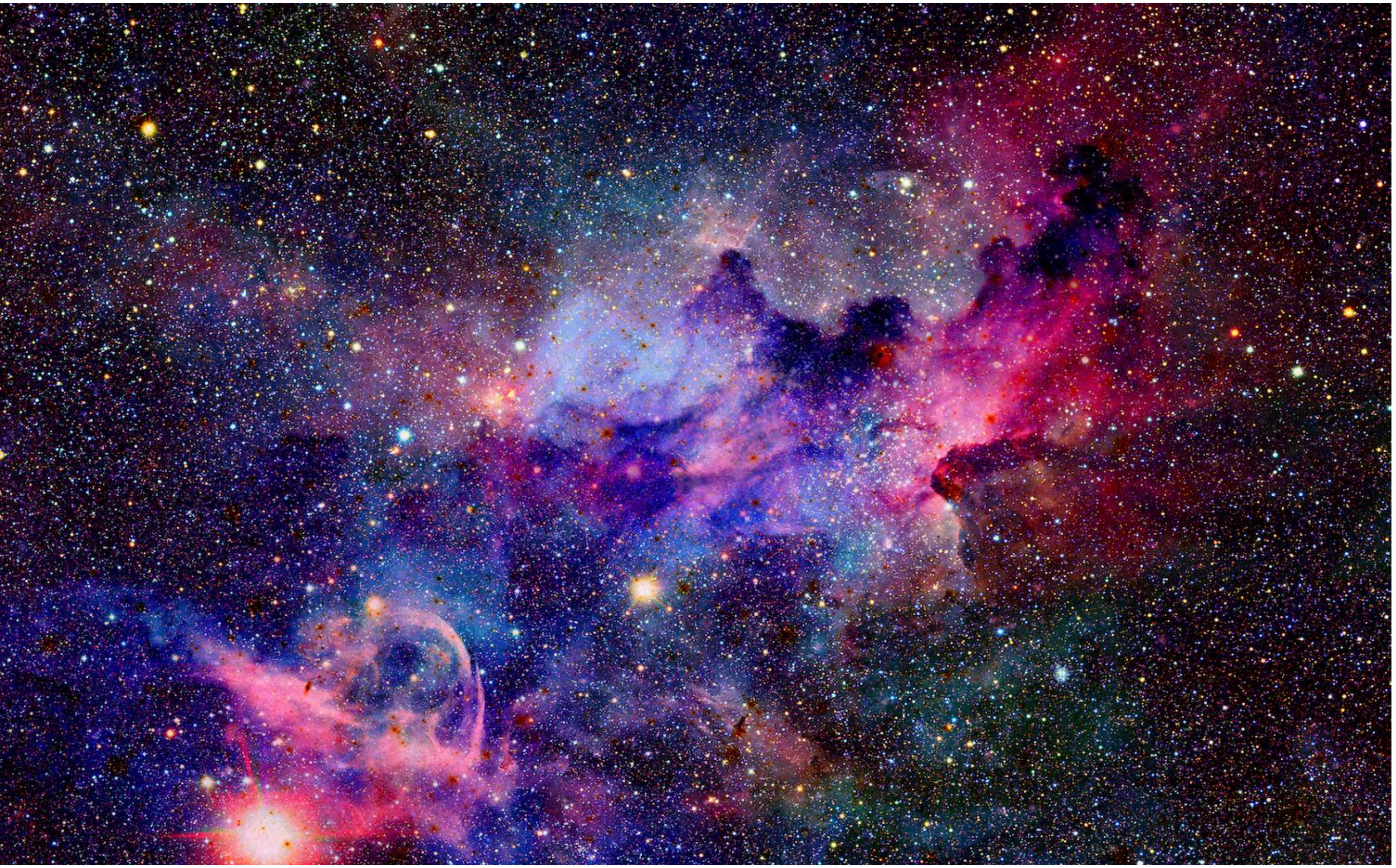

Set
Empty
Non-Empty
Finite
Finite
Infinite
Countable
Countable
Countable
Uncountable

- **Countable set** – A set is said to be countable if there can be a one to one mapping between the elements of the set and natural numbers. E.g. Set of stars, N, W, Z, Q.

- **Uncountable set** – A set is said to be uncountable if there cannot be a one to one mapping between the elements of the set and natural numbers. E.g. Set of real numbers.

- **<u>Null set / empty set</u>** - Is the unique set having no elements. its size or cardinality is zero i.e. $|\phi| = 0$. It is denoted by a symbol ϕ or {}.

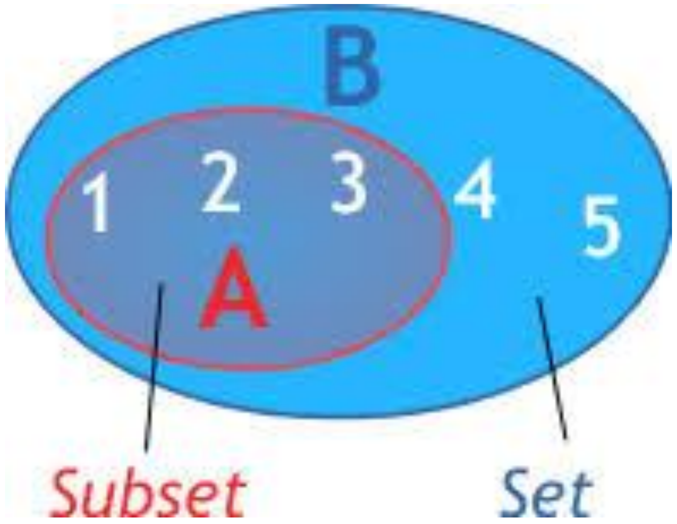
- <u>**Universal set**</u> – if all the sets under investigation are subsets of a fixed set, i.e. the set containing all objects under investigation, in Venn diagram it is represented by a rectangle, and it is denoted by U.



- **Subset of a set** – If every element of set A is also an element of set B i.e.
- $\forall x(x \in A \rightarrow x \in B)$, then A is called subset of B and is written as $A \subseteq B$. B is called the superset of A.

- E.g. A = {1,2,3} B = {1,2,3,4,5}

- Note that to show that *A* is not a subset of *B* we need only find one element $x \in A$ with $x \notin B$. To show that $A \subseteq B$, show that if $x \in A$, then $x \in B$.

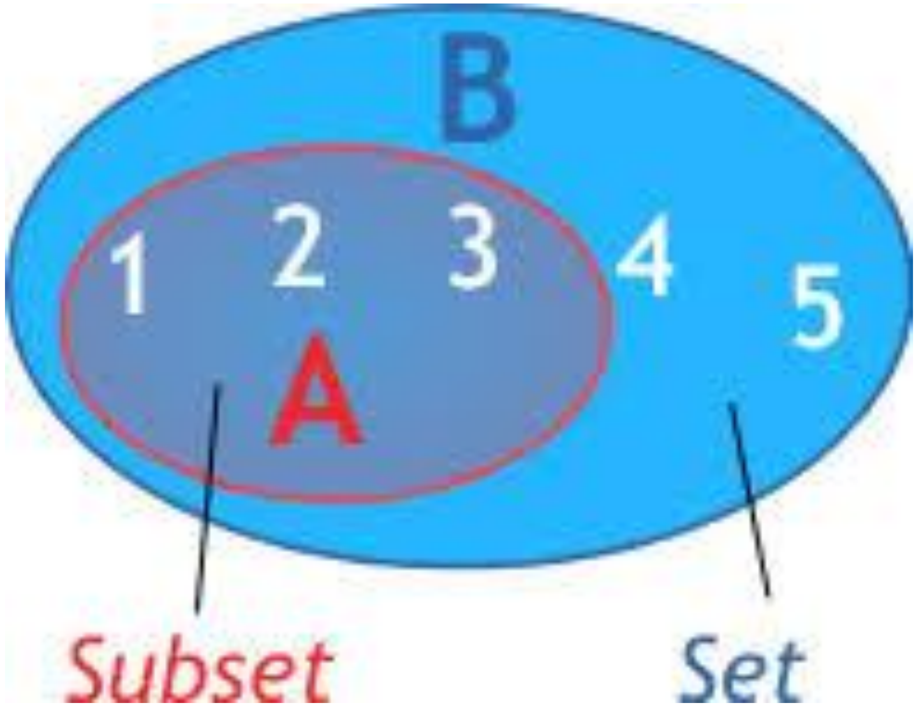


- $\phi \subseteq A$, Empty Set ϕ is a subset for every set

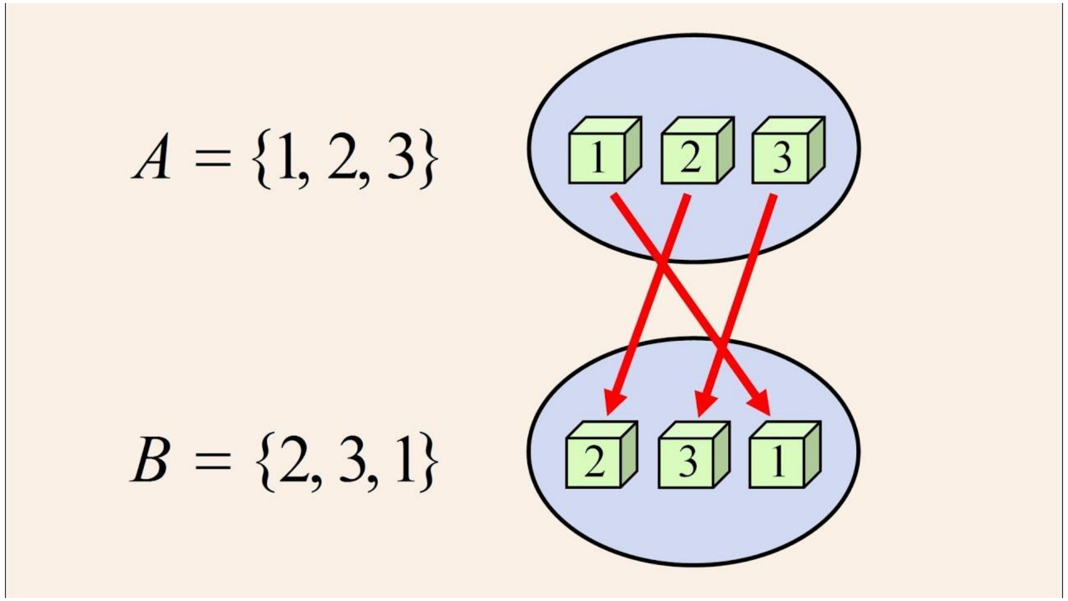
- $A \subseteq U$, Every Set is a subset of Universal set U

- $A \subseteq A$, Every Set is a subset of itself.

- **<u>Proper subset</u> –** if A is a subset of B and A ≠ B, then A is said to be a proper subset of B, i.e. there is at least one element in B which is not in A. denoted as $A \subset B$.



- **<u>Equality of sets</u>** – If two sets A and B have the same element and therefore every element of A also belong to B and every element of B also belong to A, then the set A and B are said to be equal and written as A=B.
- if $A \sqsubseteq B$ and $B \sqsubseteq A$, then A=B
- $\forall x(x \in A \leftrightarrow x \in B)$



- **Power set** – let A be any set, then the set of all subsets of A is called power set of A and it is denoted by P(A) or 2^A.

- If A= {1,2,3}, then P(A) = {ϕ, {1}, {2}, {3}, {1,2}, {2,3}, {1,3}, {1,2,3}}

- Cardinality of the power set of A is n, $|P(A)| = 2^n$

Q For any Set A, which of the following are true?

1) $\phi \in A$

2) $\phi \subseteq A$

3) $\phi \in 2^A$

4) $\phi \subseteq 2^A$

5) $A \in 2^A$

6) $A \subseteq 2^A$

Q If ϕ is an empty set. Then | P(P(P(ϕ))) | =_______?

a) 1 **b)** 2 **c)** 4 **d)** none of above

Q The cardinality of the power set of {0, 1, 2 . . ., 10} is __________.
(GATE-2015) (1 Marks)

(A) 1024 **(B)** 1023 **(C)** 2048 **(D)** 2043

Q For a set A, the power set of A is denoted by 2^A. If A = {5, {6}, {7}}, which of the following options are True. **(GATE-2015) (1 Marks)**

I) $\phi \in 2^A$

II) $\phi \subseteq 2^A$

III) $\{5, \{6\}\} \in 2^A$

IV) $\{5, \{6\}\} \subseteq 2^A$

(A) I and III only

(B) II and III only

(C) I, II and III only

(D) I, II and IV only

Q let A be a set with n elements. Let C be a collection of distinct subsets of A such that for any two subsets S_1 and S_2 in C, either S_1 is subset of S_2 or S_2 is subset of S_1. What is the maximum cardinality of C? **(GATE-2005) (2 Marks)**

a) n **b)** n+1 **c)** $2^{n-1}+1$ **d)** n!

Q Let P(S) denotes the power set of set S. Which of the following is always true? **(GATE-2000) (2 Marks)**

a) $P(P(S)) = P(S)$

(b) $P(S) \cap P(P(S)) = \{\varphi\}$

(c) $P(S) \cap S = P(S)$

(d) $S \notin P(S)$

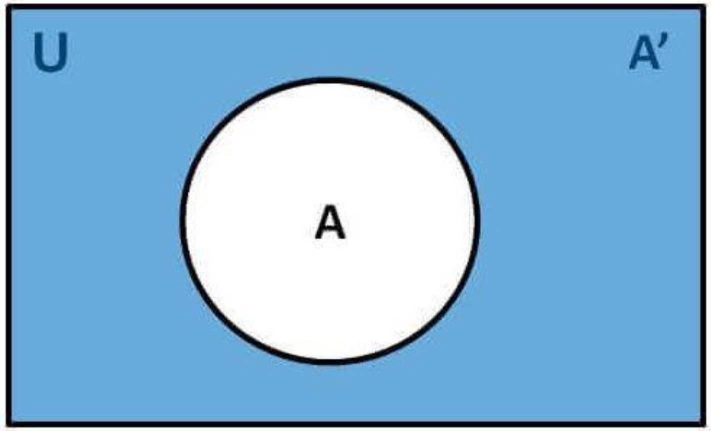
Q The number of elements in the power set P(S) of the set S= {{∅},1, {2,3}} is: **(GATE-1995) (1 Mark)**

a) 2 **b)** 4 **c)** 8 **d)** None of the above

Q Let S be a set of consisting of 10 elements. The number of tuples of the form (A,B) such that A and B are subsets of S, and A $\subseteq$ B is _____________ . **(GATE 2021) (2 MARKS)**

Operation on sets

- **Complement of set** – Set of all x such that $x \notin A$, but $x \in U$.
- $A^c = \{x \mid x \notin A \ \& \ x \in U\}$

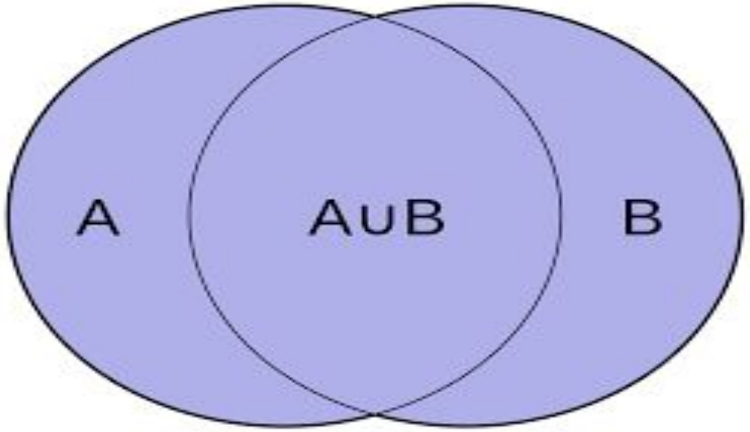


$U = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 3, 6\}$

$A^c = \{ \qquad \}$

- **<u>Union of sets</u>** – Union of two sets A and B is a set of all those elements which either belong to A or B or both, it is denoted by A U B.
- A U B = {x| x ϵ A or x ϵ B}



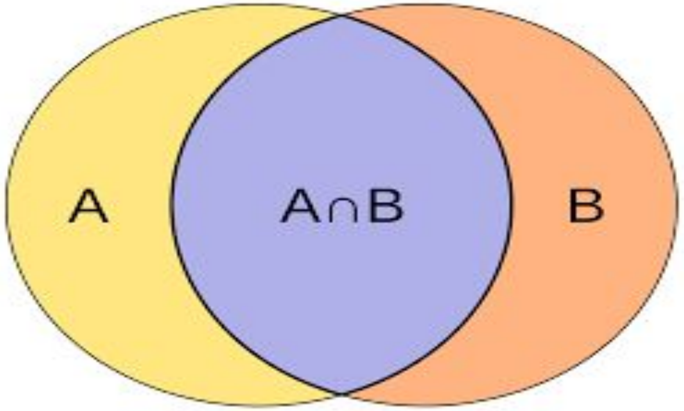
A = {1, 2, 3, 4}

B = {3, 4, 5, 6}

A U B = {   }

|A|+|B|=| A U B | ?

- **<u>Intersection of sets</u>** - Intersection of two sets A and B is a set of all those elements which belong to both A and B, and is denoted by $A \cap B$.
- $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

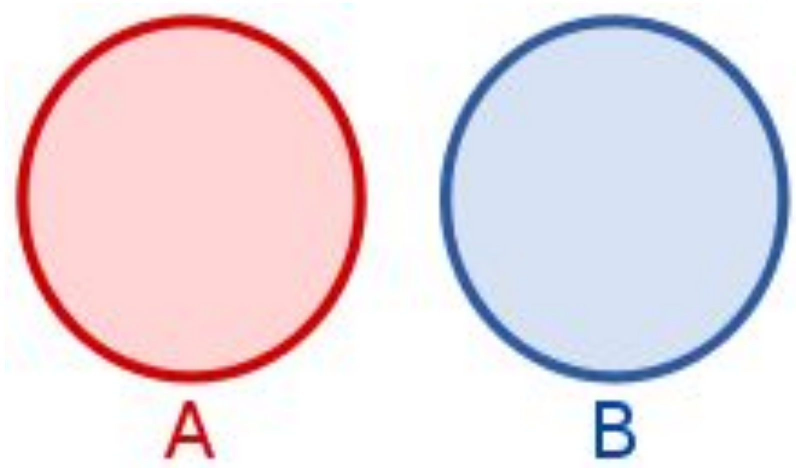


$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

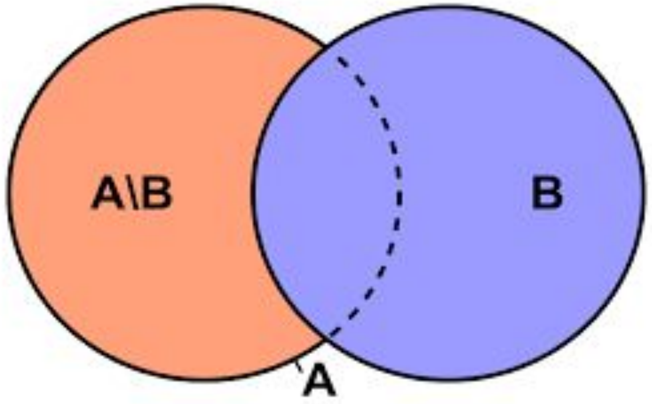
$A \cap B = \{\quad\}$

- **Disjoint sets** -- Two sets are said to be disjoint if they do not have a common element, i.e. no element in A is in B and no element in B is in A.



- $A \cap B = \phi$

- **Set difference** – the set difference of two sets A and B, is the set of all the elements which belongs to A but do not belong to B.
- $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

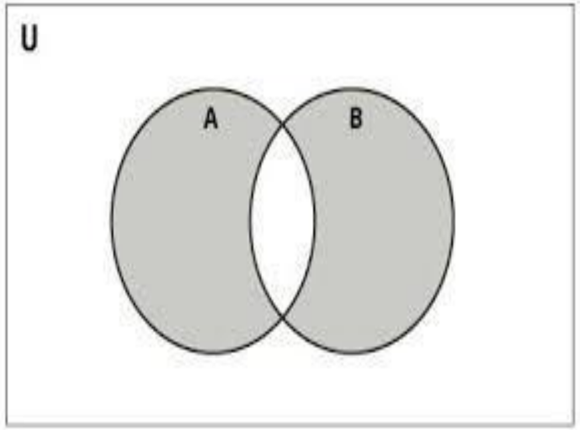


$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A - B = \{ \qquad\qquad \}$

Symmetric difference – the symmetric difference of two sets A and B is the set of all the elements that are in A or in B but not in both, denoted as. $A \oplus B = (A \cup B) - (A \cap B)$



$A \oplus B = \{x \mid (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$

$A \oplus B = (A - B) \cup (B - A)$

$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A \oplus B = \{ \qquad\qquad\qquad \}$

Q Consider the following statements?

a) Finite union of finite sets is _________(finite/infinite)

b) Finite union of Infinite sets is _________(finite/infinite)

c) Infinite union of finite sets(distinct) is _________(finite/infinite)

d) if after finite number of union result is infinite set, then at least of the input set is infinite (T / F)

e) if after finite number of union result is infinite set, then all of the input set is infinite (T / F)

f) Finite intersection of finite sets is _________(finite/infinite)

g) Finite intersection of Infinite sets is _________(finite/infinite)

h) If after finite number of intersection result is infinite set, then at least of the input set is infinite (T / F)

i) If after finite number of intersection result is infinite set, then all of the input set is infinite (T / F)

Q Let S be an infinite set $S_1, S_2, \ldots\ldots\ldots\ldots, S_n$ be Sets such that $S_1 \cup S_2 \cup \ldots \cup S_n = S$ Then, **(GATE-1993) (1 Marks)**

(a) At least one of the set S_i is a finite set

(b) Not more than one of the set S_i can be finite

(c) At least one of the sets S_i is an infinite set

(d) Not more than one of the sets S_i can be infinite

Q which of the following is not true?

a) $A - B = A \cap B^c$

b) $A - (A - B) = A \cap B$

c) $A - (A \cap B) = A - B$

d) $A - (A - B) = B$

Q If $A \subset B$, then which of the following is not true?

(a) $A \cup B = B$

(b) $A \cap B = A$

(c) $B^C \subset A^C$

(d) $B - A = \phi$

Q Which the following in not true?

a) If $A \subseteq \phi$, then $A = \phi$

b) $(A \cap B^C) \cup (A \cap B) = A$

c) $B \cup (A \cap B) = B$

d) $(A \cap B^C) \cup (A \cap B) \cup (A^C \cap B) \cup (A^C \cap B^C) = A \cap B$

Q Which of the following is true?

(i) $(A - B) - C = A - (C - B)$

(ii) $(A - B) - C = (A - C) - B$

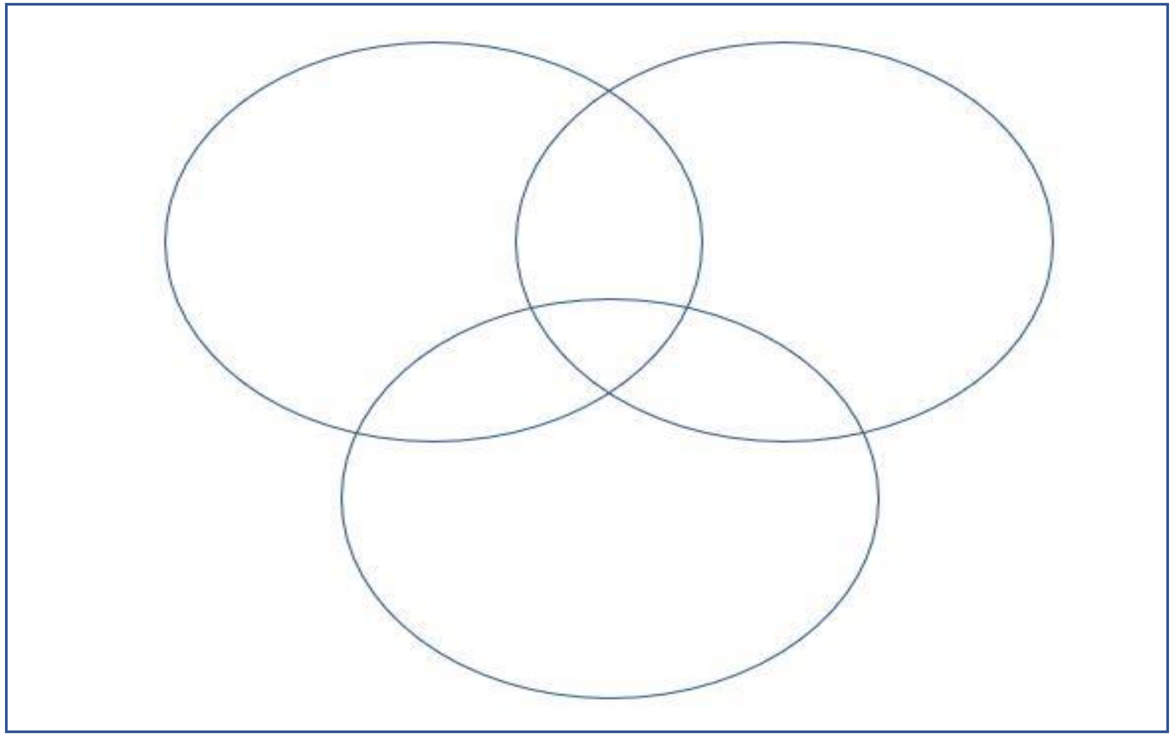
(iii) $(A - B) - C = A - (B \cap C)$

(iv) $(A \cap B) - (B \cap C) = \{A - (A \cap C)\} - (A - B)$

a) i & iii **b)** ii & iv

c) i, ii, iv **d)** ii & iii

Q In a college, there are three student clubs, Sixty students are only in the Drama club, 80 students are only in the Dance club, 30 students are only in Maths club, 40 students are in both Drama and Dance clubs, 12 students are in both Dance and Maths clubs, 7 students are in both Drama and Maths clubs, and 2 students are in all clubs. If 75% of the students in the college are not in any of these clubs, then the total number of students in the college is ______________. **(GATE-2019) (2 Mark)**



Q The number of integers between 1 and 500 (both inclusive) that are divisible by 3 or 5 or 7 is _______. **(GATE-2017) (1 Marks)**

Q Suppose U is the power set of the set S = {1,2,3,4,5,6}. For any T $\in$ U, let |T| denote the number of elements in T and T′ denote the complement of T. For any T, R $\in$ U, let T\R be the set of all elements in T which are not in R.

(GATE-2015) (2 Marks)

Which one of the following is true?

a) $\forall X \in U, (|X| = |X'|)$

B) $\exists X \in U, \exists Y \in U, (|X|=5, |Y|=5 \text{ and } X \cap Y = \phi)$

C) $\forall X \in U, \forall Y \in U, (|X|=2, |Y|=3 \text{ and } X \setminus Y = \phi)$

D) $\forall X \in U, \forall Y \in U, (X \setminus Y = Y' \setminus X')$

Q Consider the following relation on subsets of the set S of integers between 1 and 2014. For two distinct subsets U and V of S we say U < V if the minimum element in the symmetric difference of the two sets is in U. Consider the following two statements: **(GATE-2014) (2 Marks)**

S_1: There is a subset of S that is larger than every other subset.

S_2: There is a subset of S that is smaller than every other subset.

Which one of the following is CORRECT?

(A) Both S_1 and S_2 are true

(B) S_1 is true and S_2 is false

(C) S_2 is true and S_1 is false

(D) Neither S_1 nor S_2 is true

Q If P, Q, R are subsets of the universal set U, then **(GATE-2008) (1 Marks)**

$(P \cap Q \cap R) \cup (P^C \cap Q \cap R) \cup Q^C \cup R^C$

(A) $Q^c \cup R^c$

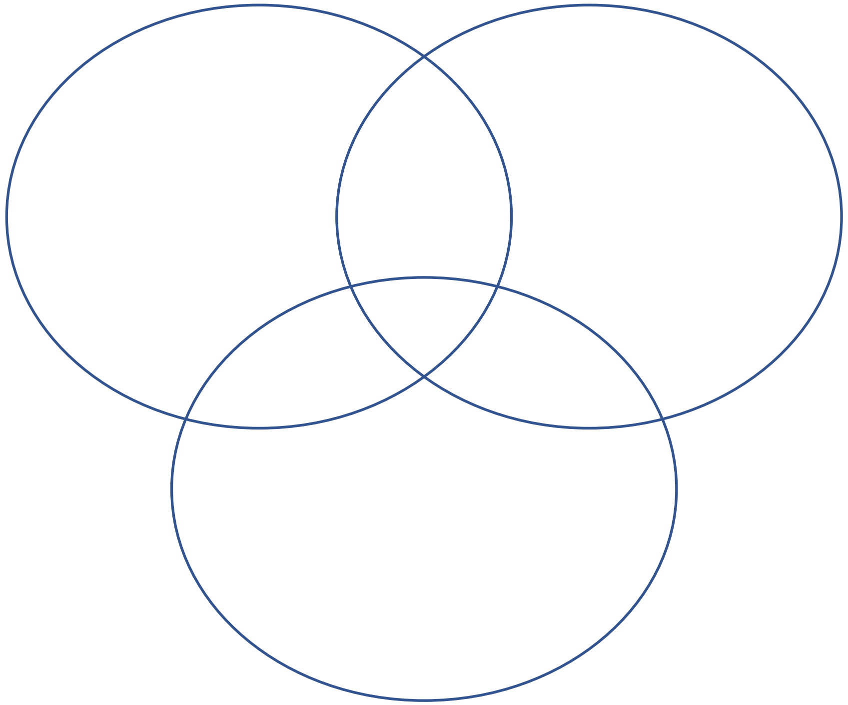
(B) $P \cup Q^c \cup R^c$

(C) $P^c \cup Q^c \cup R^c$

(D) U

Q let p, q and r be sets let @ denotes the symmetric difference operator defined as

P @ q = (p U q) – (p ∩ q)? **(GATE-2006) (2 Mark)**



I) p @ (q ∩ r) = (p @ q) ∩ (P @ r)

II) p∩ (q ∩ r) = (p ∩ q) @ (p @ r)

a) I only **b)** II only

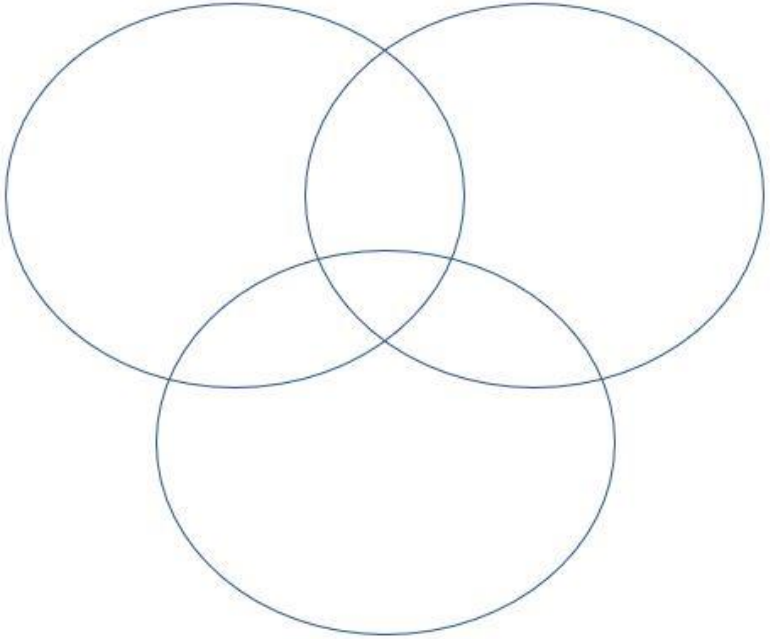
c) neither I nor II**d)** both I and II

Q Let E, F and G be finite sets.

Let $X = (E \cap F) - (F \cap G)$ and $Y = (E - (E \cap G)) - (E - F)$.

Which one of the following is true? **(GATE-2006) (2 Mark)**

(A) $X \subset Y$ **(B)** $X \supset Y$ **(C)** $X = Y$ **(D)** $X - Y \neq \varphi$ and $Y - X \neq \varphi$



Q what is the cardinality of the set of integers X defined below **(GATE-2006) (2 Mark)**

X = {n| 1<=n<=123, n is not divisible by 2,3 or 5}?

a)90

b)33

c)37

d)44

Q Let A, B and C be non-empty sets and let

$X=(A-B)-C$

$Y=(A-C)-(B-C)$

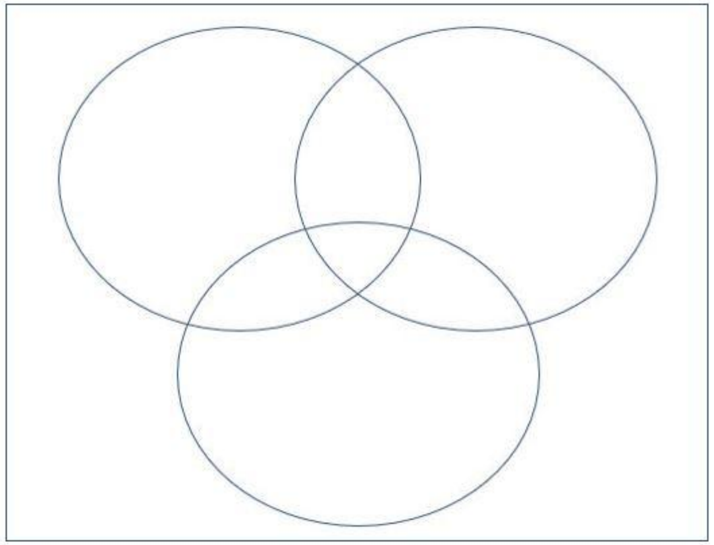
Which one of the following is TRUE? **(GATE-2005) (1 Marks)**

a) $X=Y$ **b)** $X\subset Y$ **c)** $Y\subset X$ **d)** None of these

Q In a class of 200 students, 125 students have taken Programming Language course, 85 students have taken Data Structures course, 65 students have taken Computer Organization course; 50 students have taken both Programming Language and Data Structures, 35 students have taken both Data Structures and Computer Organization; 30 students have taken both Programming Language and Computer Organization, 15 students have taken all the three courses. How many students have not taken any of the three courses? **(GATE-2004) (1 Mark)**

(A) 175 **(B)** 20 **(C)** 25 **(D)** 35

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$



Q Consider the following statements:

S_1: There exists infinite sets A, B, C such that $A \cap (B \cup C)$ is finite.

S_2: There exists two irrational numbers x and y such that $(x + y)$ is rational.

Which of the following is true about S_1 and S_2? **(GATE-2001) (2 Mark)**

(a) Only S_1 is correct

(b) Only S_2 is correct

(c) Both S_1 and S_2 are correct

(d) None of S_1 and S_2 is correct

Q Let A and B be sets and let A^c and B^c denote the complements of the sets A and B. the set $(a – b) \cup (b - a) \cup (a \cap b)$ is equal to. **(GATE- 1996) (1 Mark)**

(a) $A \cup B$ **(b)** $A^c \cup b^c$ **(c)** $A \cap B$ **(d)** $A^c \cap b^c$

Idempotent law

- $A \cup A = A$
- $A \cap A = A$

Associative law

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

Commutative law

- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

Distributive law

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De Morgan's law

- $(A \cup B)^C = A^C \cap B^C$
- $(A \cap B)^C = A^C \cup B^C$

Identity law

- $A \cup \phi = A$
- $A \cap \phi = \phi$
- $A \cup U = U$
- $A \cap U = A$

Complement law

- $A \cup A^C = U$
- $A \cap A^C = \phi$
- $U^C = \phi$
- $\phi^C = U$

Involution law

- $((A)^C)^C = A$

Cartesian Product

- Cartesian Product of two sets A and B in the set of all ordered pairs, whose first member belongs to the first set and second member belongs to the second set, denoted by $A \times B$. It is a kind of maximum relation possible, where every member of the first set belong to every member of the second set.
- $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$
- For E.g. if $A = \{a, b\}$, $B = \{1, 2, 3\}$
- $A \times B = \{ \qquad \}$

a
b

1
2
3

1. In general, commutative law does not hold good $A \times B \neq B \times A$

2. If $|A| = m$ and $|B| = n$ then $|A \times B| =$

Relation

- **Relation**: - Let A and B are sets then every possible subset of ‘A×B’ is called a relation from A to B.

- If |A| =m and |B| = n then total no of element(pair) will be m*n, every element will have two choice weather to present or not present in the subset(relation), therefore the total number of relation possible is ________

For E.g. if A = {a, b}, B = {1, 2}, A×B = {(a, 1), (a, 2), (b, 1), (b, 2)}

(a , 1)	(a , 2)	(b , 1)	(b , 2)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, 2)}
0	0	1	0	2	{(b, 1)}
0	0	1	1	3	{(b, 1), (b, 2)}
0	1	0	0	4	{(a, 2)}
0	1	0	1	5	{(a, 2), (b, 2)}
0	1	1	0	6	{(a, 2), (b, 1)}
0	1	1	1	7	{(a, 2), (b, 1), (b, 2)}
1	0	0	0	8	{(a, 1)}
1	0	0	1	9	{(a, 1), (b, 2)}
1	0	1	0	10	{(a, 1), (b, 1)}
1	0	1	1	11	{(a, 1), (b, 1), (b, 2)}
1	1	0	0	12	{(a, 1), (a, 2)}
1	1	0	1	13	{(a, 1), (a, 2), (b, 2)}
1	1	1	0	14	{(a, 1), (a, 2), (b, 1)}
1	1	1	1	15	{(a, 1), (a, 2), (b, 1), (b, 2)}

- Largest relation possible will be ________

- Smallest possible relation will be ________

- **<u>Complement of a relation</u>**: - Let R be a relation from A to B, then the complement of relation will be denoted by R’, R^C or R.
- R’ = {(a, b)|(a, b) ϵ A×B, (a, b) ϵ! R}

- R’ = (A×B) – R

- For E.g. if A×B = {(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)}

- R= {(a, 1), (a, 3), (b, 2)}

- R’ = { }

- R U R’ =

- R ∩ R’ =

- **Inverse of a relation**: - Let R be a relation from A to B, then the inverse of relation will be a relation from B to A, denoted by R^{-1}.

- $R^{-1} = \{(b, a) \mid (a, b) \in R\}$

- $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

- $R = \{(a, 1), (a, 3), (b, 2)\}$

- $R^{-1} = \{ \qquad\qquad\qquad \}$

- $|R| \qquad |R^{-1}|$

Q The number of binary relations on a set with n elements is: **(GATE-1999) (1 Marks)**

(A) n^2

(B) 2^n

(C) $2^\wedge n^2$

(D) None of the above

Q Let A be a finite set of size n. The number of elements in the power set of A × A is: **(GATE-1993) (1 Marks)**

a) $2^{(2^n)}$

b) $2^{(n^2)}$

c) 2^n

d) None of the above

- **<u>Diagonal relation</u>**: - A relation R on a set A is said to be diagonal relation if, R is a set of all ordered pair (x, x), for every $\forall x \in A$, sometimes it is also denoted by $\blacktriangle_A$

- $R = \{(x, x) \mid \forall\ x \in A\}$

	1	**2**	**3**
1	11		
2		22	
3			33

Types of a Relation

- To further study types of relations, we consider a set A with n elements, then a cartesian product $A \times A$ will have n^2 elements(pairs). Therefore, total number of relation possible is $2^{n * n}$

- **<u>Reflexive relation</u>**: - A relation R on a set A is said to be reflexive,
- If $\forall\ x \in A$
  - $(x, x) \in R$

	1	**2**	**3**
1	11		
2		22	
3			33

Q consider a set A = {1,2,3}, find which of the following relations are reflexive and Irreflexive?

	Relation	Reflexive	Irreflexive
1	$A \times A$		
2	ϕ		
3	{(1,1), (2,2), (3,3)}		
4	{(1,2), (2,3), (1,3)}		
5	{(1,1), (1,2), (2,1), (2,2)}		
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}		
7	{(1,3), (2,1), (2,3), (3,2)}		

1. Smallest reflexive relation is ________

2. Largest reflexive relation is ________

3. Total number of reflexive relations will be _________

4. If two relations R_1 and R_2 are reflexive then their union and intersection will also be reflexive (T / F).

5. Any super set of reflexive relation will also be reflexive(T / F).

6. If a relation is reflexive then its inverse R^{-1} will also be reflexive (T / F).

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	
0	0	0	1	1	
0	0	1	0	2	
0	0	1	1	3	
0	1	0	0	4	
0	1	0	1	5	
0	1	1	0	6	
0	1	1	1	7	
1	0	0	0	8	
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

Q What is the possible number of reflexive relations on a set of 5 elements? **(GATE-2010) (1 Marks)**

(A) 2^{10} **(B)** 2^{15} **(C)** 2^{20} **(D)** 2^{25}

- **<u>Irreflexive relation</u>**: - A relation R on a set A is said to be Irreflexive,

1. If $\forall x \in A$
2. $(x, x) \notin R$

Q consider a set $A = \{1,2,3\}$, find which of the following relations are reflexive and Irreflexive?

	Relation	Reflexive	Irreflexive
1	$A \times A$	Y	
2	ϕ	N	
3	$\{(1,1), (2,2), (3,3)\}$	Y	
4	$\{(1,2), (2,3), (1,3)\}$	N	
5	$\{(1,1), (1,2), (2,1), (2,2)\}$	N	
6	$\{(1,1), (2,2), (3,3), (1,3), (2,1)\}$	Y	
7	$\{(1,3), (2,1), (2,3), (3,2)\}$	N	

1. Smallest irreflexive relation is ________

2. Largest irreflexive relation is _______

3. Total number of irreflexive relation will be ________

4. If two relations R_1 and R_2 are Irreflexive then their union and intersection will also be Irreflexive (T / F).

5. If a relation R on a set A is reflexive, then R^C is Irreflexive, and vice versa (T / F).

6. Any sub set of irreflexive relation will also be irreflexive(T / F).

7. If a relation is irreflexive then its inverse R^{-1} will also be irreflexive (T / F).

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	
0	0	1	0	2	{(b, a)}
0	0	1	1	3	
0	1	0	0	4	{(a, b)}
0	1	0	1	5	
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	
1	0	0	0	8	
1	0	0	1	9	
1	0	1	0	10	
1	0	1	1	11	
1	1	0	0	12	
1	1	0	1	13	
1	1	1	0	14	
1	1	1	1	15	

- **<u>Symmetric relation</u>**: - A relation R on a set A is said to be Symmetric,

If $\forall$ a, b $\in$ A

(a, b) $\in$ R

………………………………

then (b, a) $\in$ R

………………………………

Q Consider a set A = {1,2,3}, find which of the following relations are Symmetric, Anti-Symmetric and Asymmetric?

	Relation	Symmetric	Anti-Symmetric	Asymmetric
1	$A \times A$			
2	ϕ			
3	{(1,1), (2,2), (3,3)}			
4	{(1,2), (2,3), (1,3)}			
5	{(1,1), (1,2), (2,1), (2,2)}			
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}			
7	{(1,3), (2,1), (2,3), (3,2)}			

1. Smallest symmetric relation is ________

2. Largest symmetric relation is_______

3. Total number of symmetric relation will be _________

4. If a relation on a set A is symmetric then R __ R^{-1}

5. If two relations R_1 and R_2 are symmetric then their Union and Intersection will also be symmetric. (T / F)

6. If a relation is symmetric then its superset and subset will always be symmetric. (T / F)

7. If a relation is symmetric then its complement R^c will always be symmetric. (T / F)

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	
0	0	1	1	3	
0	1	0	0	4	
0	1	0	1	5	
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	
1	0	1	1	11	
1	1	0	0	12	
1	1	0	1	13	
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

Q Let R be a relation on the set of ordered pairs of positive integers such that ((p, q), (r, s)) ∈ R if and only if p–s = q–r. Which one of the following is true about R? **(GATE-2015) (2 Marks)**

(A) Both reflexive and symmetric

(B) Reflexive but not symmetric

(C) Not reflexive but symmetric

(D) Neither reflexive nor symmetric

- **Anti-Symmetric relation**: - A relation R on a set A with cartesian product A×A is said to be Anti-Symmetric,

If $\forall$ a, b $\in$ A

(a, b) $\in$ R

(b, a) $\in$ R

……………………………

a = b

……………………………

Conclusion: Symmetry is not allowed but diagonal pairs are allowed

	Relation	Symmetric	Anti-Symmetric	Asymmetric
1	A×A	Y		
2	ϕ	Y		
3	{(1,1), (2,2), (3,3)}	Y		
4	{(1,2), (2,3), (1,3)}	N		
5	{(1,1), (1,2), (2,1), (2,2)}	Y		
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}	N		
7	{(1,3), (2,1), (2,3), (3,2)}	N		

1. Smallest Anti-symmetric relation is _________

2. Largest Anti-symmetric relation will contain ________elements

3. Total number of Anti-symmetric relation will be _________

4. A relation R on a set A is Anti-Symmetric if $(R \cap R^{-1}) \sqsubseteq \blacktriangle_A$ (T / F)

5. Sub set of a Anti-Symmetric will also be (T / F)

6. Super set of a Anti-Symmetric will also be (T / F)

7. If two relations R_1 and R_2 are Anti-symmetric then their _________need not to be Anti-symmetric but _________ will also be Anti-symmetric.

8. If a relation is Anti-symmetric then its complement R^c will always be Anti-symmetric. (T / F)

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	
0	1	1	1	7	
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	
1	1	1	1	15	

Q Consider a set A = {a, b, c} and R_1, R_2, R_3 and R_4 are relations on A which of the following is not true?

		Symmetric	Anti-Symmetric	True
1	$R_1 = \{(a, a), (c, c)\}$	Y	Y	
2	$R_2 = \{(a, b), (b, a), (a, c)\}$	N	N	
3	$R_3 = \{(a, b), (b, c), (a, c)\}$	N	Y	
4	$R_4 = \{(a, b), (b, a), (c, c)\}$	Y	N	

Q Consider the binary relation R = {(x, y), (x, z), (z, x), (z, y)} on the set {x, y, z}. Which one of the following is TRUE? **(GATE-2009) (1 Marks)**

(A) R is symmetric but NOT antisymmetric

(B) R is NOT symmetric but antisymmetric

(C) R is both symmetric and antisymmetric

(D) R is neither symmetric nor antisymmetric

- **Asymmetric relation**: - A relation R on a set A is said to be Asymmetric,

If $\forall$ a, b $\in$ A

(a, b) $\in$ R

…………………………….

(b, a) $\notin$ R

…………………………….

Conclusion: Symmetry is not allowed; even diagonal pairs are not allowed

	Relation	Symmetric	Anti-Symmetric	Asymmetric
1	A×A	Y	N	
2	ϕ	Y	Y	
3	{(1,1), (2,2), (3,3)}	Y	Y	
4	{(1,2), (2,3), (1,3)}	N	Y	
5	{(1,1), (1,2), (2,1), (2,2)}	Y	N	
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}	N	Y	
7	{(1,3), (2,1), (2,3), (3,2)}	N	N	

1. Smallest Asymmetric relation is ______

2. Largest Asymmetric relation will contain _________elements

3. Total number of Asymmetric relation will be ________

4. Every asymmetric relation is also anti-symmetric (T / F)

5. Sub set of a Asymmetric will also be Asymmetric (T / F)

6. Super set of a Asymmetric will also be Asymmetric(T / F)

7. If two relations R_1 and R_2 are Asymmetric then their Union will also be Asymmetric(T / F).

8. If two relations R_1 and R_2 are Asymmetric then their Intersection will also be Asymmetric(T / F).

9. If a relation is Asymmetric then its complement R^c will always be Asymmetric. (T / F)

For E.g. if $A = \{a, b\}$, $A \times A = \{(a, a), (a, b), (b, a), (b, b)\}$

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	
0	0	1	0	2	{(b, a)}
0	0	1	1	3	
0	1	0	0	4	{(a, b)}
0	1	0	1	5	
0	1	1	0	6	
0	1	1	1	7	
1	0	0	0	8	
1	0	0	1	9	
1	0	1	0	10	
1	0	1	1	11	
1	1	0	0	12	
1	1	0	1	13	
1	1	1	0	14	
1	1	1	1	15	

- **Transitive relation**: - A relation R on a set A is said to be Transitive,

If $\forall$ a, b, c $\in$ A

(a, b) $\in$ R

(b, c) $\in$ R

……………………………

(a, c) $\in$ R

……………………………

	Relation	Transitive
1	A×A	
2	ϕ	
3	{(1,1), (2,2), (3,3)}	
4	{(1,2), (2,3), (1,3)}	
5	{(1,1), (1,2), (2,1), (2,2)}	
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}	
7	{(1,3), (2,1), (2,3), (3,2)}	
8	{(1,2)}	
9	{(1,3), (2,3)}	
10	{(1,2), (1,3)}	
11	{(2,3), (1,2)}	

1. Smallest Transitive relation is ______

2. Largest Transitive relation will contain ___________ elements

3. If two relations R_1 and R_2 are Transitive then their ________ need not to be transitive but ____________ will also be transitive.

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	
0	1	1	1	7	
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

|A| = n	No of transitive relation
0	1
1	2
2	13
3	171
4	3994

Warshall's Algorithm: -

Q Consider a set A = {1,2,3} and a relation **R** = {(1,1), (1,3), (2,2), (3,1), (3,2)}?

	1	2	3
Column			
Row			

	1	2	3
1			
2			
3			

Q Consider a set A = {1,2,3,4} and a relation

R = {(1, 1), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)}?

	1	2	3	4
Column				
Row				

	1	2	3	4
1				
2				
3				
4				

Q A binary relation R on N x N is defined as follows:

(a, b) R (c, d) if a <= c or b <= d

Consider the following propositions:

P: R is reflexive

Q: R is transitive

Which one of the following statements is TRUE? **(GATE- 2016) (2 Marks)**

(A) Both P and Q are true.

(B) P is true and Q is false.

(C) P is false and Q is true.

(D) Both P and Q are false.

Q Let R be the relation on the set of positive integers such that a R b if and only if a and b are distinct and have a common divisor other than 1. Which one of the following statements about R is True? **(GATE-2015) (1 Marks)**

(A) R is symmetric and reflexive but not transitive

(B) R is reflexive but not symmetric and not transitive

(C) R is transitive but not reflexive and not symmetric

(D) R is symmetric but not reflexive and not transitive

Q The binary relation S = φ (empty set) on set A = {1,2,3} is **(GATE-2002) (2 Marks)**

(a) Neither reflexive nor symmetric

(b) Symmetric and reflexive

(c) Transitive and reflexive

(d) Transitive and symmetric

Q The binary relation R= {(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4)} on the set A = {1,2,3,4} is **(GATE-1998) (2 Marks)**

(a) reflexive, symmetric and transitive

(b) neither reflexive, nor irreflexive but transitive

(c) irreflexive, symmetric and transitive

(d) irreflexive and antisymmetric

Q The transitive closure of the relation {(1,2), (2,3), (3,4), (5,4)} on the set {1,2,3,4,5} is ______________. **(GATE-1989) (2 Marks)**

- **<u>Equivalence Relation</u>**: - A relation R on a set A with cartesian product A×A is said to be Equivalence, if it is

1. **Reflexive**
2. **Symmetric**
3. **Transitive**

- If two relations R_1 and R_2 are Equivalence then their union need not to be equivalence but intersection will also be Equivalence.

R_1 : (a, b) iff (a + b) is even over the set of integers

R_2 : (a, b) iff (a + b) is odd over the set of integers

R_3 : (a, b) iff a x b > 0 over the set of non-zero rational numbers

R_4 : (a, b) iff $|a - b| \leq 2$ over the set of natural numbers

Q Let S be a set of n elements. The number of ordered pairs in the largest and the smallest equivalence relations on S are **(GATE-2007) (1 Marks)**

(A) n and n

(B) n^2 and n

(C) n^2 and 0

(D) n and 1

Q Let R and S be any two equivalence relations on a non-empty set A. Which one of the following statements is TRUE? **(GATE-2005) (2 Marks)**

(A) $R \cup S$, $R \cap S$ are both equivalence relations

(B) $R \cup S$ is an equivalence relation

(C) $R \cap S$ is an equivalence relation

(D) Neither $R \cup S$ nor $R \cap S$ is an equivalence relation

Q Suppose A is a finite set with n elements. The number of elements in the largest equivalence relation of A is?

(GATE-1998) (1 Marks)

(a) n **(b)** n^2 **(c)** 1 **(d)** n + 1

Q Let R_1 and R_2 be two equivalence relations on a set. Consider the following assertions **(GATE-1998) (1 Marks)**

(i) $R_1 \cup R_2$ is an equivalence relation

(ii) $R_1 \cap R_2$ is an equivalence relation

Which of the following is correct?

a) Both assertions are true

b) Assertions (i) is true but assertions (ii) is not true

c) Assertions (ii) is true but assertions (i) is not true

d) Neither (i) nor (ii) is true

Q Let R be a non-empty relation on a collection of sets defined by A R B if and only if $A \cap B = \varphi$. Then, (pick the true statement) **(GATE-1996) (2 Marks)**

(a) R is reflexive and transitive

(b) R is symmetric and not transitive

(c) R is an equivalence relation

(d) R is not reflexive and not symmetric

- **Equivalence Class:** - of an element is denoted by [x].
  $[x] = \{y \mid y \in A \text{ and } (x, y) \in R\}$ for all $x \in A$

- We can have $[x] = [y]$, even if $x \neq y$

Q Consider A = {1, 2, 3, 4, 5} an equivalence relation R on A, R = {(1,1),(2,2),(3,3),(4,4),(5,5),(1,4),(4,1),(2,5),(5,2)} find the partition of a set A, defined by R.

[1] =

[2] =

[3] =

[4] =

[5] =

Partitions of a Set: - let A be a set, with n elements. Based on our understanding of equivalent classes, a subdivision of A into non-empty and non-overlapping subset is called a partition of A.

$$A_1 \cup A_2 \cup A_3 \cup \ldots\ldots\ldots\ldots\ldots \cup A_n = A$$

$$A_1 \cap A_2 \cap A_3 \cap \ldots\ldots\ldots\ldots\ldots \cap A_n = \phi$$

Q Consider A = {1, 2, 3, 4, 5} an equivalence relation R on A, R = {(1,1),(2,2),(3,3),(4,4),(5,5),(1,4),(4,1),(2,5),(5,2)} find the partition of a set A, defined by R.

[1] = {1, 4}

[2] = {2, 5}

[3] = {3}

[4] = {1,4}

[5] = {2, 5}

so we have partitions =

Q Let A = {1,2,3,4,5} is a set having partitions as {1, 4}, {2, 3, 5} , find the equivalence relation from which these partitions are created?

Q A relation R is defined on the set of integers as x Ry iff (x + y) is even. Which of the following statements is true? **(GATE-2000) (2 Marks)**

(a) R is not an equivalence relation

(b) R is an equivalence relation having 1 equivalence class

(c) R is an equivalence relation having 2 equivalence classes

(d) R is an equivalence relation having 3 equivalence classes

Q The number of equivalence relations of the set {1,2,3,4} is
(GATE-1997) (1 Marks)

a) 15 **b)** 16 **c)** 24 **d)** 4

Q A relation R is said to be circular if aRb and bRc together imply cRa. Which of the following options is/are correct? **(GATE 2021) (2 MARKS)**

(A) If a relation S is reflexive and symmetric, then S is an equivalence relation.

(B) If a relation S is circular and symmetric, then S is an equivalence relation.

(C) If a relation S is reflexive and circular, then S is an equivalence relation.

(D) If a relation S is transitive and circular, then S is an equivalence relation.

- **Partial Order Relation**: - A relation R on a set A with cartesian product A×A is said to be partial order, if it is

1. **Reflexive**
2. **Anti - Symmetric**
3. **Transitive**

- **<u>Partial ordering set (Poset):</u>** - a set A with partial ordering relation R defined on A is called a POSET and is denoted by [A, R]

- For e.g. [A, /], [A, <=], [P(S), $\subseteq$]

- **Total order relation:** - A Poset [A, R] is called a total order set, if every pair of elements are comparable i.e. either (a, b) or (b, a) $\in$ R, for $\forall$ a, b $\in$ A

- For e.g. A = {1, 2, 3, 6}, then Poset [A,/] is not a total order relation but A = {1,2,4,8} will be

Q A partial order P is defined on the set of natural numbers as follows. Here x/y denotes integer division. **(GATE-2007) (2 Marks)**

(1) (0,0) ∈ P **(2)** (a, b) ∈ P if and only if a % 10 <= b % 10 and (a/10, b/10) ∈ P.

Consider the following ordered pairs:

(i) (101,22) **(ii)** (22,101) **(iii)** (145,265) **(iv)** (0,153)

a) I & iii **b)** ii & iv **c)** I & iv **d)** iii & iv

Q A relation R is defined on ordered pairs of integers as follows: (x, y) R (u, v) if $x < u$ and $y > v$. Then R is **(GATR-2006) (1 Marks)**

(A) Neither a Partial Order nor an Equivalence Relation

(B) A Partial Order but not a Total Order

(C) A Total Order

(D) An Equivalence Relation

Q let R_1 be a relation from A = {1,3,5,7} to B = {2,4,6,8} and R_2 be another relation from B to C = {1,2,3,4} as defined below **(GATE-2004) (1 Marks)**

(i) an element x in A is related to an element y in B if x + y is divisible by 3

(ii) an element x in B is related to an element y in C if x + y is even but not divisible by 3.

Which is the composite relation R_1R_2 from A to C?

a) {(1,2), (1,4), (3,3), (5,4), (7,3)}

b) {(1,2), (1,3), (3,2), (5,2), (7,3)}

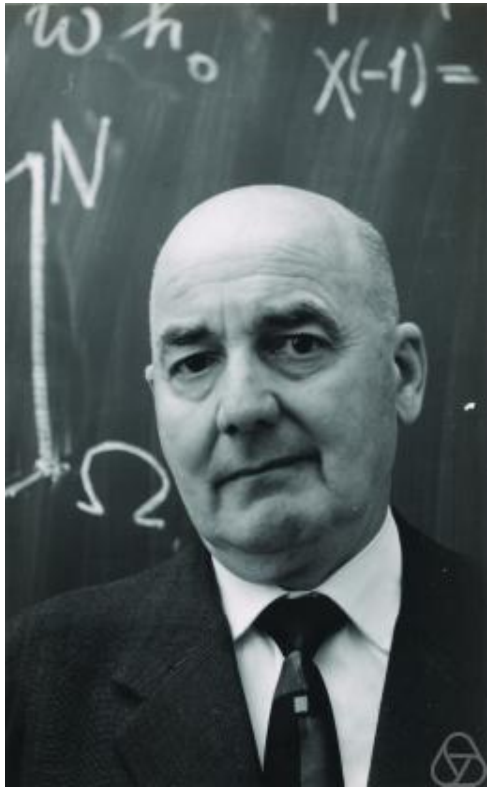
c) {(1,2), (3,2), (3,4), (5,4), (7,2)}

d) {(3,2), (3,4), (5,1), (5,3), (7,1)}

Q. Let P be the partial order defined on the set {1,2,3,4} as follows $P = \{(x, x) \mid x \in \{1,2,3,4\}\} \cup \{(1,2), (3,2), (3,4)\}$ The number of total orders on {1,2,3,4} that contain P is ___________ **(Gate 2024 CS)(1 Marks)(NAT)**

Conversion of POSET into a Hasse Diagram

- If we want to study Partial order relation further then it will be better to convert it into more convenient notation so that it can be studied easily. This graphical representation is called Hasse Diagram.
- The diagrams are named after Helmut Hasse (1898–1979).



Steps to convert partial order relation into hasse diagram

1- Draw a vertex for each element in the Set

2- If (a, b) ϵ R then draw an edge from a to b

3- Remove all Reflexive and Transitive edges

4- Remove the direction of edges and arrange them in the increasing order of heights.

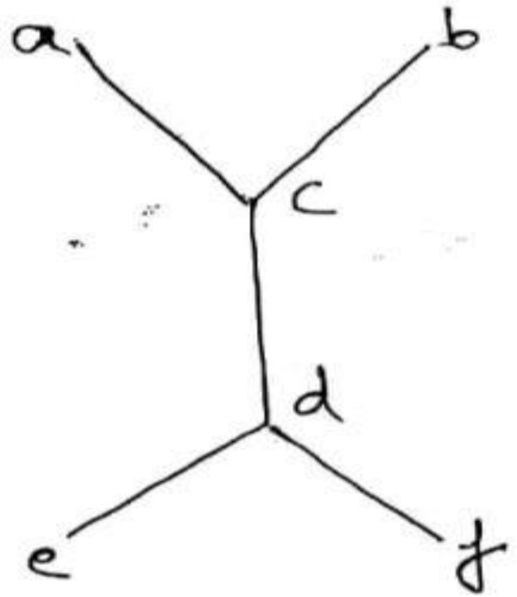
Q Consider a Partial order relation and convert it into hasse diagram?

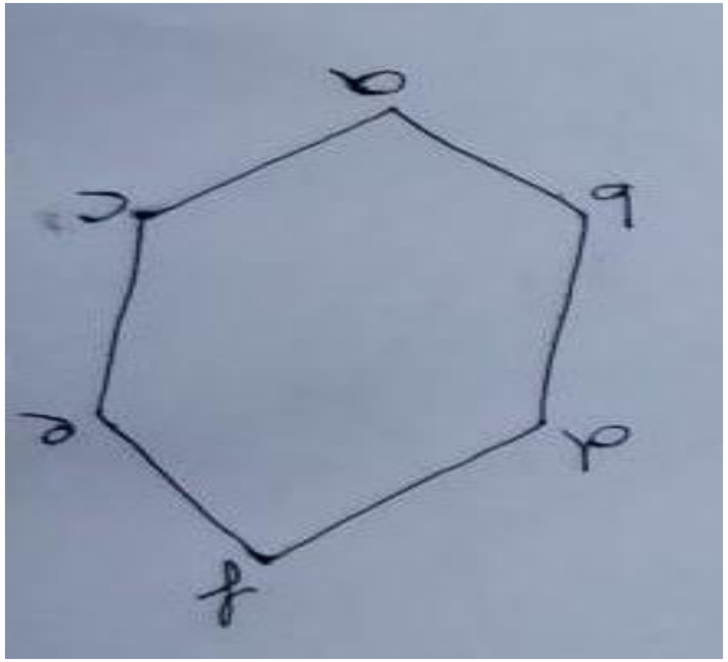
R = {(1,1), (1,2), (1,4), (1,8), (2,2), (2,4), (2,8), (4,4), (4,8), (8,8)}

Q Consider a Partial order relation and convert it into hasse diagram?

R = {(1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6), (6,6)}

Q Study the following hasse diagram and find which of the following are valid?





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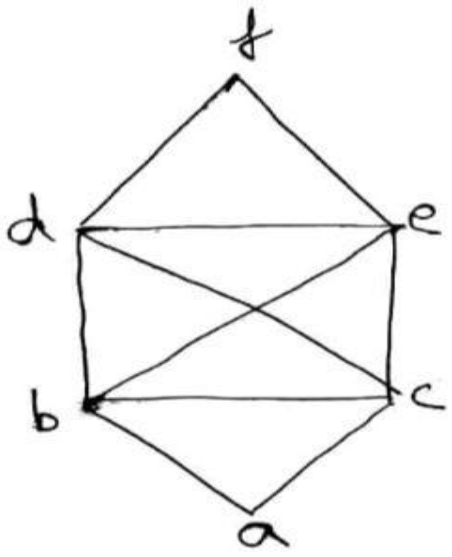
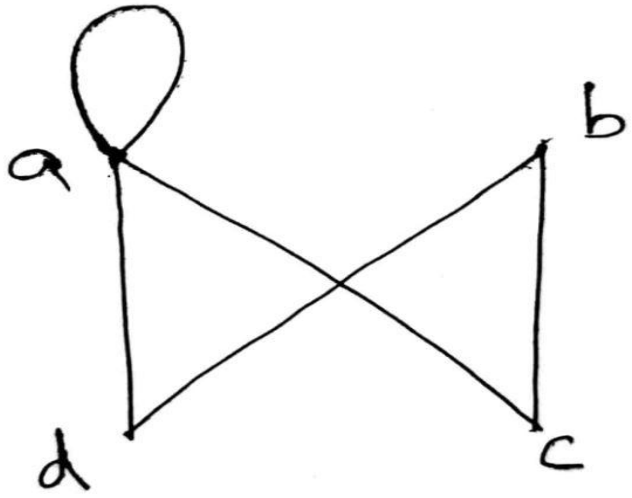
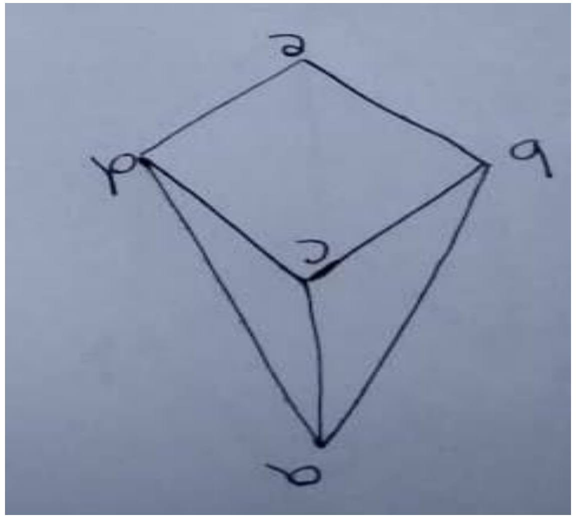
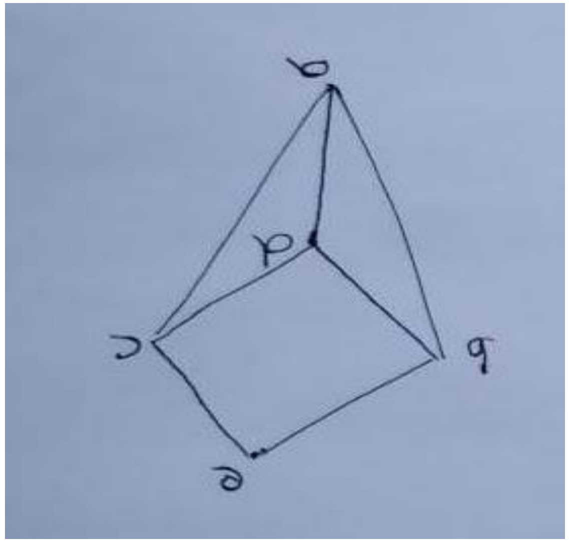
(2)







(3) **(4)** **(5)**



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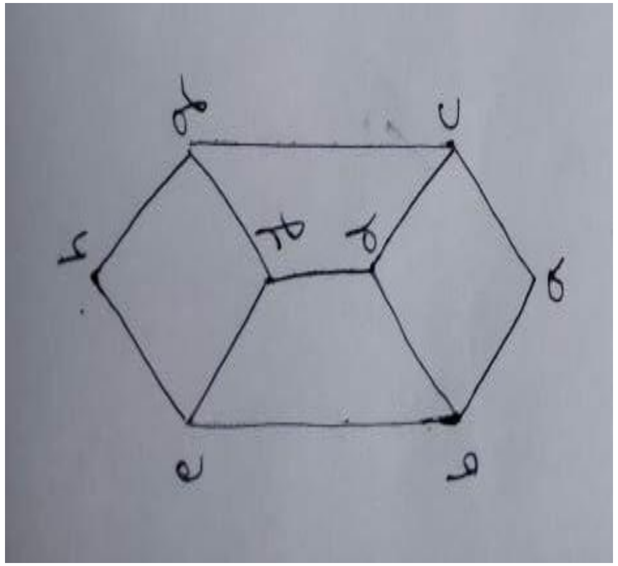
(6)



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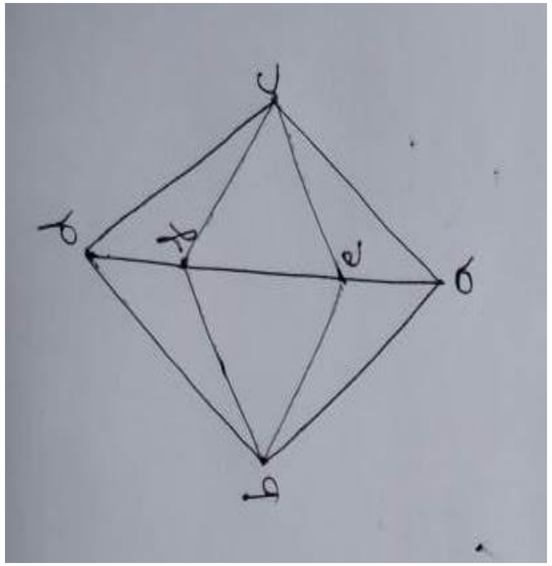
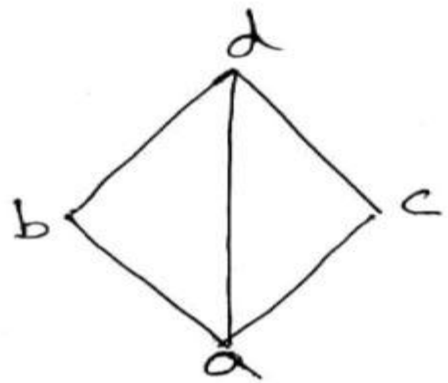
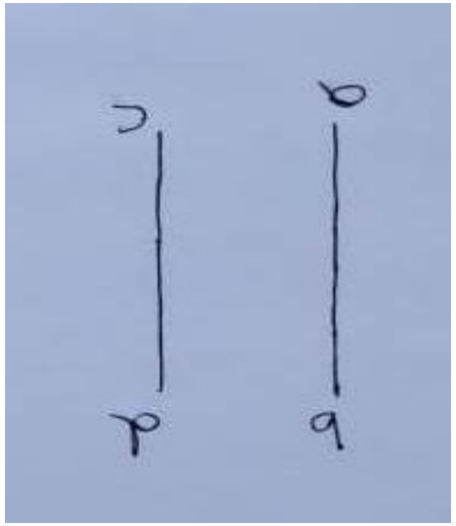






(8) **(9)** **(10)**

Conclusion

- We can not have a horizontal edge in a hasse diagram
- We can not have a reflexive and transitive edge in Hasse Diagram

Q Let X = {2,3,6,12,24}, Let ≤ be the partial order defined by X ≤ Y if x divides y. Number of edges as in the Hasse diagram of (X, ≤) is.

(GATE-1996) (1 Marks)

(a) 3 **(b)** 4 **(c)** 9 **(d)** None of the above

Least Upper Bound / LUB / Join / Supremum / $\vee$

Least value in the upper bound

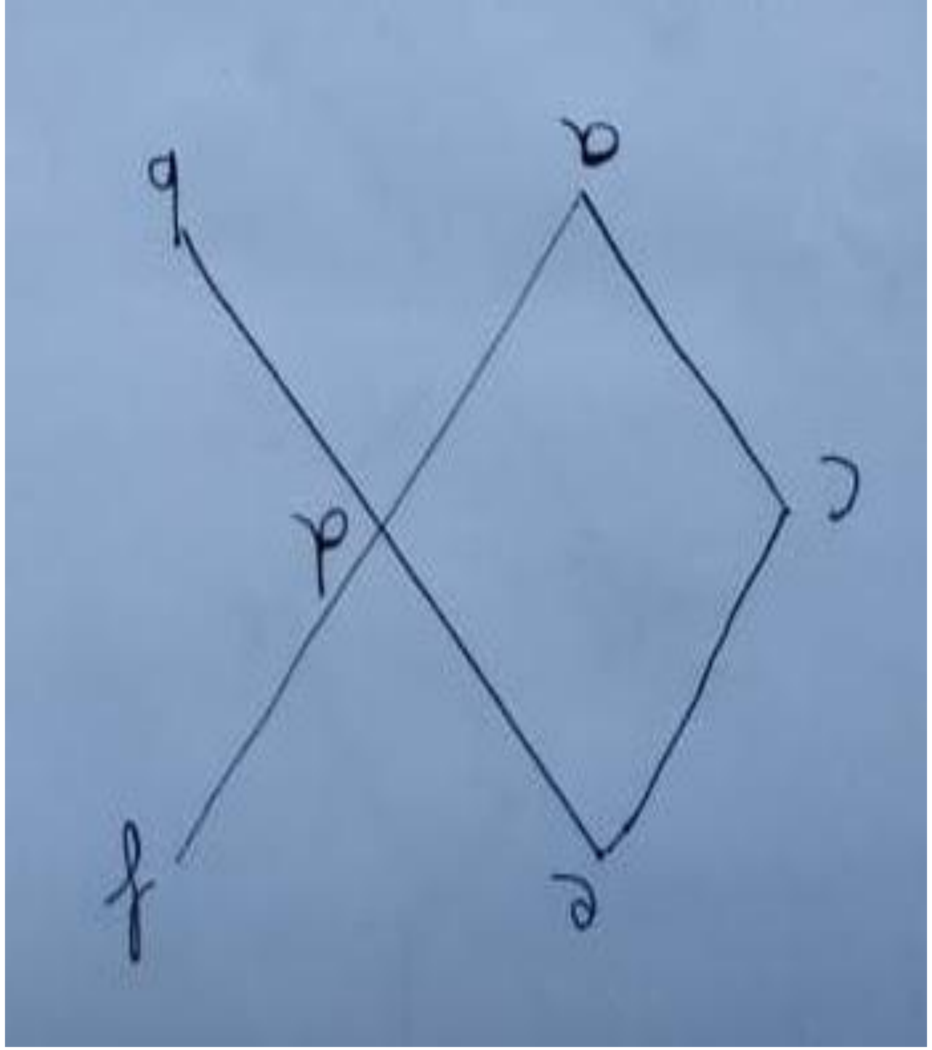
Greatest Lower Bound / GLB / Meet / Infimum / $\wedge$

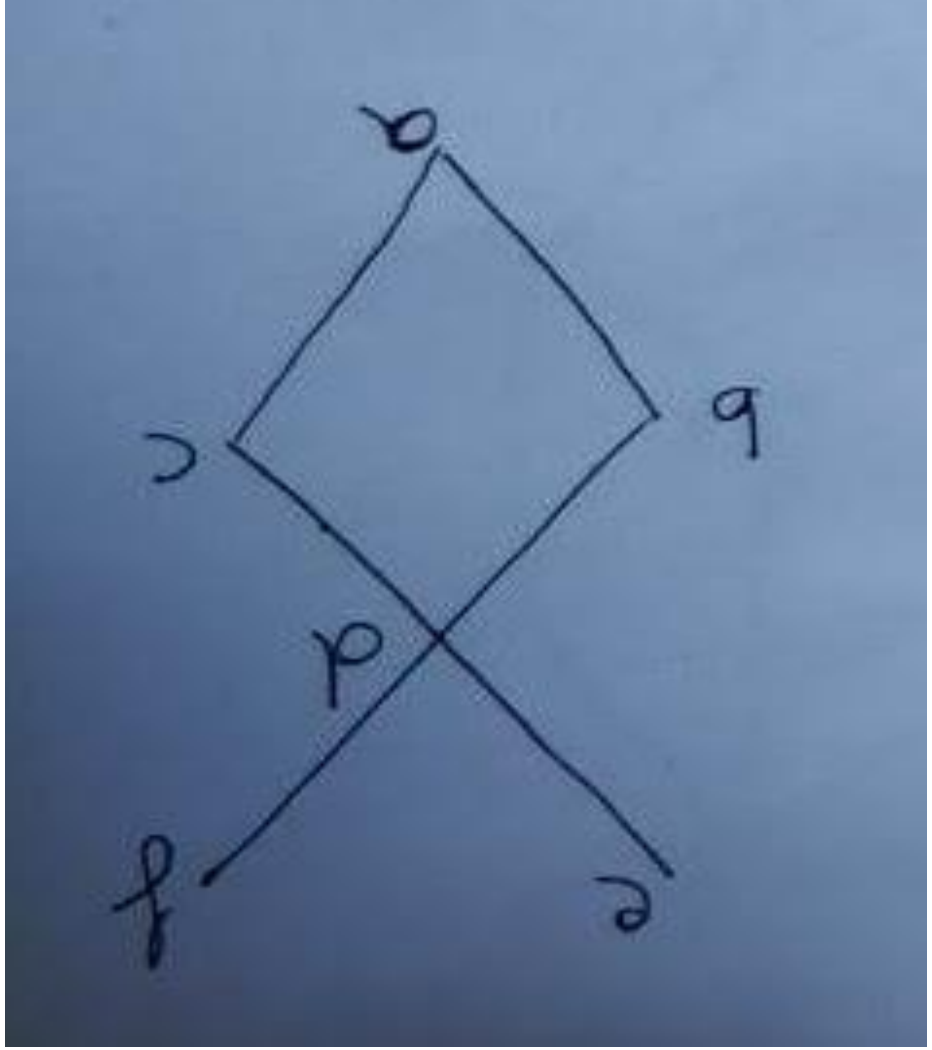
Greatest value in the lower bound

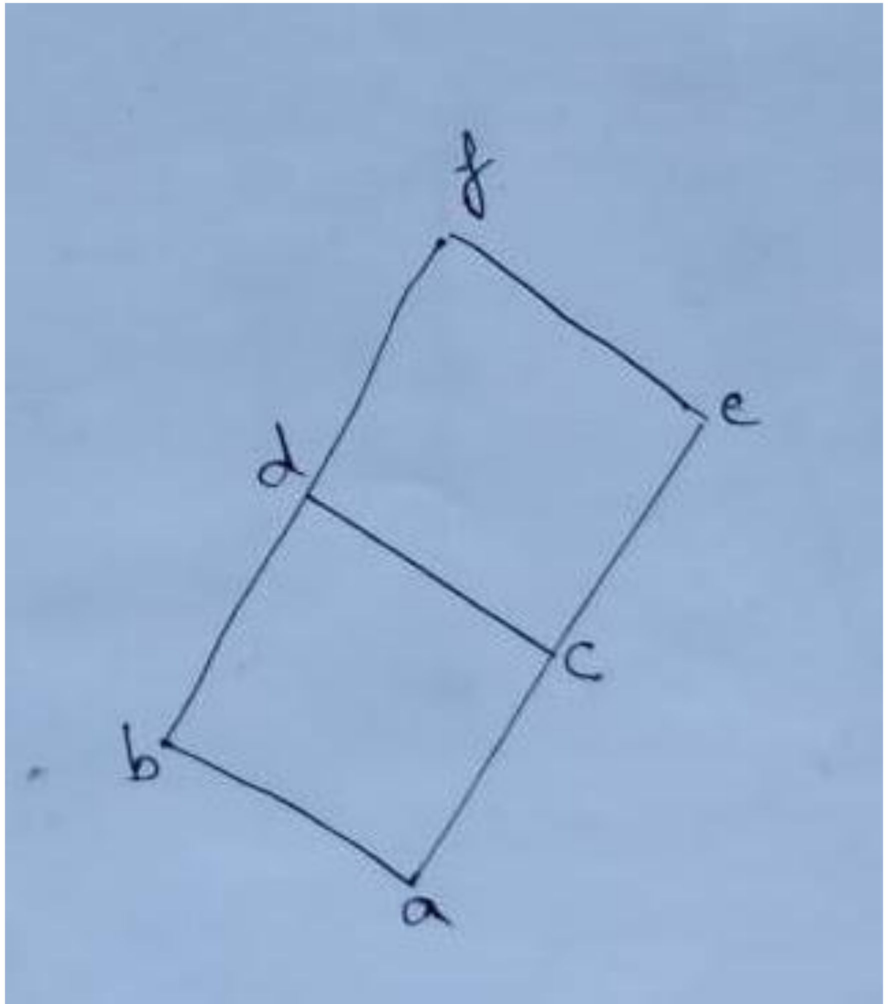
Join Semi Lattice :- A hasse diagram/Partial order relation is called Join Semi Lattice if for every elements their exists a Join.

Meet Semi Lattice :- A hasse diagram/Partial order relation is called Meet Semi Lattice if for every elements their exists a Meet.

Lattice :- A hasse diagram/Partial order relation is called Lattice if their exist a Join and Meet for every pair of element. Or A hasse diagram/Partial order relation is called Lattice if it is both Join Semi Lattice and Meet Semi Lattice.



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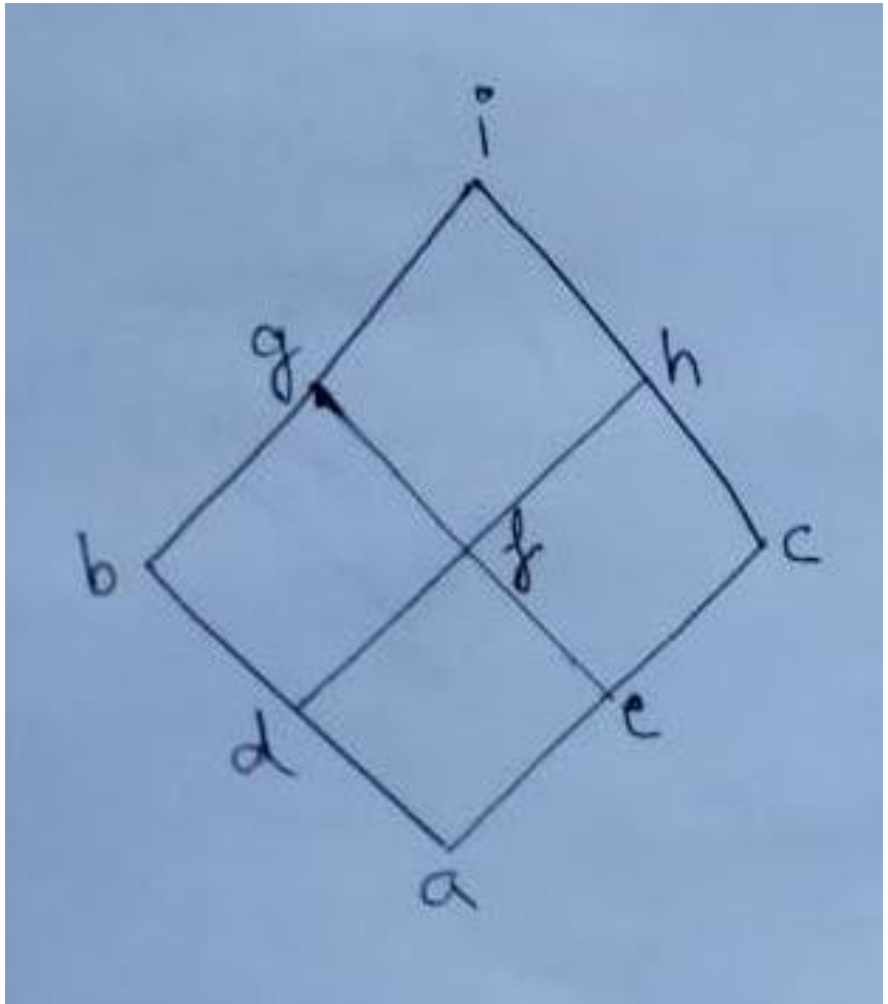
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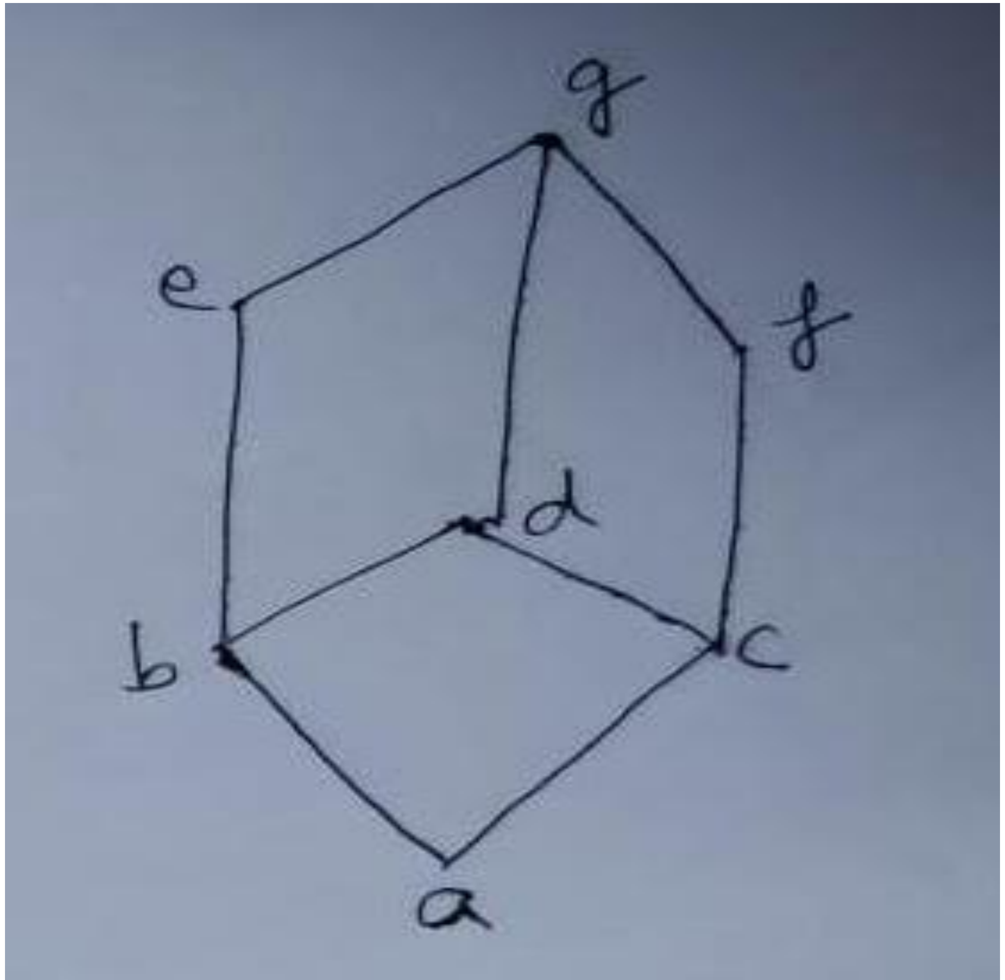
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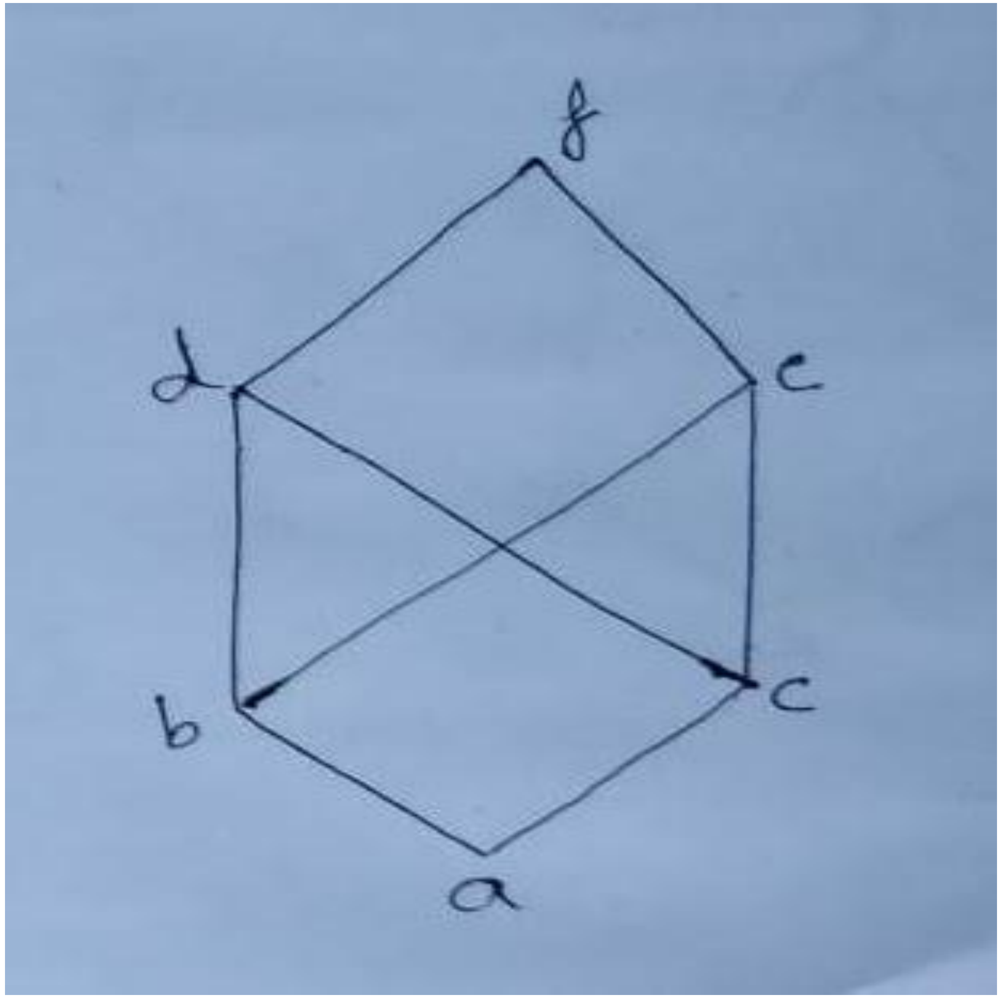
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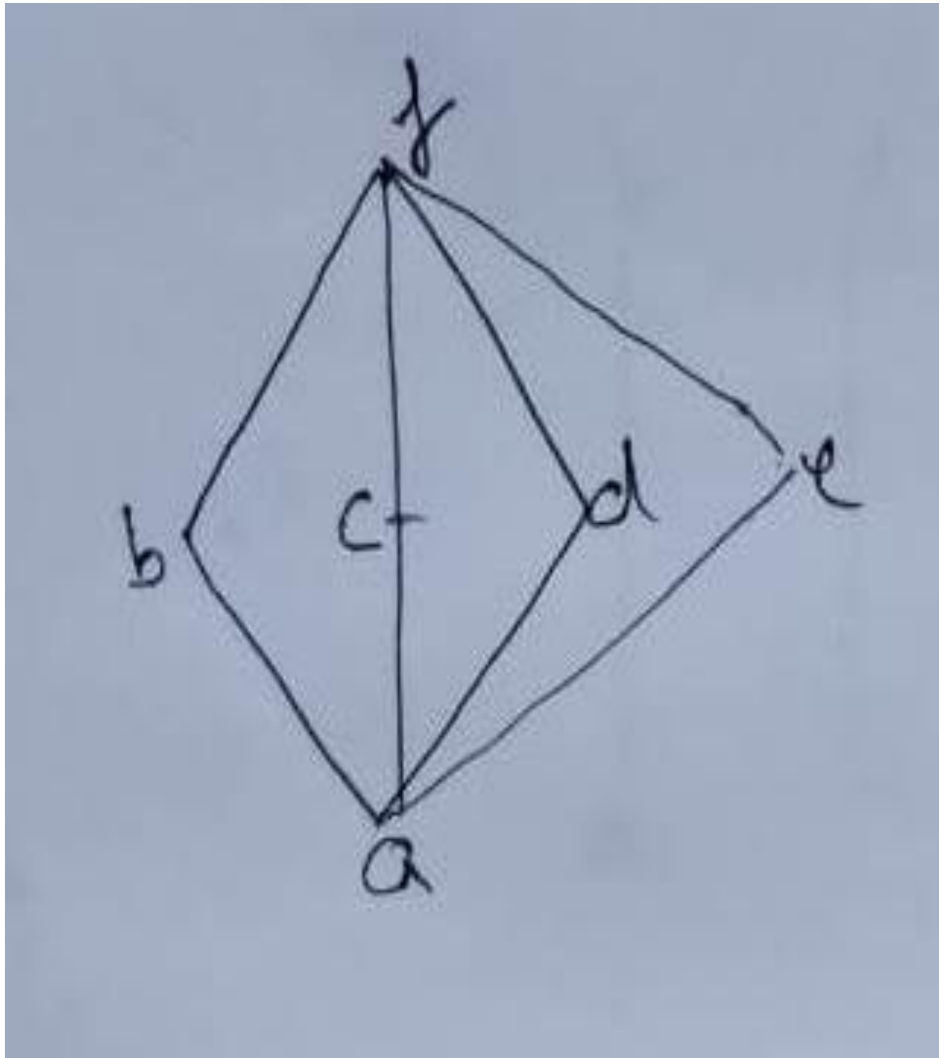
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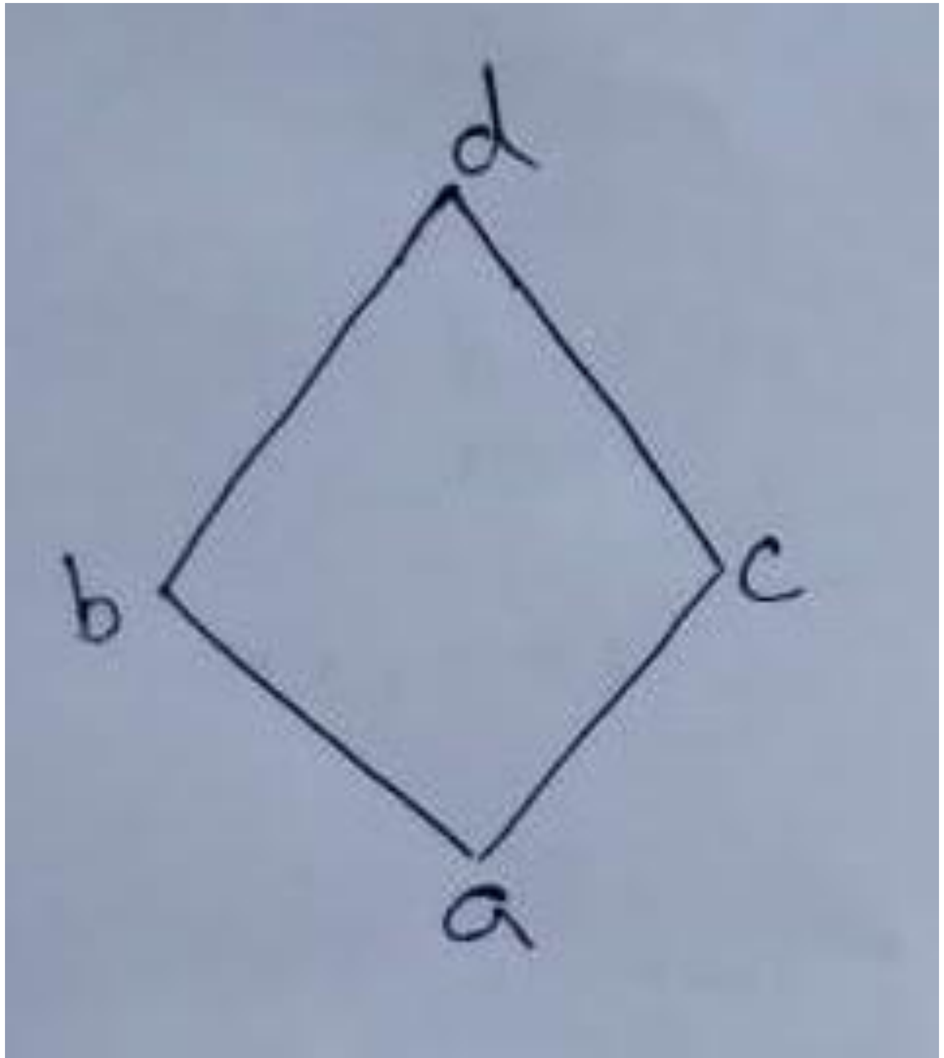
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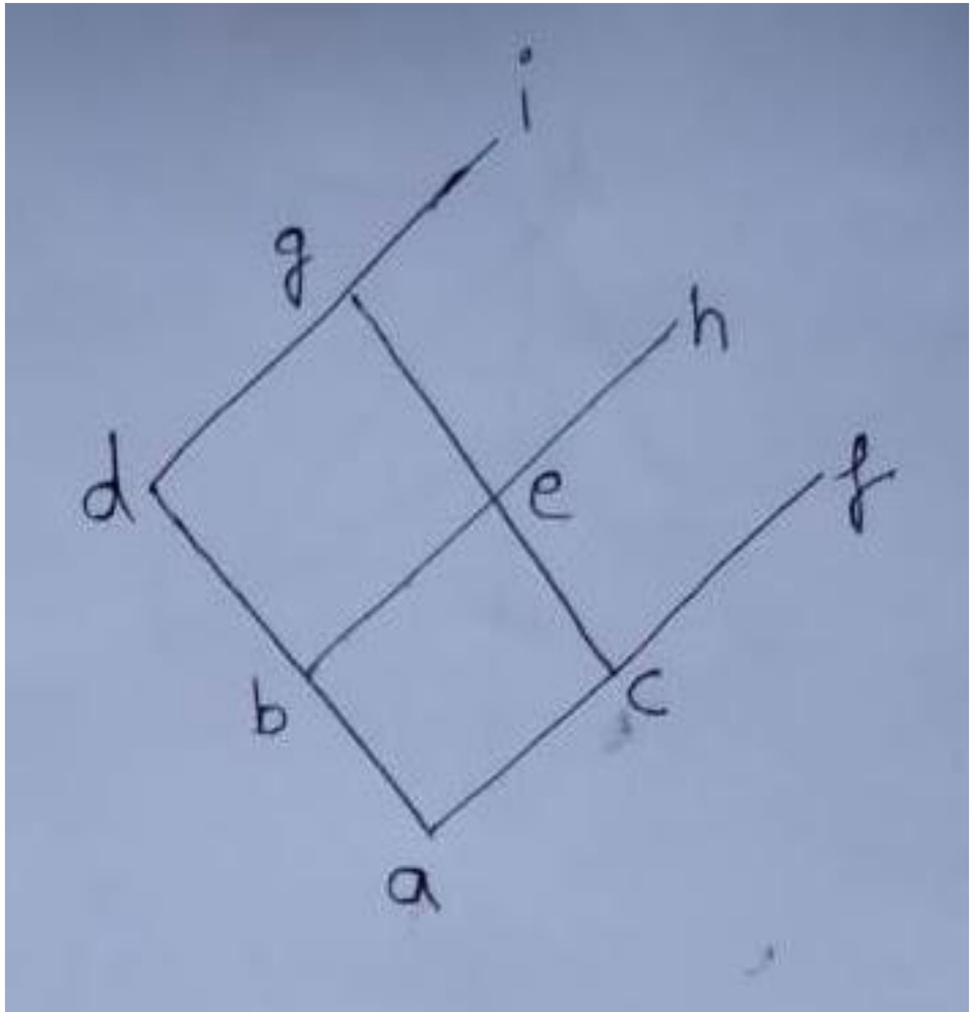
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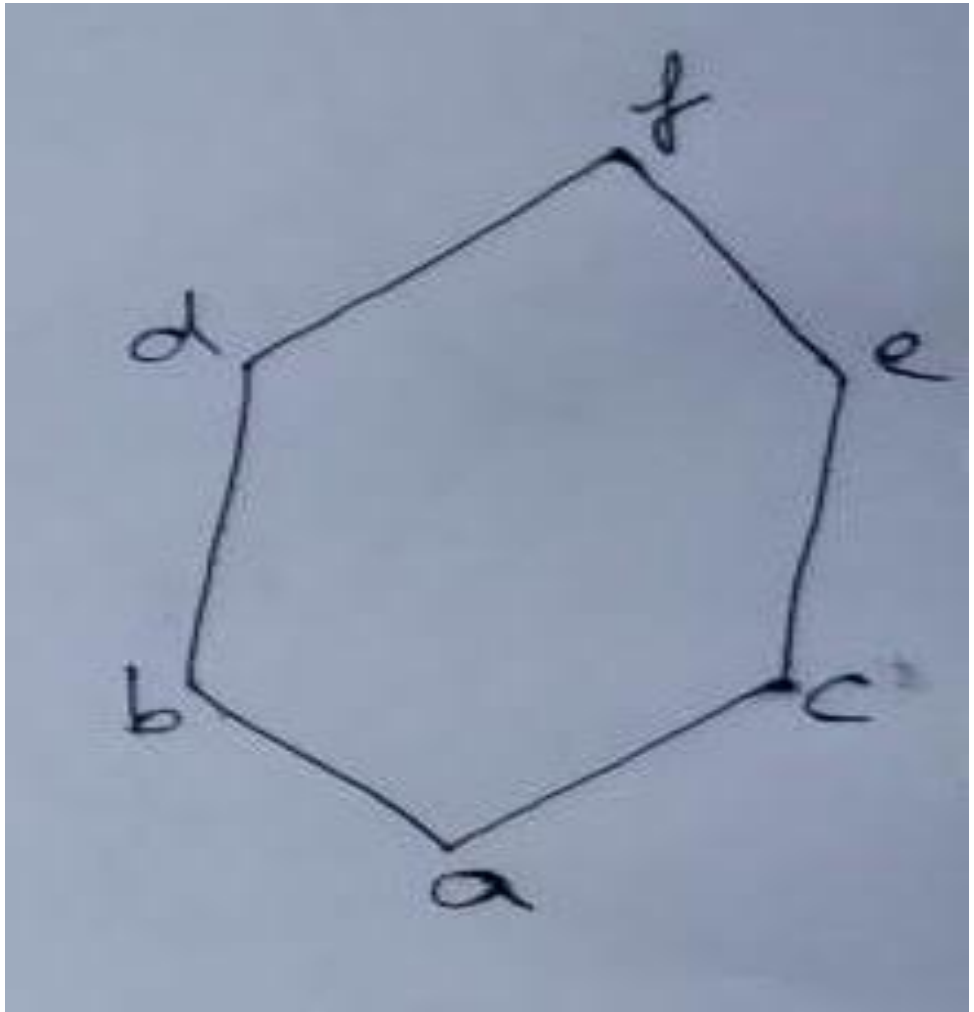
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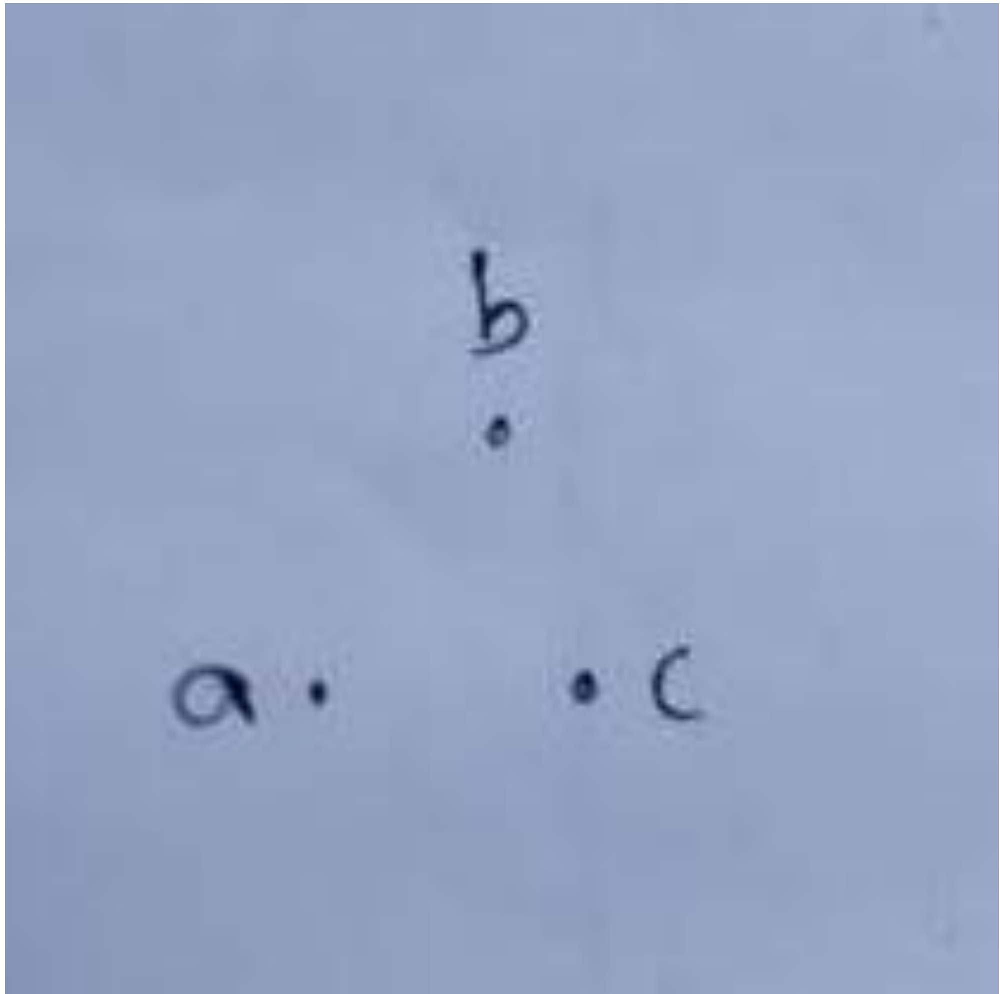
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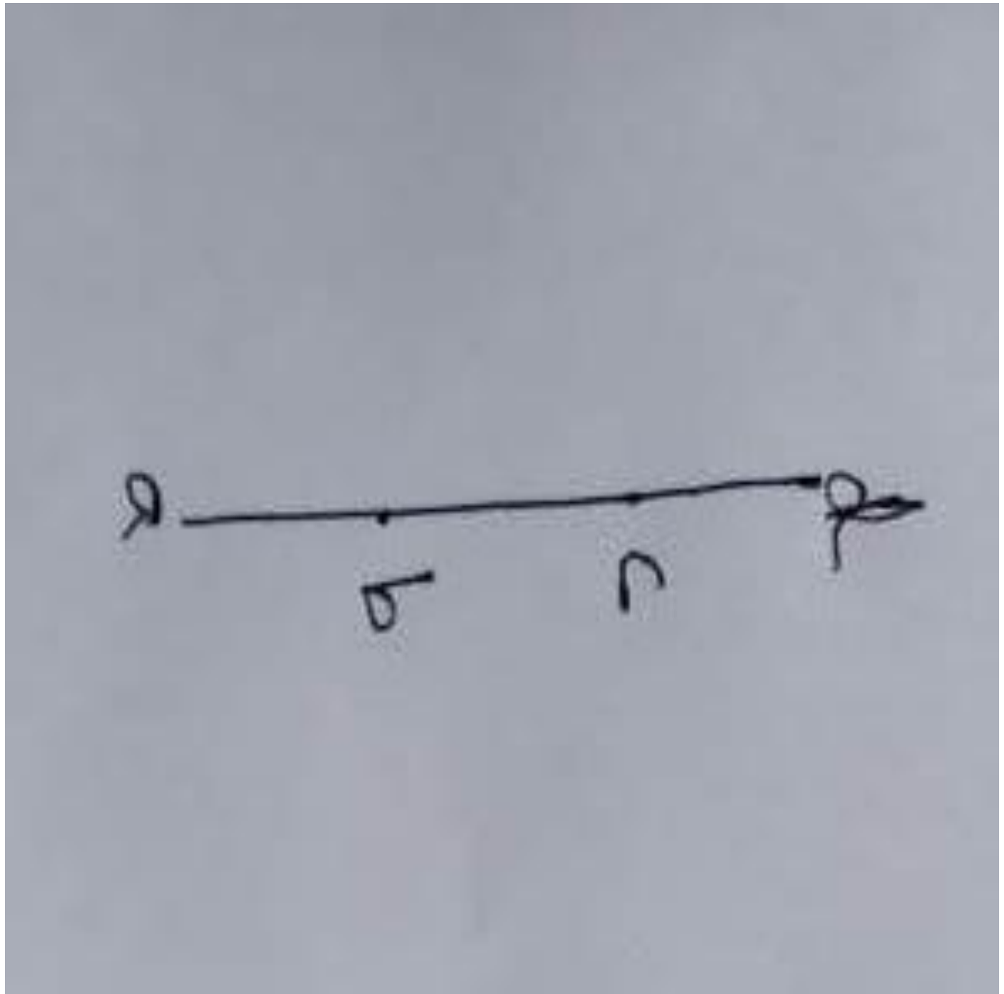

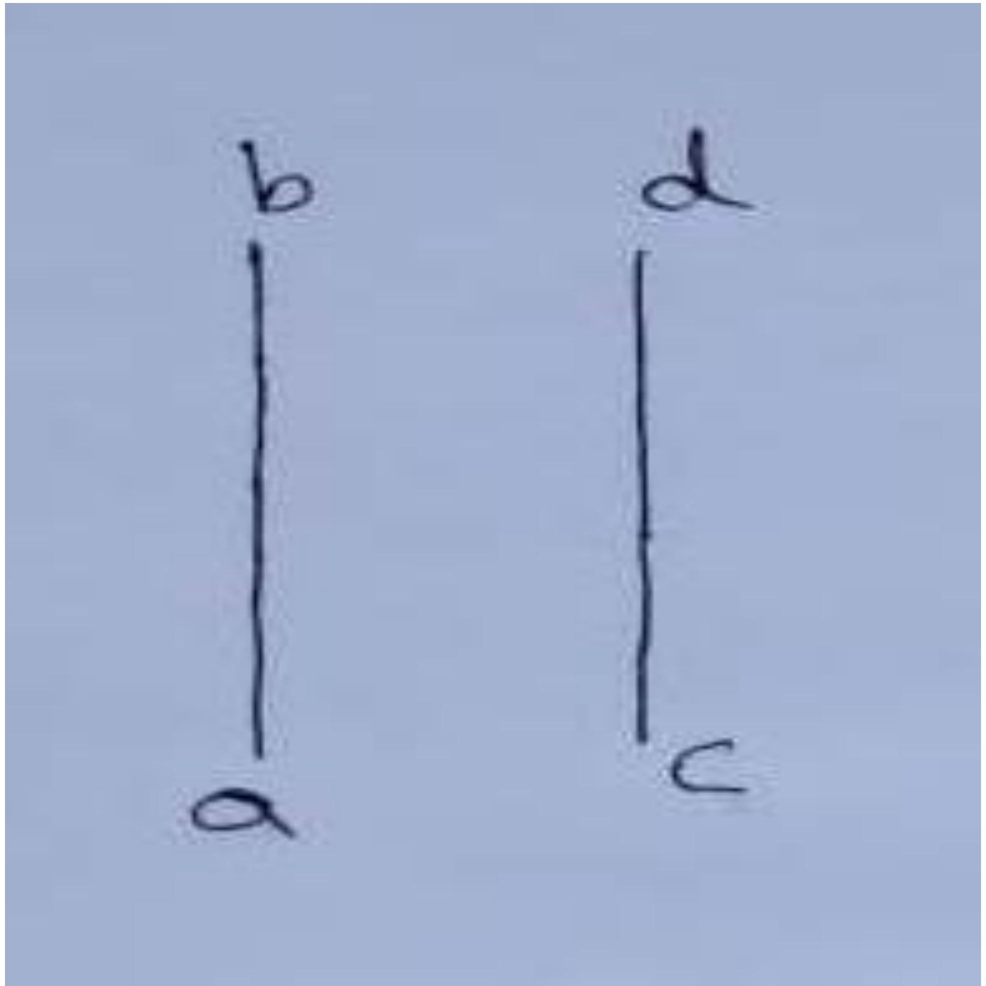
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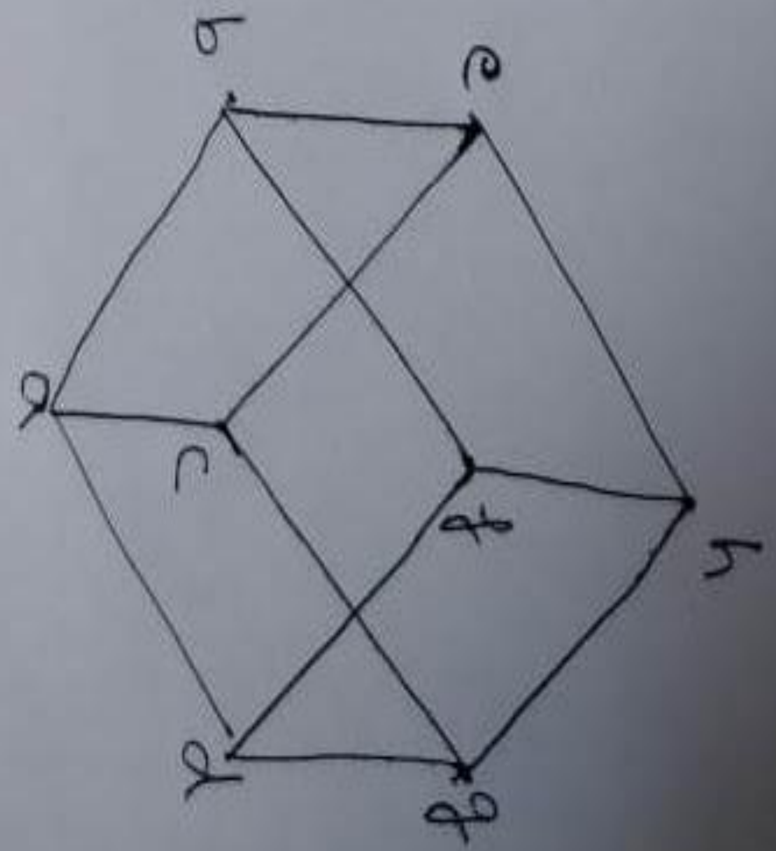

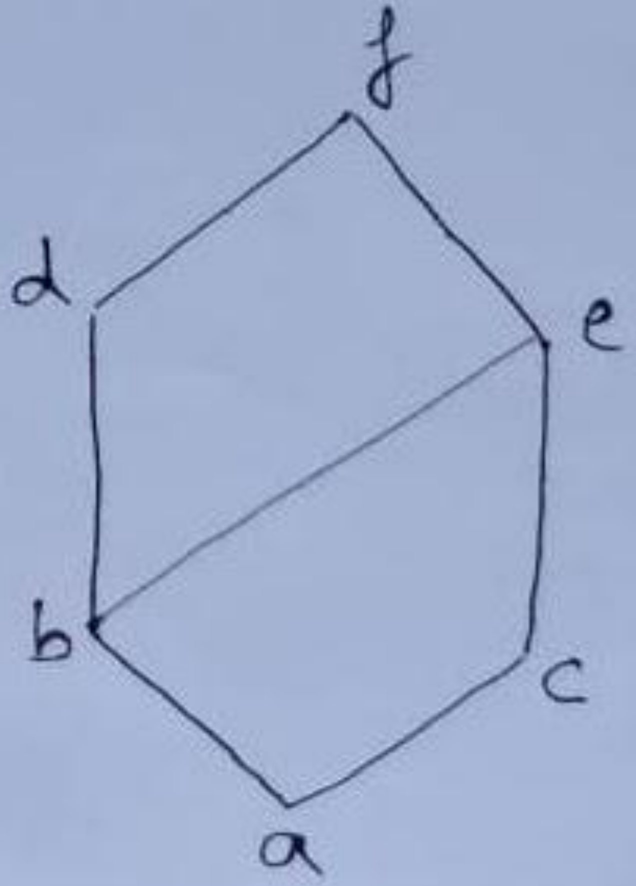

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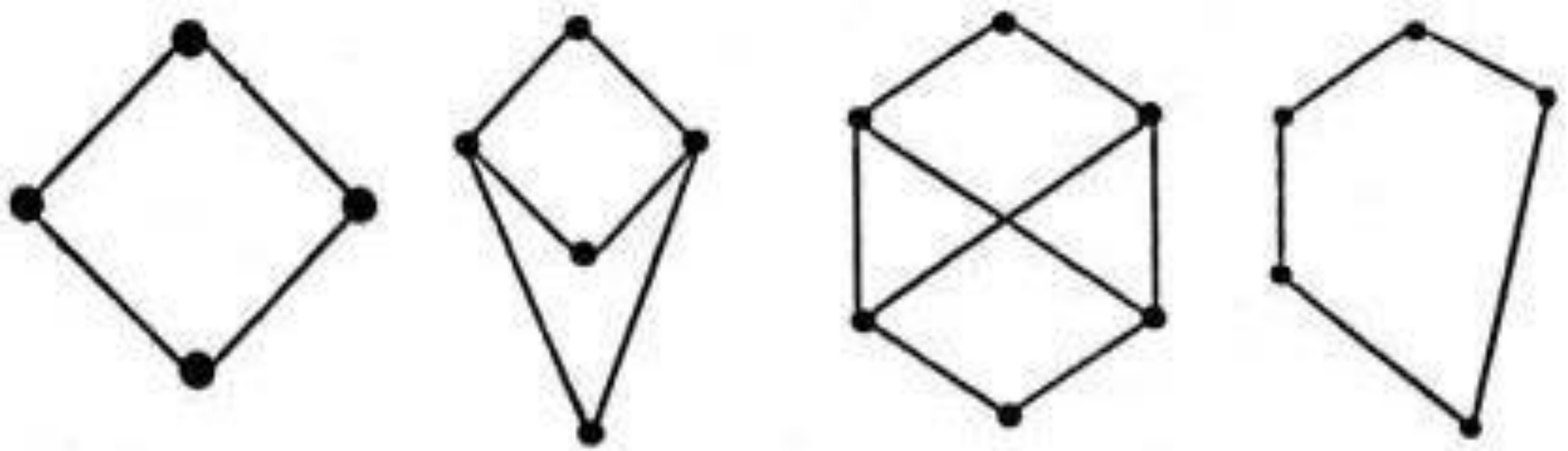
Q Consider the following Hasse diagrams

Which all of the above represent a lattice? **(GATE-2008) (2 Marks)**

(A) (i) and (iv) only **(B)** (ii) and (iii) only

(C) (iii) only **(D)** (i), (ii) and (iv) only



(i) (ii) (iii) (iv)

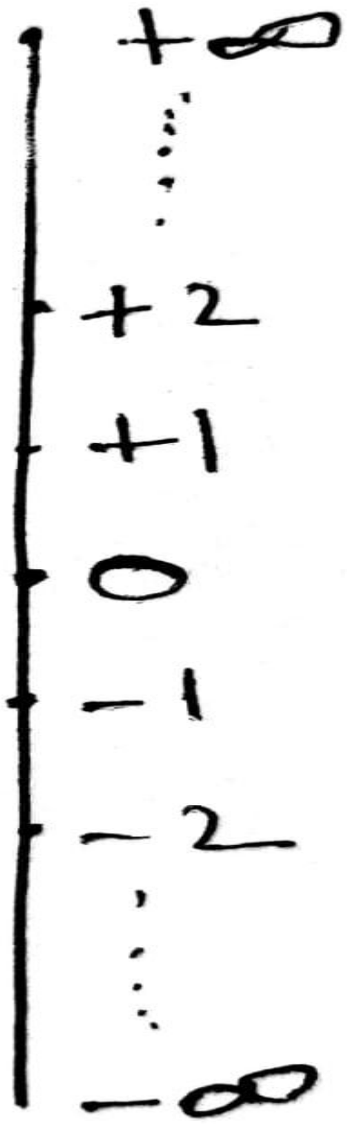
Q the inclusion of which of the following set into S = {{1,2}, {1,2,3}, {1,3,5}, {1,2,4}, {1,2,3,4,5}} is necessary and sufficient to make S a complete lattice under the partial order defined by set containment? **(GATE-2004) (2 Marks)**

a) {1}

b) {1}, {2,3}

c) {1}, {1,3}

d) {1}, {1,3}, {1,2,3,4}, {1,2,3,5}



Boolean algebra

- **Unbounded Lattice :-** If a lattice has infinite of elements then it is called Unbounded Lattice.

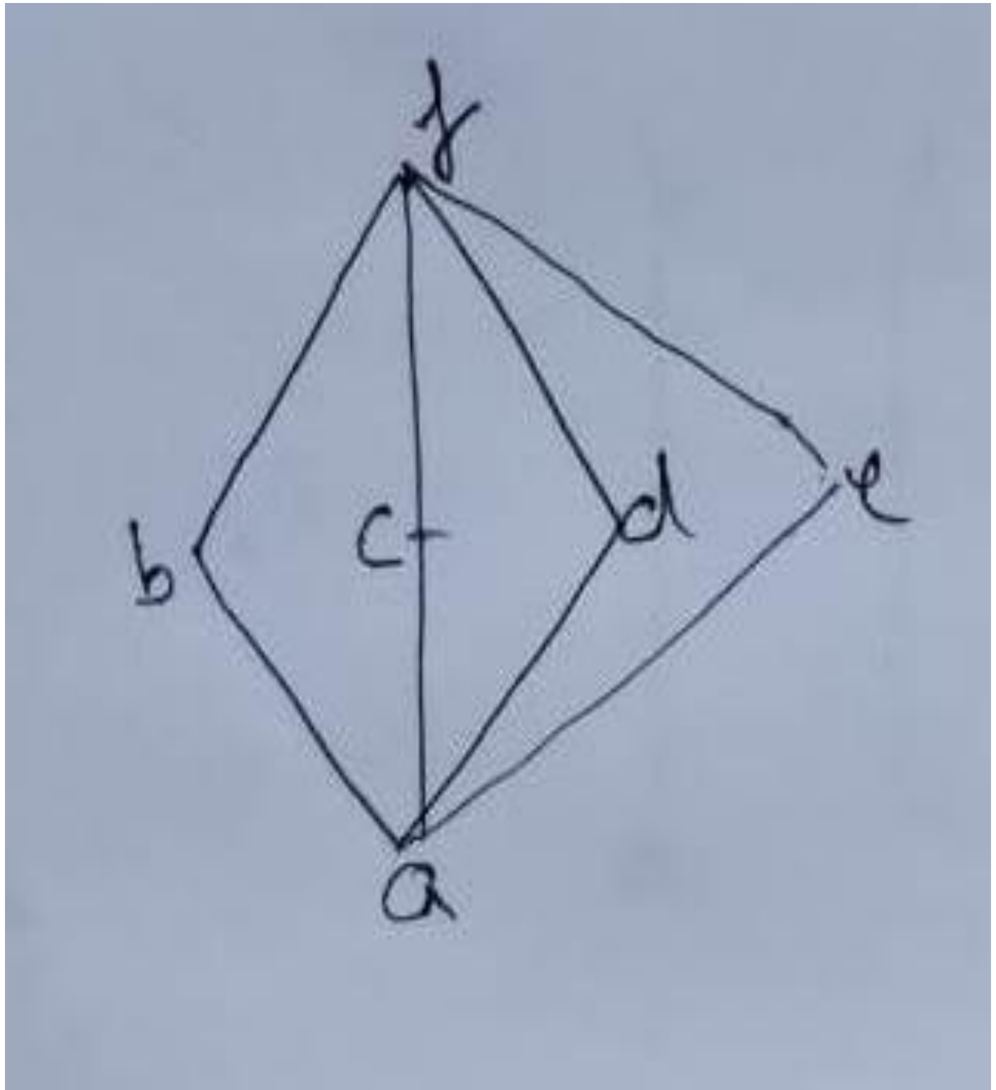
- **<u>Bounded Lattice</u> :-** If a lattice has finite number of elements then it is called Bounded lattice, there will be upper and lower bound in lattice.

- **<u>Complement of an element in a Lattice</u> :-** If two elements a and a^c, are complement of each other, then the following equations must always holds good.
  $a \vee a^c$ = Upper bound of lattice
  $a \wedge a^c$ = Lower bound of lattice

- **Distributive Lattice :-** A lattice is said to be distributed lattice. if for every element their exist at most one complement(zero or one).

- **Complement Lattice :-** A Lattice is said to be Complement lattice. if for every element their exist at least one complement(one or more).

- **Boolean Algebra :-** A Lattice is said to be Boolean Algebra, if for every element their exist exactly one complement. Or if a lattice is both complemented and distributed then it is called Boolean Algebra.



f
b
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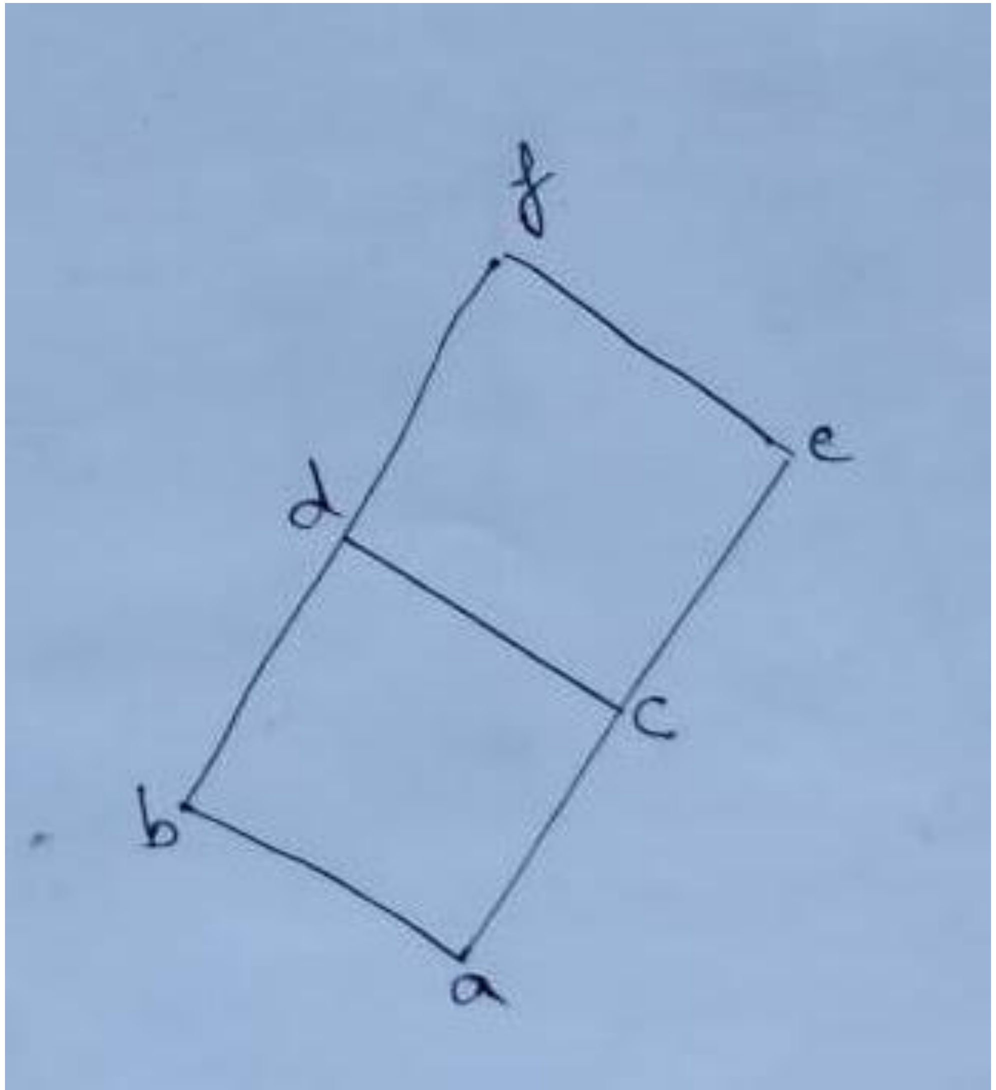

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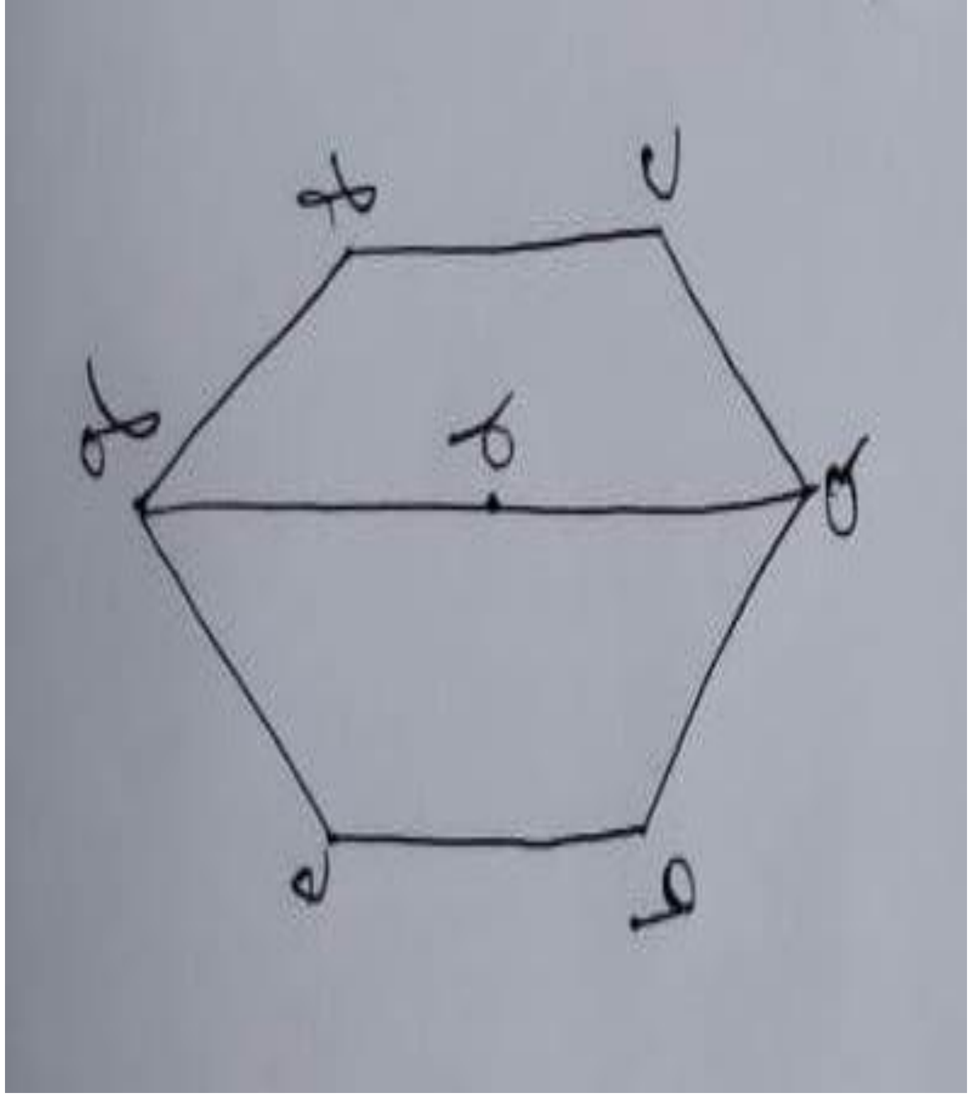

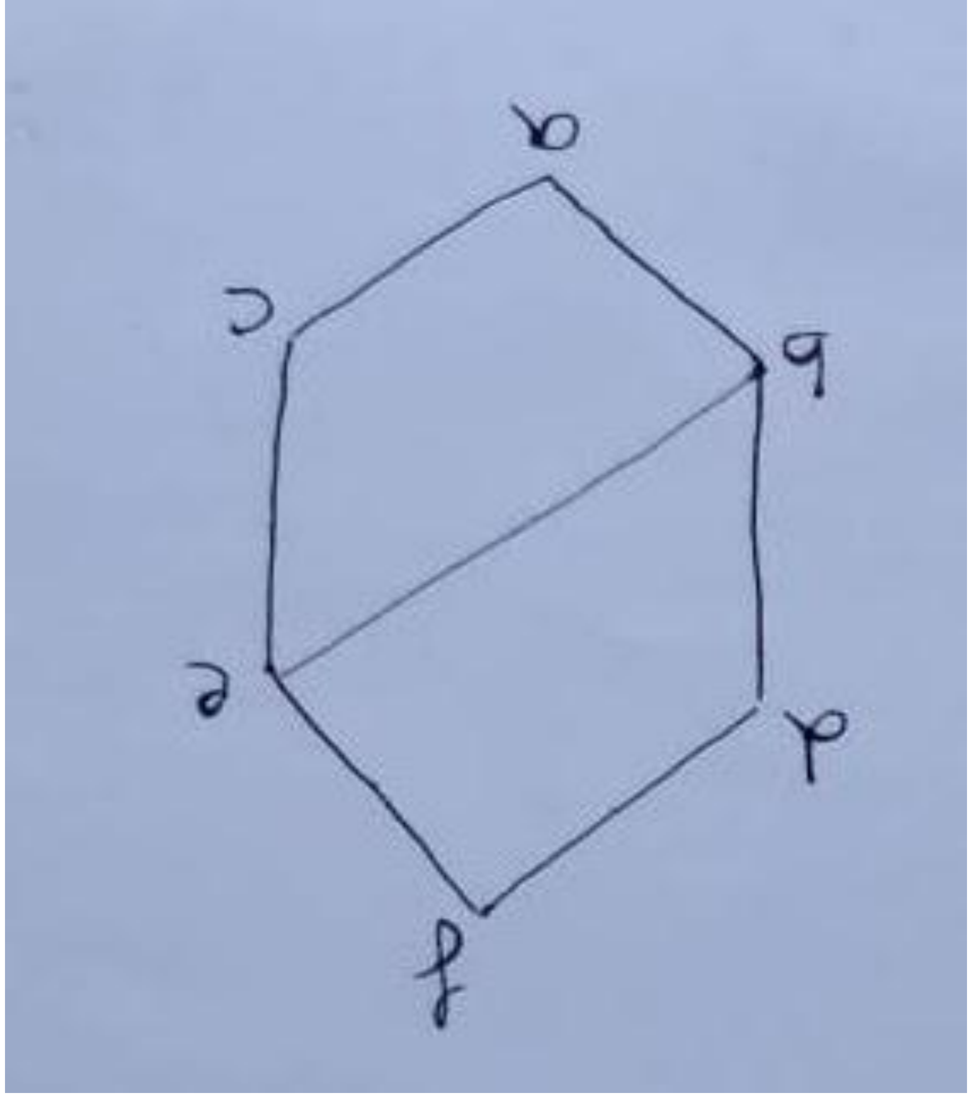

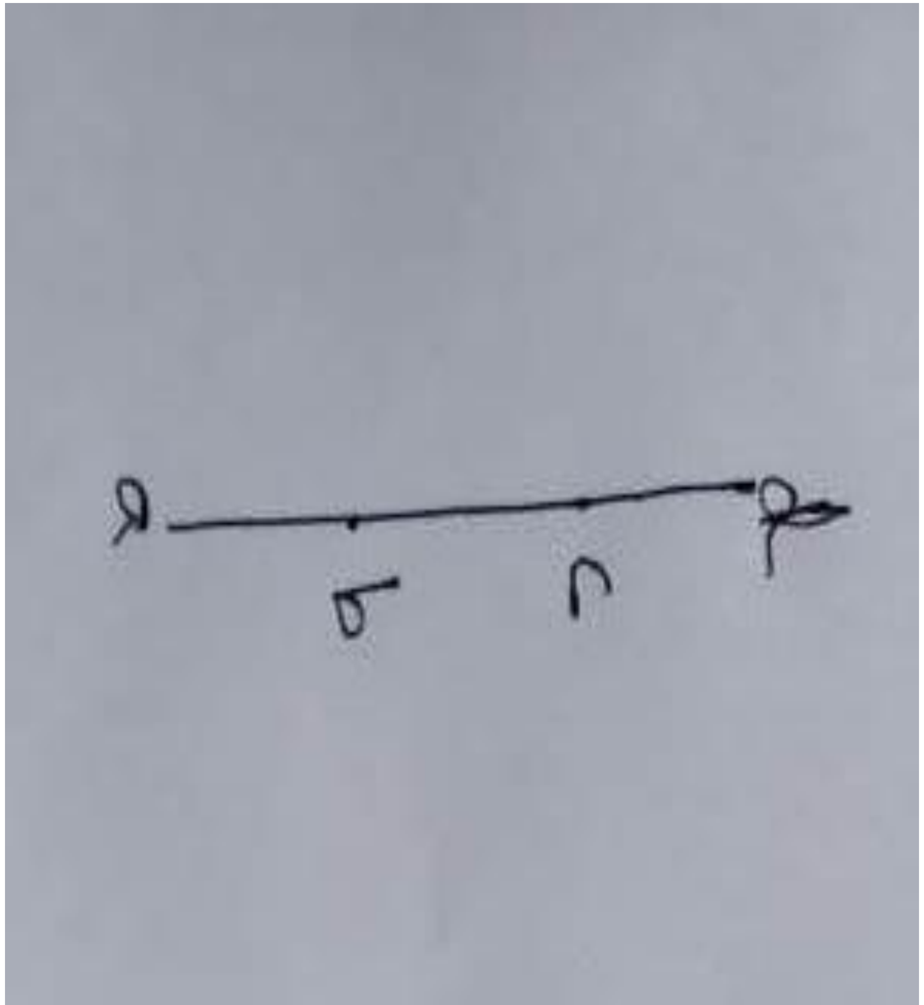

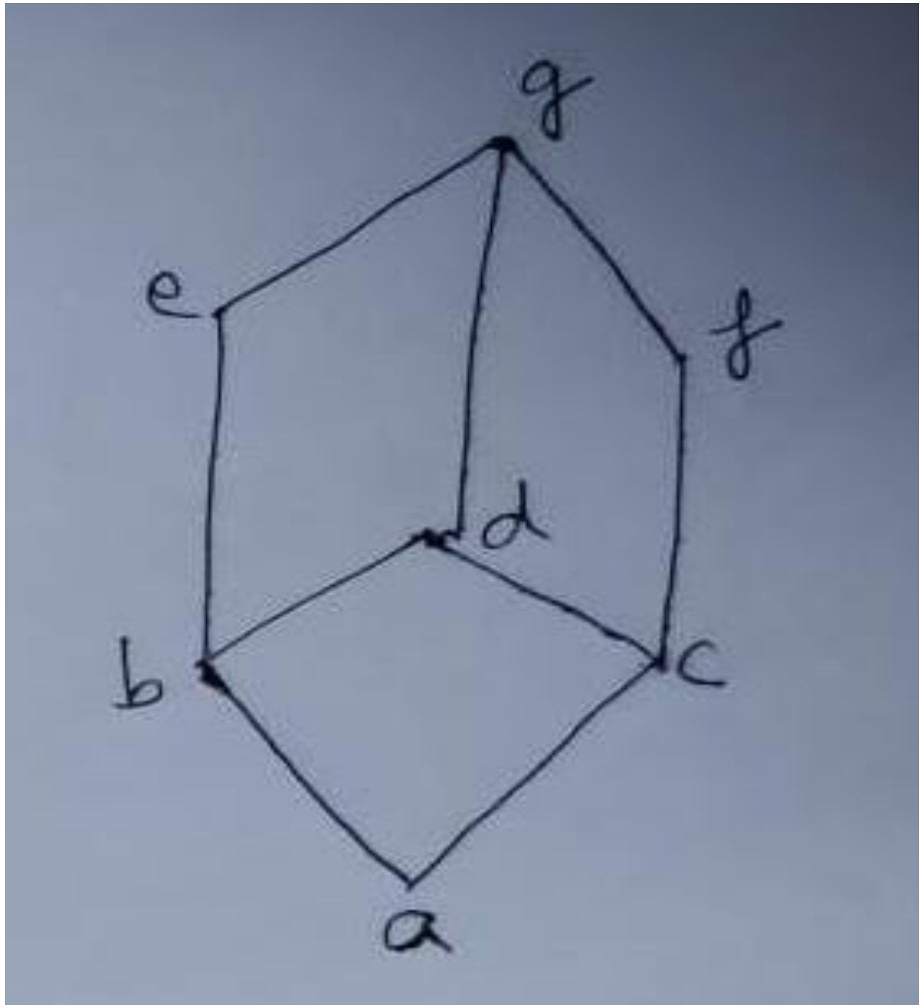
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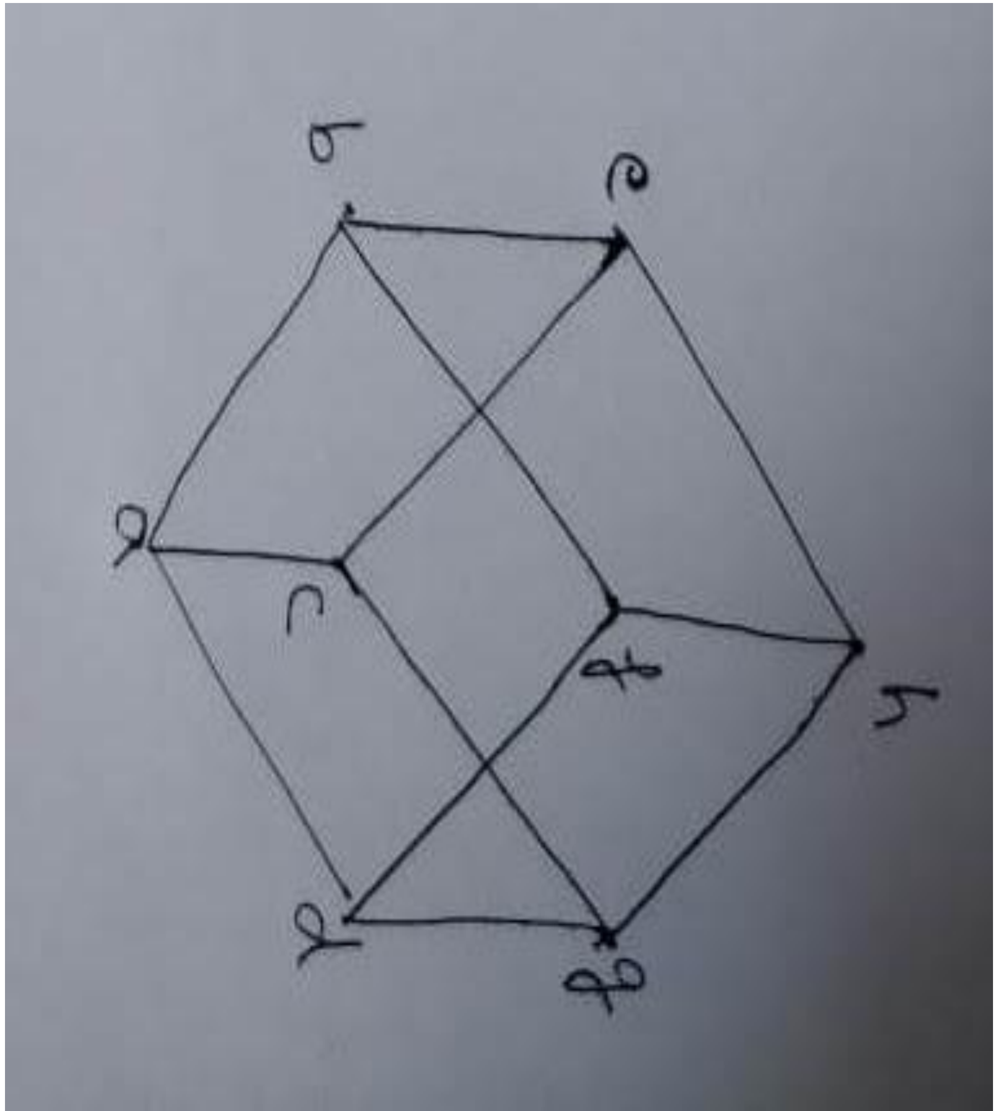
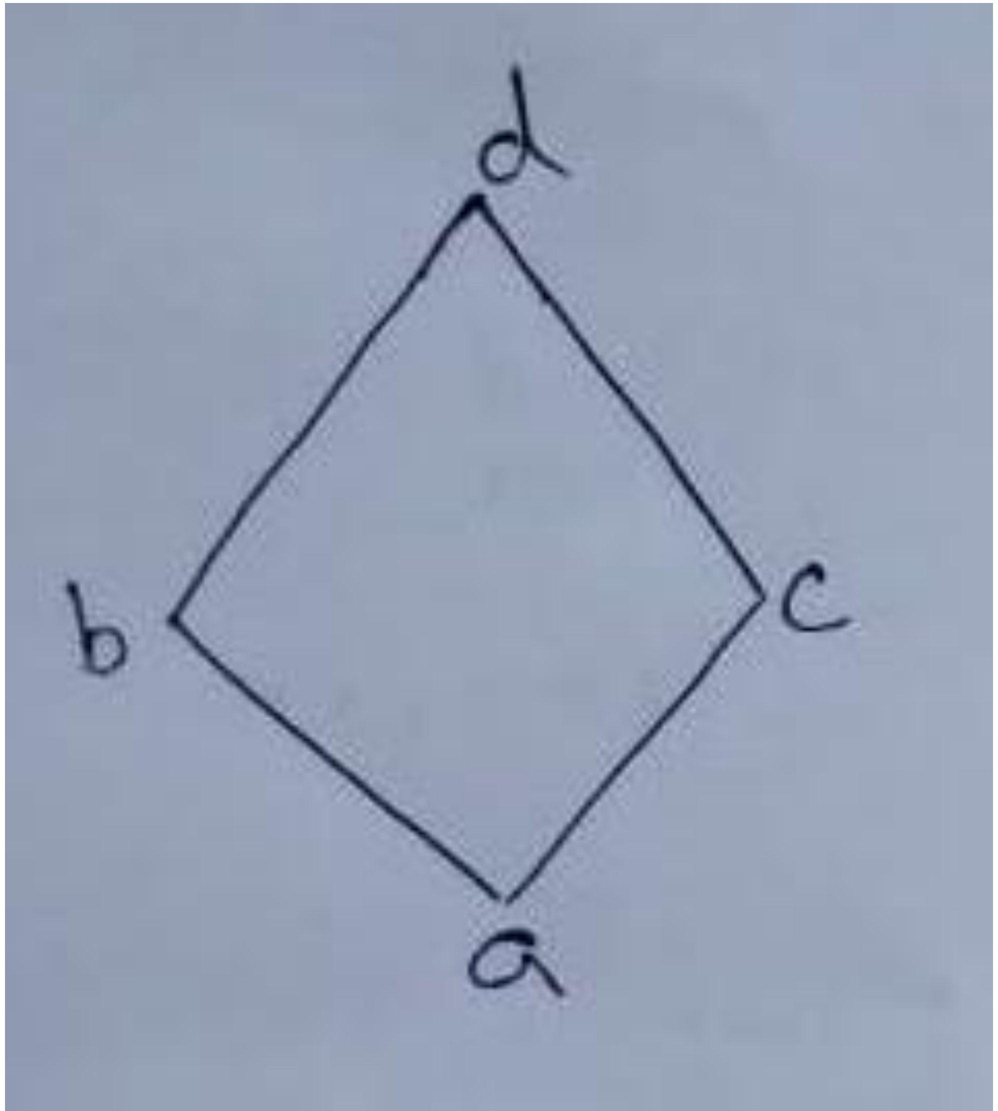



d
b
c
a

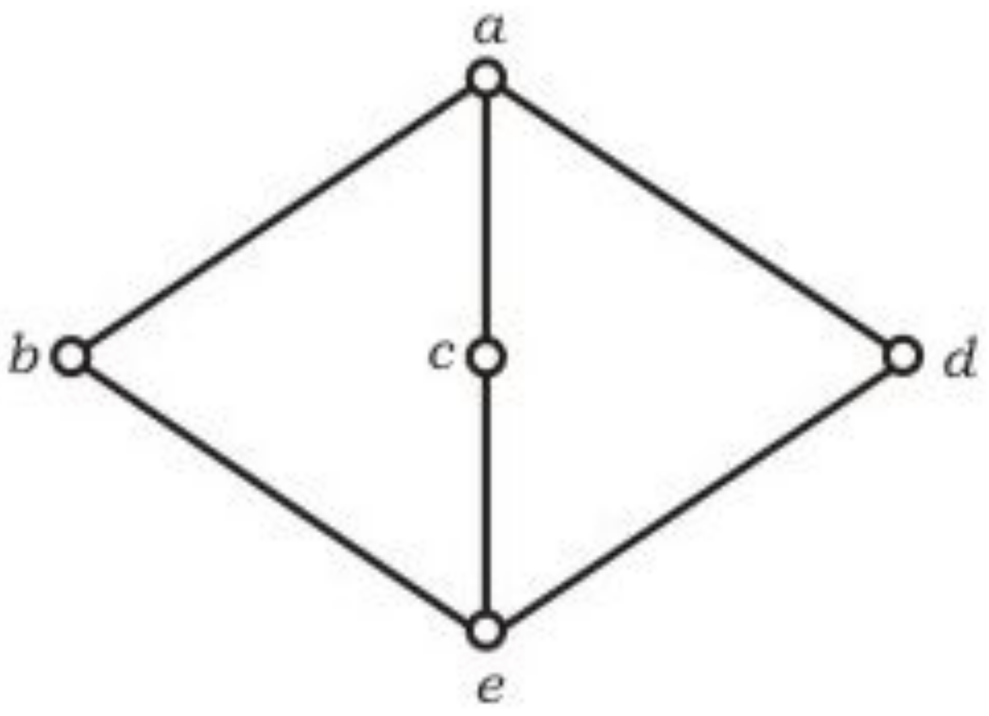
Q The following is the Hasse diagram of the Poset [{a, b, c, d, e}, ≤]
The Poset is **(GATE-2005) (1 Marks)**

(A) not a lattice

(B) a lattice but not a distributive lattice

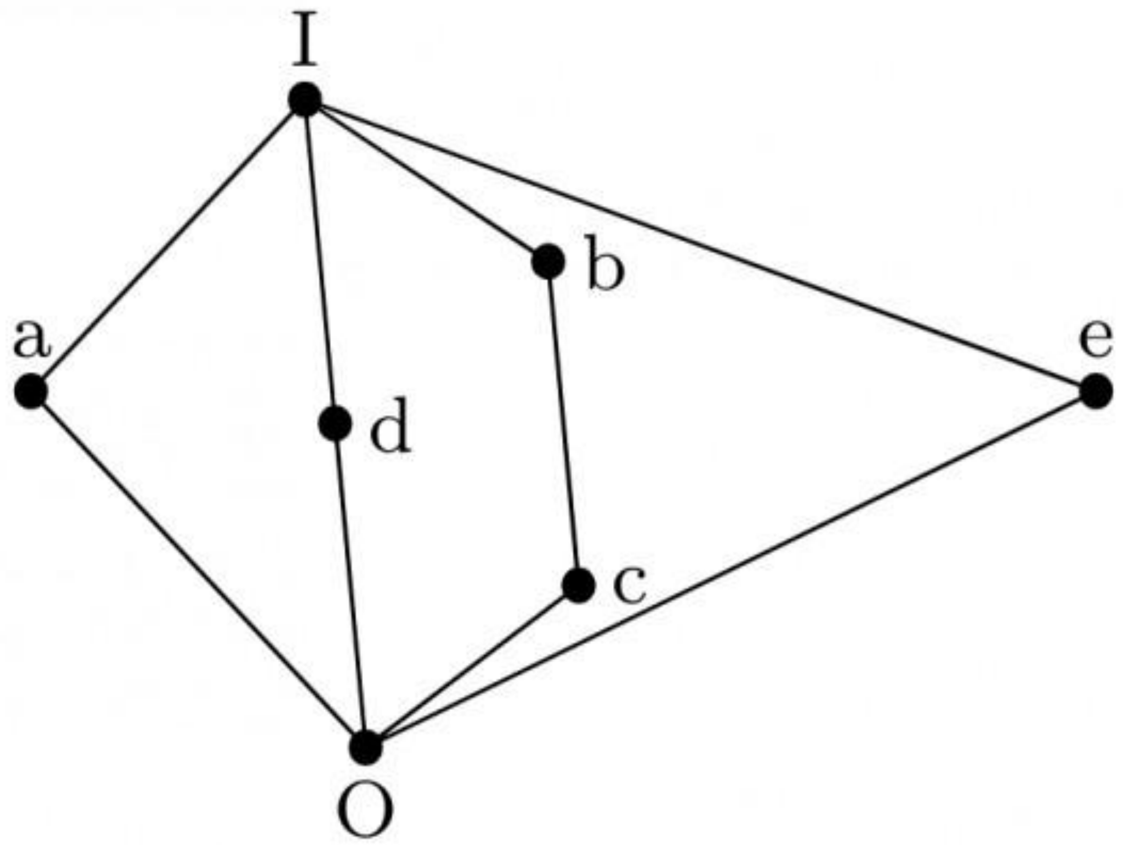
(C) a distributive lattice but not a Boolean algebra

(D) a Boolean algebra



Q The complement(s) of the element 'a' in the lattice shown in below figure is (are) ____

(GATE-1988) (2 Marks)



Q Find which of the following is a lattice and Boolean Algebra?

1) [{1,2,3,4,6,9}, /]

(2) [{2,3,4,6,12}, /]

(3) [{1,2,3,5,30}, /]

(4) [{1,2,3,6,9,18}, /]

(5) [{2,3,4,9,12,18}, /]

(6) [R, <=]

(7) [P(A), ⊆], A = {1,2,3}

Q Find which of the following is a lattice and Boolean Algebra?

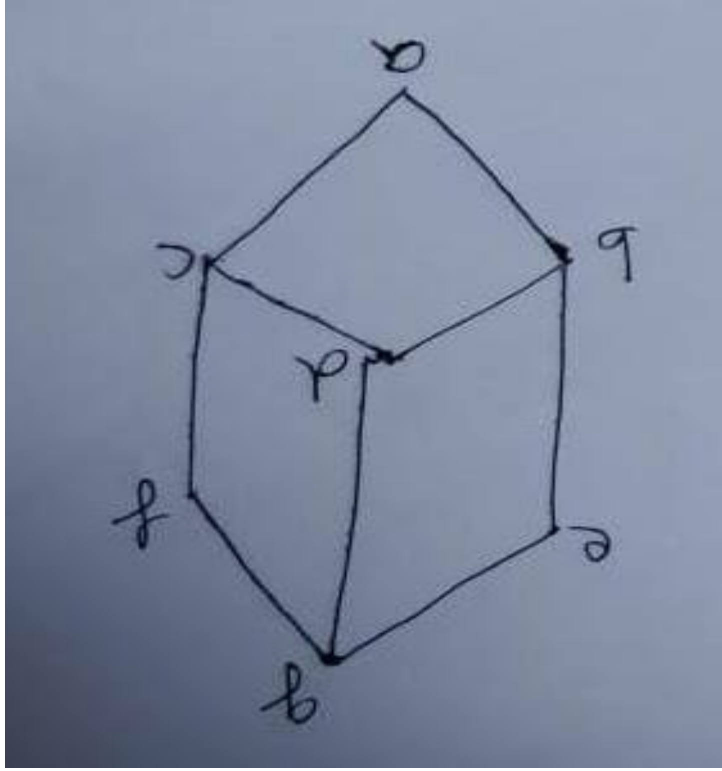
1) $[D_{10}, /]$ **(2)** $[D_{12}, /]$

(3) $[D_{30}, /]$ **(4)** $[D_{45}, /]$

(5) $[D_{64}, /]$ **(6)** $[D_{81}, /]$

(7) $[D_{91}, /]$ **(8)** $[D_{110}, /]$

Q Consider the following hasse diagram, find which of the following is true?



a) subset {a, b, c, g} is a lattice

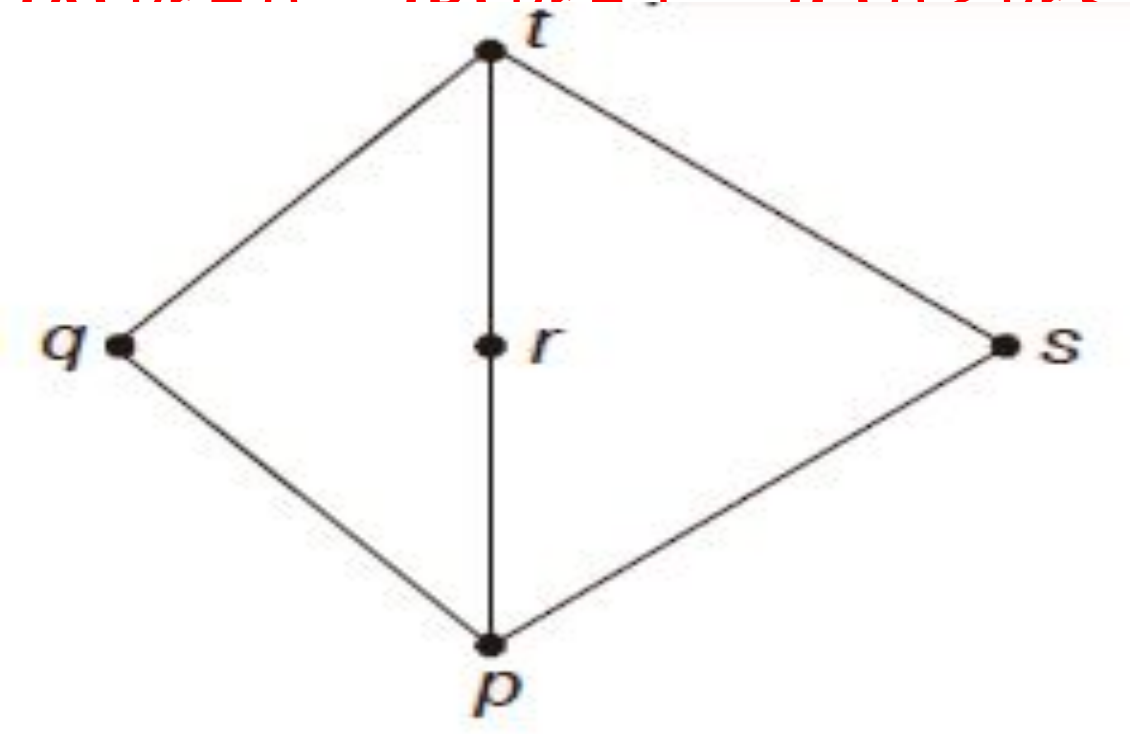
b) subset {a, b, f, g} is a lattice

c) subset {a, d, e, g} is a lattice

d) subset {a, c, e, g} is a lattice

Q Suppose L = {p, q, r, s, t} is a lattice represented by the following Hasse diagram: For any x, y ∈ L, not necessarily distinct, x ∨ y and x ∧ y are join and meet of x, y respectively. Let L^3 = {(x, y, z): x, y, z ∈ L} be the set of all ordered triplets of the elements of L. Let P_r be the probability that an element (x, y, z) ∈ L^3chosen equiprobably satisfies x ∨ (y ∧ z) = (x ∨ y) ∧ (x ∨ z). Then **(GATE-2015) (2 Marks)**

(A) Pr = 0 **(B)** Pr = 1 **(C)** 0 < Pr ≤ 1/5 **(D)** 1/5 < Pr < 1

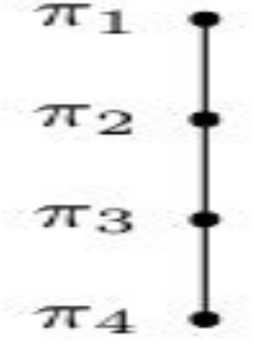


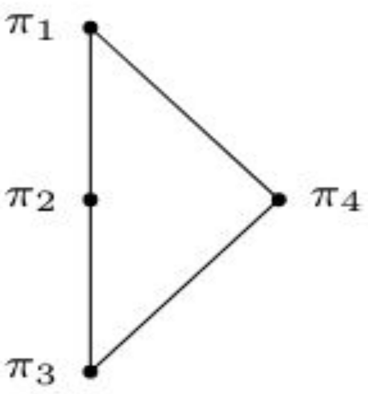
Q Consider the set S = {a, b, c, d}.

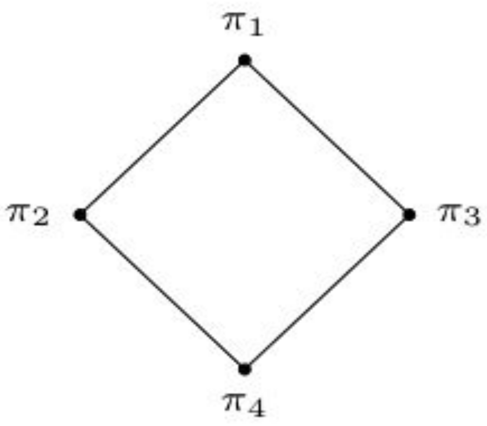
Consider the following 4 partitions π_1, π_2, π_3, π_4 on S: π_1={~~abcd~~}, π_2={~~ab~~, ~~cd~~}, π_3={~~abc~~, ~~d~~}, π_4={~~a~~, ~~b~~, ~~c~~, ~~d~~}

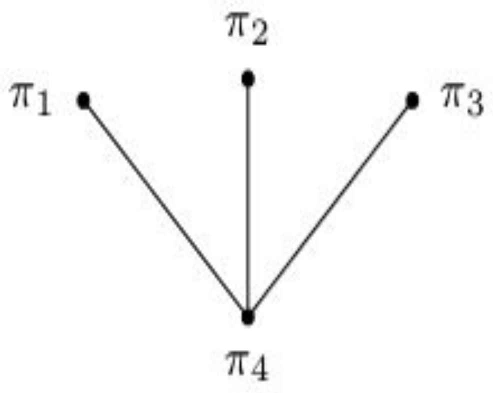
Let $\prec$ be the partial order on the set of partitions S′={π_1,π_2,π_3,π_4} defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j. The Poset diagram for (S′, $\prec$) is:

(GATE-2007) (2 Marks)





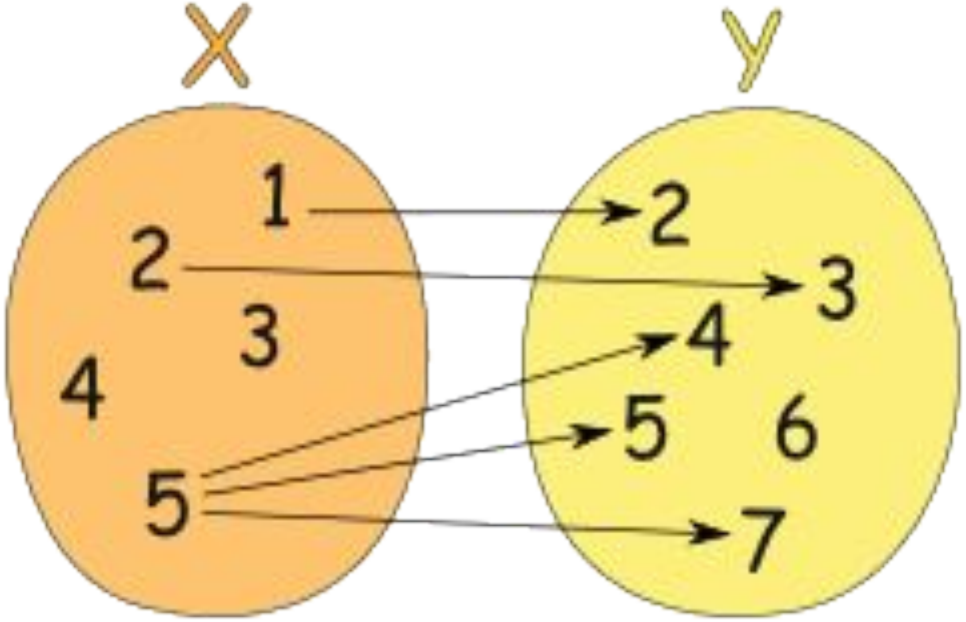




Function

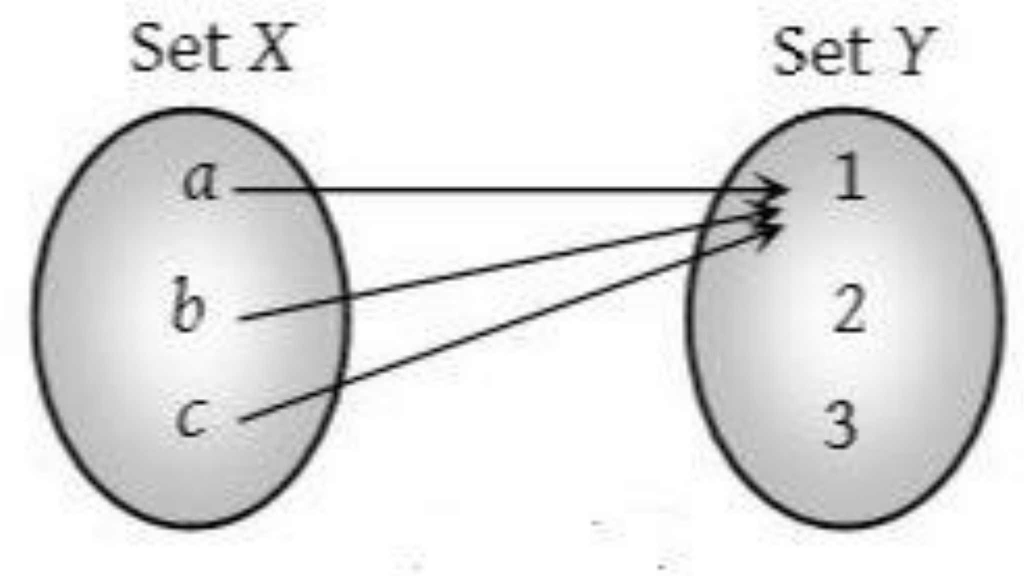
- Functions are widely used in science, and in most fields of mathematics. It has been said that functions are "the central objects of investigation" in most fields of mathematics.

- Let X = {1, 2, 3, 4, 5} and Y = {2, 3, 4, 5, 6, 7} and $R \subseteq X * Y$. Now this is a valid relation but not a function, because there is a element which is not participating in the relationship secondly 5 is relating with more than one element.



Function

- In mathematics, a function is a relation between sets that associates to every element of a first set exactly one element of the second set.
- A relation ‘f’ from a set ‘A’ to a Set ‘B’ is called a function, if each element of A is mapped with a unique element on B.
- f: A□B



- Range of fun $\subseteq$ B
- Range of f = { y | y ϵ B and (x, y) ϵ f}

- If $|A| = m$ and $|B| = n$, then number of functions possible from A to B = ?

Q The number of functions from an m element set to an n element set is **(GATE-1998) (1 Marks)**

a) $m + n$ **b)** m^n **c)** n^m **d)** $m * n$

Q Suppose X and Y are sets and $|X|$ and $|Y|$ are their respective cardinalities. It is given that there are exactly 97 functions from X to Y. From this one can conclude that **(GATE-1996) (1 Marks)**

(A) $|X|=1, |Y|=97$

(B) $|X|=97, |Y|=1$

(C) $|X|=97, |Y|=97$

(D) None of the above

Q Let X, Y, Z be sets of sizes x, y and z respectively. Let W = X x Y. Let E be the set of all subsets of W. The number of functions from Z to E is

(GATE-2006) (1 Marks)

(A) Z^{2xy} **(B)** $Z \times 2^{xy}$ **(C)** 2^{z} **(D)** 2^{xyz}

Q Let S denote the set of all functions f: $\{0,1\}^4$ -> {0,1}. Denote by N the number of functions from S to the set {0,1}. The value of Log_2Log_2N is _______. **(GATE-2014) (2 Marks)**

Q A function $f: N^+ \rightarrow N^+$, defined on the set of positive integers N^+, satisfies the following properties:

$f(n)=f(n/2)$ if n is even

$f(n)=f(n+5)$ if n is odd

Let $R=\{i \mid \exists j : f(j)=i\}$ be the set of distinct values that f takes. The maximum possible size of R is ____________. **(GATE-2016) (2 Marks)**

Q Let X and Y be finite sets and f: X -> Y be a function. Which one of the following statements is TRUE? **(GATE-2014) (1 Marks)**

a) For any subsets A and B of X, $|f(A \cup B)| = |f(A)| + |f(B)|$

b) For any subsets A and B of X, $f(A \cap B) = f(A) \cap f(B)$

c) For any subsets A and B of X, $|f(A \cap B)| = \min\{|f(A)|, |f(B)|\}$

d) For any subsets S and T of Y, $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

Q Let f: A⟶B be a function, and let E and F be subsets of A. Consider the following statements about images.

S_1: $f(E \cup F) = f(E) \cup f(F)$

S_2: $f(E \cap F) = f(E) \cap f(F)$

Which of the following is true about S1 and S2? **(GATE-2001) (2 Marks)**

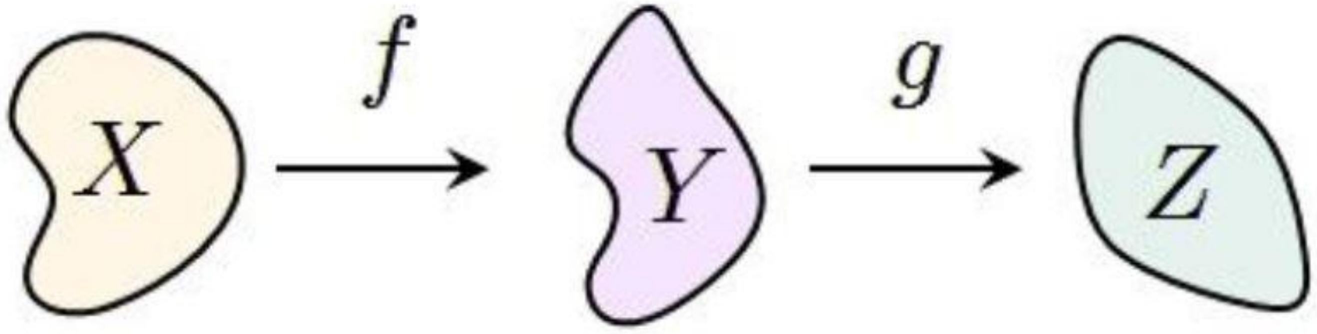
(A) Only S1 is correct **(B)** Only S2 is correct

(C) Both S1 and S2 are correct **(D)** None of S1 and S2 is correct

Function composition

- In mathematics, **function composition** is an operation that takes two functions f and g and produces a function h such that $h(x) = g(f(x))$.

- In this operation, the function g is applied to the result of applying the function f to x. That is, the functions $f: X \to Y$ and $g: Y \to Z$ are **composed** to yield a function that maps x in X to $g(f(x))$ in Z.



- fog(x) = f(g(x))
- gof(x) = g(f(x))

- Composition of functions on a finite set: If f = {(1, 3), (2, 1), (3, 4), (4, 6)}, and g = {(1, 5), (2, 3), (3, 4), (4, 1), (5, 3), (6, 2)}, then $g \circ f$ = {(1, 4), (2, 5), (3, 1), (4, 2)}.
- The composition of functions is always associative—a property inherited from the composition of relations. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$

Q If $g(x) = 1-x$ and $h(x)=x / (x-1)$, then $g(h(x)) / h(g(x))$ is: **(GATE-2015) (1 Marks)**

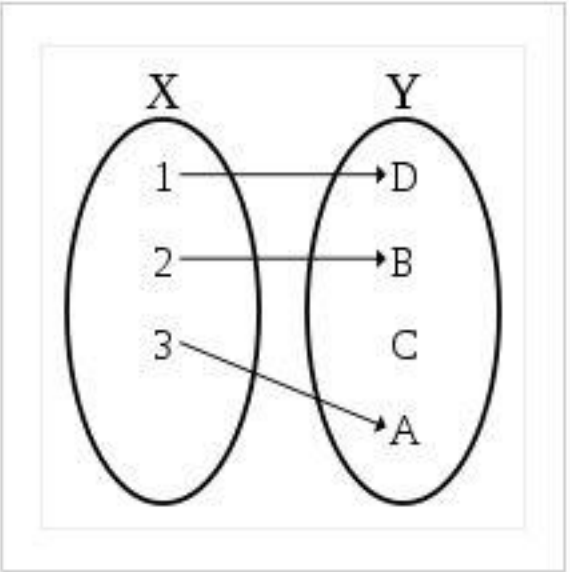
a) $h(x) / g(x)$ **b)** $-1 / x$ **c)** $g(x) / h(x)$ **d)** $x / (1-x)^2$

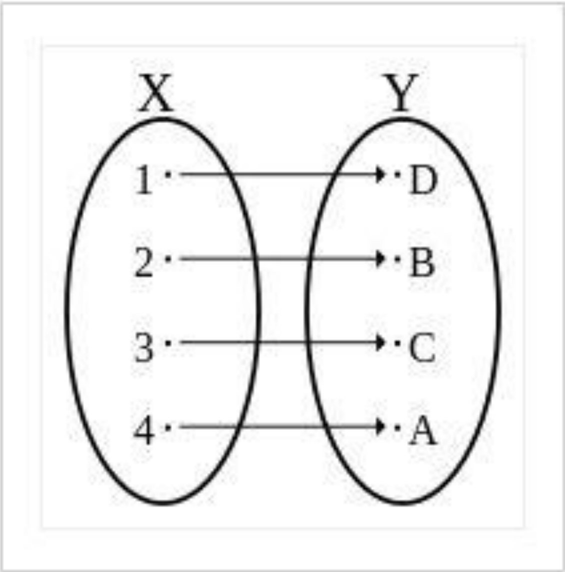
One-to-One (Injective Function)

- An **injective function** (also known as **injection**, or **one-to-one function**) is a function that maps distinct elements of its domain to distinct elements of its codomain. In other words, every element of the function's codomain is the image of *at most* one element of its domain.

One-to-One (Injective Function)

- A function F: A□B is said to be one-to-one function if every element of A has distinct image in B
- If A and B are finite set, then one-to-one from A□B is possible
  - if |A| <= |B|





- No of function possible = ?

Q Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X and Y. Let f be randomly chosen from F. The probability of f being one-to-one is __________ **(GATE-2015) (2 Marks)**

Q Let A and B be non-empty finite sets such that there exists one-to-one and onto functions

(i) from A to B

(ii) from AxA to A U B.

The number of possible values of |A| is _____________ **(Gate-2024,CS)(1 Marks)(NAT)**

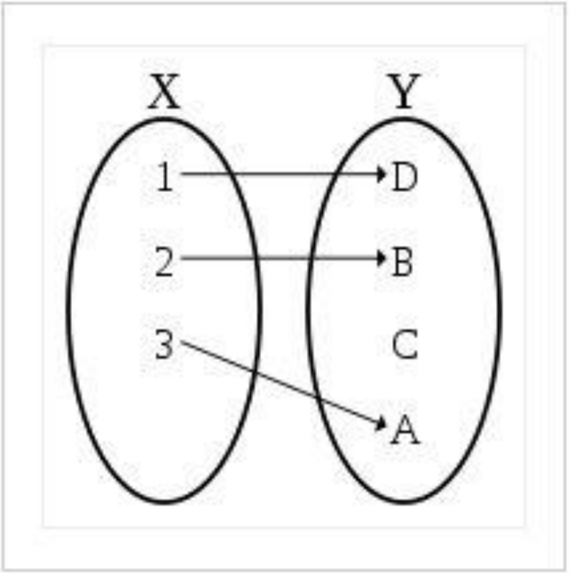
- No of function possible = ${}^{n}p_{m}$ = P (n, m)
- If |A| = |B| = n, then no of functions possible is n!

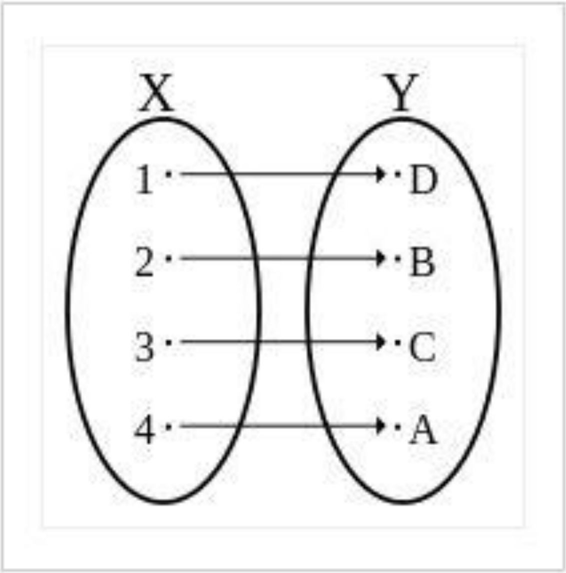
Onto (Surjective Function)

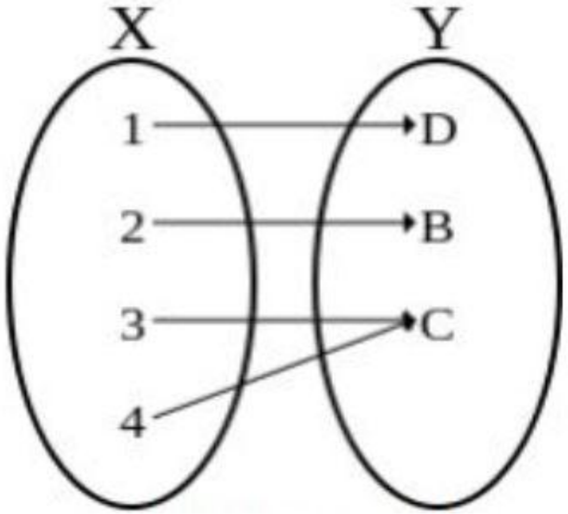
- A function f from a set X to a set Y is **surjective** (also known as **onto**, or a **surjection**), if for every element y in the co-domain Y of f, there is at least one element x in the domain X of f such that $f(x) = y$. It is not required that x be unique; the function f may map one or more elements of X to the same element of Y.

Onto (Surjective Function)

- A function f: A □ B is said to be onto if and only if every element of B is mapped by at least one element of A.
- Range of f = B







- If A and B are finite sets, then onto function from A□B is possible, |B|<=|A|
- If |A| = |B|, then every onto function from A to B is also one-to-one function.

No of onto function possible from A to B = ?

No of onto function possible from A to B

$= n^m - {}^nc_1(n-1)^m + {}^nc_2(n-2)^m - {}^nc_3(n-3)^m$ +----------+ $(-1)^n\, {}^nc_{\,n-1}\, 1^m$

Q The number of onto functions (surjective functions) from set X = {1, 2, 3, 4} to set Y = {a, b, c} is ____________________ **(GATE-2015) (2 Marks)**

Q How many onto (or surjective) functions are there from an n-element (n >= 2) set to a 2-element set? **(GATE-2014) (2 Marks)**

(A) 2^n **(B)** $2^n – 1$ **(C)** $2^n – 2$ **(D)** $2(2^n – 2)$

Q Consider the set of all functions f: {0,1, … ,2014} → {0,1, … ,2014} such that f(f(i)) = i, for all 0 ≤ i ≤ 2014. Consider the following statements: **(GATE-2014) (2 Marks)**

P. For each such function it must be the case that for every i, f(i)=i

Q. For each such function it must be the case that for some i, f(i)=i

R. Each function must be onto.

Which one of the following is CORRECT?

(A) P, Q and R are true

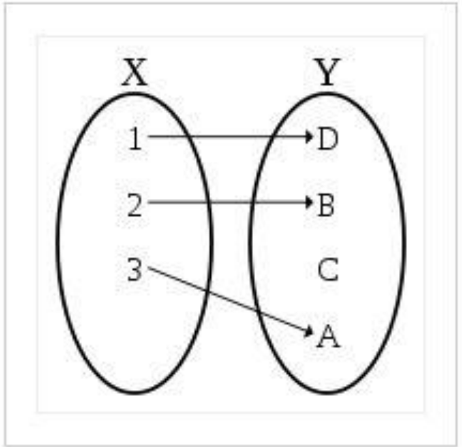
(B) Only Q and R are true

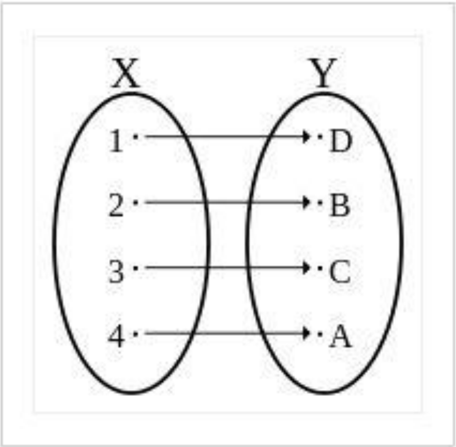
(C) Only P and Q are true

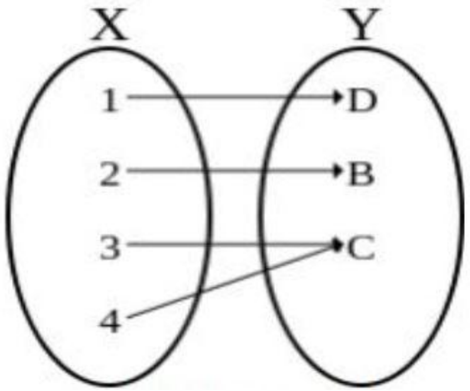
(D) Only R is true

Bijective Function

- In mathematics, a **bijection**, **bijective function**, **one-to-one correspondence**, or **invertible function**, is a function between the elements of two sets, where each element of one set is paired with exactly one element of the other set, and each element of the other set is paired with exactly one element of the first set. There are no unpaired elements.





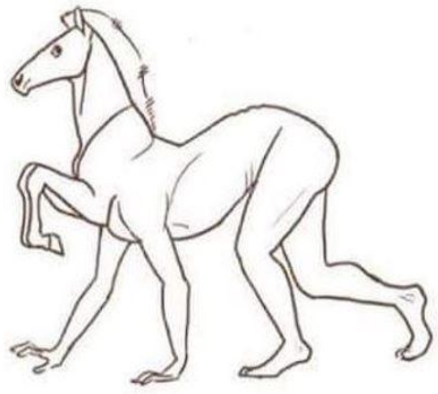


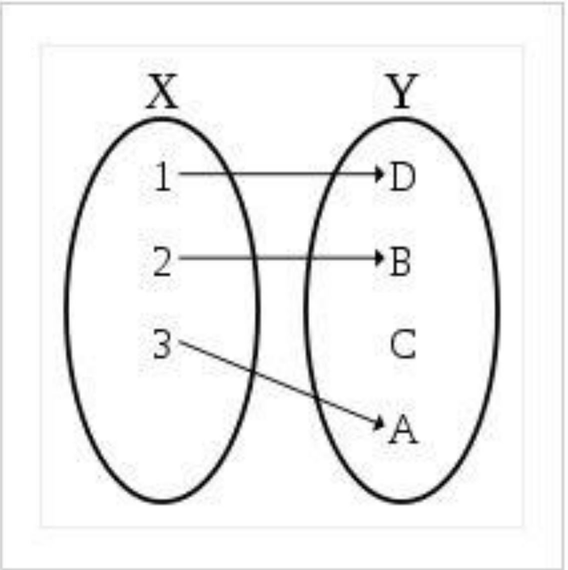
- A function f: A□B is said to be bijection if f is one-to-one and onto.
- Bijection from A and B is possible, if□|A| |B|
- No of Bijection from A to B = ?

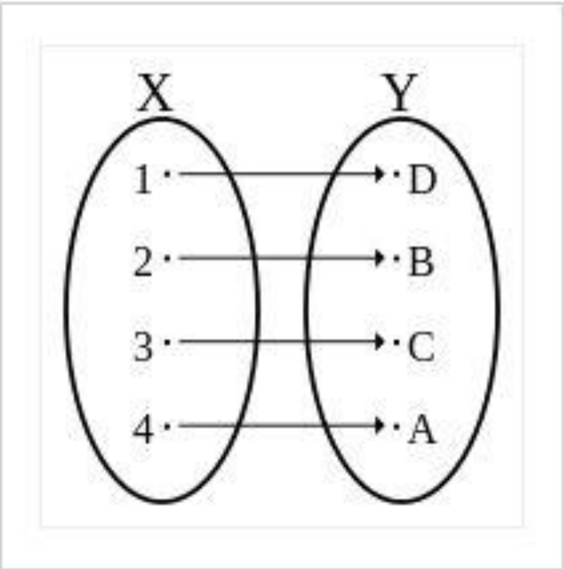
- A function $f: A \to B$ is said to be bijection if f is one-to-one and onto.
- Bijection from A and B is possible, if $|A| = |B|$
- No of Bijection from A to B = n!

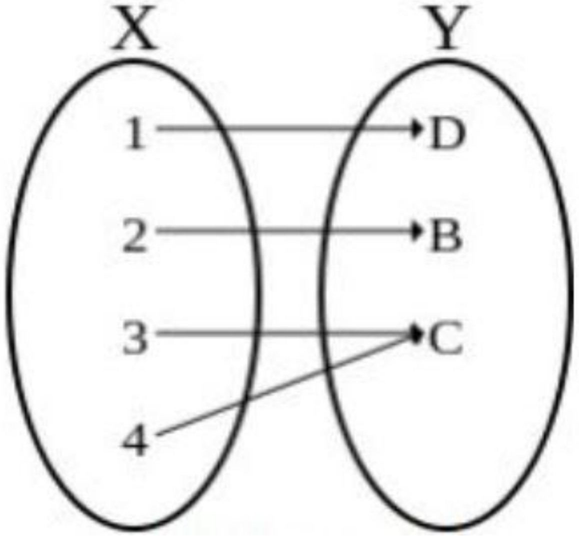
<u>Inverse of a function</u>

- In mathematics, an **inverse function** (or **anti-function**) is a function that "reverses" another function
- If the function f applied to an input x gives a result of y, then applying its inverse function f^{-1} to y gives the result x, and vice versa.
- $f(x) = y$ then $f^{-1}(y) = x$.









- Inverse of a function f: A□B exists, iff f: A□B is a bijection.

- $f(x) = 5x - 7$
- $f^{-1}(y) = (y + 7)/5$

Q Let R denote the set of real numbers. Let $f: R \times R \rightarrow R \times R$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of f is given by **(GATE-1996) (2 Marks)**

a) $f^{-1}(x, y) = (1 / (x + y), 1 / (x - y))$

b) $f^{-1}(x, y) = (x - y, x + y)$

c) $f^{-1}(x, y) = ((x + y) / 2, (x - y) / 2)$

d) $f^{-1}(x, y) = [2(x - y), 2(x + y)]$

Q Consider the following sets, where $n \geq 2$:

- **S_1:** Set of all $n \times n$ matrices with entries from the set $\{a,b,c\}$
- **S_2:** Set of all functions from the set $\{0,1,2 \ldots ,n^2-1\}$ to the set $\{0,1,2\}$

Which of the following choice(s) is/are correct? **(GATE 2021) (1 MARKS)**

(A) There does not exist a bijection from S_1 to S_2

(B) There exists a surjection from S_1 to S_2

(C) There exists a bijection from S_1 to S_2

(D) There does not exist an injection from S_1 to S_2

Q Let N be the set of natural numbers. Consider the following sets.

P: Set of Rational numbers (positive and negative)

Q: Set of functions from {0, 1} to N

R: Set of functions from N to {0, 1}

S: Set of finite subsets of N.

Which of the sets above are countable? **(GATE-2018) (1 Marks)**

(A) Q and S only **(B)** P and S only

(C) P and R only **(D)** P, Q and S only

Q Let f: B ⟶ C and g: A ⟶ B be two functions and let h = f o g. Given that h is an onto function. Which one of the following is TRUE? **(GATE-2005) (2 Marks)**

(A) f and g should both be onto functions

(B) f should be onto but g need not be onto

(C) g should be onto but f need not be onto

(D) both f and g need not be onto

Q Let f be a function from a set A to a set B, g a function from B to C, and h a function from A to C, such that h(a) = g(f(a)) for all a $\in$ A. Which of the following statements is always true for all such functions f and g? **(GATE-2005) (2 Marks)**

(A) g is onto => h is onto

(B) h is onto => f is onto

(C) h is onto => g is onto

(D) h is onto => f and g are onto

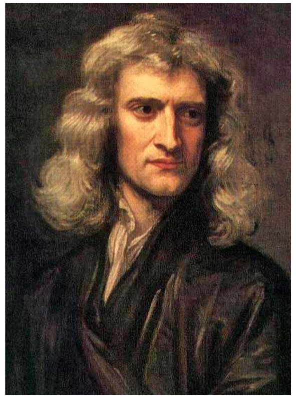
Q For the set N of natural numbers and a binary operation f: N x N ⟶ N, an element z ∈ N is called an identity for f, if f (a, z) = a = f (z, a), for all a ∈ N. Which of the following binary operations have an identity? **(GATE-2006) (1 Marks)**

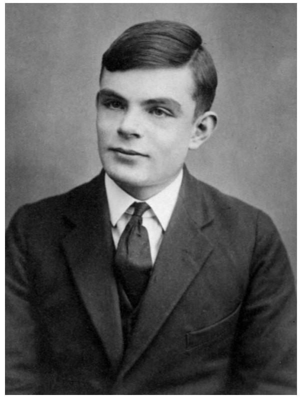
1. $f(x, y) = x + y - 3$ **2.** $f(x, y) = \max(x, y)$ **3.** $f(x, y) = x^y$

(A) I and II only
(B) II and III only
(C) I and III only
(D) None of these

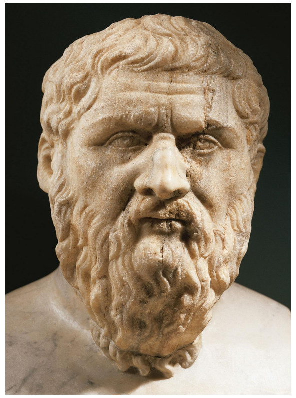
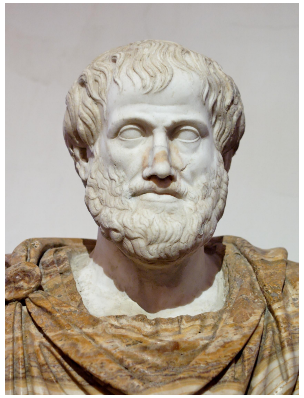
Proposition

- First we must look at the difference between Scientist and Philosopher. Philosopher give an idea or theory which may have different interpretation from person to person. It depends on the wisdom of a person.

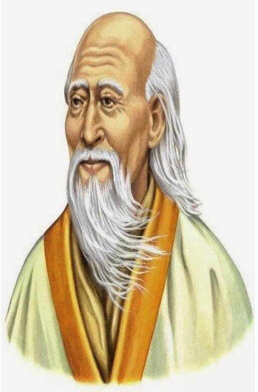




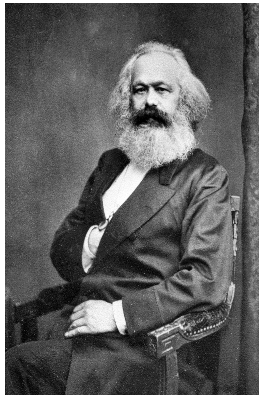
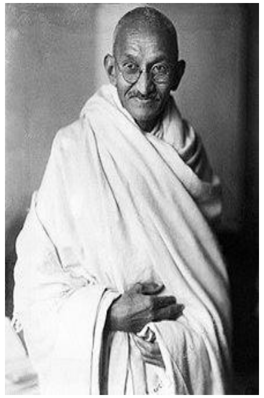



Confucius



Laozi

- **Logic in Reasoning:** Developed by Aristotle, it's a precise method for reasoning.
- **Beyond Propositions:** Other reasoning methods exist for problem-solving.
- **Role of Logic:** Central to mathematical statements, automated reasoning, and computer science.
- **Proofs and Theorems:** Correct arguments in math called proofs; proven statements are theorems.
- **Propositional Calculus:** A section of logic, also known as propositional or predicate logic, formalized by Aristotle.
- **Generating Propositions:** George Boole discussed methods in "The Laws of Thought" (1854).

- A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.
  1. Delhi is the capital of USA

  2. How are you doing

  3. 5<= 11

  4. Temperature is less than 10 C

  5. It is cold today

  6. Read this carefully

  7. X + y = z

- Premises is a statement that provides reason or support for the conclusion(proposition). Premises(proposition) is always considered to be true.
- If a set of Premises(P) yield another proposition Q(Conclusion), then it is called an Argument.
- An argument is said to be valid if the conclusion Q can be derived from the premises by applying the rules of inference.

$\{P_1, P_2, P_3 \ldots, P_N\} \vdash Q$	P_1 P_2 P_3 . . . P_N …… Q …….	$\{P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_N\}$ $\vdash Q$

- **<u>Law of contradiction</u> -** the law of non-contradiction (LNC) (also known as the law of contradiction, principle of non-contradiction (PNC), or the principle of contradiction) states that, ”Contradictory propositions cannot both be true in the same sense at the same time”.
  - e.g. the two propositions "*A is B*" and "*A is not B*" are mutually exclusive.

- **<u>Law of excluded middle</u> -** The law of excluded middle (or the principle of excluded middle) states that for any proposition, either that proposition is true or its negation is true.

Types of proposition

- We use letters to denote **propositional variables** to represent propositions, just as letters are used to denote numerical variables. The conventional letters used for propositional variables are *p, q, r, s.*

- Many mathematical statements are constructed by combining one or more propositions. New propositions, called **compound propositions**, are formed from existing propositions using logical operators.

Operators / Connectives

1. **Negation: -** let p be a proposition, then negation of p new proposition, denoted by $\neg p$, is the statement "it is not the case that p".

Negation	
P	¬P
F	
T	

Conjunction

- Let *p* and *q* be propositions. The *conjunction* of *p* and *q*, denoted by $p \land q$, is the proposition "*p* and *q*." The conjunction $p \land q$ is true when both *p* and *q* are true and is false otherwise.

Conjunction		
p	q	$p \land q$
F	F	
F	T	
T	F	
T	T	

Q Consider the following arguments and find which of them are valid?

1	
P_1	$(p \land q)$
Q	p

2	
P_1	P
Q	$p \land q$

3	
P_1	P
P_2	q
Q	$p \land q$

4	
P_1	$\neg(p \land q)$
P_2	P
Q	$\neg q$

5	
P_1	$\neg (p \land q)$
P_2	q
Q	$\neg p$

6	
P_1	$\neg (p \land q)$
P_2	$\neg p$
Q	q

7	
P_1	$\neg (p \land q)$
P_2	$\neg p$
Q	$\neg q$

Disjunction

- Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition. "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Disjunction		
p	q	$p \vee q$
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1

P_1	$(p \wedge q)$
Q	$p \vee q$

2

P_1	$p \vee q$
Q	$(p \wedge q)$

3

P_1	$\neg(p \vee q)$
Q	$\neg p$

4

P_1	$(p \vee q)$
Q	$\neg p$

5

P_1	$(p \vee q)$
P_2	$\neg p$
Q	q

6

P_1	$(p \vee q)$
P_2	$\neg q$
Q	p

7

P_1	$(p \vee q)$
P_2	p
Q	$\neg q$

8

P_1	$(p \vee q)$
P_2	p
Q	q

Implication

1. Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q". The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise.
2. In conditional statement $p \rightarrow q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion.

Implication		
p	q	$p \rightarrow q$
F	F	
F	T	
T	F	
T	T	

- Let p be the statement "Tori learns discrete mathematics" and q the statement "Tori will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

1. "If Tori learns discrete mathematics, then she will find a good job."

2. "Tori will find a good job when she learns discrete mathematics."

3. "For Tori to get a good job, it is sufficient for her to learn discrete mathematics."

p	q	$P \to q$	$\neg p$	$\neg q$	$\neg P \to \neg q$	$q \to p$	$\neg q \to \neg p$	$\neg p \vee q$
F	F							
F	T							
T	F							
T	T							

- $p \rightarrow q$ *implication*
- $q \rightarrow p$ *converse*
- $\neg p \rightarrow \neg q$ *inverse*
- $\neg q \rightarrow \neg p$ *contra positive*
- $p \rightarrow q = \neg q \rightarrow \neg p$
- $p \rightarrow q$ *will be true if either* p *is false or* q *is true,* $p \rightarrow q = \neg p \vee q$

Q Consider the following arguments and find which of them are valid?

Modus Ponens	
P_1	$p \longrightarrow q$
P_2	P
Q	Q

Modus Tollens	
P_1	$p \longrightarrow q$
P_2	¬Q
Q	¬p

1	
P_1	¬p
Q	$p \longrightarrow q$

2	
P_1	q
Q	$p \longrightarrow q$

3	
P_1	$\neg(p \longrightarrow q)$
Q	¬q

4	
P_1	$\neg(p \longrightarrow q)$
Q	p

Bi-conditional

- Let *p* and *q* be propositions. The bi*conditional statement* $p \leftrightarrow q$ is the proposition.
  - " *p if and only* q".
  - "*p* is necessary and sufficient for *q*"
  - "if *p* then *q*, and conversely"
  - "*p* iff *q*."
  - $p \longleftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$
- The biconditional statement $p \leftrightarrow q$ *is true when p and q have the same values, and false otherwise.*

Bi-conditional		
p	q	$P \leftrightarrow q$
F	F	
F	T	
T	F	
T	T	

Q Consider the following arguments and find which of them are valid?

1	
P_1	$p \rightarrow q$
P_2	$q \rightarrow r$
Q	$p \rightarrow r$

2	
P_1	$p \vee q$
P_2	$p \rightarrow r$
P_3	$q \rightarrow r$
Q	r

3	
P_1	$p \vee q$
P_2	$p \rightarrow r$
P_3	$q \rightarrow s$
Q	$r \vee s$

4	
P_1	$p \rightarrow r$
P_2	$q \rightarrow s$
P_3	$\neg r \vee \neg s$
Q	$\neg p \vee \neg q$

Q Consider the following arguments and find which of them are valid?

5	
P_1	p
P_2	q
Q	r

6	
P_1	p
P_2	¬p
Q	q

7	
P_1	
Q	q

Type of cases

- **Tautology/valid: -** A propositional function which is always having truth in the last column, is called tautology. E.g. $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
F	T	
T	F	

- **<u>Contradiction/ Unsatisfiable</u>: -** A propositional function which is always having false in the last column, is called Contradiction. E.g. $\boldsymbol{p \wedge \neg p}$

p	¬ p	p ⋀ ¬ p
F	T	
T	F	

- **Contingency:** - A propositional function which is neither a tautology nor a contradiction, is called Contingency. E.g. $p \lor q$

p	q	p ∨ q
F	F	
F	T	
T	F	
T	T	

- **<u>Satisfiable</u>: -** A propositional function which is not contradiction is satisfiable. i.e. it must have at least one truth value in the final column e.g. $\boldsymbol{p} \vee \boldsymbol{q}$

- **<u>Functionality Complete Set</u>: -** A set of connectives is said to be functionally complete if it is able to write any propositional function.
  - $\{\wedge, \neg\}$
  - $\{\vee, \neg\}$

Q Consider two well-formed formulas in propositional logic

F_1: $P \Rightarrow \neg P$ **F_2**: $(P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which one of the following statements is correct? **(GATE-2001) (1 Marks)**

(NET-Jan-2017)

A) F_1 is satisfiable, F_2 is valid

B) F_1 unsatisfiable, F_2 is satisfiable

C) F_1 is unsatisfiable, F_2 is valid

D) F_1 and F_2 are both satisfiable

Q Let p, q, and r be the propositions and the expression (p -> q) -> r be a contradiction. Then, the expression (r -> p)-> q is **(GATE-2017) (2 Marks)**

(A) a tautology

(B) a contradiction

(C) always TRUE when p is FALSE

(D) always TRUE when q is TRUE

Q the statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statement below? **(GATE-2017) (1 Marks)**

1) $p \Rightarrow q$ **2)** $q \Rightarrow p$ **3)** $(\neg q) \lor (p)$ **4)** $(\neg p) \lor q$

a) 1 only **b)** 1 and 4 only **c)** 2 only **d)** 2 and 3 only

Q Let **p**, **q**, **r** denotes the statement “It is raining”, “It is cold”, and “It is pleasant”, respectively. Then the statement “It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold” is represented by: **(GATE-2017) (1 Marks)**

(A) $(\neg p \land r) \land ((\neg r \rightarrow (p \land q))$

(B) $(\neg p \land r) \land ((p \land q) \rightarrow \neg r)$

(C) $(\neg p \land r) \lor ((p \land q) \rightarrow \neg r)$

(D) $(\neg p \land r) \lor ((r \rightarrow (p \land q))$

Q consider the following expression: **(GATE-2016) (1 Marks)**

i) false **ii)** Q **iii)** true **iv)** $P \lor Q$ **v)** $\neg Q \lor P$

The number of expressions given above that are logically implied by $P \land (P \Longrightarrow Q)$ is____

Q Let p, q, r, s represents the following propositions. **(GATE-2016) (1 Marks)**

p: $x \in \{8, 9, 10, 11, 12\}$

q: x is a composite number

r: x is a perfect square

s: x is a prime number

The integer $x \geq 2$ which satisfies $\neg((p \Rightarrow q) \land (\neg r \lor \neg s))$ is ______________.

Q Consider the following two statements.

S_1: If a candidate is known to be corrupt, then he will not be elected.

S_2: If a candidate is kind, he will be elected.

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic? **(GATE-2015) (1 Marks)**

(A) If a person is known to be corrupt, he is kind

(B) If a person is not known to be corrupt, he is not kind

(C) If a person is kind, he is not known to be corrupt

(D) If a person is not kind, he is not known to be corrupt

Q Which one of the following is NOT equivalent to $p \leftrightarrow q$? **(GATE-2015) (1 Marks)**

a) $(\neg p \lor q) \land (p \lor \neg q)$

b) $(\neg p \lor q) \land (q \rightarrow p)$

c) $(\neg p \land q) \lor (p \land \neg q)$

d) $(\neg p \land \neg q) \lor (p \land q)$

Q In propositional logic $P \leftrightarrow Q$ is equivalent to (Where ~ denotes NOT) **(GATE-2015) (1 Marks)**

a) $\sim(P \lor Q) \land \sim(Q \lor P)$

b) $(\sim P \lor Q) \land (\sim Q \lor P)$

c) $(P \lor Q) \land (Q \lor P)$

d) $\sim(P \lor Q) \rightarrow \sim(Q \lor P)$

Q In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking the person replies the following

"The result of the toss is head if and only if I am telling the truth"

Which of the following options is correct? **(Gate-2015)(2 Marks)**

a) The result is head

b) The result is tail

c) If the person is of Type 2, then the result is tail

d) If the person is of Type 1, then the result is tail

Q Consider the following statements:

P: Good mobile phones are not cheap

Q: Cheap mobile phones are not good

L: P implies Q

M: Q implies P

N: P is equivalent to Q

Which one of the following about L, M, and N is CORRECT? **(GATE-2014) (1 Marks)**

(A) Only L is TRUE.

(B) Only M is TRUE.

(C) Only N is TRUE.

(D) L, M and N are TRUE

Q Which one of the following Boolean expressions is NOT a tautology?

(GATE-2014) (2 Marks)

A) $((a \rightarrow b) \land (b \rightarrow c)) \rightarrow (a \rightarrow c)$

B) $(a \rightarrow c) \rightarrow (\sim b \rightarrow (a \land c))$

C) $(a \land b \land c) \rightarrow (c \lor a)$

D) $a \rightarrow (b \rightarrow a)$

Q Which one of the following propositional logic formulas is TRUE only when exactly two of p, q and r are TRUE? **(GATE-2014) (2 Marks)**

a) $((p \leftrightarrow q) \land r) \lor (p \land q \land \sim r)$

b) $(\sim (p \leftrightarrow q) \land r) \lor (p \land q \land \sim r)$

c) $((p \rightarrow q) \land r) \lor (p \land q \land \sim r)$

d) $(\sim (p \leftrightarrow q) \land r) \land (p \land q \land \sim r)$

Q Consider the following logical inferences.

I_1: If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.

I_2: If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.

Which of the following is **TRUE**? **(GATE-2012) (1 Marks)**
(A) Both I_1 and I_2 are correct inferences
(B) I_1 is correct but I_2 is not a correct inference
(C) I_1 is not correct but I_2 is a correct inference
(D) Both I_1 and I_2 are not correct inferences

Q The binary operation $\odot$ is defined as follows

Which one of the following is equivalent to $P \vee Q$? **(GATE-2009) (2 Marks)**

P	Q	$P \odot Q$
T	T	T
T	F	T
F	T	F
F	F	T

a) $(\sim Q \odot \sim P)$

b) $(P \odot \sim Q)$

c) $(\sim P \odot Q)$

d) $(\sim P \odot \sim Q)$

Q P and Q are two propositions. Which of the following logical expressions are equivalent? **(GATE-2008) (2 Marks)**

I) $P \lor \neg Q$

II) $\neg(\neg P \land Q)$

III) $(P \land Q) \lor (P \land \neg Q) \lor (\neg P \land \neg Q)$

IV) $(P \land Q) \lor (P \land \neg Q) \lor (\neg P \land Q)$

(A) Only I and II
(B) Only I, II and III
(C) Only I, II and IV
(D) All of I, II, III and IV

Q Consider the following propositional statements: **(GATE-2006) (2 Marks)**

$$\mathbf{P_1} : ((A \land B) \to C)) \equiv ((A \to C) \land (B \to C))$$

$$\mathbf{P_2} : ((A \lor B) \to C)) \equiv ((A \to C) \lor (B \to C))$$

Which one of the following is true?

(A) P_1 is a tautology, but not P_2

(B) P_2 is a tautology, but not P_1

(C) P_1 and P_2 are both tautologies

(D) Both P_1 and P_2 are not tautologies

Q A logical binary relation ⊙, is defined as follows: **(GATE-2006) (2 Marks)**

Let ~ be the unary negation (NOT) operator, with higher precedence then ⊙.

Which one of the following is equivalent to $A \wedge B$?

a) (~A⊙B) **b)** ~(A⊙~B) **c)** ~(~A⊙~B) **d)** ~(~A⊙B)

A	B	A ⊙ B
T	T	T
T	F	T
F	T	F
F	F	T

Q Let P, Q and R be three atomic prepositional assertions. Let X denote $(P \vee Q) \rightarrow R$ and Y denote $(P \rightarrow R) \vee (Q \rightarrow R)$. Which one of the following is a tautology? **(GATE-2005) (2 Marks)**

(A) $X \equiv Y$ **(B)** $X \rightarrow Y$ **(C)** $Y \rightarrow X$ **(D)** $\neg Y \rightarrow X$

Q The following propositional statement is **(GATE-2004) (2 Marks)**

$(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$

(A) satisfiable but not valid

(B) valid

(C) a contradiction

(D) none of the above

Q Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\neg p \lor q) \land (r \rightarrow s) \land (p \lor r)] \rightarrow (\neg s \rightarrow q)$

Q: $[(\neg p \land q) \land [q \rightarrow (p \rightarrow r)]] \rightarrow \neg r$

R: $[[(q \land r) \rightarrow p] \land (\neg q \lor p)] \rightarrow r$

S: $[p \land (p \rightarrow r) \land (q \lor \neg r)] \rightarrow q$

Which of the above arguments are valid? **(GATE-2004) (2 Marks)**

a) P and Q only

b) P and R only

c) P and S only

d) P, Q, R and S

Q Let a, b, c, d be propositions. Assume that the equivalences $a \leftrightarrow (b \lor \neg b)$ and $b \leftrightarrow c$ hold. Then the truth value of the formula $(a \land b) \rightarrow (a \land c) \lor d$ is always. **(GATE-2003) (2 Marks)**

(A) True

(B) False

(C) Same as the truth value of b

(D) Same as the truth value of d

Q The following resolution rule is used in logic programming:

Derive clause $(P \lor Q)$ from clauses $(P \lor R)$, $(Q \lor \neg R)$

Which of the following statements related to this rule is FALSE? **(GATE-2003) (2 Marks)**

(A) $((P \lor R) \land (Q \lor \neg R)) \Rightarrow (P \lor Q)$ is logically valid

(B) $(P \lor Q) \Rightarrow ((P \lor R) \land (Q \lor \neg R))$ is logically valid

(C) $(P \lor Q)$ is satisfiable if and only if $(P \lor R) \land (Q \lor \neg R)$ is satisfiable

(D) $(P \lor Q) \Rightarrow$ FALSE if and only if both P and Q are unsatisfiable

Q "If X, then Y unless Z" is represented by which of the following formulae in propositional logic? **(GATE-2002) (1 Marks)**

(A) $(X \wedge \neg Z) \rightarrow Y$

(B) $(X \wedge Y) \rightarrow \neg Z$

(C) $(X \rightarrow (Y \wedge \neg Z)$

(D) $(X \rightarrow Y (\wedge \neg Z)$

Q Which of the following is false? Read $\wedge$ as AND, $\vee$ as OR, $\neg$ as NOT, $\rightarrow$ as one-way implication and $\leftrightarrow$ as two-way implication **(GATE-1996) (2 Marks)**

a) $((x \rightarrow y) \wedge x) \rightarrow y$

b) $((\neg x \rightarrow y) \wedge (\neg x \rightarrow \neg y)) \rightarrow x$

c) $(x \rightarrow (x \vee y))$

d) $((x \vee y) \leftrightarrow (\neg x \rightarrow \neg y))$

Q If the proposition $\neg p \rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \rightarrow q)$, where $\neg$ is negation, $\vee$ is inclusive OR and $\rightarrow$ is implication, is **(GATE-1995) (2 Marks)**

a) True

b) Multiple Values

c) False

d) Cannot be determined

Q The proposition $p \wedge (\sim p \vee q)$ is: **(GATE-1993) (1 Marks)**

a) a tautology

b) logically equivalent to $p \wedge q$

c) logically equivalent to $p \vee q$

d) a contradiction

e) none of the above

Q Which of the following is/are a tautology? **(GATE-1992) (1 Marks)**

a) $a \lor b \to b \land c$

b) $a \land b \to b \lor c$

c) $a \lor b \to (b \to c)$

d) $a \to b \to (b \to c)$

Q Indicate which of the following well-formed formulae are valid:
(GATE-1990) (2 Marks)

a) $[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$

b) $(P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q)$

c) $(P \wedge (\neg P \vee \neg Q)) \Rightarrow Q$

d) $(P \Rightarrow R) \vee (Q \Rightarrow R) \Rightarrow ((P \vee Q) \Rightarrow R)$

Q Let p and q be two propositions. Consider the following two formulae in propositional logic.

S_1: $(\neg p \wedge (p \vee q)) \rightarrow q$

S_2: $q \rightarrow (\neg p \wedge (p \vee q))$

Which one of the following choices is correct? **(GATE 2021)**

(a) Both S_1 and S_2 are tautologies.

(b) S_1 is a tautology but S_2 is not a tautology

(c) S_1 is not a tautology but S_2 is a tautology

(d) Neither S_1 nor S_2 is a tautology

Q Choose the correct choice(s) regarding the following proportional logic assertion S:

$S:((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow (Q \rightarrow R))$

(GATE 2021) (1 MARKS)

(A) S is neither a tautology nor a contradiction

(B) S is a tautology

(C) S is a contradiction

(D) The antecedent of S is logically equivalent to the consequent of S

Q consider the following argument

I_1: if today is Gandhi ji's birthday, then today is oct 2nd

I_2: today is oct 2nd

C: today is Gandhi ji's birthday

Q consider the following argument

I_1: if Canada is a country, then London is a city

I_2: London is not a city

C: Canada is not a country

Find out whether these arguments are valid or invalid?

1	
P_1	$p \rightarrow q$
P_2	$q \rightarrow r$
P_3	$\neg r$
Q	$\neg p$

2	
P_1	$r \rightarrow s$
P_2	$p \rightarrow q$
P_3	$r \vee p$
Q	$s \vee q$

3	
P_1	$(p \rightarrow (q \rightarrow s))$
P_2	$\neg r \vee p$
P_3	q
P_4	p
Q	s

4	
P_1	$(p \rightarrow (r \rightarrow s))$
P_2	$\neg r \rightarrow \neg p$
P_3	p
Q	s

5	
P_1	$\neg p \rightarrow \neg r$
P_2	$\neg S$
P_3	$P \rightarrow w$
P_4	$R \vee s$
Q	W

6	
P_1	$\neg x \rightarrow y$
P_2	$\neg x \wedge \neg y$
Q	x

First order Predicate Logic

- Sometime propositional logic cannot derive any meaningful information even though, we as human can understand that argument is meaningful or not.

- P_1: Every Indian like cricket
- P_2: Sunny is an Indian
- Q: Sunny Likes cricket

- The reason propositional logic fails here because using only inference system we can not conclude Q from P_1 and P_2.

- In first order logic we understand, a new approach of subject and predicate to extract more information from a statement
  - 1 is a natural number (1 is subject, natural number is predicate)
  - we can write FOPL (short hand notation) for this as NatNo(1) = 1 is natural number
  - Similarly, we can understand the meaning of NatNo(2) as 2 is a natural number
  - NatNo(x): x is a natural number

- Sometime subject is not a single element but representing the entire group.
  - Every Indian like Cricket.
  - We can have a propositional function Cricket(x): x likes Cricket.
  - We can fix domain of discussion or universe of discourse as, x is an Indian.

- If i say four Indian are there I_1, I_2, I_3, I_4

- I_1 likes cricket $\wedge$ I_2 likes cricket $\wedge$ I_3 likes cricket $\wedge$ I_4 likes cricket

- Cricket(I_1) $\wedge$ Cricket(I_2) $\wedge$ Cricket(I_3) $\wedge$ Cricket(I_4)

- But problem with this notation is as there is 130+ corers Indian this formula will become very long and in some case we actually do not know how many subjects are there in the universe of discourse. so, we again need a short hand formula.

- $\forall_x$ Cricket(x), if we confine x to be Indian then it means every x like cricket.

Q. Let p and q be the following propositions:**(Gate 2024 CS)(1Mark)(MCQ)**
p: Fail grade can be given.
q: Student scores more than 50% marks.
Consider the statement: "Fail grade cannot be given when student scores more than 50% marks." Which one of the following is the CORRECT representation of the above statement in propositional logic?

(a) $q \rightarrow \neg p$

(b) $q \rightarrow p$

(c) $p \rightarrow q$

(d) $\neg p \rightarrow q$

- **<u>Universal quantifiers</u>**: - The universal quantification of a propositional function is the proposition that asserts

- P(x) is true for all values of x in the universe of discourse.

- The universe of discourse specifies the possible value of x.

- $\forall_x$ P(x), i.e. for all value of a P(x) is true

- Let try some other statement ‘Some Indian like samosa’
  - if i say four Indian are there I_1, I_2, I_3, I_4
  - I_1 like samosa $\vee$ I_2 like samosa $\vee$ I_3 like samosa $\vee$ I_4 like samosa
  - $Samosa(I_1) \vee Samosa(I_2) \vee Samosa(I_3) \vee Samosa(I_4)$
  - $\exists_x$ Samosa(x), if we confine x to be Indian then it means some x likes samosa.

- **<u>Existential quantifiers</u>**: - with existential quantifier of a propositional that is true if and only if P(x) is true for at least one value of x in the universe of discourse.

- There exists an element x is the universe of discourse such that P(x) is true.

- $\exists_x$ P(x), i.e. for at least one value of a P(x) is true

- let’s change the universe of discourse from Indian to human
  - if human is Indian then it likes cricket
  - Indian(x): x is an Indian
  - Cricket(x): x likes Cricket

  - if I_1 is Indian then likes cricket $\wedge$ if I_2 is Indian then likes cricket $\wedge$ if I_3 is Indian then likes cricket $\wedge$ if I_4 is Indian then likes cricket

  - $[\text{Indian}(I_1) \rightarrow \text{cricket}(I_1)] \wedge [\text{Indian}(I_2) \rightarrow \text{cricket}(I_2)] \wedge [\text{Indian}(I_3) \rightarrow \text{cricket}(I_3)] \wedge [\text{Indian}(I_4) \rightarrow \text{cricket}(I_4)]$

  - $\forall_x [\text{Indian}(x) \rightarrow \text{cricket}(x)]$

- let's change the universe of discourse from Indian to human
  - if human is Indian then it likes samosa
  - Indian(x): x is an Indian
  - Samosa(x): x likes Samosa

  - if I_1 is Indian then likes samosa $\vee$ if I_2 is Indian then likes samosa $\vee$ if I_3 is Indian then likes samosa $\vee$ if I_4 is Indian then likes samosa

  - $[\text{Indian}(I_1) \wedge \text{samosa}(I_1)] \vee [\text{Indian}(I_2) \wedge \text{samosa}(I_2)] \vee [\text{Indian}(I_3) \wedge \text{samosa}(I_3)] \vee [\text{Indian}(I_4) \wedge \text{samosa}(I_4)]$

  - $\exists_x [\text{Indian}(x) \wedge \text{samosa}(x)]$

- let check validity of a statement “Some Indians like samosa” = $\exists_x$ [Indian(x) □samosa(x)], x is human
- let human contains four elements $I_{1,}$ $I_{2,}$ $I_{3,}$ I_4 out of which $I_{1,}$ I_2 are Indian while $I_{3,}$ I_4 are not Indian
- Suppose $I_{1,}$ $I_{2,}$ I_3 do not likes samosa
  - [Indian(I_1) □ samosa(I_1)] $\vee$ [Indian(I_2) □ samosa(I_2)] $\vee$ [Indian(I_3) □ samosa(I_3)]
  - [T □ F] $\vee$ [T □ F] $\vee$ [F □ F]
  - [F] $\vee$ [F] $\vee$ [T]
  - T
- conclusion $\exists_x$ is not used with □

Negation

- $\neg [\forall_x P(x)] = \exists_x \neg P(x)$
- $\neg [\exists_x P(x)] = \forall_x \neg P(x)$

Let L(x, y): x like y, which means x likes y or y is liked by x

1- $\forall_x \forall_y L(x, y)$

2- $\forall_y \forall_x L(x, y)$

3- $\exists_x \exists_y L(x, y)$

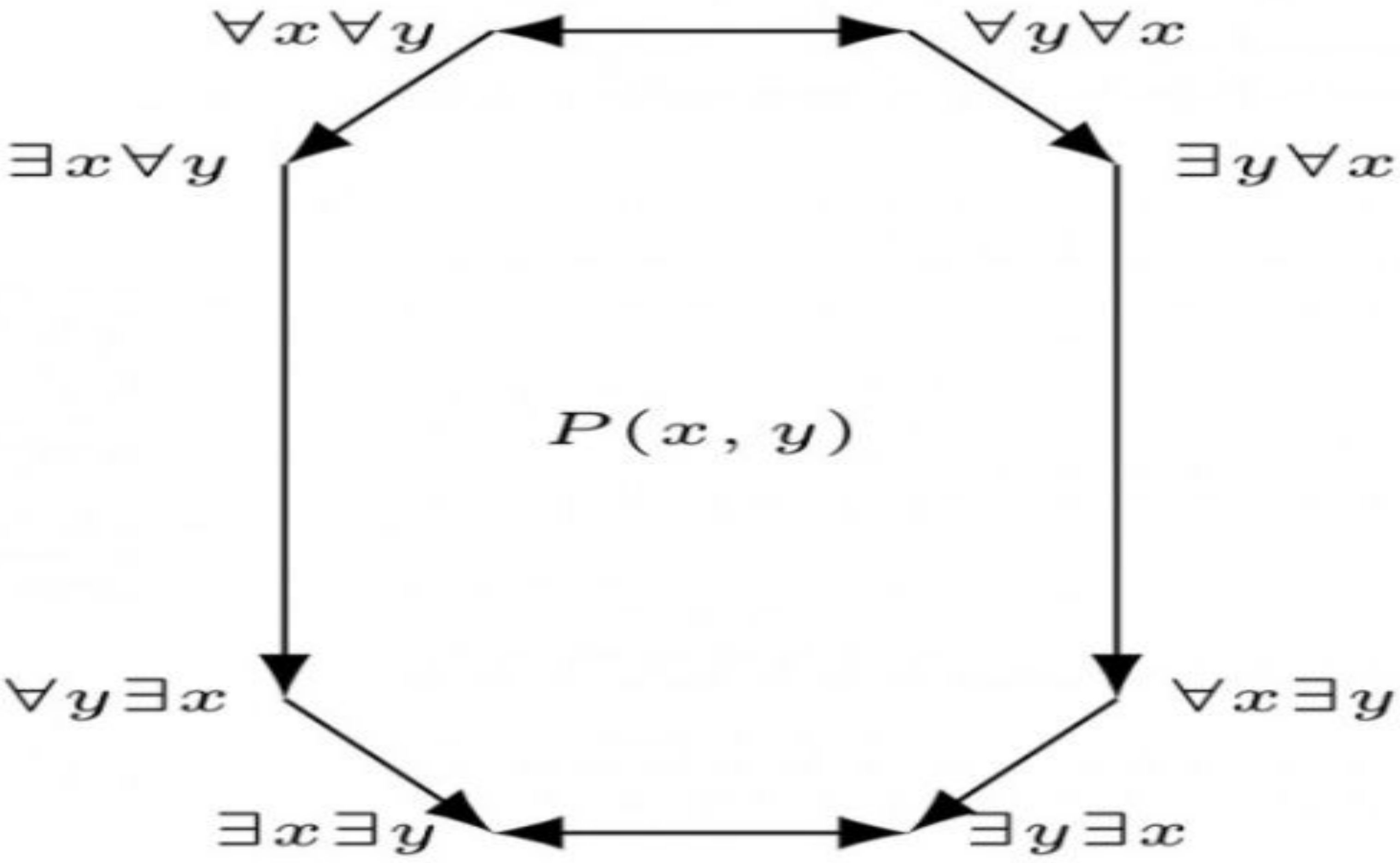
4- $\exists_y \exists_x L(x, y)$

5- $\forall_x \exists_y L(x, y)$

6- $\exists_y \forall_x L(x, y)$

7- $\forall_y \exists_x L(x, y)$

8- $\exists_x \forall_y L(x, y)$



$\forall x \forall y$
$\forall y \forall x$
$\exists x \forall y$
$\exists y \forall x$
$P(x, y)$
$\forall y \exists x$
$\forall x \exists y$
$\exists x \exists y$
$\exists y \exists x$

	1
P_1	$\exists_x P(x) \lor \exists_x Q(x)$
Q	$\exists_x (P(x) \lor Q(x))$

	2
P_1	$\exists_x (P(x) \lor Q(x))$
Q	$\exists_x P(x) \lor \exists_x Q(x)$

	3
P_1	$\exists_x P(x) \land \exists_x Q(x)$
Q	$\exists_x (P(x) \land Q(x))$

	4
P_1	$\exists_x (P(x) \land Q(x))$
Q	$\exists_x P(x) \land \exists_x Q(x)$

	1
P_1	$\forall_x P(x) \lor \forall_x Q(x)$
Q	$\forall_x (P(x) \lor Q(x))$

	2
P_1	$\forall_x (P(x) \lor Q(x))$
Q	$\forall_x P(x) \lor \forall_x Q(x)$

	3
P_1	$\forall_x P(x) \land \forall_x Q(x)$
Q	$\forall_x (P(x) \land Q(x))$

	4
P_1	$\forall_x (P(x) \land Q(x))$
Q	$\forall_x P(x) \land \forall_x Q(x)$

Q consider the statement $\exists_x [P(x) \land \neg Q(x)]$, Which of the following is equivalent?

a) $\forall_x [P(x) \rightarrow Q(x)]$

b) $\forall_x [\neg P(x) \rightarrow Q(x)]$

c) $\neg \{\forall_x [P(x) \rightarrow Q(x)]\}$

d) $\neg \{\forall_x [\neg P(x) \rightarrow Q(x)]\}$

Q negation of the statement

$\exists_x \forall_y [F(x, y) \rightarrow \{G(x, y) \vee H(x, y)\}] = \forall_x \exists_y [F(x, y) \wedge \{\neg G(x, y) \wedge \neg H(x, y)\}]$?

Q let in a set of all integers

G (x, y): x is greater than y

"for any given positive integer, there is a greater positive integer"

a) $\forall_x \exists_y G(x, y)$

b) $\exists_y \forall_x G(x, y)$

c) $\forall_y \exists_x G(x, y)$

d) $\exists_x \forall_y G(x, y)$

Q let in a set of all humans

L (x, y): x likes y

"there is someone, whom no one like"

a) $\forall_x \exists_y \{\neg L(x, y)\}$

b) $\{\neg \forall_x \exists_y L(x, y)\}$

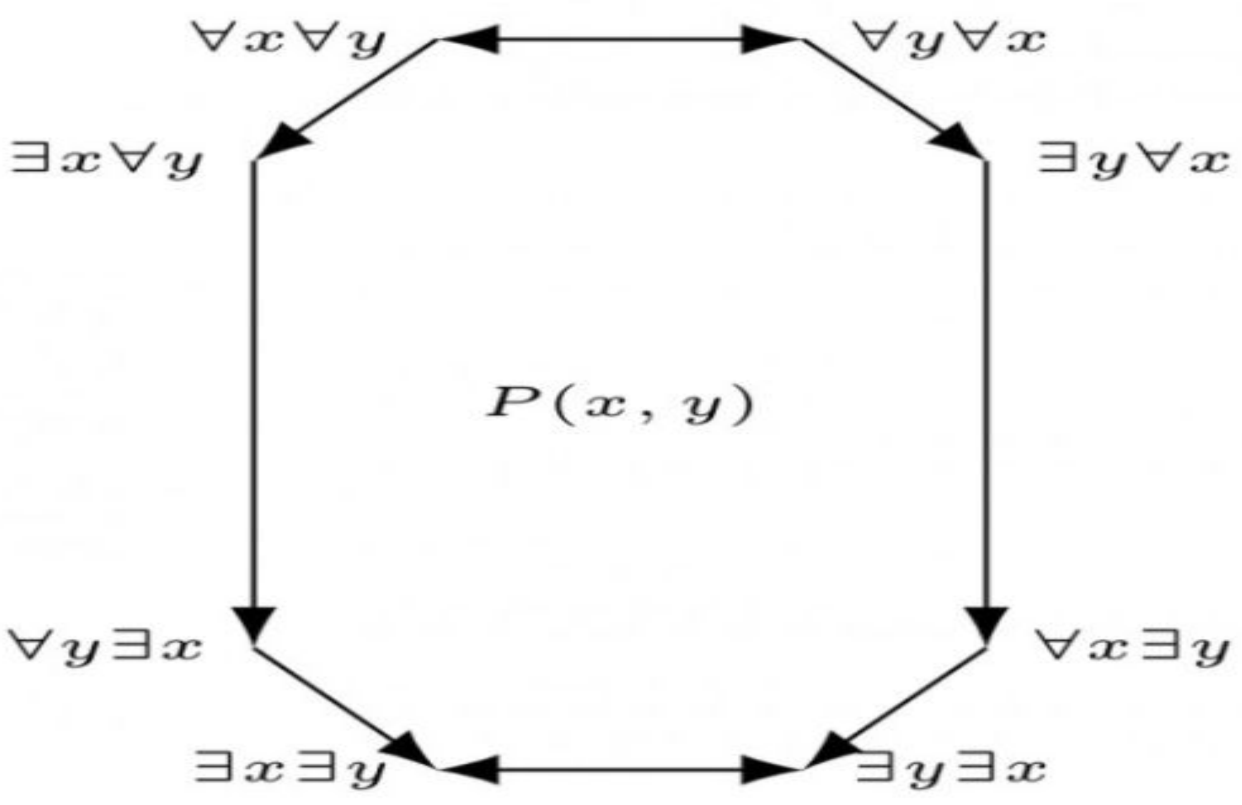
c) $\neg \{\forall_y \exists_x L(x, y)\}$

d) $\neg \{\exists_Y \forall_X L(x, y)\}$

Q Consider the first-order logic sentence
F: $\forall_x (\exists_y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are implied by F? **(GATE-2017) (1 Marks)**

I. $\exists_y (\exists_x R(x, y))$ II. $\exists_y (\forall_x R(x, y))$ III. $\forall_y (\exists_x R(x, y))$ IV. $\sim \exists_x (\forall_y \sim R(x, y))$

(A) IV only
(B) I and IV only
(C) II only
(D) II and III only



Q Which one of the following well-formed formulae in predicate calculus is **NOT** valid? **(GATE-2016) (2 Marks)**

a) $(\forall_x p(x) \Rightarrow \forall_x q(x)) \Rightarrow (\exists_x \neg p(x) \lor \forall_x q(x))$

b) $(\exists_x p(x) \lor \exists_x q(x)) \Rightarrow \exists_x (p(x) \lor q(x))$

c) $\exists_x (p(x) \land q(x)) \Rightarrow (\exists_x p(x) \land \exists_x q(x))$

d) $\forall_x (p(x) \lor q(x)) \Rightarrow (\forall_x p(x) \lor \forall_x q(x))$

Q Which one of the following well-formed formulae is a tautology? **(GATE-2015) (2 Marks)**

a) $\forall_x \exists_y R(x, y) \leftrightarrow \exists_y \forall_x R(x, y)$

b) $(\forall_x [\exists_y R(x, y) \rightarrow S(x, y)]) \rightarrow \forall_x \exists_y S(x, y)$

c) $[\forall_x \exists_y (P(x, y) \rightarrow R(x, y))] \leftrightarrow [\forall_x \exists_y (\neg P(x, y) \vee R(x, y))]$

d) $\forall_x \forall_y P(x, y) \rightarrow \forall_x \forall_y P(y, x)$

Q The CORRECT formula for the sentence, **"not all rainy days are cold"** is **(GATE-2014) (2 Marks)**

a) $\forall_d (Rainy(d) \wedge \sim Cold(d))$

b) $\forall_d (\sim Rainy(d) \rightarrow Cold(d))$

c) $\exists_d (\sim Rainy(d) \rightarrow Cold(d))$

d) $\exists_d (Rainy(d) \wedge \sim Cold(d))$

Q Consider the statement

"Not all that glitters is gold"

Predicate glitters(x) is true if x glitters and predicate gold(x) is true if x is gold. Which one of the following logical formulae represents the above statement? **(GATE-2014) (1 Marks)**

a) $\forall_x$: glitters(x) ⇒ ¬gold(x)

b) $\forall_x$: gold(x) ⇒ glitters(x)

c) $\exists_x$: gold(x) ∧ ¬glitters(x)

d) $\exists_x$: glitters(x) ∧ ¬gold(x)

Q Which one of the following is **NOT** logically equivalent to $\neg\exists x(\forall y(\alpha) \land \forall z(\beta))$? **(GATE-2013) (2 Marks)**

a) $\forall_x(\exists_z(\neg\beta) \rightarrow \forall_y(\alpha))$

b) $\forall_x(\forall_z(\beta) \rightarrow \exists_y(\neg\alpha))$

c) $\forall_x(\forall_y(\alpha) \rightarrow \exists_z(\neg\beta))$

d) $\forall_x(\exists_y(\neg\alpha) \rightarrow \exists_z(\neg\beta))$

Q What is the logical translation of the following lowing statement? **(GATE-2013) (2 Marks)**

"None of my friends are perfect."

A) $\exists_x(F(x) \land \neg P(x))$

B) $\exists_x(\neg F(x) \land P(x))$

C) $\exists_x(\neg F(x) \land \neg P(x))$

D) $\neg \exists_x(F(x) \land P(x))$

Q What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational" (GATE-2012) (1 Marks)

a) $\exists_x (\text{real}(x) \lor \text{rational}(x))$

b) $\forall_x (\text{real}(x) \rightarrow \text{rational}(x))$

c) $\exists_x (\text{real}(x) \land \text{rational}(x))$

d) $\exists_x (\text{rational}(x) \rightarrow \text{real}(x))$

Q Which one of the following options is CORRECT given three positive integers x, y and z, and a predicate? **(GATE-2011) (2 Marks)**

$$P(x) = \neg(x=1) \wedge \forall_y (\exists_z (x=y*z) \Rightarrow (y=x) \vee (y=1))$$

(A) P(x) being true means that x is a prime number

(B) P(x) being true means that x is a number other than 1

(C) P(x) is always true irrespective of the value of x

(D) P(x) being true means that x has exactly two factors other than 1 and x

Q Suppose the predicate F (x, y, t) is used to represent the statement that person x can fool person y at time t. which one of the statements below expresses best the meaning of the formula $\forall_x \exists_y \exists_t (\neg F(x, y, t))$? **(GATE-2010) (2 Marks)**

(A) Everyone can fool some person at some time

(B) No one can fool everyone all the time

(C) Everyone cannot fool some person all the time

(D) No one can fool some person at some time

Q Consider the following well-formed formulae:

1) $\neg \forall x(P(x))$ **2)** $\neg \exists x(P(x))$ **3)** $\neg \exists x(\neg P(x))$ **4)** $\exists x(\neg P(x))$

Which of the above are equivalent? **(GATE-2009) (2 Marks)**

a) I and III

b) I and IV

c) II and III

d) II and IV

Q Which one of the following is the most appropriate logical formula to represent the statement? **"Gold and silver ornaments are precious".** The following notations are used:

G(x): x is a gold ornament

S(x): x is a silver ornament

P(x): x is precious **(GATE-2009) (2 Marks)**

(A) $\forall_x (P(x) \rightarrow (G(x) \land S(x)))$

(B) $\forall_x ((G(x) \land S(x)) \rightarrow P(x))$

(C) $\exists_x ((G(x) \land S(x)) \rightarrow P(x)$

(D) $\forall_x ((G(x) \lor S(x)) \rightarrow P(x))$

Q Let fsa and pda be two predicates such that fsa(x) means x is a finite state automaton, and pda(y) means that y is a pushdown automaton. Let equivalent be another predicate such that equivalent (a, b) means a and b are equivalent. Which of the following first order logic statements represents the following. Each finite state automaton has an equivalent pushdown automaton. **(GATE-2008) (1 Marks)**

a) $(\forall_x \text{ fsa}(x)) \Rightarrow (\exists y \text{ pda}(y) \land \text{equivalent}(x, y))$

b) $\neg \forall_y(\exists x \text{ fsa}(x) \Rightarrow \text{pda}(y) \land \text{equivalent}(x, y))$

c) $\forall_x \exists_y (\text{fsa}(x) \land \text{pda}(y) \land \text{equivalent}(x, y))$

d) $\forall_x \exists_y(\text{fsa}(y) \land \text{pda}(x) \land \text{equivalent}(x, y))$

Q Which of the following first order formula is logically valid? Here $\alpha(x)$ is a first order formula with x as a free variable, and β is a first order formula with no free variable. **(GATE-2008) (2 Marks)**

(A) $[\beta \rightarrow (\exists x, \alpha(x))] \rightarrow [\forall x, \beta \rightarrow \alpha(x)]$

(B) $[\exists x, \beta \rightarrow \alpha(x)] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]$

(C) $[(\exists x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

(D) $[(\forall x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

Q Let Graph(x) be a predicate which denotes that x is a graph. Let Connected(x)be a predicate which denotes that x is connected. Which of the following first order logic sentences DOES NOT represent the statement: **“Not every graph is connected”** ? **(GATE-2007) (2 Marks)**

(A) $\neg \forall_x (\text{Graph}(x) \rightarrow \text{Connected}(x))$

(B) $\exists_x (\text{Graph}(x) \land \neg\text{Connected}(x))$

(C) $\neg \forall_x (\neg\text{Graph}(x) \lor \text{Connected}(x))$

(D) $\forall_x (\text{Graph}(x) \rightarrow \neg\text{Connected}(x))$

Q Which one of these first-order logic formulae is valid? **(GATE-2007) (2 Marks)**

(A) $\forall x(P(x) \Rightarrow Q(x)) \Rightarrow (\forall x P(x) \Rightarrow \forall x Q(x))$

(B) $\exists x(P(x) \vee Q(x)) \Rightarrow (\exists x P(x) \Rightarrow \exists x Q(x))$

(C) $\exists x(P(x) \wedge Q(x)) \Leftrightarrow (\exists x P(x) \wedge \exists x Q(x))$

(D) $\forall x \exists y\, P(x, y) \Rightarrow \exists y \forall x\, P(x, y)$

Q Which one of the first order predicate calculus statements given below correctly express the following English statement? **(GATE-2006) (2 Marks)**

"Tigers and lions attack if they are hungry or threatened"

a) $\forall_x[(\text{tiger}(x) \land \text{lion}(x)) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x)) \rightarrow \text{attacks}(x)]$

b) $\forall_x[(\text{tiger}(x) \lor \text{lion}(x)) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x)) \land \text{attacks}(x)]$

c) $\forall_x[(\text{tiger}(x) \lor \text{lion}(x)) \rightarrow \text{attacks}(x) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x))]$

d) $\forall_x[(\text{tiger}(x) \lor \text{lion}(x)) \rightarrow (\text{hungry}(x) \lor \text{threatened}(x)) \rightarrow \text{attacks}(x)]$

Q What is the first order predicate calculus statement equivalent to the following?

Every teacher is liked by some student (GATE-2005) (2 Marks)

A) $\forall_{(x)}$ [teacher (x) $\rightarrow$ $\exists_{(y)}$[student (y) $\rightarrow$ likes (y, x)]]

(B) $\forall_{(x)}$ [teacher (x) $\rightarrow$ $\exists_{(y)}$[student (y) ^ likes (y, x)]]

(C) $\exists_{(y)}$ $\forall_{(x)}$ [teacher (x) $\rightarrow$ [student (y) ^ likes (y, x)]]

(D) $\forall_{(x)}$ [teacher (x) ^ $\exists_{(y)}$[student (y) $\rightarrow$ likes (y, x)]]

Q Let P(x) and Q(x) be arbitrary predicates. Which of the following statements is always TRUE? **(GATE-2005) (2 Marks)**

(A) $((\forall x(P(x) \lor Q(x)))) \Rightarrow ((\forall xP(x)) \lor (\forall xQ(x)))$

(B) $(\forall x(P(x) \Rightarrow Q(x))) \Rightarrow ((\forall xP(x)) \Rightarrow (\forall xQ(x)))$

(C) $(\forall x(P(x)) \Rightarrow \forall x(Q(x))) \Rightarrow (\forall x(P(x) \Rightarrow Q(x)))$

(D) $(\forall x(P(x)) \Leftrightarrow (\forall x(Q(x)))) \Rightarrow (\forall x(P(x) \Leftrightarrow Q(x)))$

Q Let a(x, y), b(x, y,) and c(x, y) be three statements with variables x and y chosen from some universe. Consider the following statement: **(GATE-2004) (2 Marks)**

$(\exists x)(\forall y)\ [(a(x, y) \land b(x, y)) \land \neg c(x, y)]$

Which one of the following is its equivalent?

a) $(\forall x)(\exists y)\ [(a(x, y) \lor b(x, y)) \rightarrow c(x, y)]$ **b)** $(\exists x)(\forall y)[(a(x, y) \lor b(x, y)) \land \neg c(x, y)]$

c) $\neg(\forall x)(\exists y)[(a(x, y) \land b(x, y)) \rightarrow c(x, y)]$ **d)** $\neg(\forall x)(\exists y)[(a(x, y) \lor b(x, y)) \rightarrow c(x, y)]$

Q Identify the correct translation into logical notation of the following assertion.

Some boys in the class are taller than all the girls

Note: taller (x, y) is true if x is taller than y. **(GATE-2004) (1 Marks)**

(A) $(\exists_x)\ (boy(x) \rightarrow (\forall_y)\ (girl(y) \wedge taller\ (x, y)))$

(B) $(\exists_x)\ (boy(x) \wedge (\forall_y)\ (girl(y) \wedge taller\ (x, y)))$

(C) $(\exists_x)\ (boy(x) \rightarrow (\forall_y)\ (girl(y) \rightarrow taller\ (x, y)))$

(D) $(\exists_x)\ (boy(x) \wedge (\forall_y)\ (girl(y) \rightarrow taller\ (x, y)))$

Q Which of the following is a valid first order formula? (Here α and β are first order formulae with x as their only free variable) **(GATE-2003) (2 Marks)**

a) $\{(\forall x)[\alpha] \Rightarrow (\forall x)[\beta]\} \Rightarrow \{(\forall x)[\alpha \Rightarrow \beta]\}$

b) $(\forall x)[\alpha] \Rightarrow (\exists x)[\alpha \wedge \beta]$

c) $\{(\forall x)[\alpha \vee \beta]\} \Rightarrow \{(\exists x)[\alpha]) \Rightarrow (\forall x)[\alpha]\}$

d) $(\forall x)[\alpha \Rightarrow \beta] \Rightarrow ((\forall x)[\alpha]) \Rightarrow (\forall x)[\beta])$

Q Which of the following predicate calculus statements is/are valid? **(GATE-1992) (1 Marks)**

a) $(\forall(x))P(x) \lor (\forall(x))Q(x) \Rightarrow (\forall(x))(P(x) \lor Q(x))$

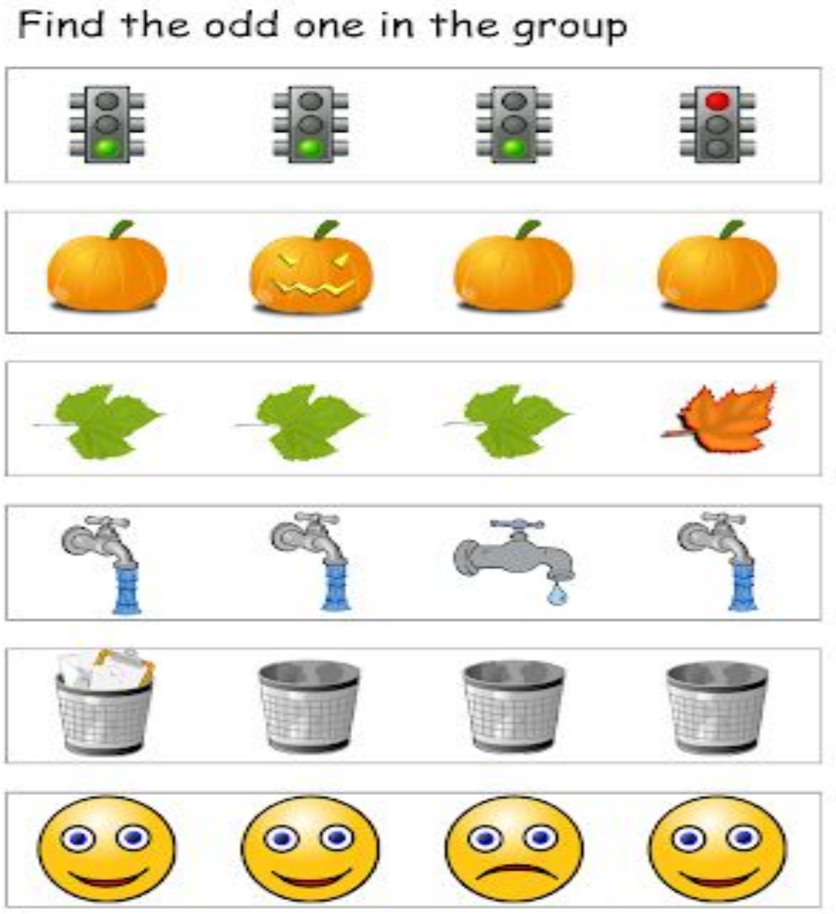
b) $(\exists(x))P(x) \land (\exists(x))Q(x) \Rightarrow (\exists(x))(P(x) \land Q(x))$

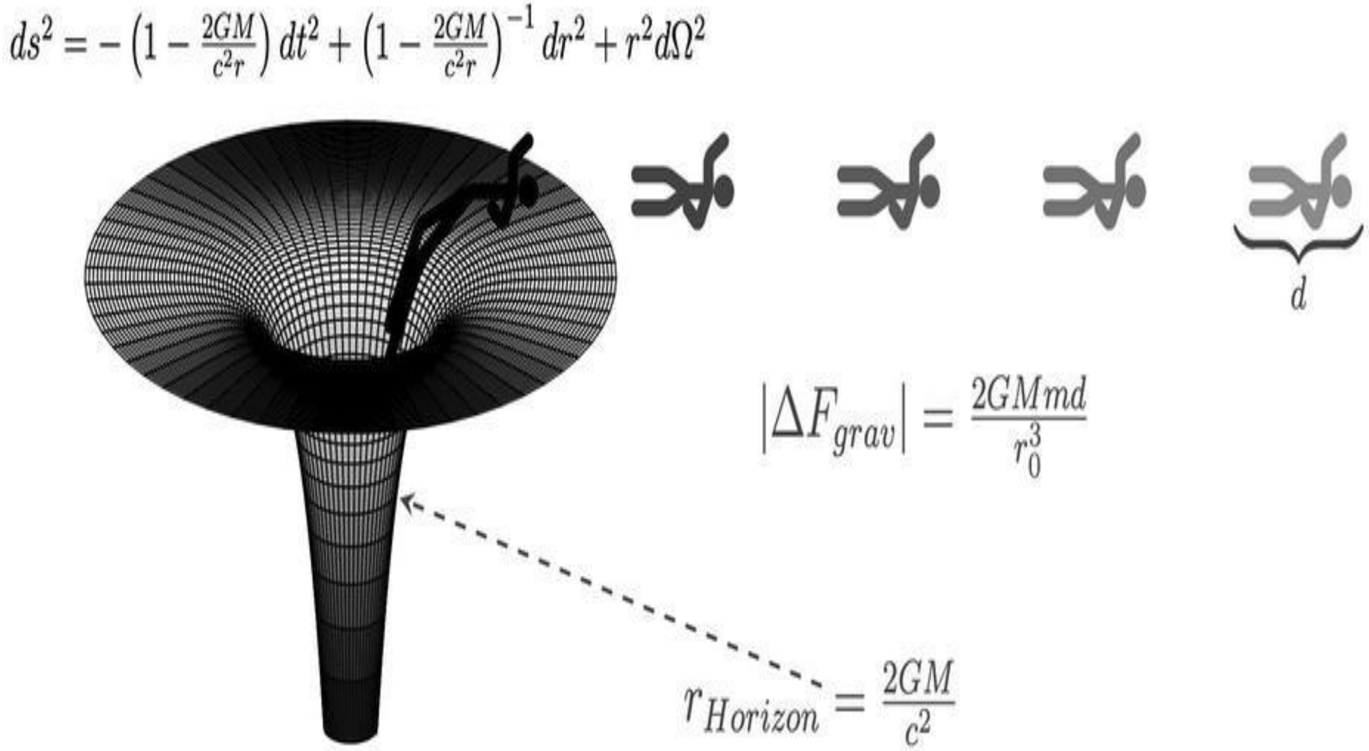
c) $(\forall(x))(P(x) \lor Q(x)) \Rightarrow (\forall(x))P(x) \lor (\forall(x))Q(x)$

d) $(\exists(x))(P(x) \lor Q(x)) \Rightarrow \sim(\forall(x))P(x) \lor (\exists(x))Q(x)$

Group Theory

- Group theory is very important mathematical tool which is used in a number of areas in research and application. Using group theory, we can estimate the strength of a set with respect to an operator. This idea will further help us in research field to identify the correct mathematical system to work in a particular research area. E.g. can we use natural numbers in complex problem area like soft computing or studying black holes.





1. **Closure property: -** Consider a non-empty set A and a binary operation * on A. A is said to be closed with respect to *, if ∀ a, b ∈ A, then a*b ∈ A.

2. **Algebraic Structure: -** A non-empty set A is said to be an algebraic structure with respect to a binary operation *, if A satisfy closure property with respect to *.

		Algebraic Structure
1	(N, +)	
2	(N, -)	
3	(N, /)	
4	(N, x)	
5	(Z, +)	
6	(Z, -)	
7	(Z, /)	
8	(Z, x)	
9	(R, +)	
10	(R, -)	
11	(R, /)	
12	(R, x)	
13	(M, +)	
14	(M, x)	
15	(E, +)	
16	(E, x)	
17	(O, +)	
18	(O, x)	
19	(R-0, x)	
20	(R-0, /)	
21	(Non-Singular Matrix, x)	

1. **Associative property: -** Consider a non-empty set A and a binary operation * on A. A is said to be associative with respect to *, if $\forall$ a, b, c $\in$ A, then (a*b) *c = a*(b*c)

2. **Semi-Group: -** A non-empty set A is said to be a Semi-group with respect to a binary operation *, if A satisfy closure, Associative property with respect to *.

		Algebraic Structure	Semi Group
1	(N, +)	Y	
2	(N, -)	N	
3	(N, /)	N	
4	(N, x)	Y	
5	(Z, +)	Y	
6	(Z, -)	Y	
7	(Z, /)	N	
8	(Z, x)	Y	
9	(R, +)	Y	
10	(R, -)	Y	
11	(R, /)	N	
12	(R, x)	Y	
13	(M, +)	Y	
14	(M, x)	Y	
15	(E, +)	Y	
16	(E, x)	Y	
17	(O, +)	N	
18	(O, x)	Y	
19	(R-0, x)	Y	
20	(R-0, /)	Y	
21	(Non-Singular Matrix, x)	Y	

Q. Consider the operators () and □ defined by

a () b = a + 2b

a □ b = ab,

for positive integers, Which of the following statements is/are TRUE? **(Gate 2024,CS)(2 Marks)(MSQ)**

(a) Operator () obeys the associative law

(b) Operator □ obeys the associative law

(c) Operator () over the operator □ obeys the distributive law

(d) Operator □ over the operator () obeys the distributive law

1. **<u>Identity property: -</u>** Consider a non-empty set A and a binary operation * on A. A is said to satisfy identity property with respect to *, if ∀ a ∈ A, there must be unique e ∈ A, such that
   a*e = e*a = a

2. There is exactly one Identity element in the set and will be same for all element in the set.

3. **<u>Monoid: -</u>** A non-empty set A is said to be a Monoid with respect to a binary operation *, if A satisfy closure, Associative, identity property with respect to *.

		Algebraic Structure	Semi Group	Monoid
1	(N, +)	Y	Y	
2	(N, -)	N	N	
3	(N, /)	N	N	
4	(N, x)	Y	Y	
5	(Z, +)	Y	Y	
6	(Z, -)	Y	N	
7	(Z, /)	N	N	
8	(Z, x)	Y	Y	
9	(R, +)	Y	Y	
10	(R, -)	Y	N	
11	(R, /)	N	N	
12	(R, x)	Y	Y	
13	(M, +)	Y	Y	
14	(M, x)	Y	Y	
15	(E, +)	Y	Y	
16	(E, x)	Y	Y	
17	(O, +)	N	N	
18	(O, x)	Y	Y	
19	(R-0, x)	Y	Y	
20	(R-0, /)	Y	N	
21	(Non-Singular Matrix, x)	Y	Y	

1. **Inverse property: -** Consider a non-empty set A and a binary operation * on A. A is said to satisfy inverse property with respect to *, if ∀ a ∈ A, there must be unique element a^{-1} ∈ A, such that $a * a^{-1} = a^{-1} * a = e$

2. Every element has a exactly one unique inverse which is also present in the same set.

3. If a is the inverse of b, then b will be invers one a.

4. No two elements can have the same inverse

5. Identity element is its own inverse.

6. **Group: -** A non-empty set A is said to be a group with respect to a binary operation *, if A satisfy closure, Associative, identity, inverse property with respect to *.

		AS	Semi Group	Monoid	Group
1	(N, +)	Y	Y	N	
2	(N, -)	N	N	N	
3	(N, /)	N	N	N	
4	(N, x)	Y	Y	Y	
5	(Z, +)	Y	Y	Y	
6	(Z, -)	Y	N	N	
7	(Z, /)	N	N	N	
8	(Z, x)	Y	Y	Y	
9	(R, +)	Y	Y	Y	
10	(R, -)	Y	N	N	
11	(R, /)	N	N	N	
12	(R, x)	Y	Y	Y	
13	(M, +)	Y	Y	Y	
14	(M, x)	Y	Y	Y	
15	(E, +)	Y	Y	Y	
16	(E, x)	Y	Y	N	
17	(O, +)	N	N	N	
18	(O, x)	Y	Y	Y	
19	(R-0, x)	Y	Y	Y	
20	(R-0, /)	Y	N	N	
21	(Non-Singular Matrix, x)	Y	Y	Y	

1. If the total number of elements in a group is even then there exists at least one element in the group who is the inverse of itself.

2. Some time it is also possible that every element is inverse of itself in a group.

3. In a group $(a * b)^{-1} = b^{-1} * a^{-1}$ for $\forall$ $a, b \in A$

4. Cancelation law holds good

    1. $a * b = a * c$ □ $b = c$

    2. $a * c = b * c$ □ $a = b$

1. **Commutative property: -** Consider a non-empty set A and a binary operation * on A. A is said to satisfy commutative property with respect to *, if ∀ a, b ∈ A, such that a* b = b*a

2. **Abelian Group: -** A non-empty set A is said to be a group with respect to a binary operation *, if A satisfy closure, Associative, identity, inverse, commutative property with respect to *.

		AS	SG	Monoid	Group	Abelian Group
1	(N, +)	Y	Y	N	N	
2	(N, -)	N	N	N	N	
3	(N, /)	N	N	N	N	
4	(N, x)	Y	Y	Y	N	
5	(Z, +)	Y	Y	Y	Y	
6	(Z, -)	Y	N	N	N	
7	(Z, /)	N	N	N	N	
8	(Z, x)	Y	Y	Y	N	
9	(R, +)	Y	Y	Y	Y	
10	(R, -)	Y	N	N	N	
11	(R, /)	N	N	N	N	
12	(R, x)	Y	Y	Y	N	
13	(M, +)	Y	Y	Y	Y	
14	(M, x)	Y	Y	Y	N	
15	(E, +)	Y	Y	Y	Y	
16	(E, x)	Y	Y	N	N	
17	(O, +)	N	N	N	N	
18	(O, x)	Y	Y	Y	N	
19	(R-0, x)	Y	Y	Y	Y	
20	(R-0, /)	Y	N	N	N	
21	(Non-Singular Matrix, x)	Y	Y	Y	Y	

Q Consider a set of natural numbers N, with respect to *, such that a * b = a^b which of the following is true?

a) semi group but not monoid **b)** A monoid but not a group

c) A group **d)** not a semi group

Q let {p, q, r, s} be the set. A binary operation * is defined on the set and is given by the following table:Which of the following is true about the binary operation?

*	p	q	r	s
P	p	r	s	p
q	p	q	r	s
r	p	q	p	r
S	p	q	q	q

a) it is commutative but not associative

b) it is associative but not commutative

c) it is both associative and commutative

d) it is neither associative nor commutative

Q Consider a set of integers Z, with respect to *, such that a * b = max (a, b) which of the following is true?

a) Algebraic structure

b) semi-group

c) Monoid

d) group

Q which of the following is not a group?

a) {……. -6, -4, -2, 0, 2, 4, 6, ……}, +

b) {…. -3k, -2k, -k, 0, k, 2k, 3k, ….}, + [k $\in$ Z]

c) {2^n, n $\in$ Z}, X

d) set of complex number, X

Q Consider the set of all integers(Z) with the operation defined as $m * n = m + n + 2$, $m, n \in Z$

if (z, *) forms a group, then determine the identity element

a) 0 **b)** -1 **c)** -2 **d)** 2

Q Consider a set of positive rational number with respect to an operation *, such that $a*b = (aXb)/3$, it is known that the it is an abelian group, which of the following is not true?

a) identity element $e = 3$

b) inverse of $a = 9/a$

c) inverse of $2/3 = 6$

d) inverse of $3 = 3$

Q A binary operation α on a set of integers is defined as $x * y = x^2 + y^2$. Which one of the following statements is TRUE about *? **(GATE-2013)** **(1 Marks)**

(A) Commutative but not associative

(B) Both commutative and associative

(C) Associative but not commutative

(D) Neither commutative nor associative

Q The binary operator ≠ is defined by the following truth table **(GATE-2015) (1 Marks)**

Which one of the following is true about the binary operator ≠?

p	q	P q
0	0	0
0	1	1
1	0	1
1	1	0

(A) Both commutative and associative

(B) Commutative but not associative

(C) Not commutative but associative

(D) Neither commutative nor associative

Q Which one of the following in NOT necessarily a property of a Group?

(GATE-2009) (2 Marks)

(A) Commutativity

(B) Associativity

(C) Existence of inverse for every element

(D) Existence of identity

Q Consider the set $\sum^*$ of all strings over the alphabet $\sum = \{0, 1\}$. $\sum^*$ with the concatenation operator for strings **(GATE-2003) (1 Marks)**

(A) does not form a group

(B) forms a non-commutative group

(C) does not have a right identity element

(D) forms a group if the empty string is removed from $\sum^*$

Q Which of the following is true? **(GATE-2002) (2 Marks)**

(A) The set of all rational negative numbers forms a group under multiplication.

(B) The set of all non-singular matrices forms a group under multiplication.

(C) The set of all matrices forms a group under multiplication.

(D) Both (2) and (3) are true.

Q Which of the following statements is FALSE? **(GATE-1996) (1 Marks)**

a) The set of rational numbers is an abelian group under addition

b) The set of integers in an abelian group under addition

c) The set of rational numbers form an abelian group under multiplication

d) The set of real numbers excluding zero is an abelian group under multiplication

Q Let A be the set of all non-singular matrices over real number and let $*$ be the matrix multiplication operation. Then **(GATE-1994) (2 Marks)**

a) A is closed under $*$ but $\langle A,* \rangle$ is not a semigroup.

b) $\langle A,* \rangle$ is a semigroup but not a monoid.

c) $\langle A,* \rangle$ is a monoid but not a group.

d) $\langle A,* \rangle$ is a group but not an abelian group.

- **<u>Finite Group</u>: -** A group with finite number of elements is called a finite group.

- **<u>Order of group</u>: -** Order of a group is denoted by O(G) = no of elements in G
  - If there is only one element in the Group, it must be an identity element.

Q Check out which of the following is a finite group?

1- {0}, +

+	0
0	

2- {1}, X

*	1
1	

3- {0,1}, +

+	0	1
0		
1		

4- {0,1}, X

*	0	1
0		
1		

5- {-1, 0, 1}, +

+	-1	0	1
-1			
0			
+1			

6- {-1, 0, 1}, X

*	-1	0	1
-1			
0			
1			

Q Check out which of the following is a finite group?

7- {-1, 1}, +

+	-1	1
-1		
1		

8- {-1, 1}, X

*	-1	1
-1		
1		

9- {-2, -1, 0, 1, 2}, +

+	-2	-1	0	1	2
-2					
-1					
0					
1					
2					

Q Check out which of the following is a finite group?

10- {-2, -1, 0, 1, 2}, X

*	-2	-1	0	1	2
-2					
-1					
0					
1					
2					

11- {1, ω, $ω^2$}, X

*	1	ω	$ω^2$
1			
ω			
$ω^2$			

12- {-1, 1, i, -i}, X

*	-1	1	i	-i
-1				
1				
i				
-i				

1. **Conclusion: -** it is very difficult to design finite group as with number greater than 2 closure property fails with simple addition and multiplication operation.

2. So we will try to develop new modified addition and multiplication operators with which closure and other properties can be satisfied.

- **Addition modulo**: **-** addition modulo is a binary operator denoted by $+_m$ such that

- $a +_m b = a + b \quad$ if $(a + b < m)$

- $a +_m b = a + b - m \quad$ if $(a + b >= m)$

$\{0,1,2,3\}, +_4$

$+_4$	0	1	2	3
0				
1				
2				
3				

- **Multiplication modulo**: - Multiplication modulo is a binary operator denoted by $*_m$ such that

- $a \, X_m \, b = a \, X \, b$ if $(a \, X \, b < m)$

- $a \, X_m \, b = (a \, X \, b) \% m$ if $(a \, X \, b >= m)$

$\{1,2,3,4\}, X_5$

X_5	1	2	3	4
1				
2				
3				
4				

Q Check out which of the following is a group?

1- {0,1,2,3}, $+_4$

$+_4$	0	1	2	3
0				
1				
2				
3				

2- {0,1,2,3}, X_4

$*_4$	0	1	2	3
0				
1				
2				
3				

3- {1,2,3}, $+_4$

$+_4$	1	2	3
1			
2			
3			

4- {1,2,3}, X_4

$*_4$	1	2	3
1			
2			
3			

5- {0,1,2,3,4,5,6}, $+_7$

$+_7$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

6- {0,1,2,3,4,5,6}, X_7

$*_7$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

7-

$+_7$	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

8- {1,2,

$*_7$	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

13- {1,3,5,7}, X_8

$*_8$	1	3	5	7
1				
3				
5				
7				

14- {1,2,4,7,8,11,13,14}, X_{15}

$*_{15}$	1	2	4	7	8	11	13	14
1								
2								
4								
7								
8								
11								
13								
14								

15- $\{1,2,3, 4........, p-1\}, X_p$

16- $\{0,1,2,3, 4........, p-1\}, X_p$

17- $\{1,2,3, 4........, p-1\}, +_p$

18- $\{0,1,2,3, 4........, p-1\}, +_p$

Q The set {1, 2, 3, 5, 7, 8, 9} under multiplication modulo 10 is not a group. Given below are four plausible reasons. Which one of them is false? **(GATE-2006) (1 Marks)**

(A) It is not closed

(B) 2 does not have an inverse

(C) 3 does not have an inverse

(D) 8 does not have an inverse

X_{10}	1	2	3	5	7	8	9
1							
2							
3							
5							
7							
8							
9							

Q The set {1, 2, 4, 7, 8, 11, 13, 14} is a group under multiplication modulo 15. The inverses of 4 and 7 are respectively **(GATE-2005) (2 Marks)**

(A) 3 and 13 **(B)** 2 and 11 **(C)** 4 and 13 **(D)** 8 and 14

X_{15}	1	2	4	7	8	11	13	14
1								
2								
4								
7								
8								
11								
13								
14								

Q The following is the incomplete operation table a 4-element group. **(GATE-2004) (2 Marks)**

The last row of the table is

(A) c a e b **(B)** c b a e **(C)** c b e a **(D)** c e a b

X	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b				
c				

Q Consider the binary operation $\oplus$ over set $Z_n = \{0, 1, 2, \ldots\ldots, n-1\}$

$a \oplus b = a + b \quad \text{if } (a + b < n)$

$a \oplus b = a + b - n \quad \text{if } (a + b >= n)$

a) it is closed

b) it does not form a group

c) it forms a group but not an abelian group

d) it is an abelian group

<u>Sub Group</u>

1. The subset of a group may or may not be a group.

2. When the subset of a group is also a group then it is called sub group.

3. The identity element of a group and its sub group is always same.

4. Union of two subgroup may or may not be a subgroup.

5. Intersection of two subgroup is always a subgroup.

6. <u>Lagrange's theorem</u>: - the order of a group is always exactly divisible by the order of a sub group.

Q consider a group G = {1,3,5,7}, X_8 which of the following sub set of this set does not form is sub group?

a) {1,3} **b)** {1,5} **c)** {1,7} **d)** {1,3,7}

$*_8$	1	3
1		
3		

$*_8$	1	5
1		
5		

$*_8$	1	7
1		
7		

$*_8$	1	3	7
1			
3			
7			

Q Let G be a group with 15 elements. Let L be a subgroup of G. It is known that L != G and that the size of L is at least 4. The size of L is ___________. **(GATE-2014) (1 Marks)**

(A) 3 **(B)** 5 **(C)** 7 **(D)** 9

Q Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is ________ . **(GATE-2014) (1 Marks)**

Q let (A, X) be a group of prime order, how many proper-subgroups are possible for A?

a) 0 **b)** 1 **c)** P-1 **d)** P

<u>Order of an element</u>: - (A, *) be a group, then $\forall$ a $\in$ A, order of a is denoted by O(a).

1. O(a) =n (smallest positive integer), such that $a^n = e$

2. Order of identity element is always one.

3. Order of an element and its inverse is always same.

4. Order of an element in an infinite group does not exist or infinite expect identity.

5. Order of a group is always divisible by order of every element of the group.

Q consider a group $\{0,1,2,3\}, +_4$ and find the order of each element?

Q consider a set on cube root of unity $\{1, \omega, \omega^2\}$, X and find the order of each element?

Q consider a set on forth root of unity {-1, 1, I, -i}, X and find the order of each element?

Q. Let Z_n be the group of integers {0, 1, 2, … , n − 1} with addition modulo n as the group operation. The number of elements in the group $Z_2 \times Z_3 \times Z_4$ that are their own inverses is __________ **(Gate 2024 CS)(2 Marks)(NAT)**

We're interested in the number of elements in the group $Z_2 \times Z_3 \times Z_4$ that are their own inverses. The elements of this group are ordered triples (a, b, c), where $a \in Z_2$, $b \in Z_3$, and $c \in Z_4$. An element (a, b, c) is its own inverse if each of its components is its own inverse in its respective group.

Generating element or Generator: - A element ‘a’ is said to be a generating element, if every element of A is an integral power of a, i.e. every element of A can be represented using power of a.

$A = \{a^1, a^2, a^3, a^4, a^5 \ldots.\}$

<u>Cyclic group</u>: - A group (A, *) is said to be a cyclic group if it contains at least one generator.

1. In a cyclic group if an element is a generator than its inverse will also be a generator.
2. The order of a cyclic group is always the order of the generating element of G.
3. Cyclic group is always alelian group.
4. Every group of order prime no is always always cyclic group where every number expect identity is generator.

Q consider a group {1,2,4,7,8,11,13,14}, X_{15} and find the order of each element?

Q For the composition table of a cyclic group shown below

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Which one of the following choices is correct?

(GATE-2009) (2 Marks)

(A) a, b are generators

(B) b, c are generators

(C) c, d are generators

(D) d, a are generators

Number of generators

Lagrange's theorem: - let A be a cyclic group of order n, number of Generator in A is denoted by $\phi(n) = \{n(p_1-1)(p_2-1)(p_3-1) \ldots\ldots\ldots\ldots (p_k-1)\} / (p_1 p_2 p_3 \ldots\ldots\ldots p_k)$

Q let G be a cyclic group, O(G) = 8, number of generators in G =?

Q let G be a cyclic group, O(G) = 70, number of generators in G =?

Q let G be a cyclic group, O(G) = 23100, number of generators in G =?

Q Let s = set of all integers. A binary operation * is defined by

$a * b = a + b + 3$

consider the following statements

S_1: $(S,*)$ is a group

S_2: -3 is identity element of $(S, *)$

S_3: the inverse of -6 is 0

which of the following are true

a) Only S_1 and S_2

b) Only S_2 and S_3

c) Only S_1 and S_3

d) Only S_1 ,S_2 and S_3

Q $(D_{12}, *)$ where $a*b$ = g.c.d of (a, b) $\forall_{a, b} \in D_{12}$ then $(D_{12}, *)$ is

a) a semigroup but not monoid

b) a monoid but not a group

c) a group

d) not a semi group

Gcd(a, b)	1	2	3	4	6	12
1						
2						
3						
4						
6						
12						

Q In a group (G, *) if a*a = a, then proof that a = e, where e is identity element of a?

Q In a group if $x' = x$ for $\forall\ a \in G$ in G, then G is an abelian group?

Q In a group (G, *) , if $(a*b)^2 = a^2 * b^2$, then prove that G is an abelian group?

Q Which of the following statements is/are TRUE for a group G? **(GATE 2022) (1 MARKS)**

(A) If for all $x, y \in G$, $(xy)^2 = x^2y^2$, then G is commutative.

(B) If for all $x \in G$, $x^2=1$, then G is commutative. Here, 1 is the identity element of G.

(C) If the order of G is 2, then G is commutative.

(D) If G is commutative, then a subgroup of G need not be commutative.

Q Let G be a group of order 6, and H be a subgroup of G such that $1<|H|<6$. Which one of the following options is correct? **(GATE 2021) (2 MARKS)**

(a) Both G and H are always cyclic

(b) G may not be cyclic, but H is always cyclic

(c) G is always cyclic, but H may not be cyclic

(d) Both G and H may not be cyclic

Q Let G be an arbitrary group. Consider the following relations on G:

- **R_1:** $\forall a, b \in G$, aR_1b if and only if $\exists g \in G$ such that $a = g^{-1}bg$
- **R_2:** $\forall a, b \in G$, aR_2b if and only if $a = b^{-1}$

Which of the above is/are equivalence relation/relations? **(GATE-2019) (2 Marks)**

(A) R_1 and R_2 **(B)** R_1 only **(C)** R_2 only **(D)** Neither R_1 nor R_2

Q There are two elements x, y in a group (G, $*$) such that every element in the group can be written as a product of some number of x's and y's in some order. It is known that $x * x = y * y = x * y * x * y = y * x * y * x = e$ where e is the identity element. The maximum number of elements in such a group is ___________. **(GATE-2014) (2 Marks)**

Q Consider the set S = {1, ω, ω^2}, where ω and w^2 are cube roots of unity. If * denotes the multiplication operation, the structure (S, *) forms **(GATE-2010) (1 Marks)**

(A) A group **(B)** A ring **(C)** An integral domain **(D)** A field

A **ring** is a set R equipped with two binary operations[a] + (addition) and · (multiplication) satisfying the following three sets of axioms, called the **ring axioms**

1. R is an abelian group under addition, meaning that:
   - $(a + b) + c = a + (b + c)$ for all a, b, c in R (that is, + is associative).
   - $a + b = b + a$ for all a, b in R (that is, + is commutative).
   - There is an element 0 in R such that $a + 0 = a$ for all a in R (that is, 0 is the additive identity).
   - For each a in R there exists $-a$ in R such that $a + (-a) = 0$ (that is, $-a$ is the additive inverse of a).
2. R is a monoid under multiplication, meaning that:
   - $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all a, b, c in R (that is, · is associative).
   - There is an element 1 in R such that $a \cdot 1 = a$ and $1 \cdot a = a$ for all a in R (that is, 1 is the multiplicative identity).[b]
3. Multiplication is distributive with respect to addition, meaning that:
   - $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ for all a, b, c in R (left distributivity).
   - $(b + c) \cdot a = (b \cdot a) + (c \cdot a)$ for all a, b, c in R (right distributivity).

$(Z,+,\times)$

A ring becomes an **<u>integral domain</u>** when it meets two additional criteria:

.<u>No Zero Divisors</u>: There are no pairs of non-zero elements that multiply to zero. In other words, if a x b = 0 in the ring, then either a = 0 or b = 0 (or both).

.<u>Commutativity of Multiplication</u>: The multiplication operation in the ring is commutative, meaning that for any two elements a and b in the ring, a x b = b x a.

.<u>And there's one more thing</u>: The ring must have an identity element for multiplication (often denoted as 1), such that for any element a in the ring, a x 1 = a and 1 x a = a.

A classic example of a ring that is not an integral domain is the ring of 2 x 2 matrices with real number entries, denoted as R^{2x2}. This ring is not an integral domain mainly because it contains zero divisors.

Definition: A zero divisor is a non-zero element a in a ring such that there exists another non-zero element b where a x b = 0 or b × a = 0.

Example in R^{2x2}:

Consider the matrices:

Both A and B are non-zero matrices, but their product is the zero matrix:

Since we have found non-zero elements whose product is zero, A and B are zero divisors, which means R^{2x2} contains zero divisors and therefore cannot be an integral domain.

An integral domain becomes a field when every non-zero element in it has a multiplicative inverse. It must satisfy the following additional criterion:

- **Existence of Multiplicative Inverses**: For every non-zero element a in D, there exists an element b in D such that a x b = b x a = 1, where 1 is the multiplicative identity in D.

An example of an integral domain that is not a field is the set of integers, denoted as Z. Integral Domain Properties of Z

- **No Zero Divisors**: In Z, if the product of two integers is zero, then at least one of the integers must be zero. This means Z has no zero divisors.
- **Commutativity of Multiplication**: The multiplication of integers is commutative, satisfying another requirement of an integral domain.
- **Additive Identity and Inverses**: Z has an additive identity (0) and every integer has an additive inverse (-a for any integer a).

Why Z Is Not a Field

- **Lack of Multiplicative Inverses**: For an integral domain to be a field, every non-zero element must have a multiplicative inverse within the same set. In Z, this is not the case. For example, the integer 2 does not have a multiplicative inverse in Z because there is no integer x such that 2x = 1. This lack of multiplicative inverses for every non-zero element is what prevents Z from being a field, despite it being an integral domain.

(Q,+,×)

Q Which one of the following is false? **(GATE-1996) (2 Marks)**

(A) The set of all bijective functions on a finite set forms a group under function composition.

(B) The set {1, 2, ………………….., p-1} forms a group under multiplication mod p where p is a prime number

(C) The set of all strings over a finite alphabet $\sum$ forms a group under concatenation

(D) A subset $S \neq \emptyset$ of G is a subgroup of the group if and only if for any pair of element $a, b \in S$, $a * b^{-1} \in S$

Q Some group (G,o) is known to be abelian. Then, which one of the following is true for G? **(GATE-1994) (2 Marks)**

A) $g = g^{-1}$ for every $g \in G$

B) $g=g^2$ for every $g \in G$

C) $(goh)^2=g^2oh^2$ for every $g, h \in G$

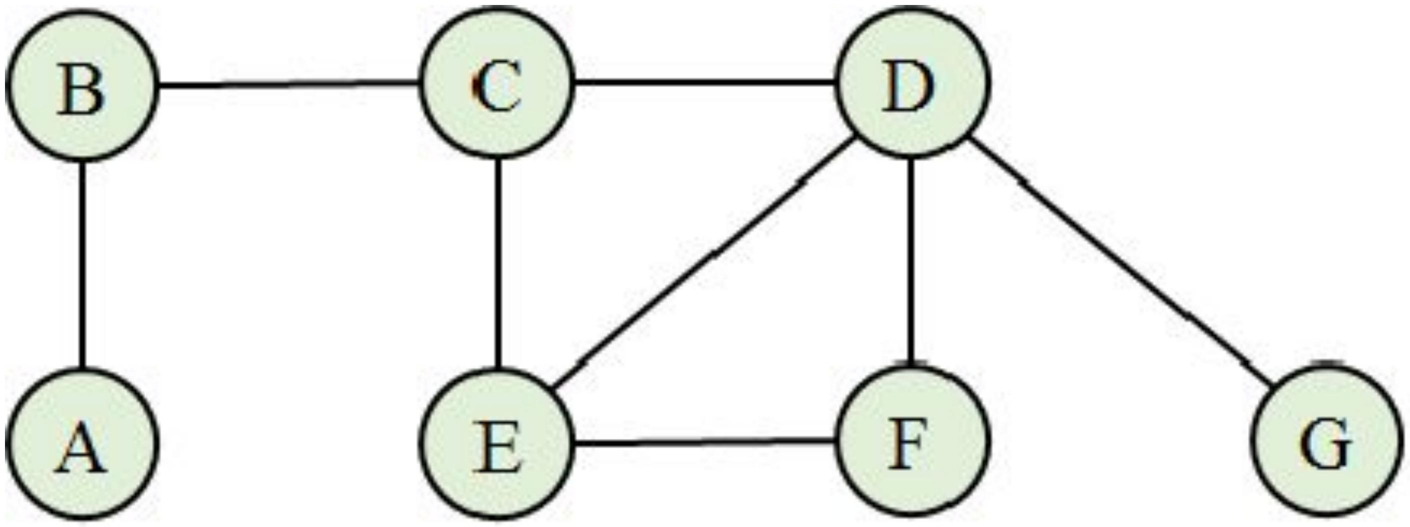
D) G is of finite order

Graph Theory

1. A graph G (V, E) consist of a set off objects $V = \{V_1, V_2, V_3 \dots, V_N\}$ called vertices and another set $E = \{E_1, E_2, E_3, \dots, E_n\}$ whose elements are called edges.

2. Each edge e_k is identified with an unordered pair (v_i, v_j) of vertices.

3. The vertices $v_{i,}$ v_j associated with edge e_k are called the end vertices of e_k.

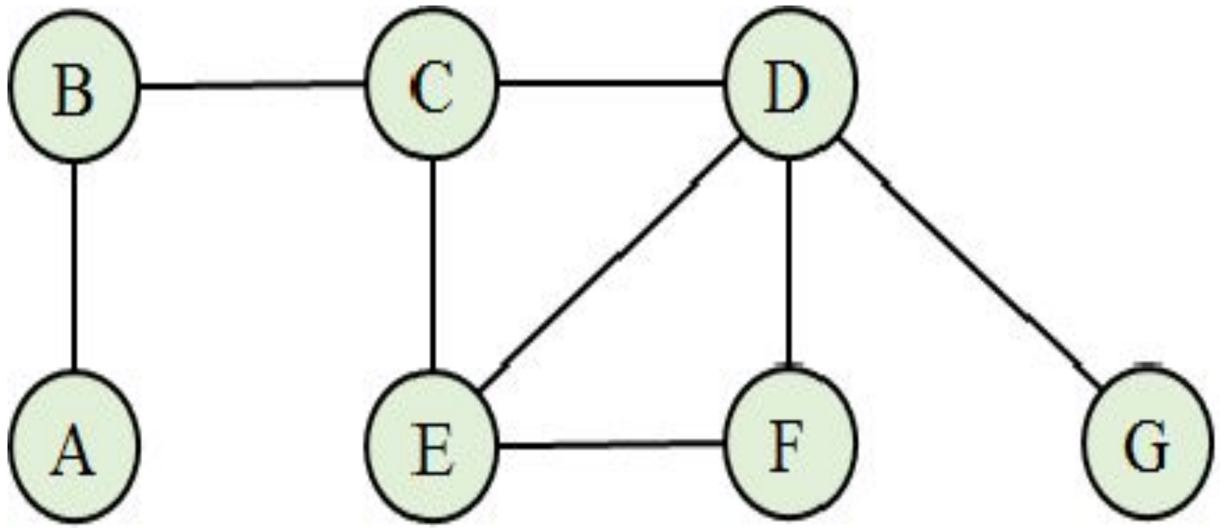


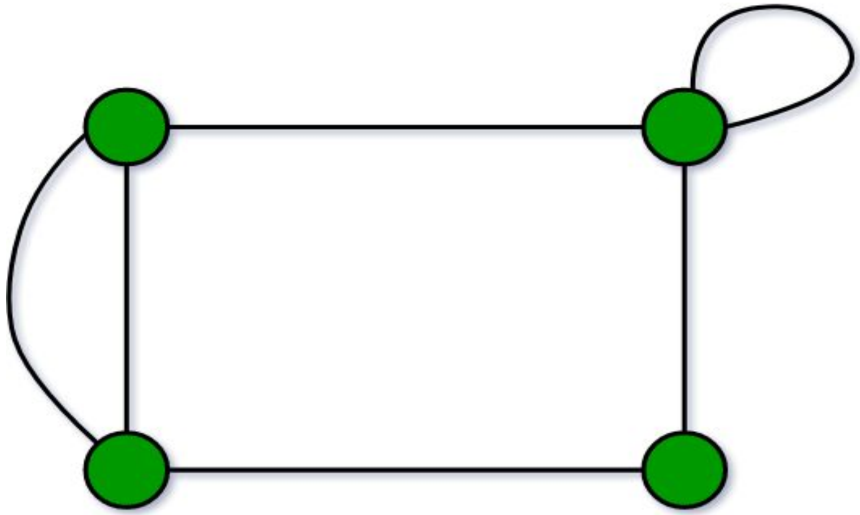
1. **Self-Loop:** Edge having the same vertex (v_i, v_i) as both its end vertices is called self-loop.

2. **Parallel Edge:** When more than one edge associated with a given pair of vertices such edges are referred as parallel edges.

3. **Adjacent Vertices:** If two vertices are joined by the same edges, they are called adjacent vertices.

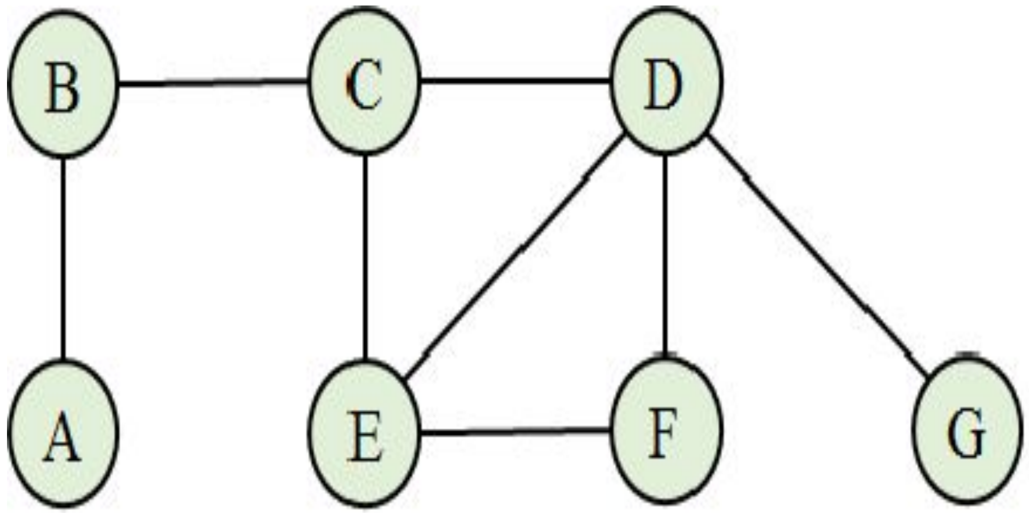
4. **Adjacent Edges:** If two edges are incident on some vertex, they are called adjacent edges.

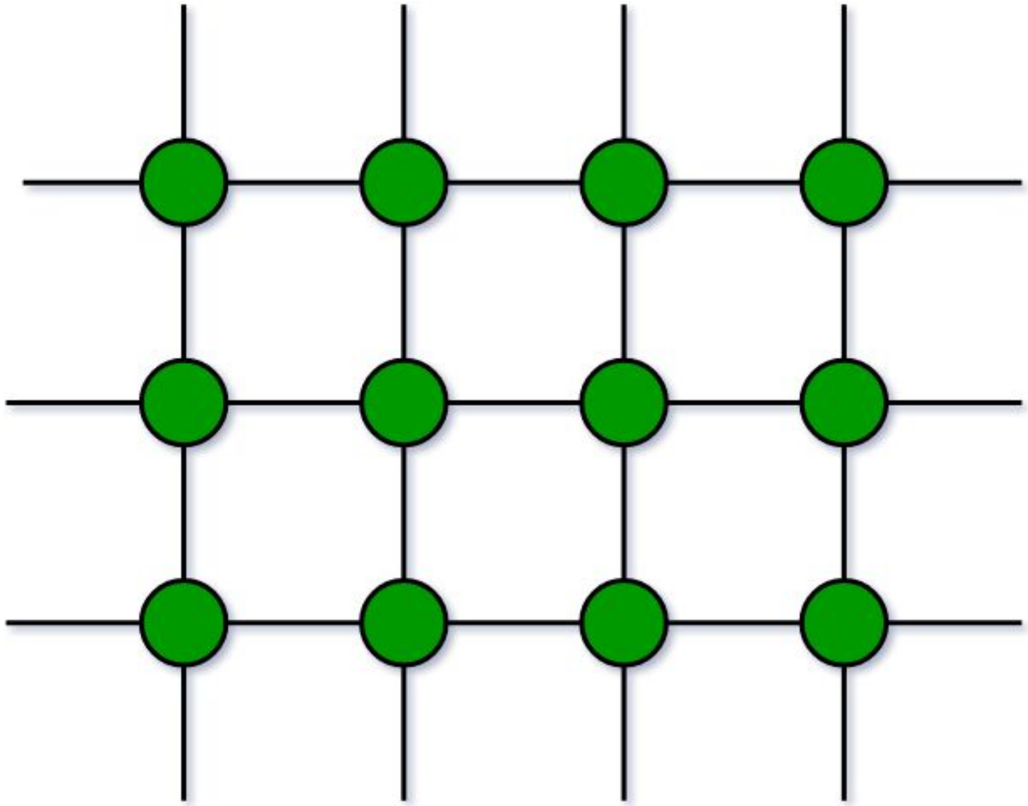




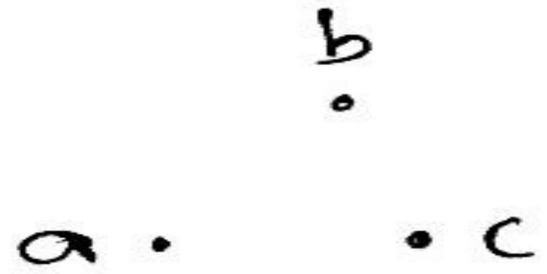
1. **<u>Finite graph</u>**: - A graph with finite number of vertices as well as the finite number of edges is called a finite graph.

2. For simple graph we can say if the number of vertices are finite then number of edges will also be finite.

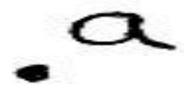




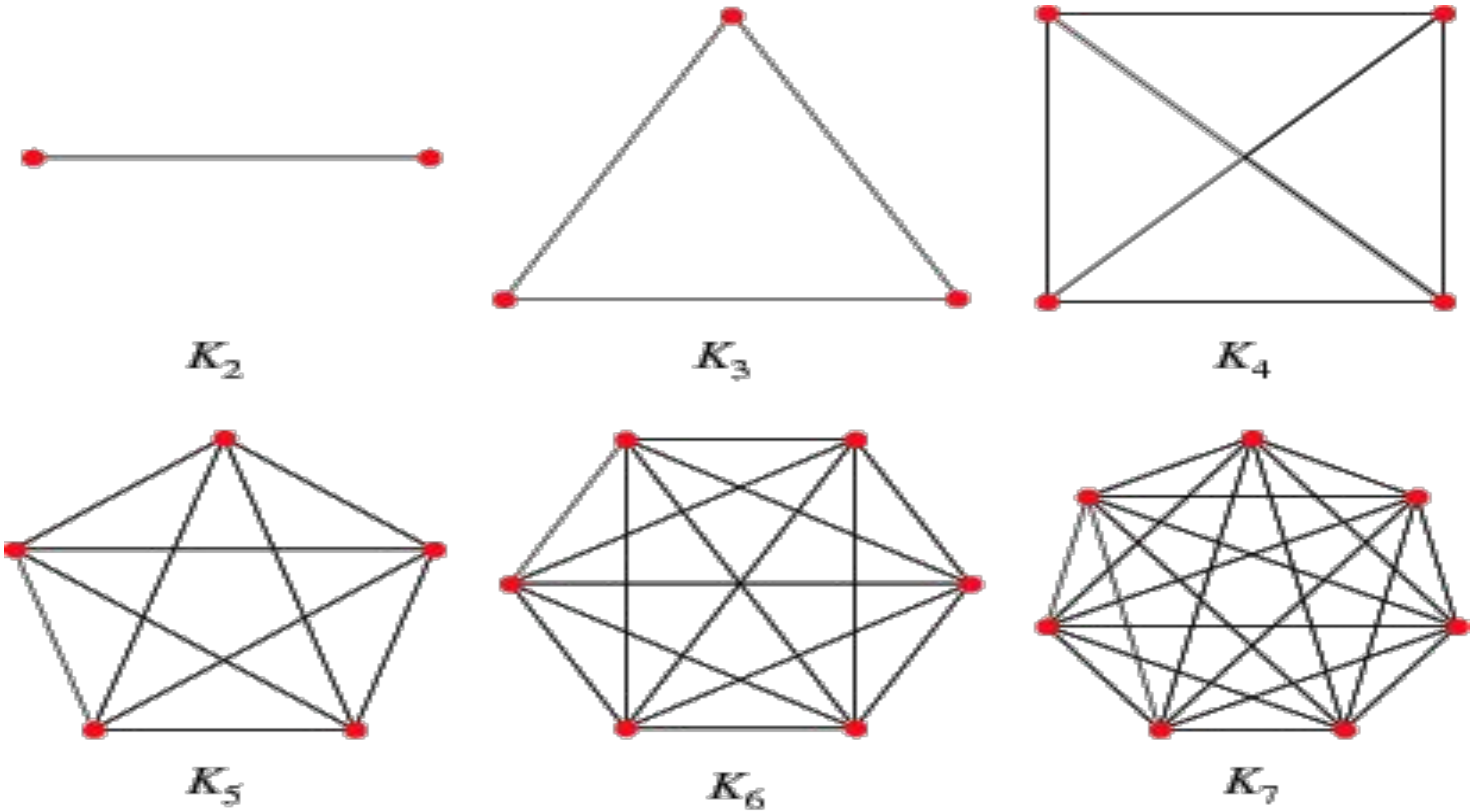
1. **Null Graph:** A graph is said to be null if edge set is empty E = {}, that is a graph with only vertices but no edges.



2. **Trivial Graph:** A graph with only one vertex without an edge is called trivial graph. It is the smallest possible.



- **Complete or Full Graph:** In a simple graph there exist an edge between each and every pair of vertices i.e. every vertex are adjacent to each other, then the graph is said to be a complete graph, denoted by $K_{n.}$



1. A simple graph with maximum number of edges are called Complete Graph.

2. Number of edges in a simple graph is n(n-1)/2

Q Maximum number of edges in a n node undirected graph without self-loops is **(GATE-2002) (1 Marks) (NET-DEC-2011)**

(A) n^2 **(B)** $n(n - 1)/2$ **(C)** $n - 1$ **(D)** $(n + 1)(n)/2$

Q Number of simple graph possible with n vertices?

Q Number of simple graph possible with n vertices and e edges?

Q The number of distinct simple graphs with up to three nodes is **(GATE-1994) (1 Marks)**

a) 15 **b)** 11 **c)** 8 **d)** 9

Q How many undirected graphs (not necessarily connected) can be constructed out of a given set $V = \{v_1, v_2, \dots v_n\}$ of n vertices?

(GATE-2001) (2 Marks)

(A) $n(n-1)/2$ **(B)** $2n$ **(C)** $n!$ **(D)** $2^{n(n-1)/2}$

Q Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to **(GATE-2012) (2 Marks)**

(A) 15 **(B)** 30 **(C)** 90 **(D)** 360

Q Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $\{(i, j): 1 <= i <= 12, 1 <= j <= 12\}$. There is an edge between (a, b) and (c, d) if $|a − c| <= 1$ and $|b − d| <= 1$. **(GATE-2014) (2 Marks)** **(NET-AUG-2016)** The number of edges in this graph is ____________.

Q Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is 1/2. What is the expected number of unordered cycles of length three? **(GATE-2013) (1 Marks)**

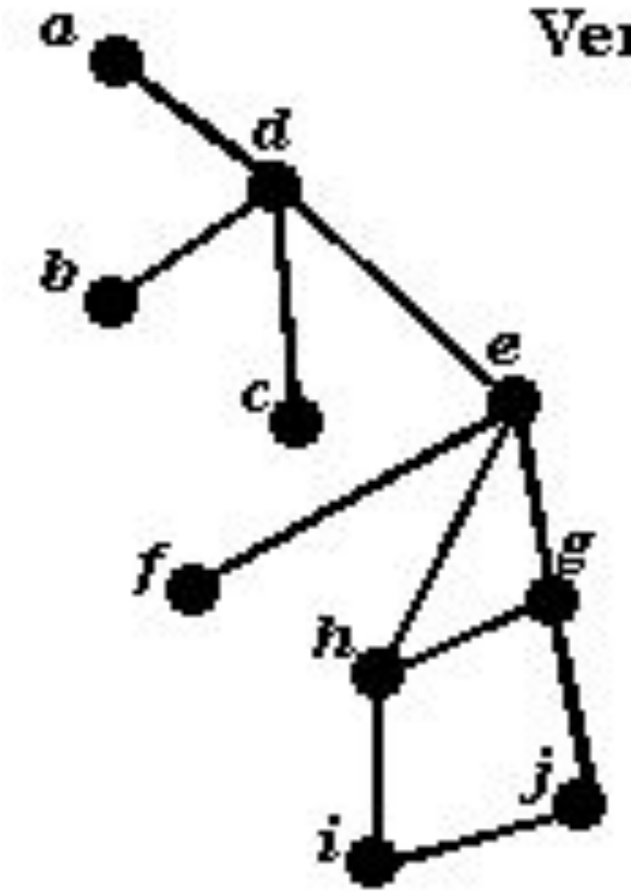
(A) 1/8 **(B)** 1 **(C)** 7 **(D)** 8

Q The 2^n vertices of a graph G corresponds to all subsets of a set of size n, for n >= 6. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements. The number of vertices of degree zero in G is **(GATE-2006) (2 Marks)**

(A) 1 **(B)** n **(C)** n+1 **(D)** 2n

Degree

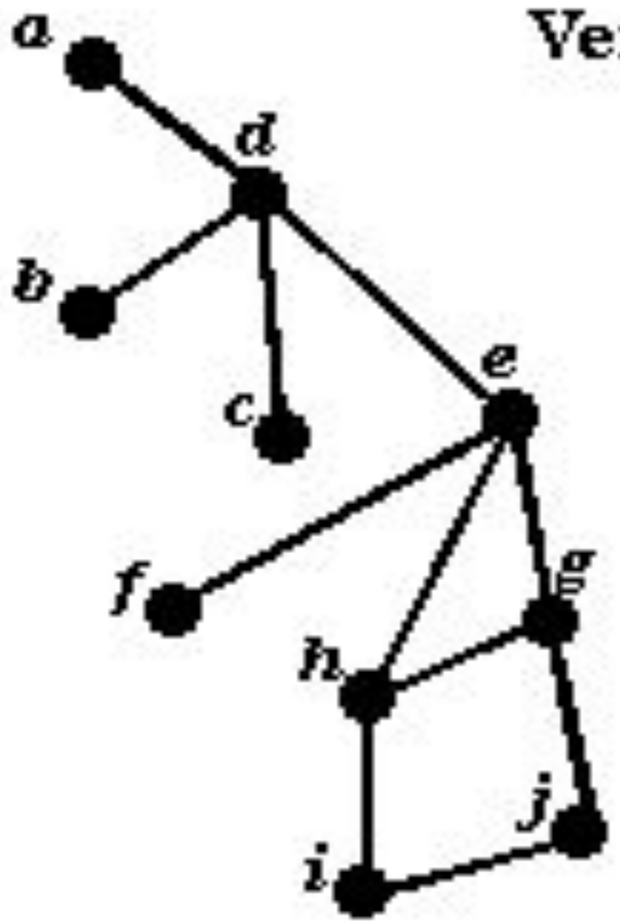
- **Degree of a Vertex:** The degree of a vertex in an undirected graph is the number of edges associated with it, denoted by $\deg(v_i)$.



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

- **<u>Isolated vertex:</u>** A vertex with degree zero is called isolated vertex.

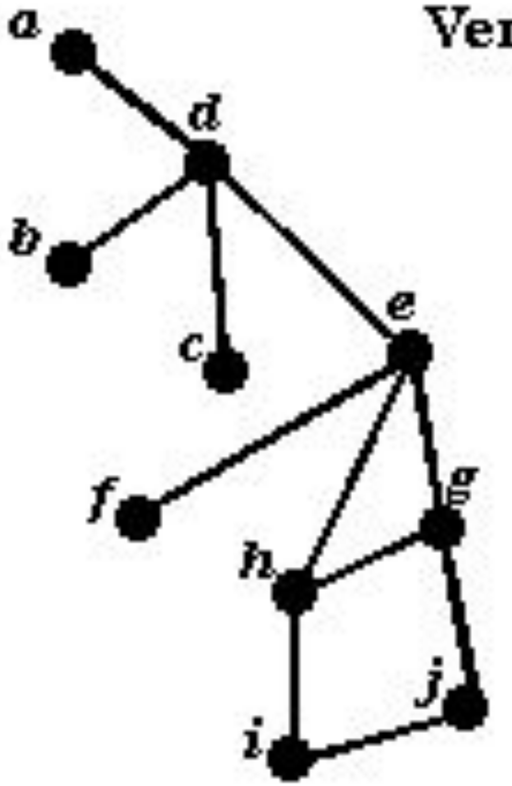
- **<u>Pendant vertex:</u>** A vertex with degree one is called pendant vertex.



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

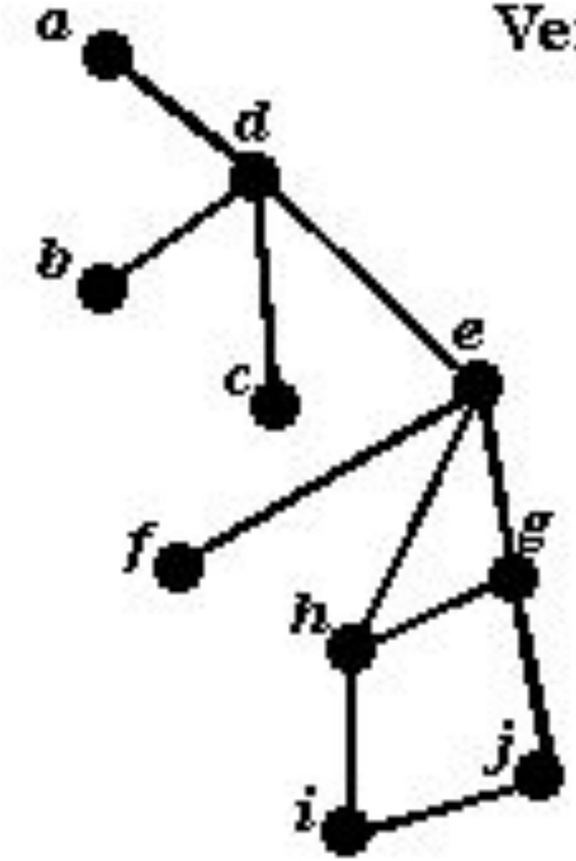
Hand Shaking Algorithm: Since each edge contribute two degrees the graph, the sum of the degree of all the vertices in G is twice the number of edges in g.

$degG(v)=2|E(G)|$



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

The no of vertices f odd degree in a graph is always even.



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

Q Which of the following statements is/are TRUE for undirected graphs? **(GATE-2013) (1 Marks)**

P: Number of odd degree vertices is even.

Q: Sum of degrees of all vertices is even.

a) P only **b)** Q only

c) Both P and Q **d)** Neither P nor Q

Q Which one of the following is TRUE for any simple connected undirected graph with more than 2 vertices?**(GATE-2009) (1 Marks)**

(A) No two vertices have the same degree.

(B) At least two vertices have the same degree.

(C) At least three vertices have the same degree.

(D) All vertices have the same degree.

Q A simple graph G contains 21 edges, 3 vertices of degree 4 and all the remaining vertices are of degree 2. Then number of vertices |v| is?

Q A simple non-directed graph G has 24 edges and degree of each vertex is 4, then find the $|v|$?

Q Consider a simple graph with 35 edges such that 4 vertex of degree 5, 5 vertex of degree 4, 4 vertex of degree 3, find the number of vertex with degree 2?

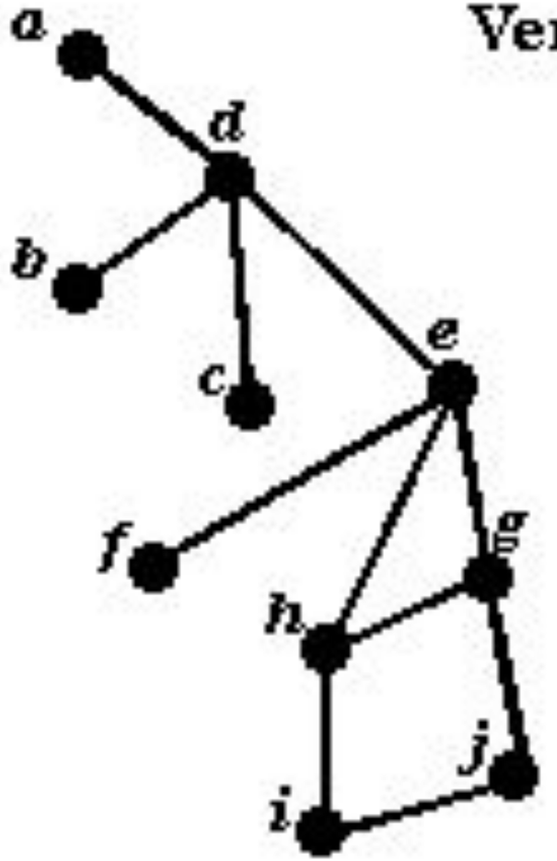
Q What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3? **(GATE-2004) (2 Marks)**

(A) 10 **(B)** 11 **(C)** 18 **(D)** 19

Q Simple non-directed graph G has 24 edges and degree of each vertex is K, the which of the following is possible no of vertices?

a) 20 **b)** 15 **c)** 10 **d)** 8

- δ(G) is the minimum possible degree of any vertex in a graph
- Δ(G) is the maximum possible degree of any vertex in a graph.



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

$$\boxed{\delta(G) * |V(G)| <= 2|E|} <= \Delta(G)*|V(G)|$$

Q G is undirected graph with n vertices and 25 edges such that each vertex has degree at least 3. Then the maximum possible value of n is _________ **(GATE-2017) (2 Marks)**

Q Maximum number of vertices possible in a simple graph if 35 edges and degree of each vertex is at least 3 is ______?

Q Minimum number of vertices possible in a simple graph if 41 edges and degree of each vertex is at most 5?

1. The **Havel–Hakimi algorithm** is an algorithm in graph theory solving the graph realization problem. That is, it answers the following question: Given a finite list of nonnegative integers, is there a simple graph such that its degree sequence is exactly this list.

2. Here, the "degree sequence" is a list of numbers that for each vertex of the graph states how many neighbors it has. For a positive answer the list of integers is called *graphic*.

3. The algorithm constructs a special solution if one exists or proves that one cannot find a positive answer. This construction is based on a recursive algorithm. The algorithm was published by Havel (1955), and later by Hakimi (1962).

Q Which of the following degree sequence represent a simple non-directed graph?

1) {2, 3, 3, 4, 4, 5}

2) {2, 3, 4, 4, 5}

3) {3, 3, 3, 1}

4) {1, 3, 3, 4, 5, 6, 6}

5) {2, 3, 3, 3, 3}

6) {6, 6, 6, 6, 4, 3, 3, 0}

7) {6, 5, 5, 4, 3, 3, 2, 2, 2}

Q An ordered n-tuple (d1, d2, …, dn) with d1 >= d2 >= ⋯ >= dn is called graphic if there exists a simple undirected graph with n vertices having degrees d1, d2, …, dn respectively. Which of the following 6-tuples is NOT graphic? **(GATE-2014) (2 Marks)**

(A) (1, 1, 1, 1, 1, 1)

(B) (2, 2, 2, 2, 2, 2)

(C) (3, 3, 3, 1, 0, 0)

(D) (3, 2, 1, 1, 1, 0)

Q The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences cannot be the degree sequence of any graph? **(GATE-2010) (2 Marks)**

I. 7, 6, 5, 4, 4, 3, 2, 1

II. 6, 6, 6, 6, 3, 3, 2, 2

III. 7, 6, 6, 4, 4, 3, 2, 2

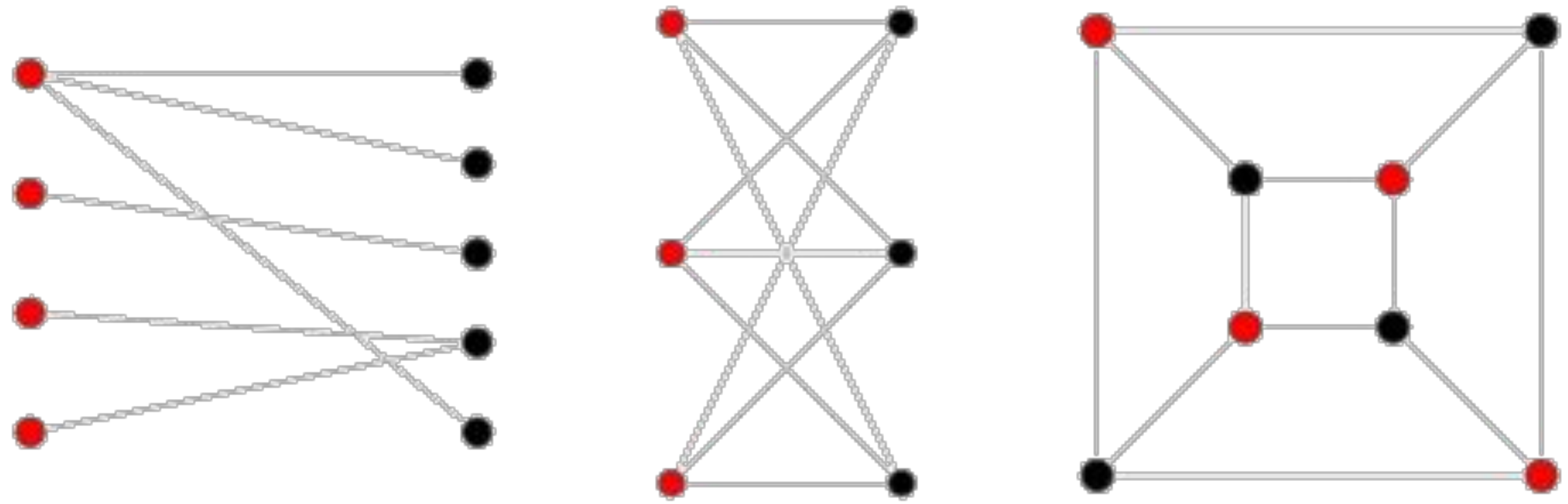
IV. 8, 7, 7, 6, 4, 2, 1, 1

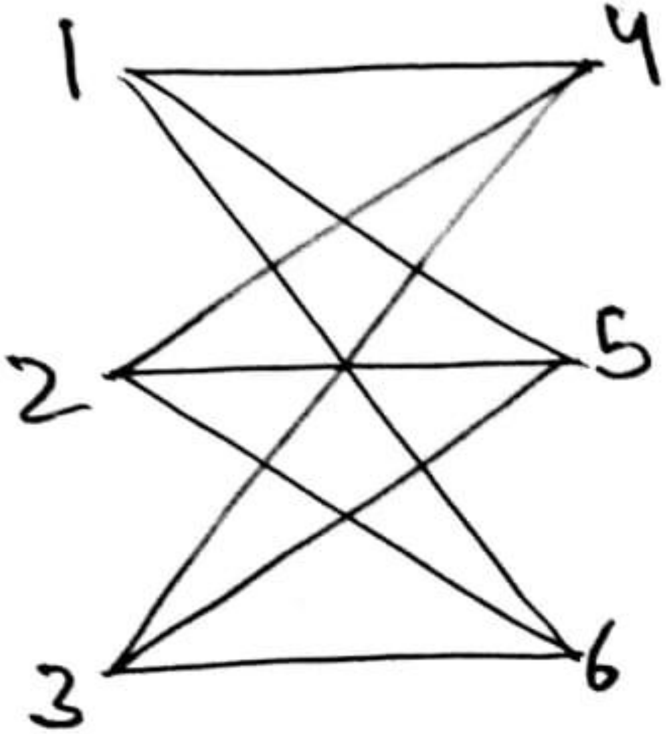
(A) I and II **(B)** III and IV **(C)** IV only **(D)** II and IV

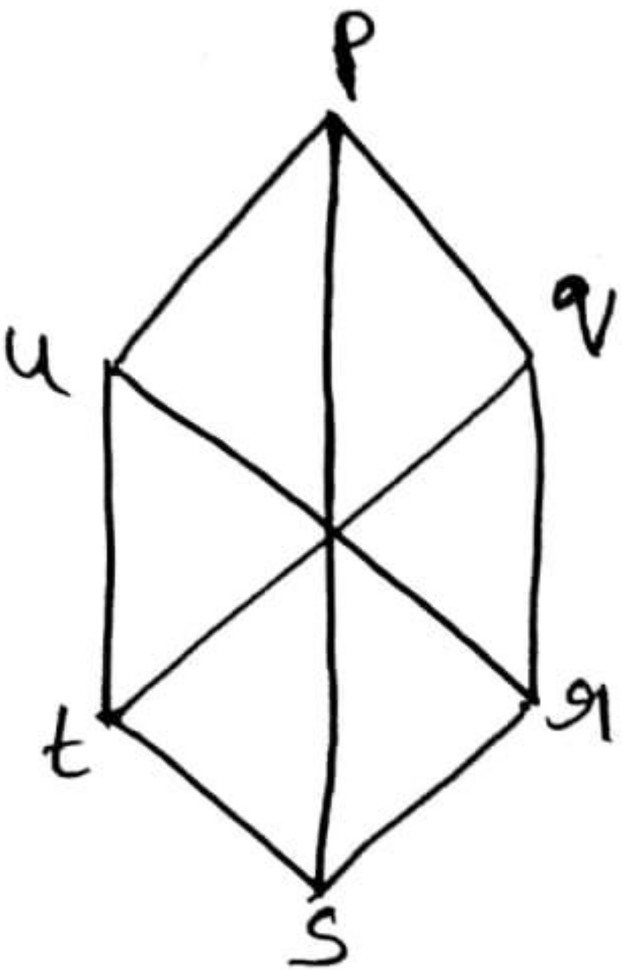
Some Popular Graph

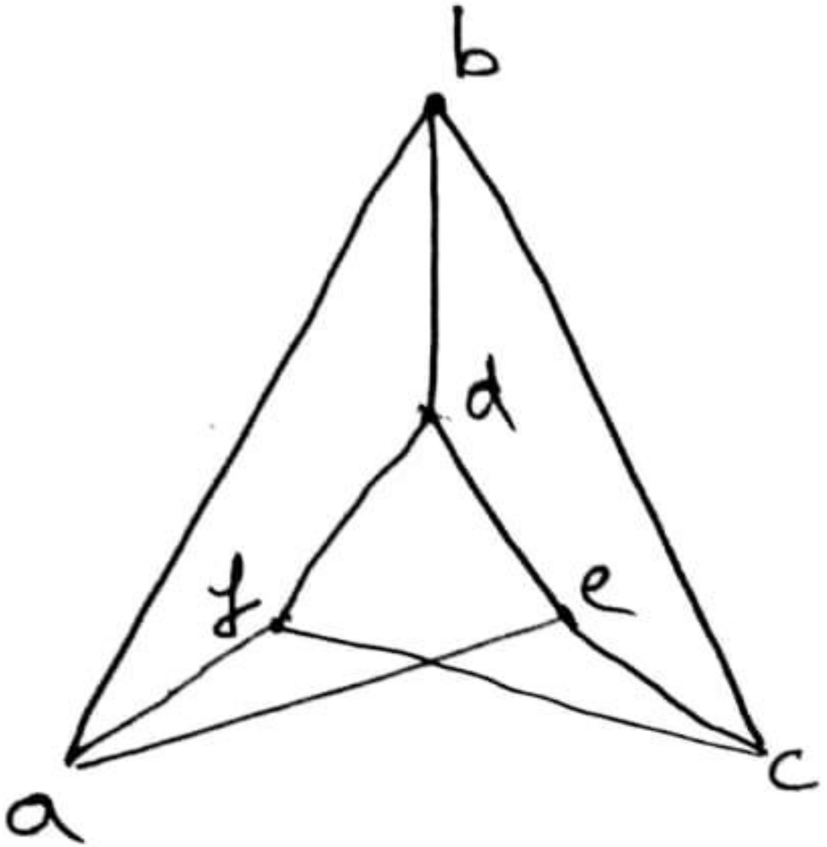
1. **Bi-partite graph:** - A graph G(V, E) is called bi-partite graph if it's vertex set V(G) can be partitioned into two non-empty disjoint subset V_1(G) and V_2(G) in such a way that each edge e ϵ E(G) has it's one end point in V_1(g) and other end point in V_2(g). The partition V = V_1 U V_2 is called bipartition of G.

2. **Complete Bi-partite graph:** - A Bi-partite graph G(V, E) is called Complete bi-partite graph if every vertex in the first partition is connected to every vertex in the second partition, denoted by $K_{m,n}$.



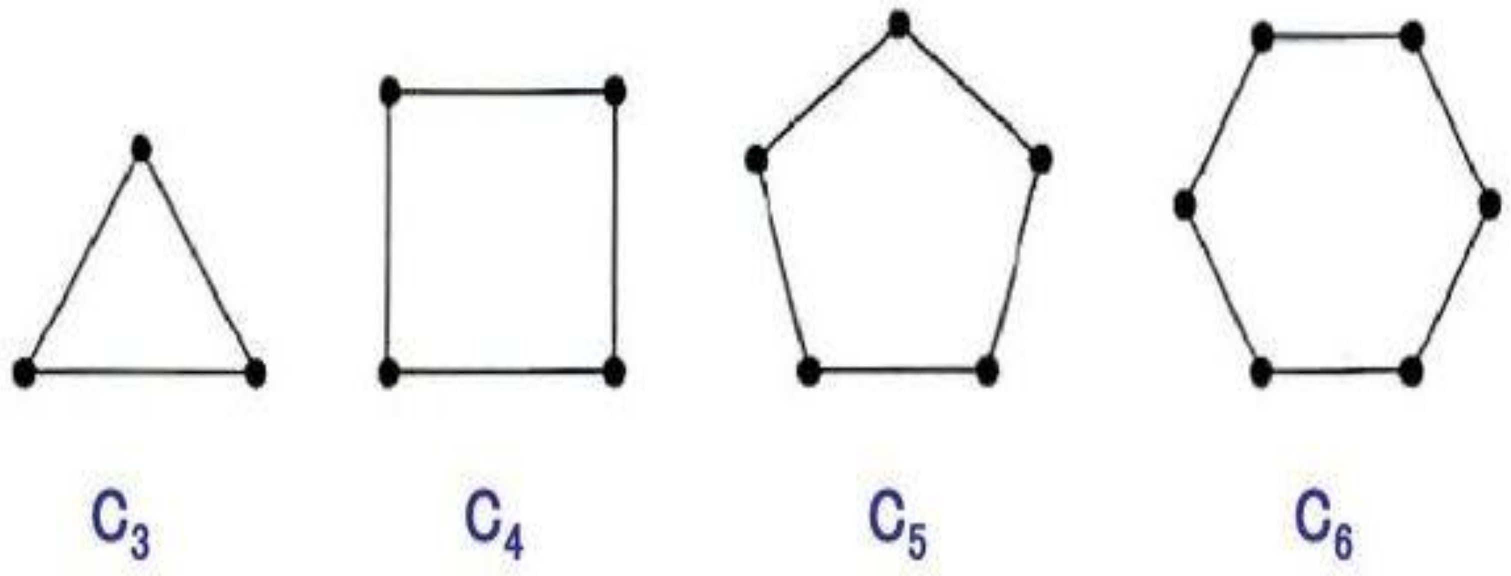

1
4
2
5
3
6


p
u
q
t
r
s

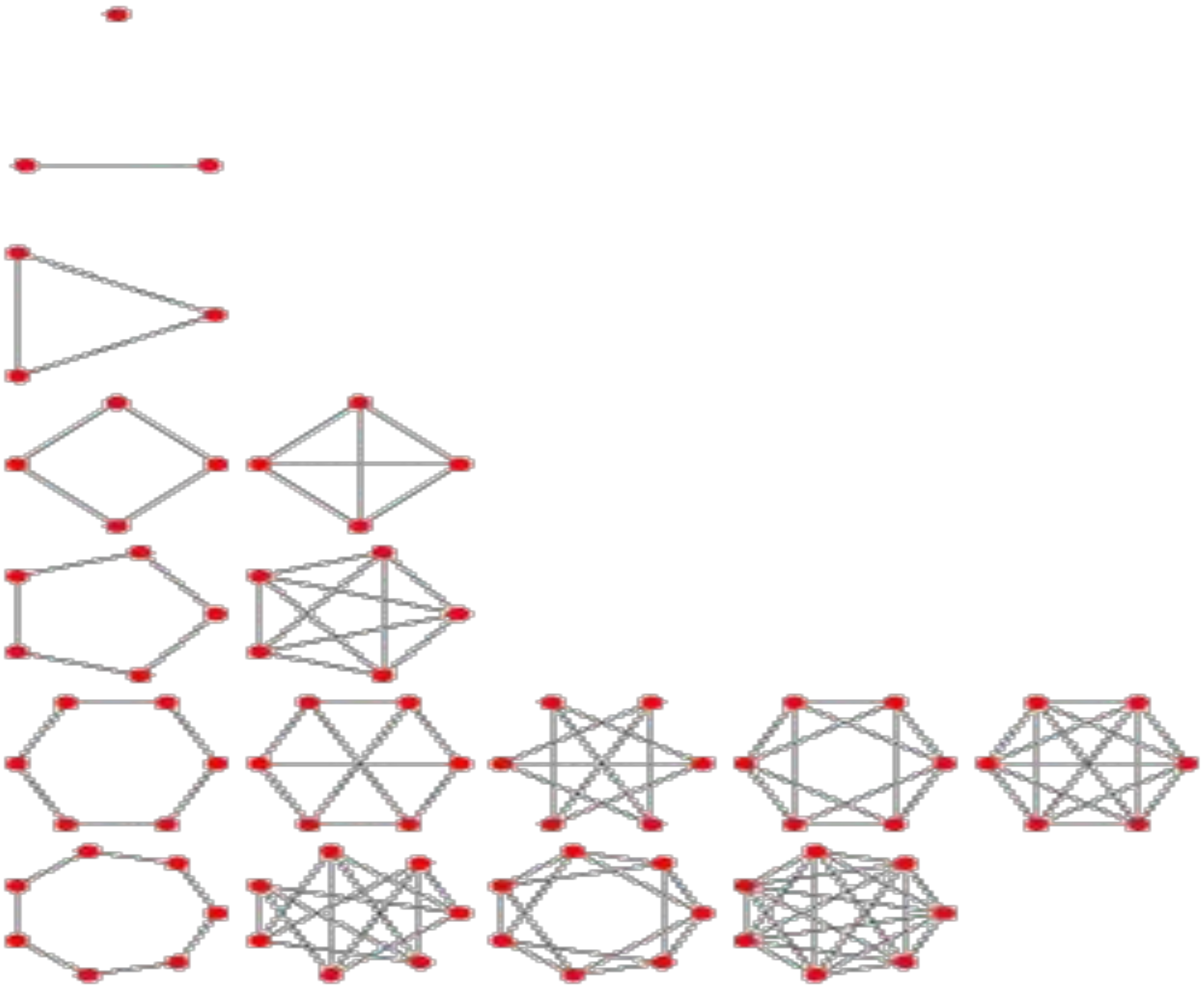

b
d
f
e
a
c

Q The maximum number of edges in a bipartite graph on 12 vertices is ______________________________. **(GATE-2014) (1 Marks)**

- **<u>Cycle Graph:</u>** - A cycle graph or circular graph is a graph that consists of a single cycle, or in other words, some number of vertices (at least 3) connected in a closed chain. The cycle graph with *n* vertices is called C_n. The number of vertices in C_n equals the number of edges, and every vertex has degree 2; that is, every vertex has exactly two edges incident with it.

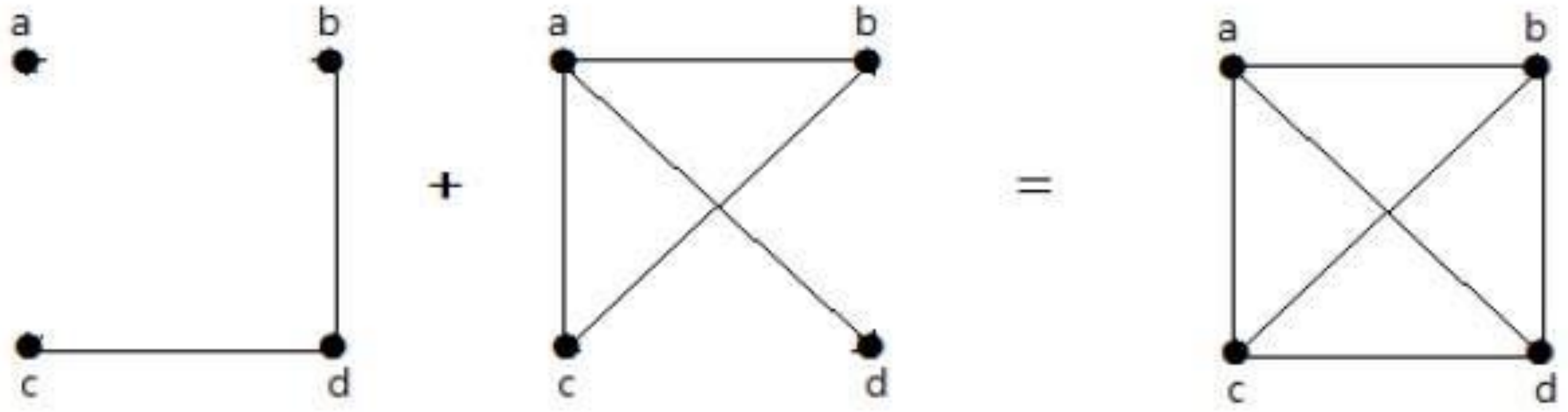


- **Regular graph:** - A graph in which all the vertices are of equal degree is called a regular graph. E.g. 2-regular graph, 3-regular graph.



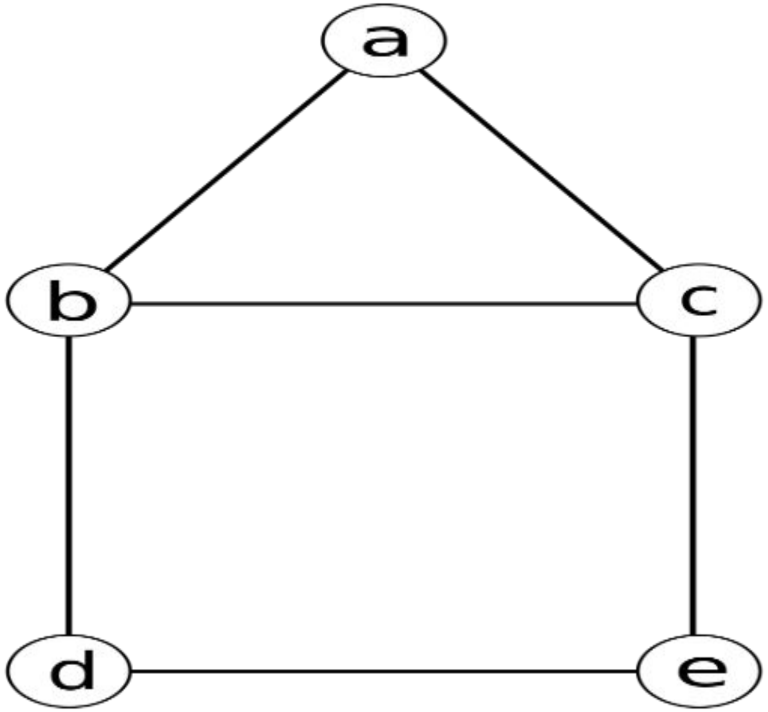
Complement of a Graph

1. The complement of a simple graph G (V, E) is a graph G^c (V, E^c) on the same vertices set as of G, such that there will be an edge between two vertices u, v in G^c if ang only if there is no edge between u, v in G. i.e. two vertices of Gc are adjacent iff they are not adjacent in G.
2. $V(G) = V(G^c)$
3. $E(G^c) = \{(u, v) \mid (u, v) \notin E(G)\}$
4. $E(G^c) = E(K_n) - E(G)$



I

II


a
b
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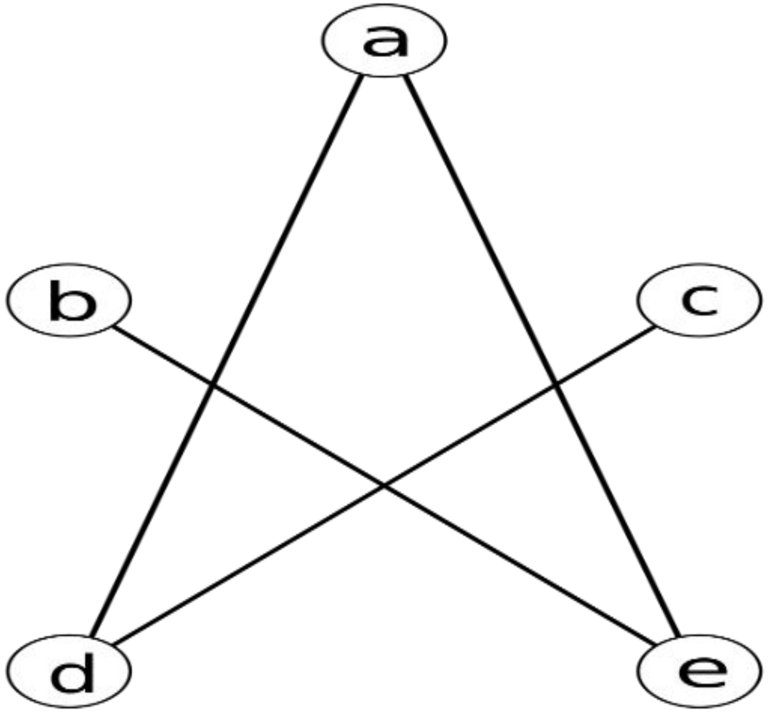

a
b
c
d
e

Properties

1. $G \cup G^c = K_n$

2. $G \cap G^c =$ null graph

3. $|E(G)| + |E(G^c)| = E(K_n) = n(n-1)/2$

Q A simple graph G has 30 edges and G^c has 36 edges, the number of vertices in G will be?

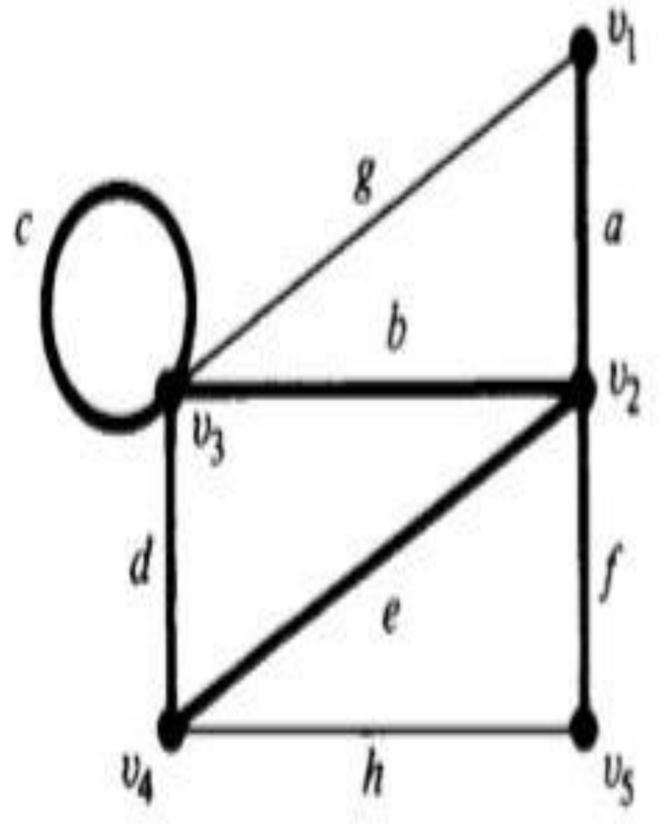
Q A simple graph G has 56 edges and G^c has 80 edges, the number of vertices in G will be?

Q A simple graph G has $|v|=8$ and $|E|=12$, find number of edges in $|E(G^c)|$?

Traversal

- **Walk / Edge Train / Chain**: -A Walk is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it. Both vertex and edges may appear more than once.

- An **open walk** in graph theory is a walk that starts and ends at different vertices, whereas a **closed walk** starts and ends at the same vertex.
- An open walk becomes a path when it does not revisit any vertices Number of edges in a path is called length of a path.



Traversal	Walk	Open Walk	Closed Walk	Path
V_1 g V_3 b V_2 e V_4 d V_3 b V_2				
V_1 a V_2 e V_4 d V_3 b V_2 f V_5				
V_1 g V_3 c V_3 b V_2 a V_1				
V_1 a V_2 b V_3 d V_4 h V_5				

- **<u>Connected Graph:</u>** A graph is said to be connected if there is at least one path between every pair of vertices in G.
- A graph with n vertices can be connected with minimum n -1 edges.
- A graph with n vertices will necessary be connected if it has more than (n - 1) (n - 2)/2 edges.
- if a graph (connected or disconnected) has exactly two vertices of odd degree, there must be a path joining these two vertices

Q Which condition is necessarily for a graph to be connected?

a) A graph with 6 vertices and 10 edges

b) A graph with 7 vertices and 14 edges

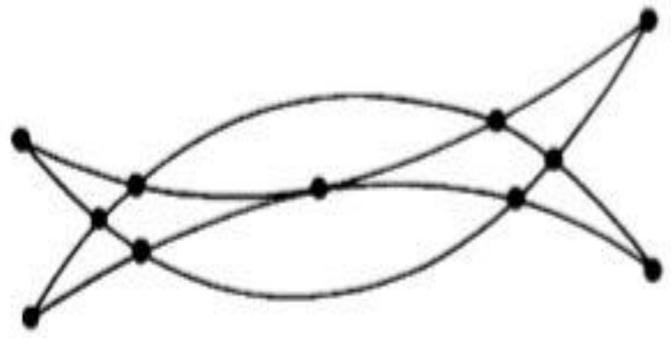
c) A graph with 8 vertices and 22 edges

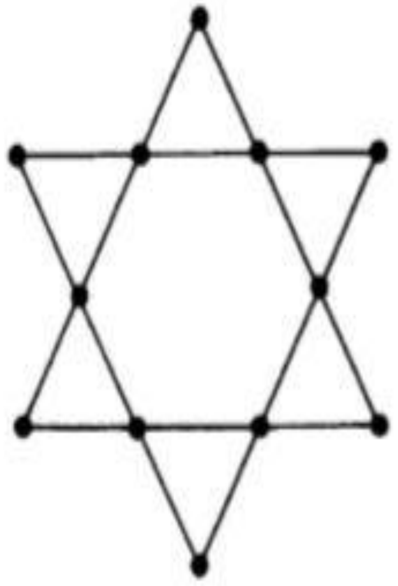
d) A graph with 9 vertices and 28 edges

Q Consider a simple undirected graph of 10 vertices. If the graph is disconnected, then the maximum number of edges it can have is _______________. **(GATE 2022) (1 MARKS)**

Euler Graph

- **Euler Graph:** - If some closed walk in a graph contains all the edges of the graph(connected), then the walk is called a Euler line and the graph a Euler Graph.
- A given connected graph G is a Euler graph if and only if all vertices of G are of even degree.





Q G is a simple undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is sure to be **(GATE-2008) (2 Marks)**

(A) regular **(B)** Complete **(C)** Hamiltonian **(D)** Euler

Q Which of the following graphs has a Eulerian circuit? **(GATE-2007) (2 Marks)**

(A) Any k-regular graph where k is an even number.

(B) A complete graph on 90 vertices

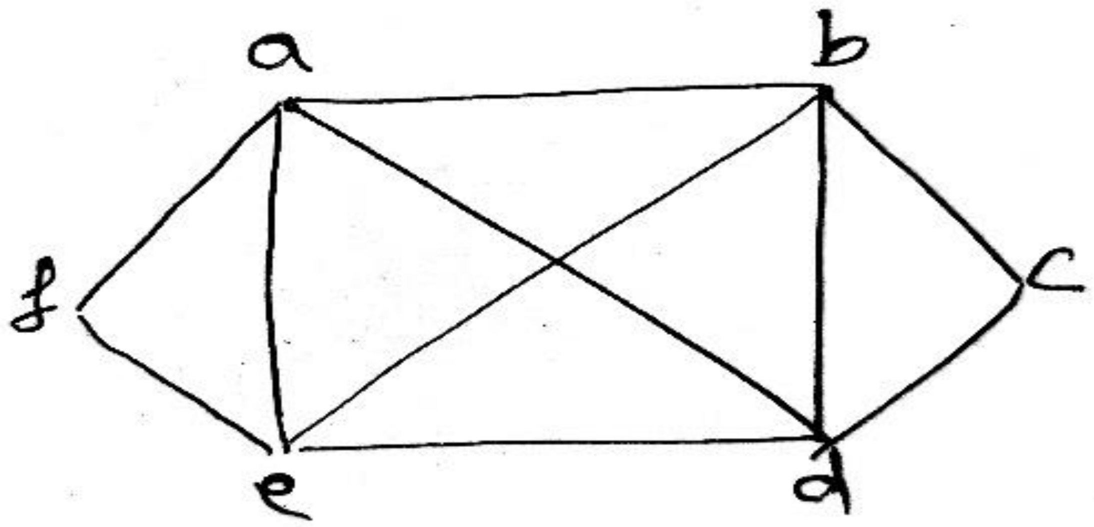
(C) The complement of a cycle on 25 vertices

(D) None of the above

Hamiltonian

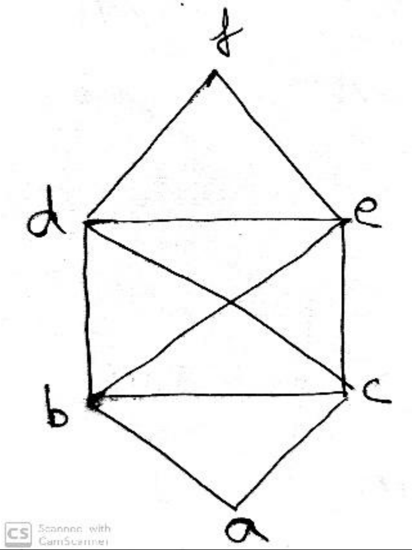
1. **Hamiltonian Graph:** - A Hamiltonian circuit in a connected graph is defined as a closed walk that traverses every vertex of G exactly once, except of course the starting vertex, at which the walk also terminates. A graph containing Hamiltonian circuit is called Hamiltonian graph.

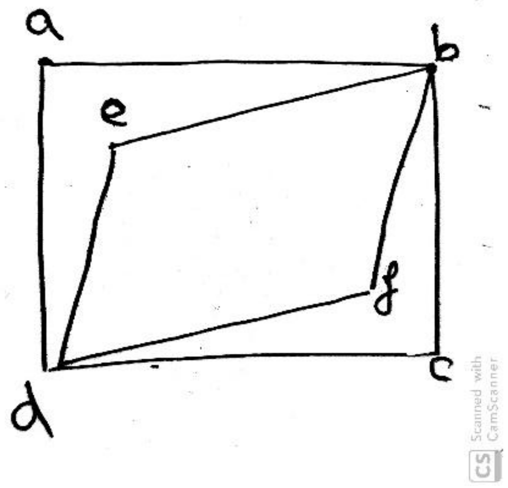
2. Finding weather a graph is Hamiltonian or not is a NPC problem.

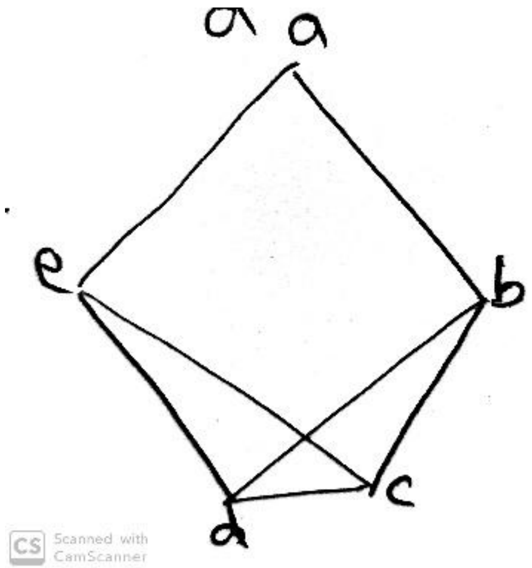


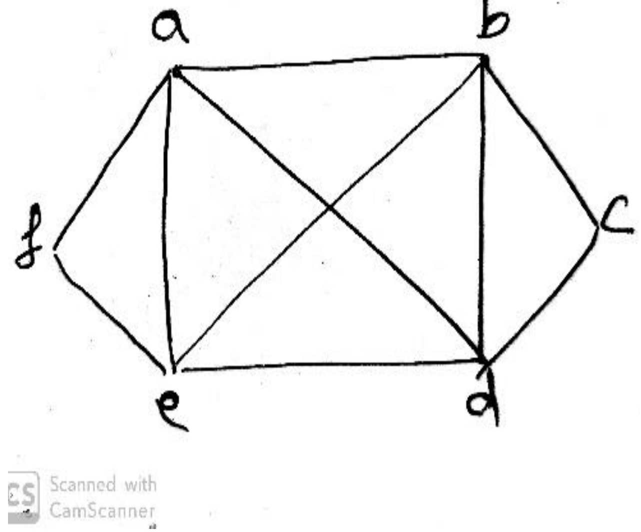
- A sufficient (but by no means necessary) condition for a simple graph G to have a Hamiltonian circuit is that the degree of every vertex in G be at least n/2, where n is the number of vertices in G. (if this condition satisfy graph will be Hamiltonian but to be a Hamiltonian graph this condition is not required to be true)

1. If we remove any one edge from a Hamiltonian circuit, we are left with a path. This path is called a Hamiltonian path.
2. If a graph has Hamiltonian circuit then it also has Hamiltonian path, but vice versa is not true.
3. In a complete graph with n vertices there are (n - 1)!/2 Hamiltonian circuits, if n >= 3.
4. In a complete graph with n vertices there are (n - 1)/2 edge-disjoint Hamiltonian circuits, if n is odd number >= 3.

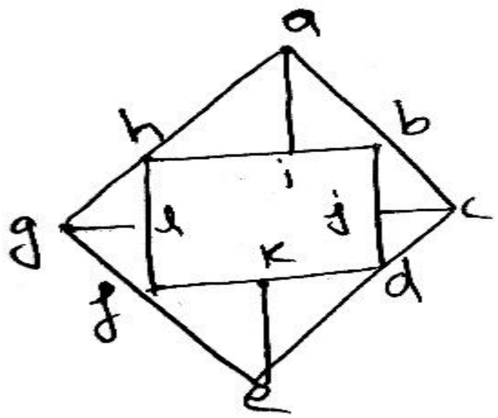


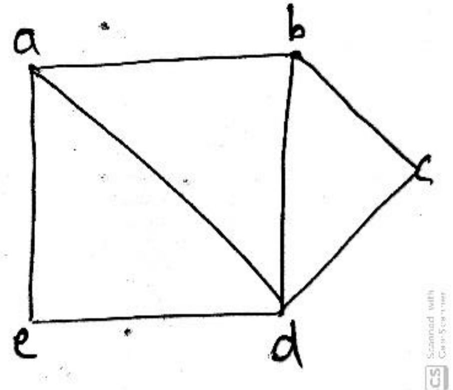


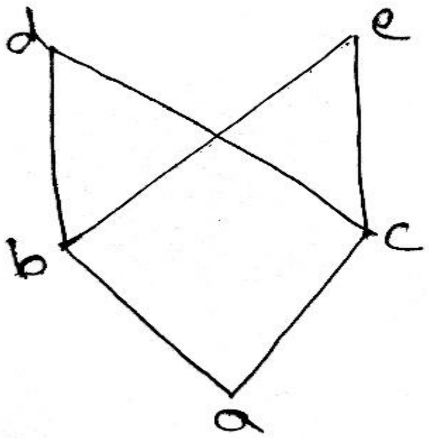


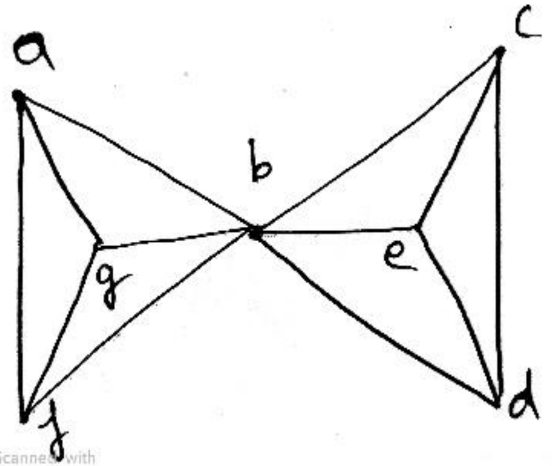


Euler Graph				
Hamiltonian Graph				

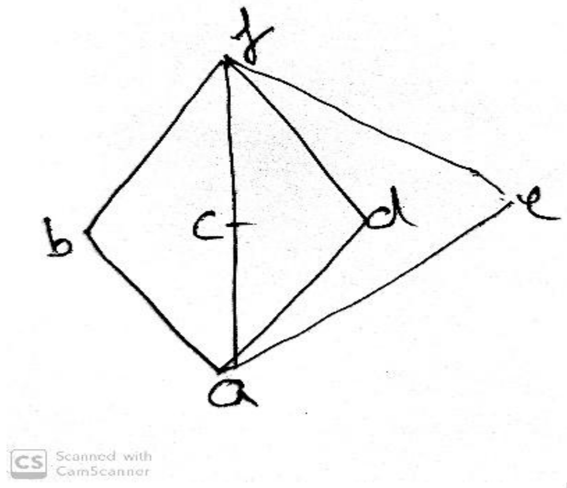


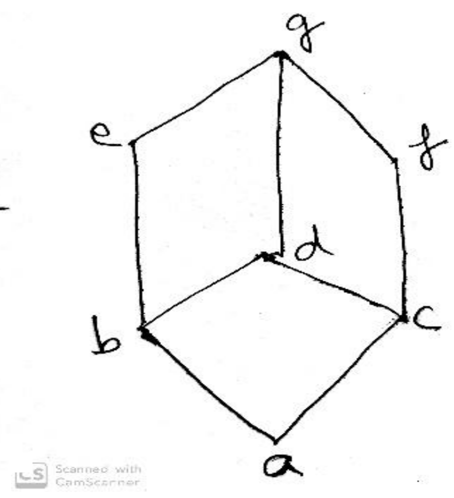


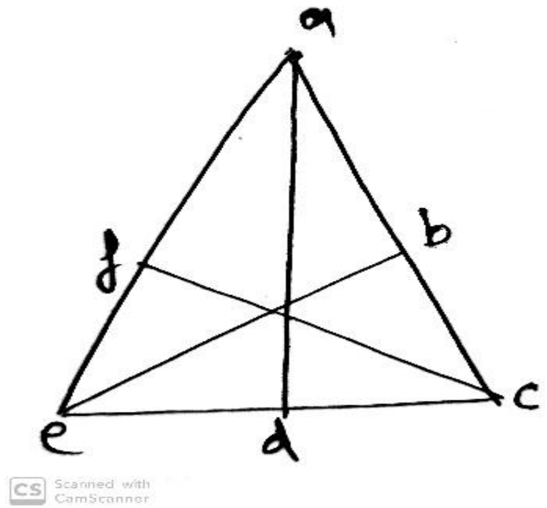


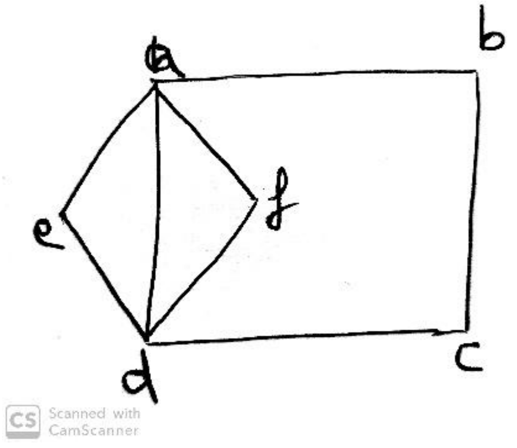


Euler Graph				
Hamiltonian Graph				









Euler Graph				
Hamiltonian Graph				

Q Let G be an undirected complete graph on n vertices, where n > 2. Then, the number of different Hamiltonian cycles in G is equal to

(GATE-2019) (2 Marks)

(A) n! **(B)** n – 1! **(C)** 1 **(D)** (n-1)! / 2

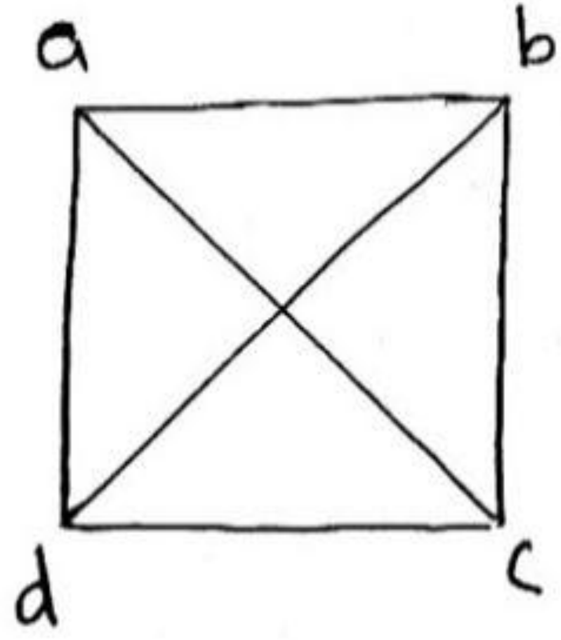
Q Let G be an undirected complete graph on n vertices, where $n > 2$. Then, the number of different Hamiltonian cycles in G is equal to **(GATE-2019) (2 Marks)**

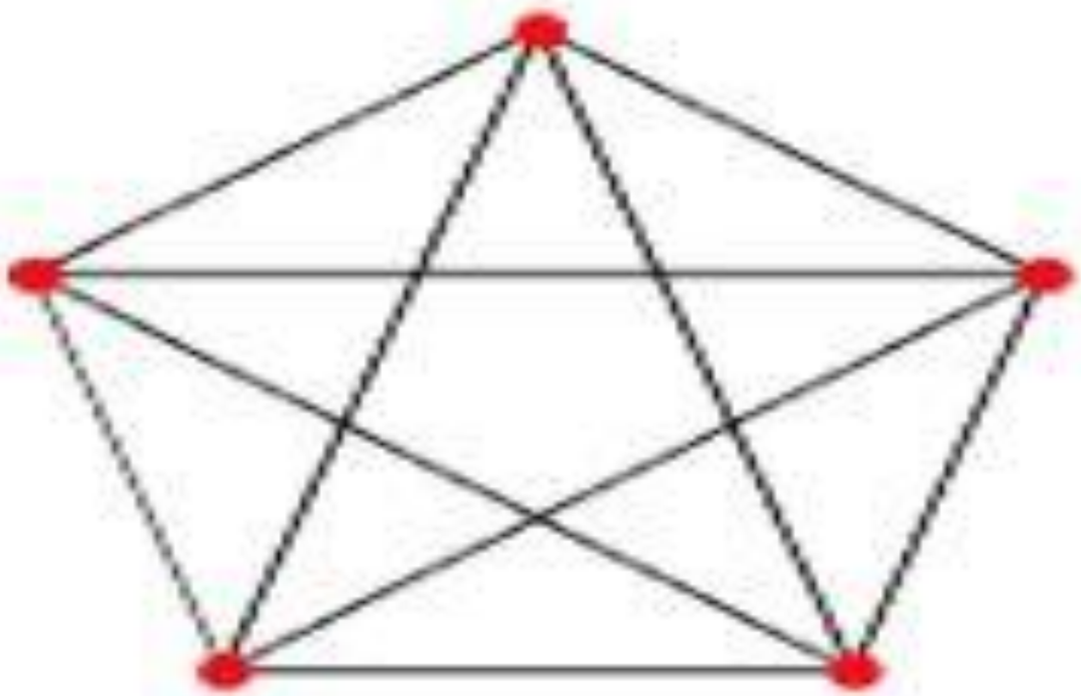
(A) $n!$ **(B)** $n - 1!$ **(C)** 1 **(D)** $(n-1)! / 2$

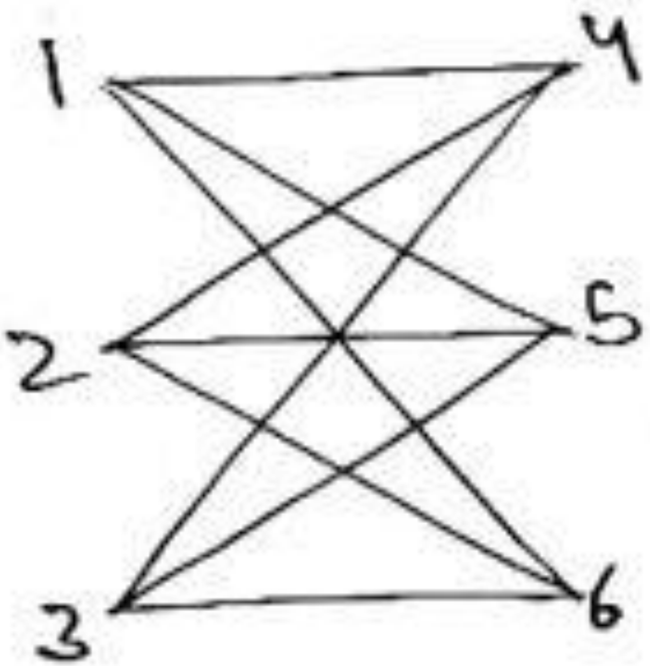
A simple circuit in a graph G that passes through every vertex exactly once is called a Hamiltonian circuit.
In an undirected complete graph on n vertices, there are n permutations are possible to visit every node. But from these permutations, there are: n different places (i.e., nodes) you can start; 2 (clockwise or anticlockwise) different directions you can travel.
So any one of these n! cycles is in a set of 2n cycles which all contain the same set of edges. So there are,
$= (n)! / (2n)$
$= (n-1)! / 2$ distinct Hamilton cycles.

Planer Graph

Planer Graph: - A graph is called a planer graph if it can be drawn on a plan in such a way that no edges cross each other, otherwise it is called non-planer. Application: civil engineering, circuit designing



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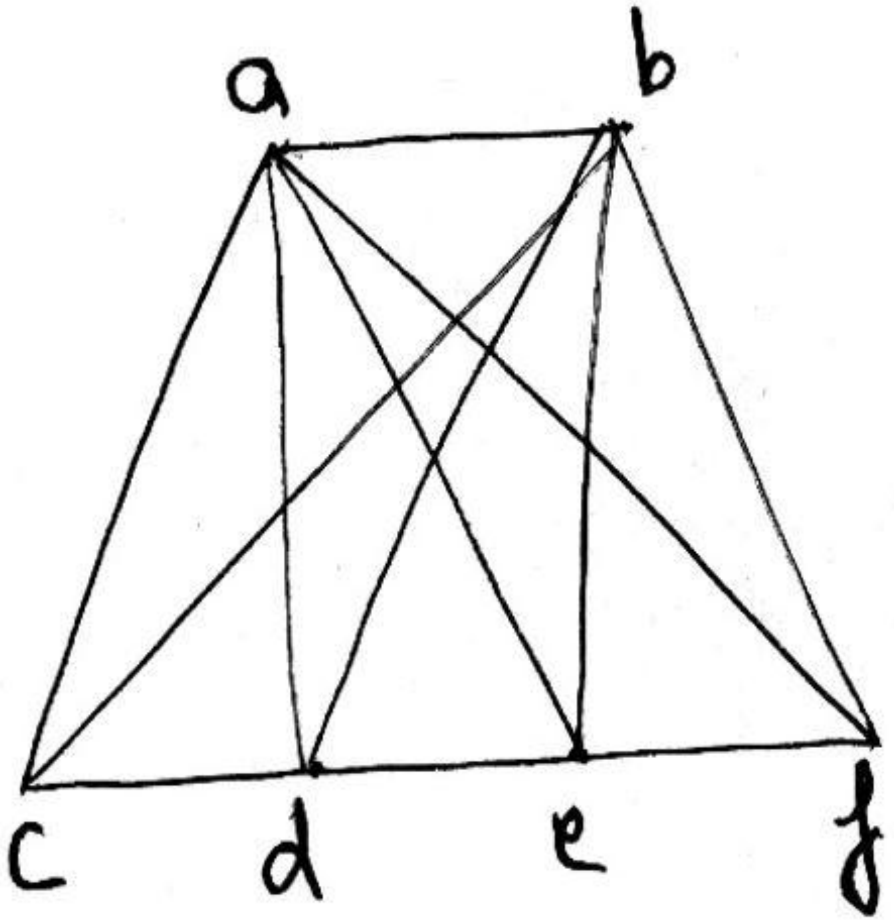
Simplest Non-Planer Graphs

1. **Kuratowski's case I:** - K_5
2. **Kuratowski's case II:** - $K_{3,3}$
3. Both are simplest non-planer graph
4. Both are regular graph
5. If we delete either an edge or a vertex from any of the graph, they will become planer

- **Kazimierz Kuratowski** (2 February 1896 – 18 June 1980) was a Polish mathematician and logician.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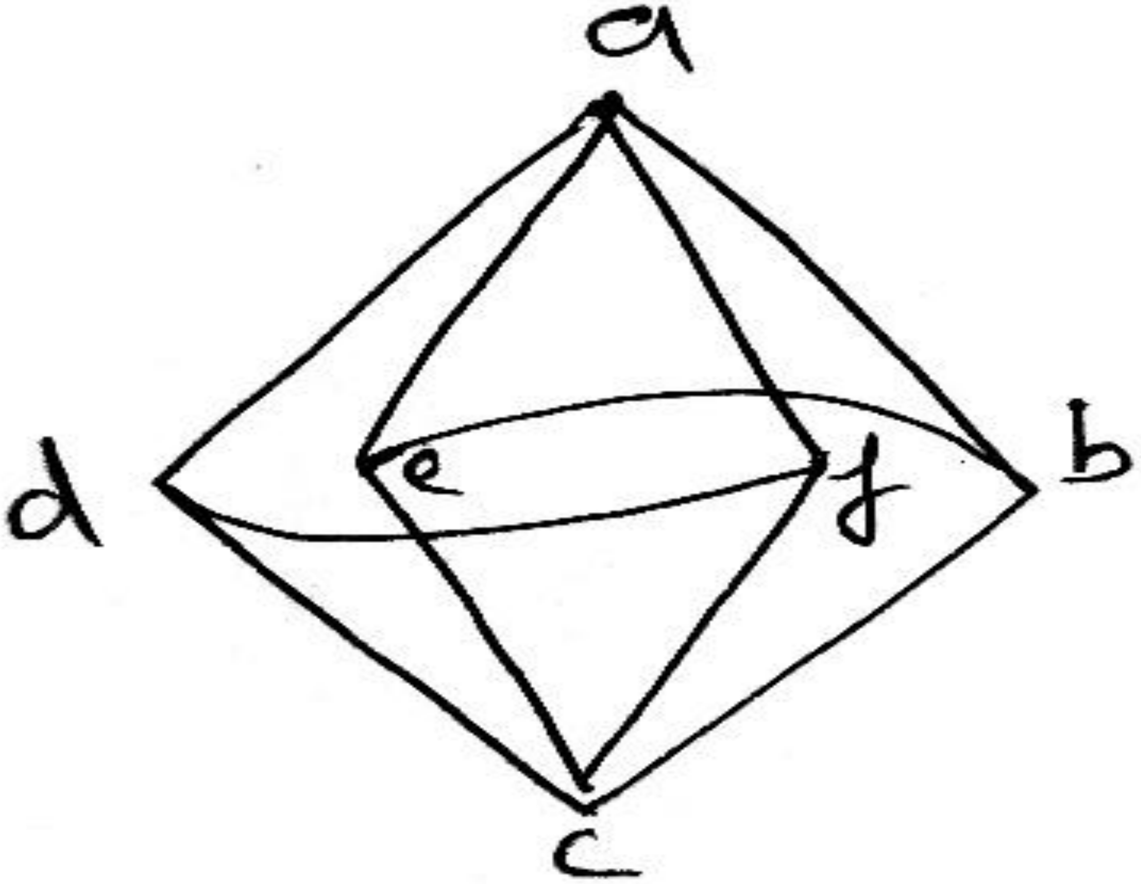
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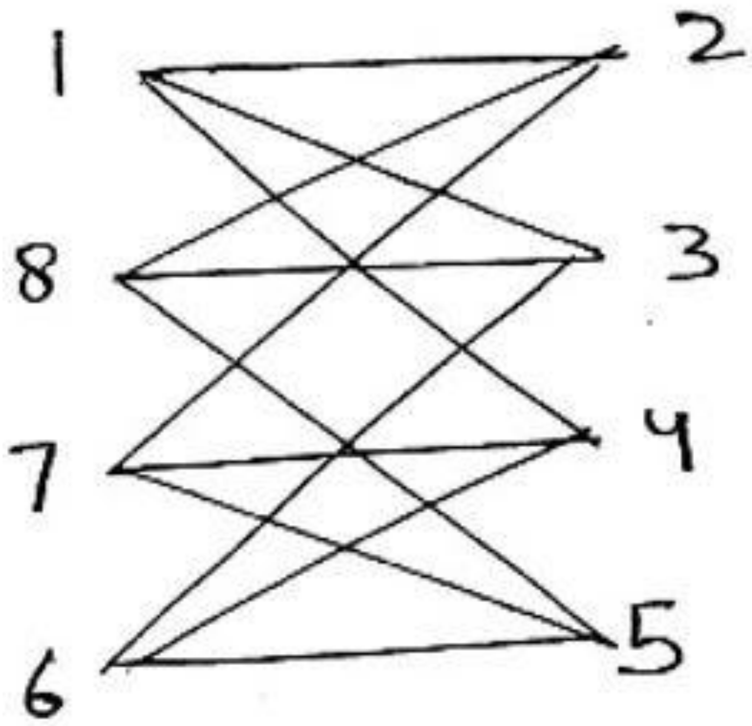
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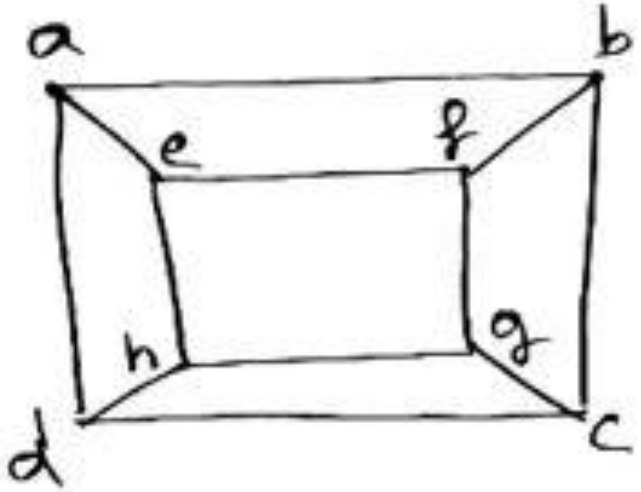
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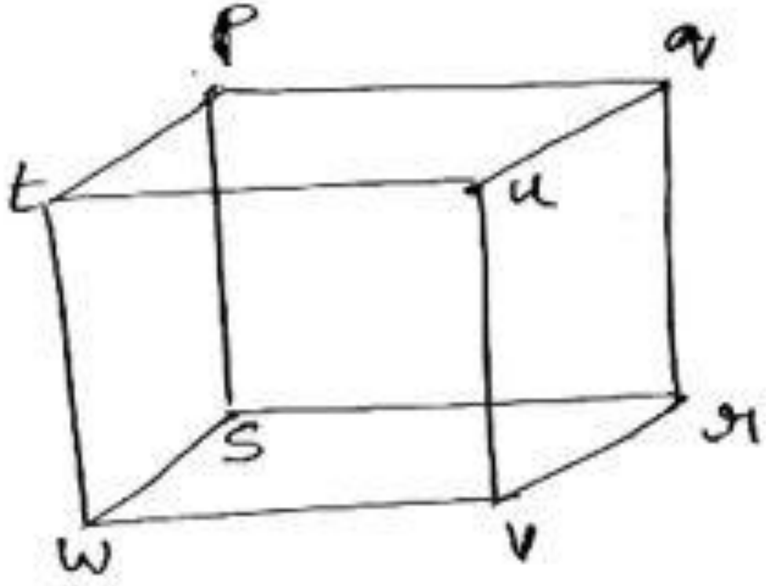
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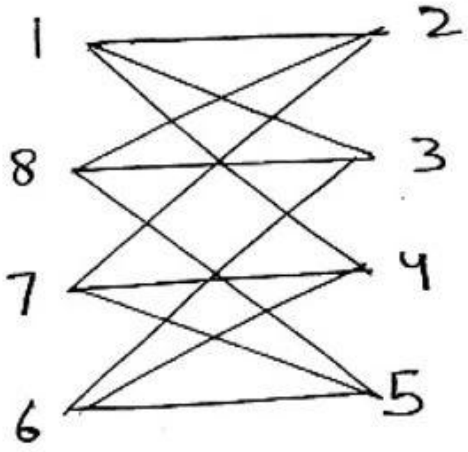
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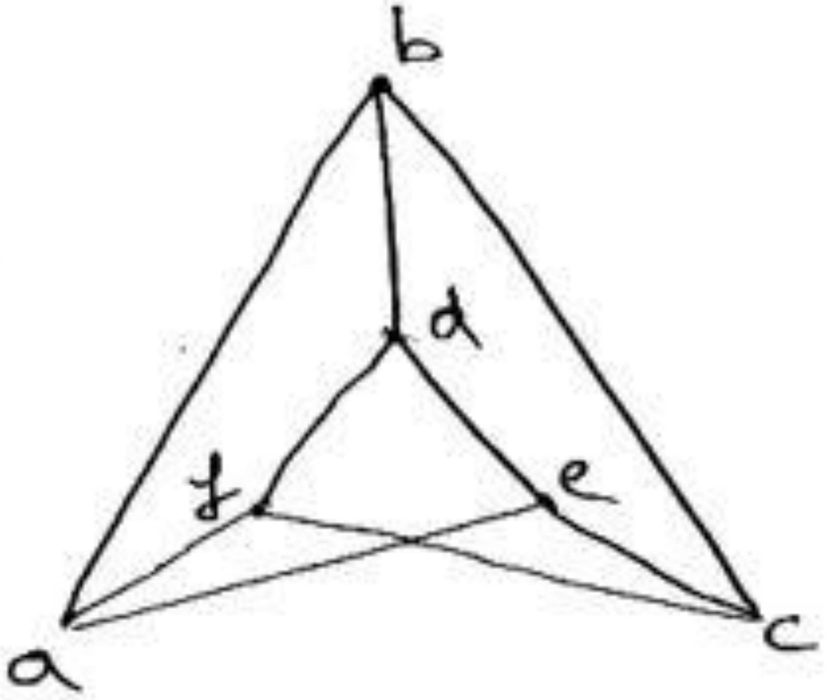
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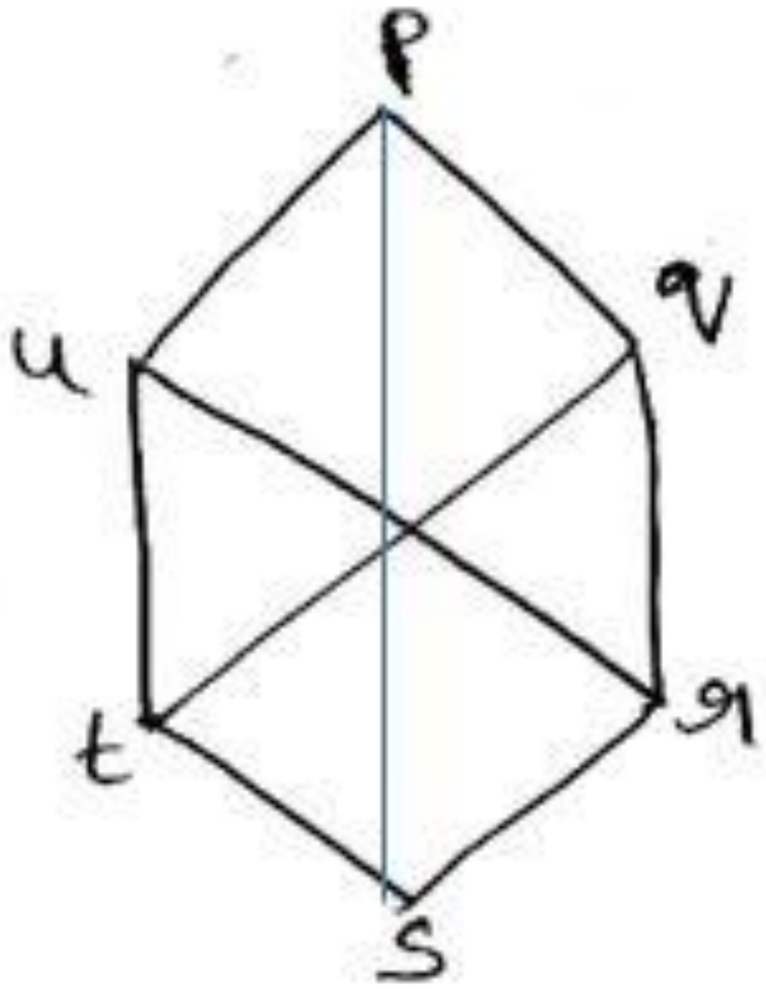
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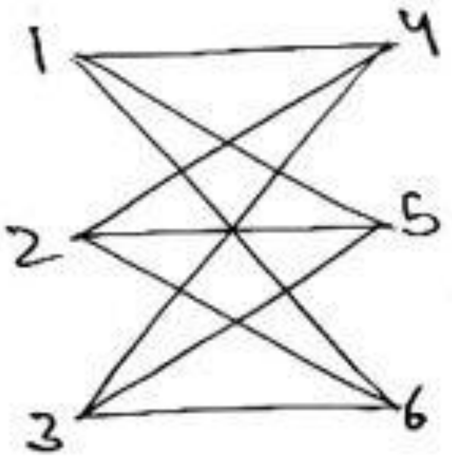
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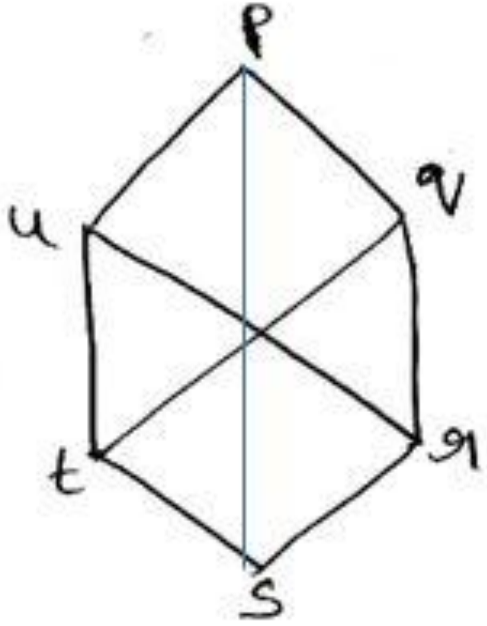
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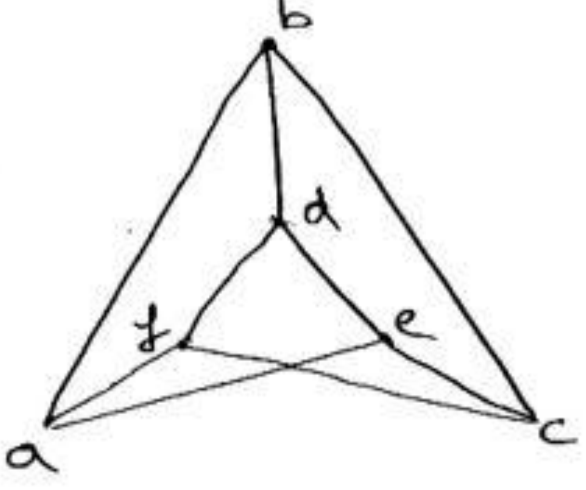
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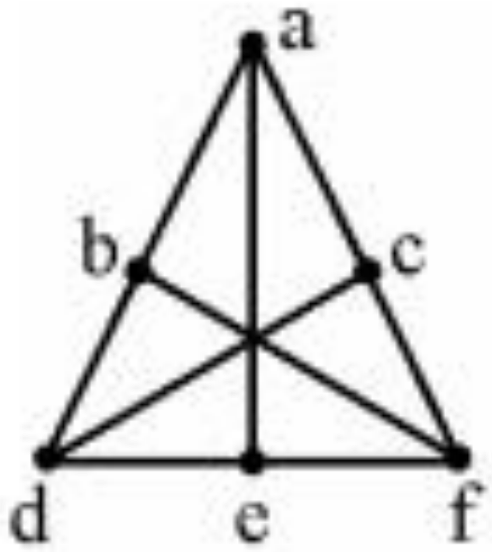
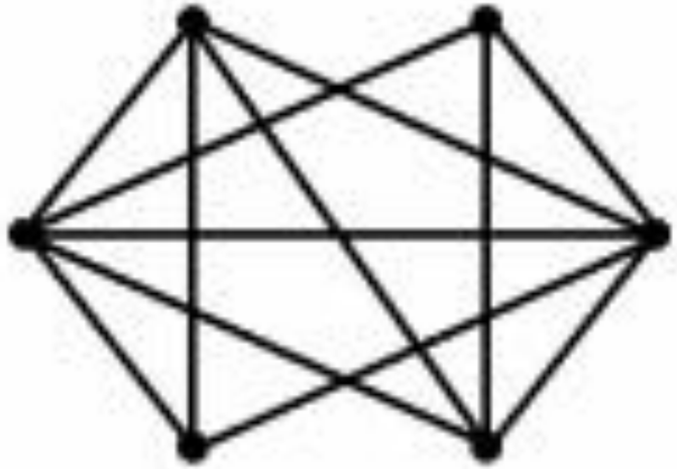
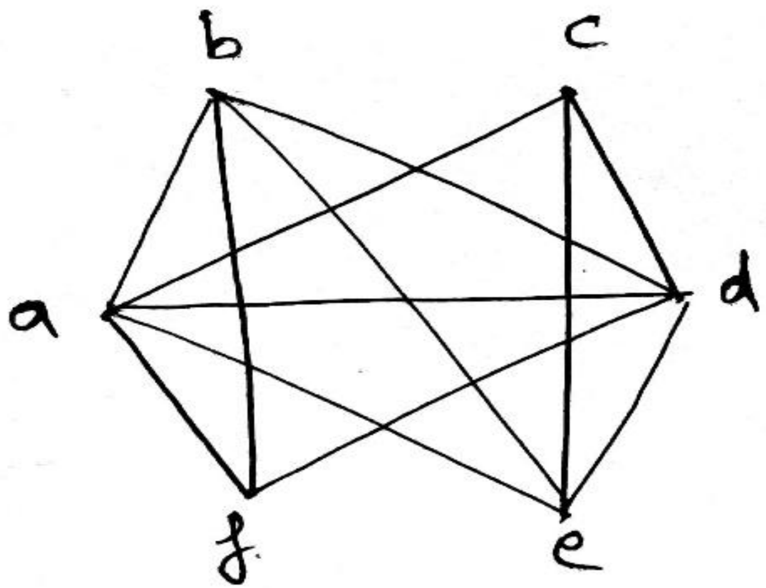


G1



G2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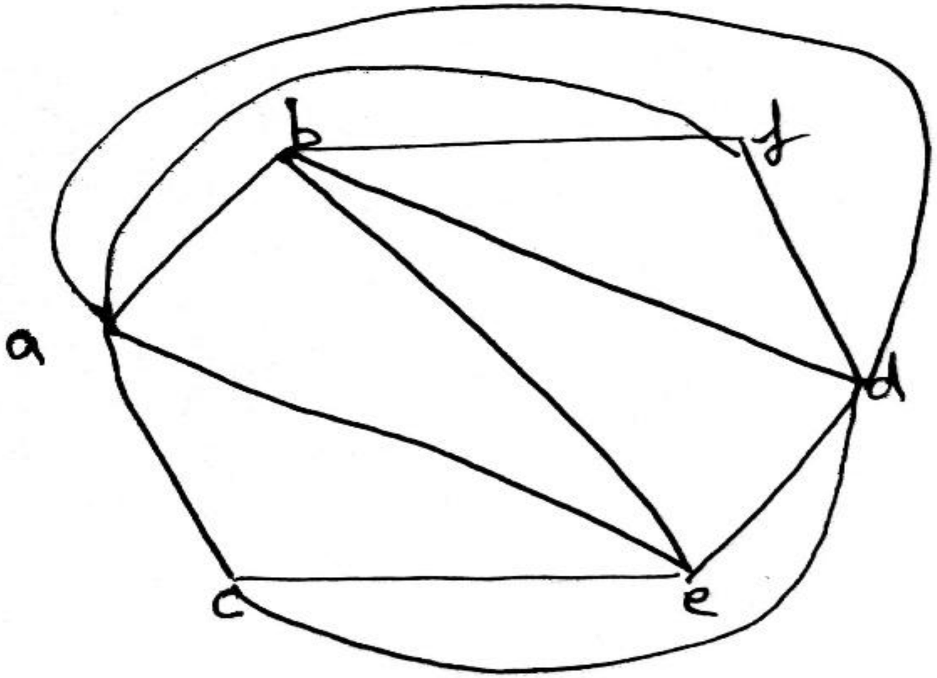
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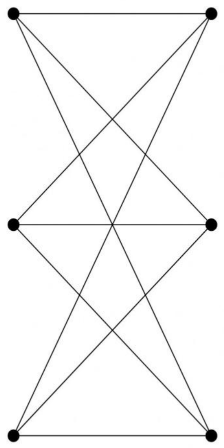
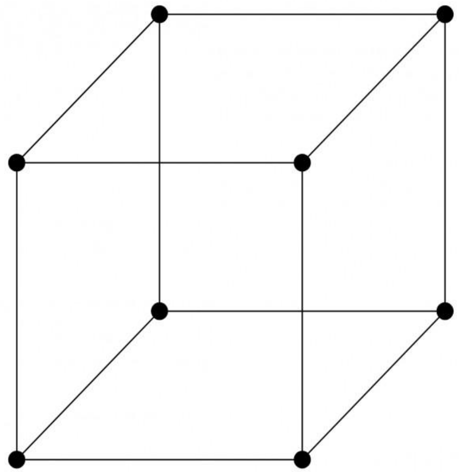
- How to find weather a graph is planer or non-planer
  - A finite graph is planar if and only if it does not contain a subgraph that is a subdivision(homorphism) of the complete graph K_5 or the complete bipartite graph. In practice, it is difficult to use Kuratowski's criterion to quickly decide whether a given graph is planar.

- This is a well-studied problem in computer science for which many practical algorithms have emerged, many taking advantage of novel data structures. Most of these methods operate in O(n) time (linear time), where n is the number of edges (or vertices) in the graph, which is asymptotically optimal.

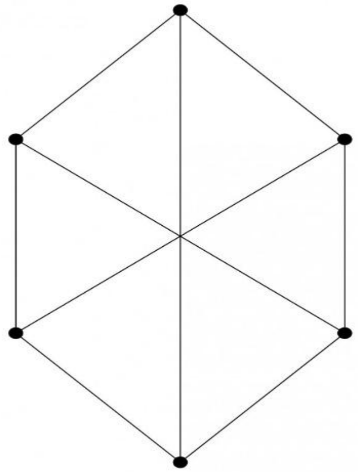
Q Which of the following graphs is/are planner? **(GATE-19) (2 Marks)**



G_1



G_2



G_3

a) G1 only

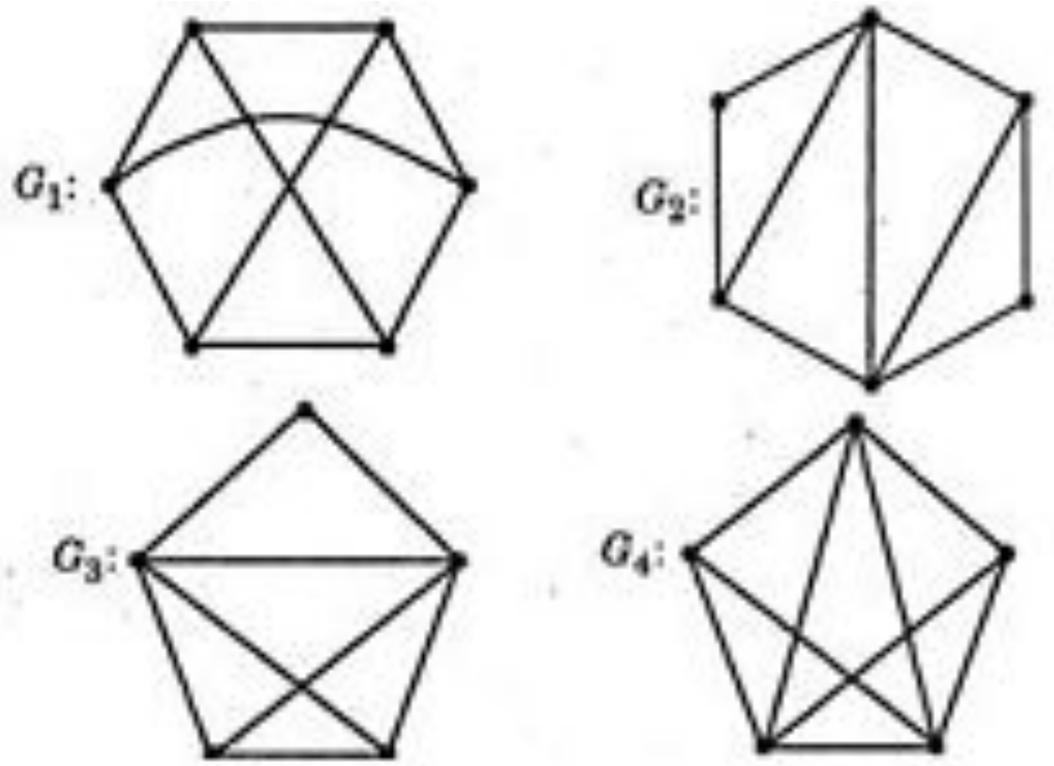
b) G1 and G2

c) G2 only

d) G2 and G3

Q Which one of the following graphs is NOT planar? **(GATE-2005) (2 Marks)**

(A) G_1



(B) G_2

(C) G_3

(D) G_4

Q (GATE-2010) (2 Marks)

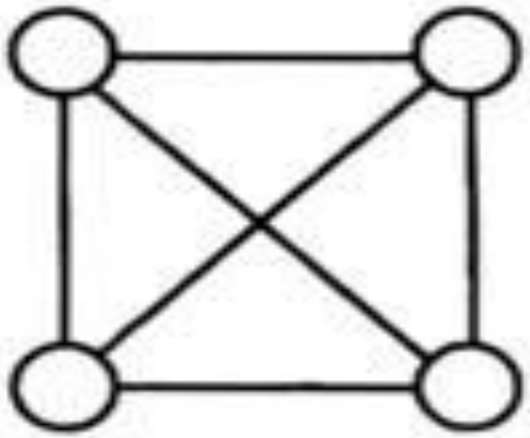
(A) K_4 is planar while Q_3 is not

(B) Both K_4 and Q_3 are planar

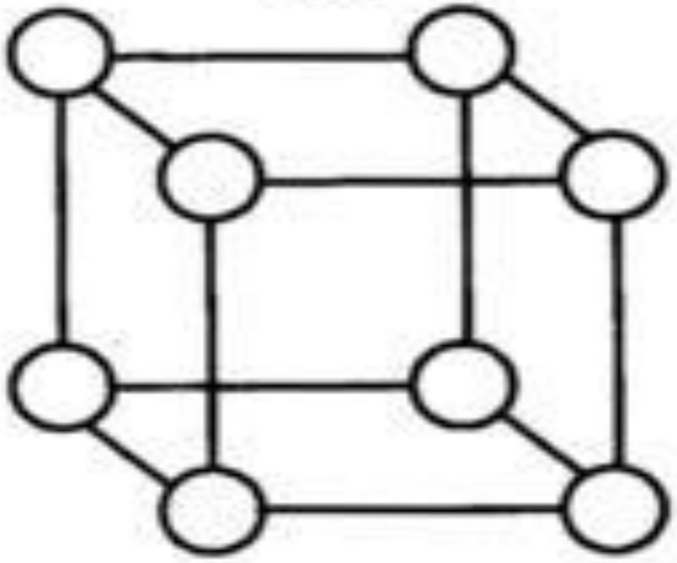
(C) Q_3 is planar while K_4 is not

(D) Neither K_4 nor Q_3 are planar

K4



Q3



Q Let G be the non-planar graph with the minimum possible number of edges. Then G has **(GATE-1992) (1 Marks) (GATE-2007) (1 Marks)**

(A) 9 edges and 5 vertices

(B) 9 edges and 6 vertices

(C) 10 edges and 5 vertices

(D) 10 edges and 6 vertices

Q A graph is planar if and only if, **(GATE-1990) (2 Marks)**

a) It does not contain subgraphs homeomorphic to k5 and k3,3.

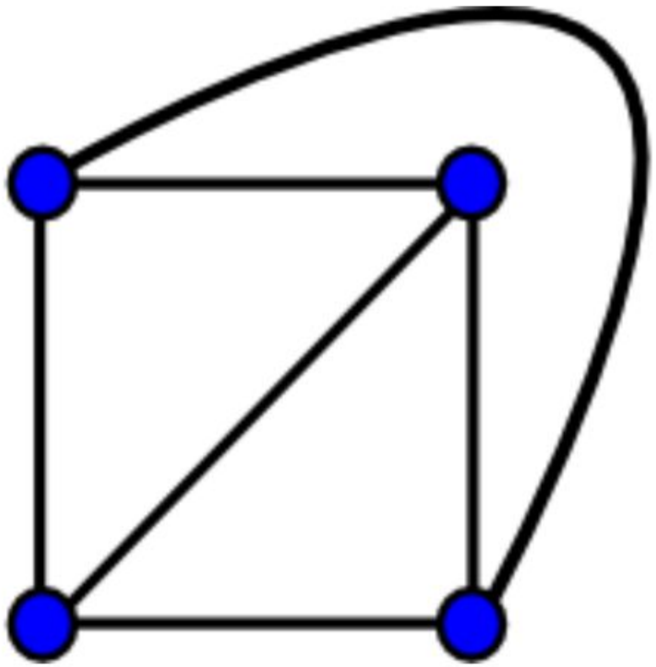
b) It does not contain subgraphs isomorphic to k5 or k3,3.

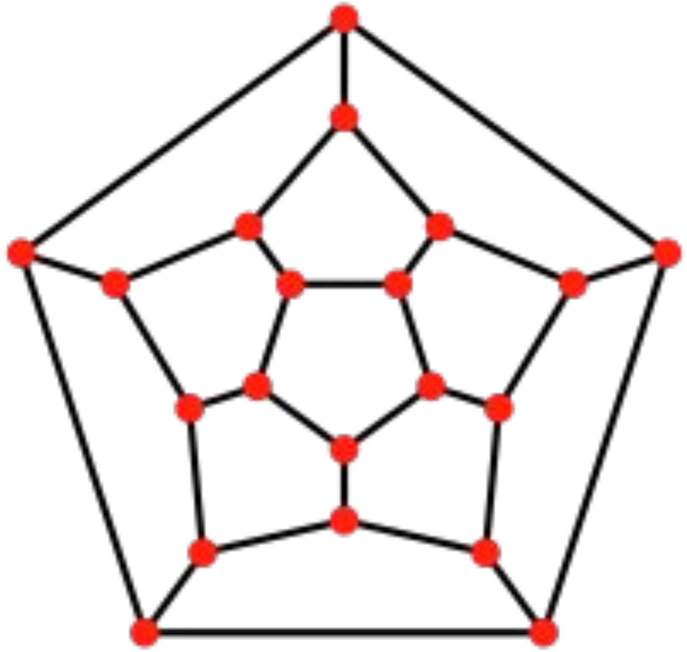
c) It does not contain a subgraph isomorphic to k5 or k3,3

d) It does not contain a subgraph homeomorphic to k5 or k3,3.

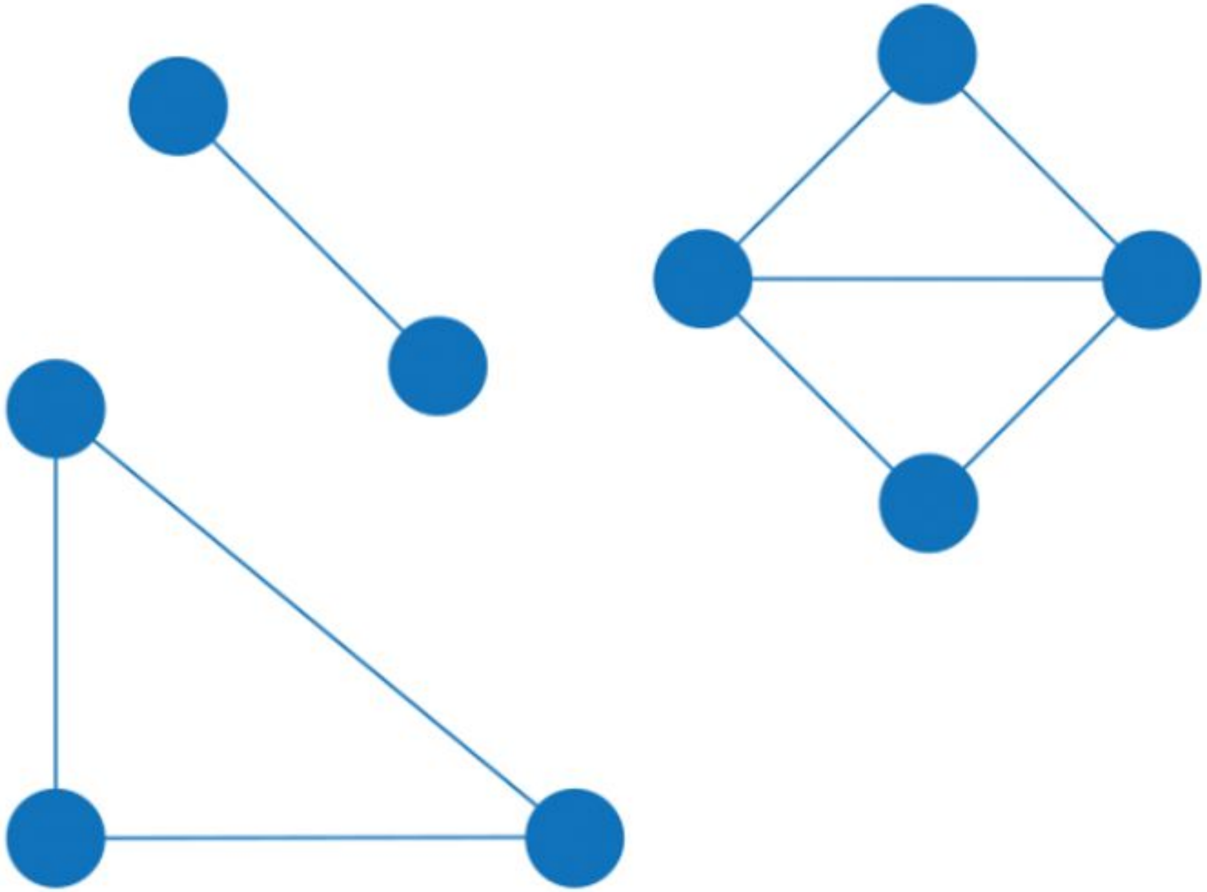
Euler's formula

- A planer graph divides the plane into number or regions (faces, planer embedding), which are further divided into bounded(internal) and unbounded region(external).
- **Euler's formula** states that if a finite, connected, planar graph with *v* is the number of vertices, *e* is the number of edges and *r* is the number of faces (regions bounded by edges, including the outer, infinitely large region), then
- r = e – v + 2
- Euler's formula can be proved by mathematical induction





- Euler's formula (Disconnected graph): $V - e + r - k = 1$



Q In an undirected connected planar graph G, there are eight vertices and five faces. The number of edges in G is __________. **(GATE 2021)**

(a) 10

(b) 11

(c) 12

(d) 6

Q Let G be a simple connected planar graph with 13 vertices and 19 edges. Then, the number of faces in the planar embedding of the graph is **(GATE-2005) (2 Marks)**

(A) 6 **(B)** 8 **(C)** 9 **(D)** 13

Q Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to **(GATE-2012) (1 Marks)**

(A) 7 (B) 4 (C) 5 (D) 6

Other formula derived from Euler's formula

- Connected planar graphs with more than one edge obey the inequality $2e>=3r$, because each face has at least three face-edge incidences and each edge contribute exactly two incidences.

- Degree of the region is number of edges covering the region. Sum of degree of regions = $2|E|$

Using r = e – v + 2 and 3r<=2e
eliminating r we get, $e \leq 3v - 6$

Using r = e – v + 2 and 3r<=2e
Eliminating e we get, $r \leq 2v - 4$

Q Maximum number of edges in a planar graph with n vertices **(GATE-1992) (1 Marks)**

Q A graph G = (V, E) satisfies $|E| \leq 3\ |V| - 6$. The min-degree of G is defined as

$$\min_{v \in V} \{\text{degree}(v)\}$$

Therefore, min-degree of G cannot be **(GATE-2003) (2 Marks)**

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

Q Let δ denote the minimum degree of a vertex in a graph. For all planar graphs on n vertices with $\delta \geq 3$, which one of the following is TRUE? **(GATE-2014) (2 Marks)**

(A) In any planar embedding, the number of faces is at least $n/2 + 2$

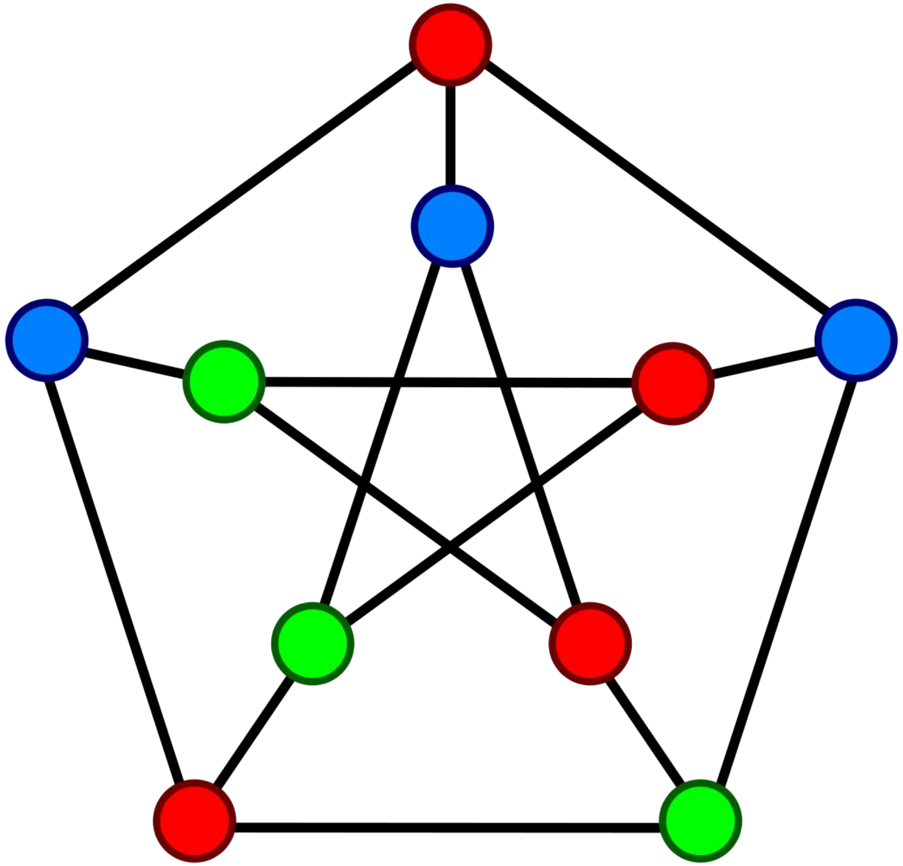
(B) In any planar embedding, the number of faces is less than $n/2 + 2$

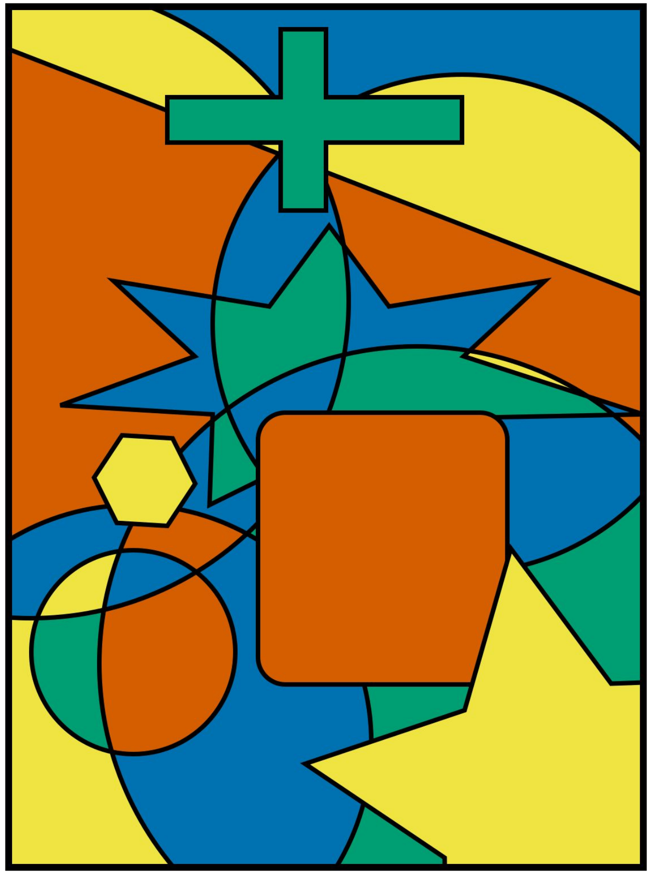
(C) There is a planar embedding in which the number of faces is less than $n/2 + 2$

(D) There is a planar embedding in which the number of faces is at most $n/(\delta + 1)$

Graph Coloring

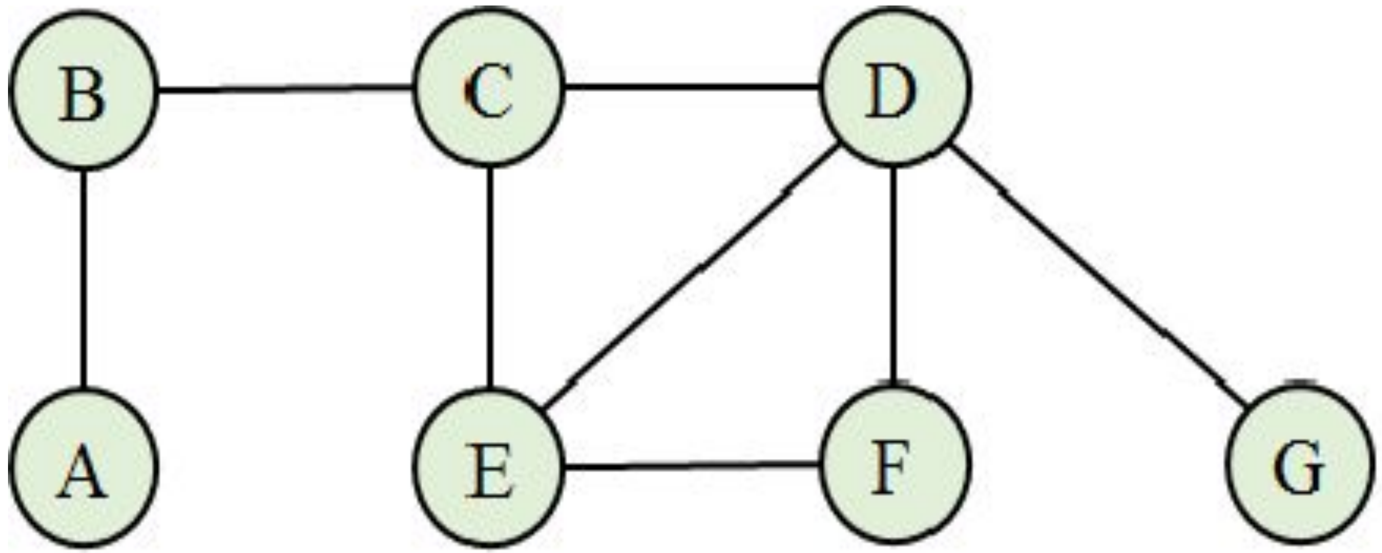
- Graph coloring can be of two types vertex coloring and region coloring.



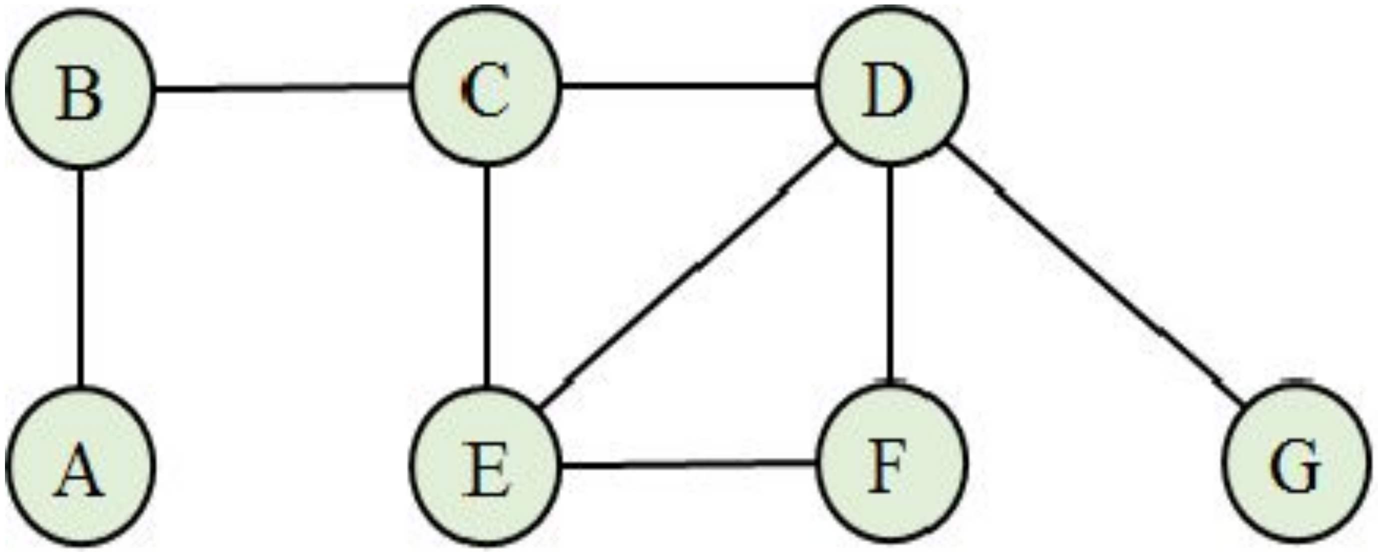


- Associating a color with each vertex of the graph is called vertex coloring.

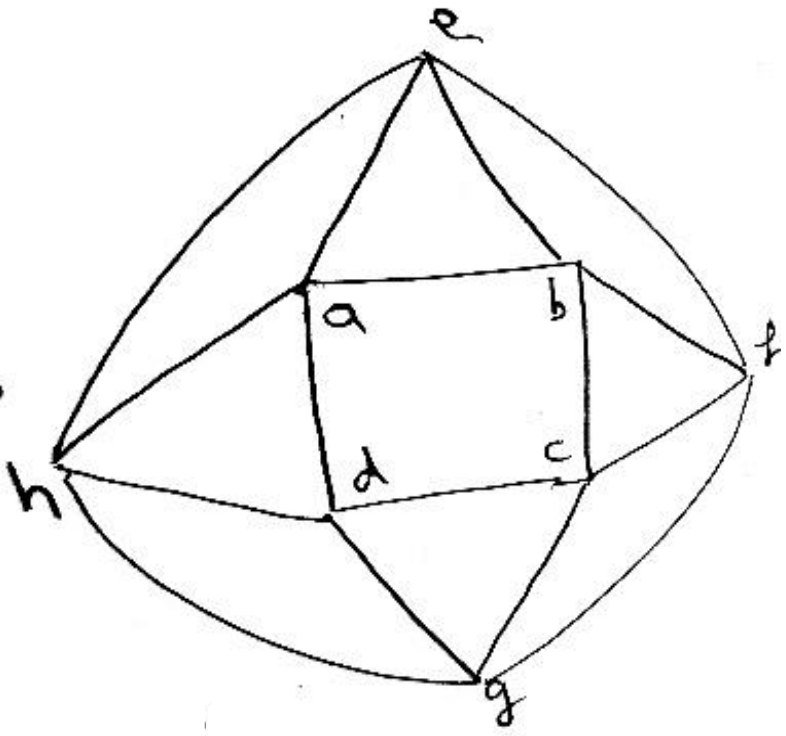
- **<u>Proper Vertex coloring:</u>** - Associating all the vertex of a graph with colors such that no two adjacent vertices have the same color is called proper vertex coloring.



- **<u>Chromatic number of the graph:</u>** - Minimum number of colors required to do a proper vertex coloring is called the chromatic number of the graph, denoted by $\chi(G)$. the graph is called K-chromatic or K-colorable.



- Cost of finding chromatic number is an NPC problem and there exists no polynomial algorithm to do that. There exists some greedy approach which try to solve it in P time, but they do not guarantee optimal solution.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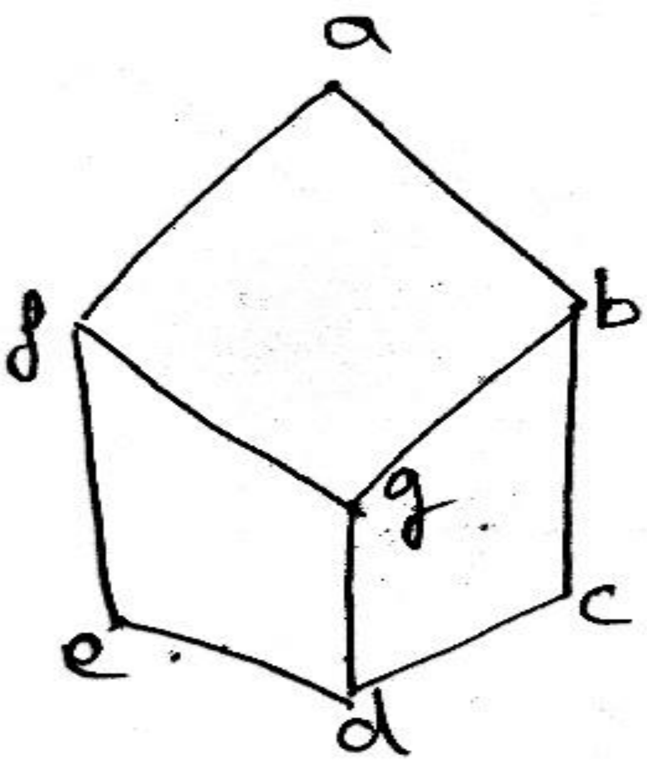
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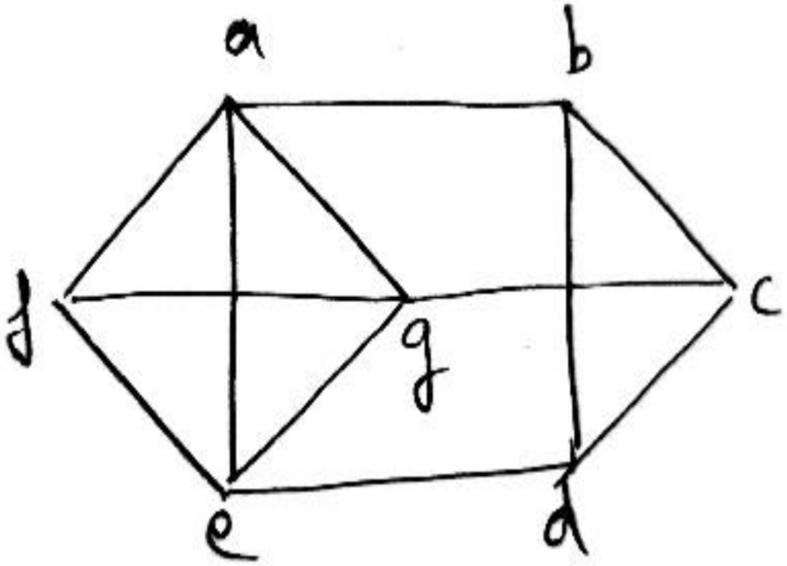
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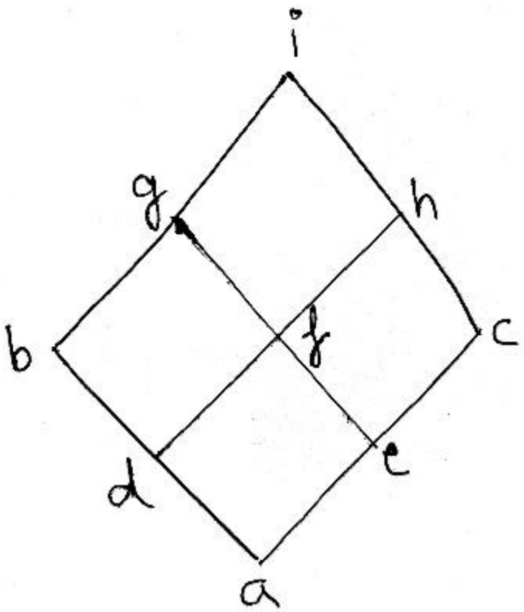
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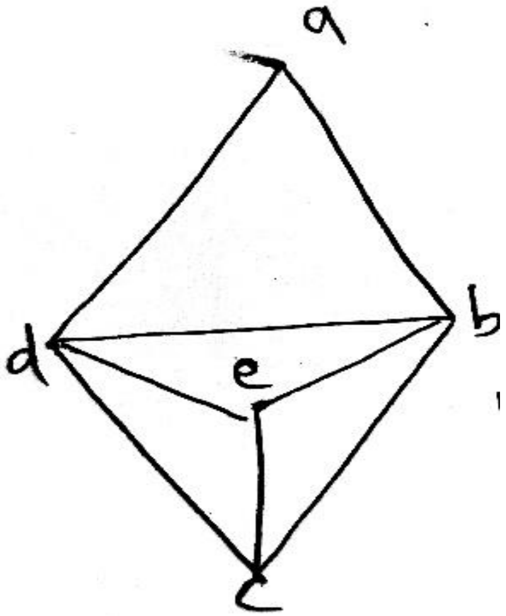
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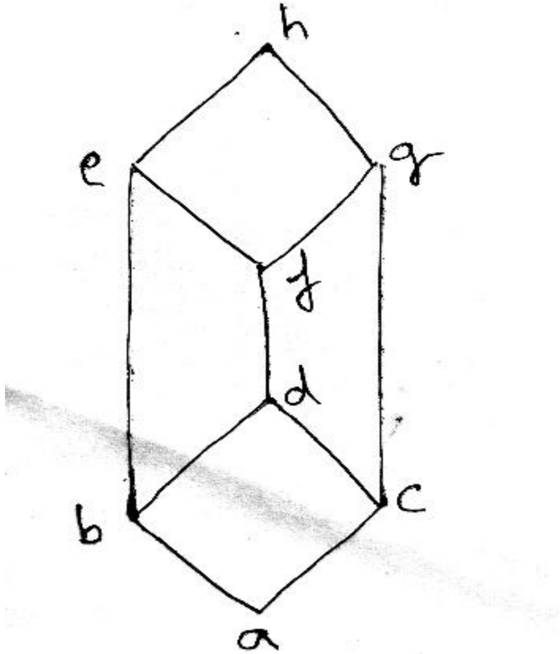
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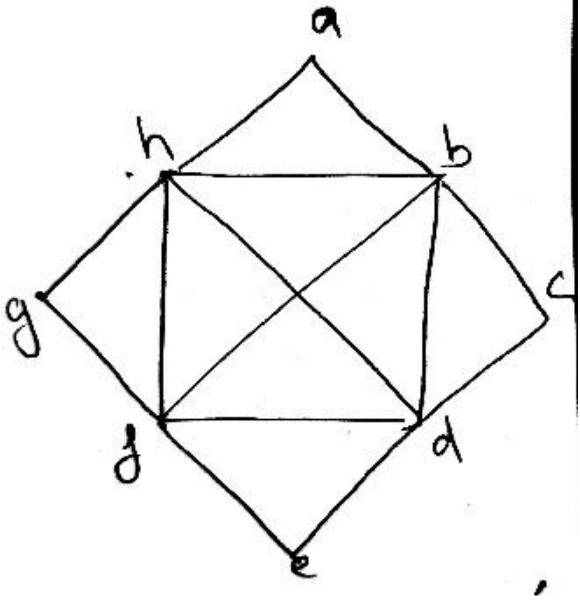
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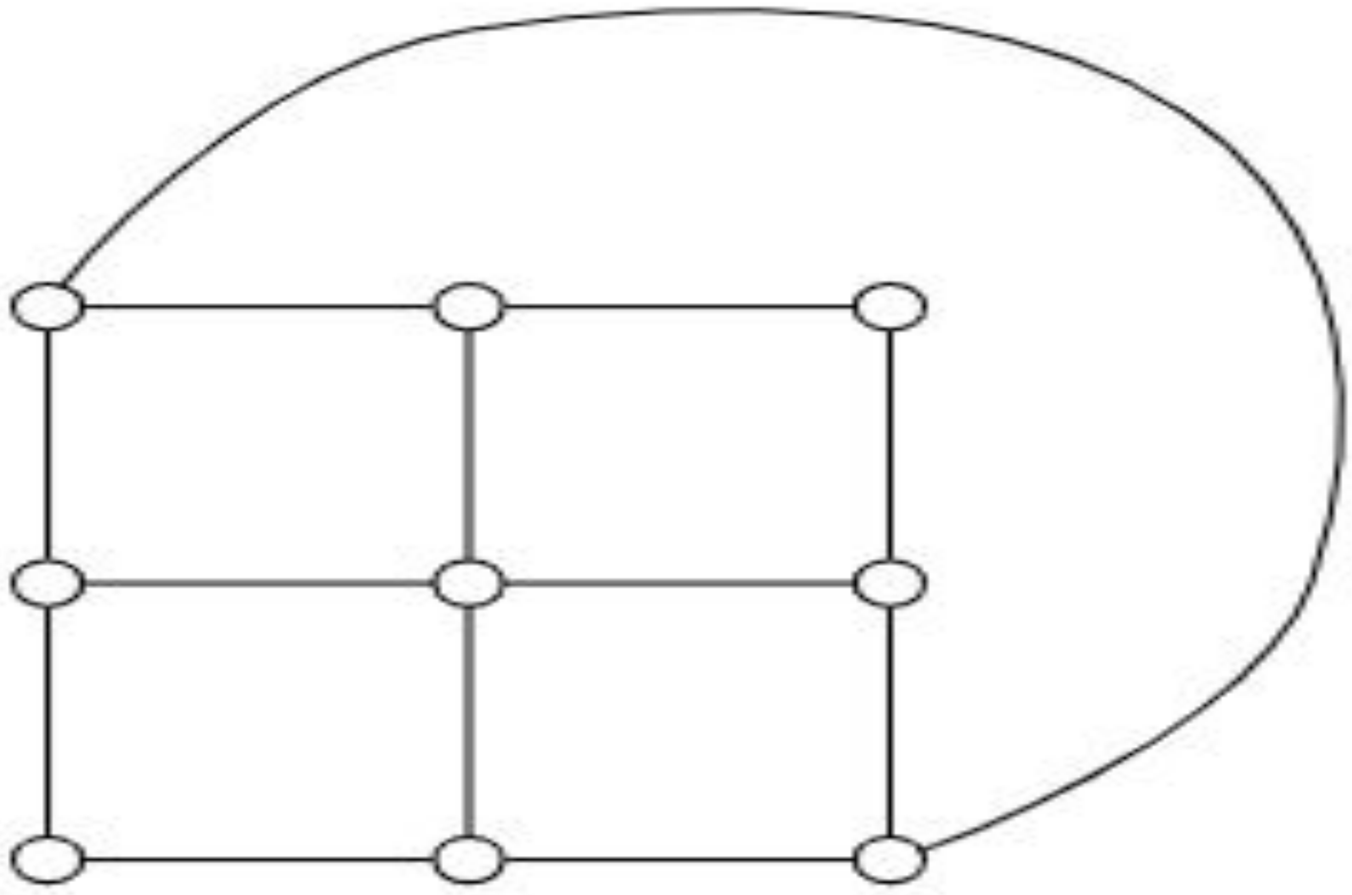
Q What is the chromatic number of the following graph? **(GATE-2008) (1 Marks)**

(A) 2

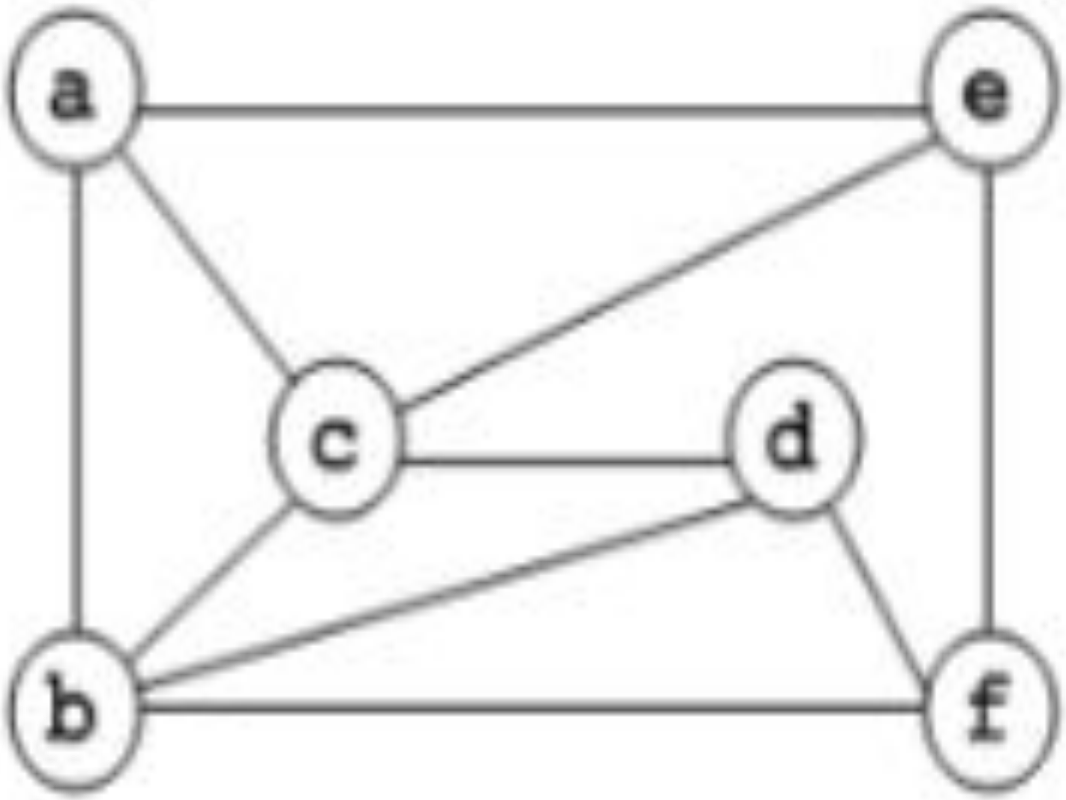
(B) 3

(C) 4

(D) 5

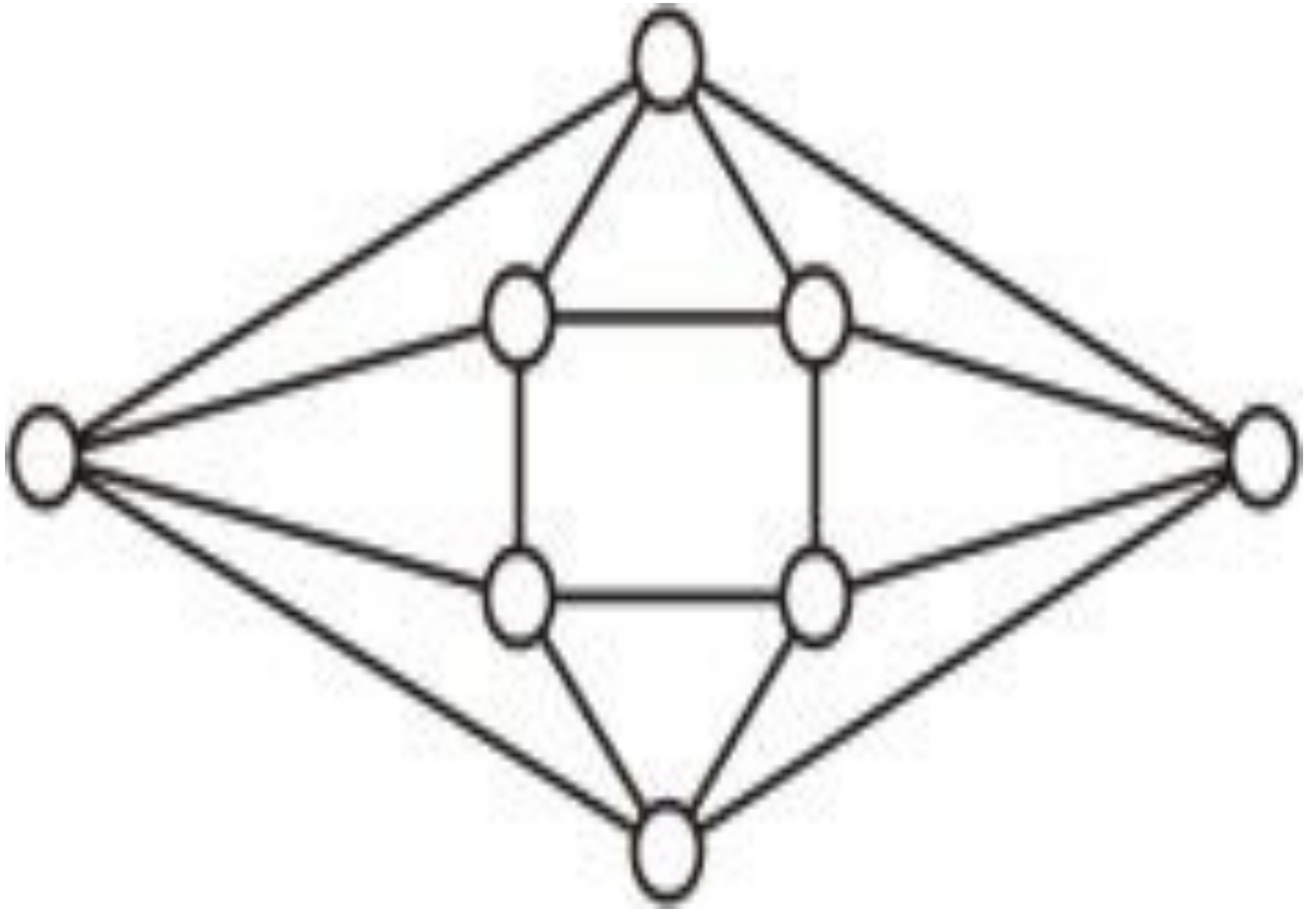


Q The chromatic number of the following graph is _________**(GATE-2018) (2 Marks)**



Q The minimum number of colors required to color the following graph, such that no two adjacent vertices are assigned the same color, is **(GATE-2004) (2 Marks)**

(A) 2



(B) 3

(C) 4

(D) 5

Q The minimum number of colors that is sufficient to vertex color any planar graph is ______________________**(GATE-2016) (1 Marks)**

Q What is the chromatic number of an n-vertex simple connected graph which does not contain any odd length cycle? Assume n >= 2. **(GATE-2009) (1 Marks)**

(A) 2 **(B)** 3 **(C)** n-1 **(D)** n

Q The minimum number of colors required to color the vertices of a cycle with η nodes in such a way that no two adjacent nodes have the same color is

(GATE-2002) (1 Marks)

(A) 2 **(B)** 3 **(C)** 4 **(D)** $n - 2\lfloor n/2 \rfloor + 2$

Q.The chromatic number of a graph is the minimum number of colours used in a proper coloring of the graph. Let G be any graph with n vertices and chromatic number k. Which of the following statements is/are always TRUE? **(Gate 2024,CS) (2 Marks)(MSQ)**

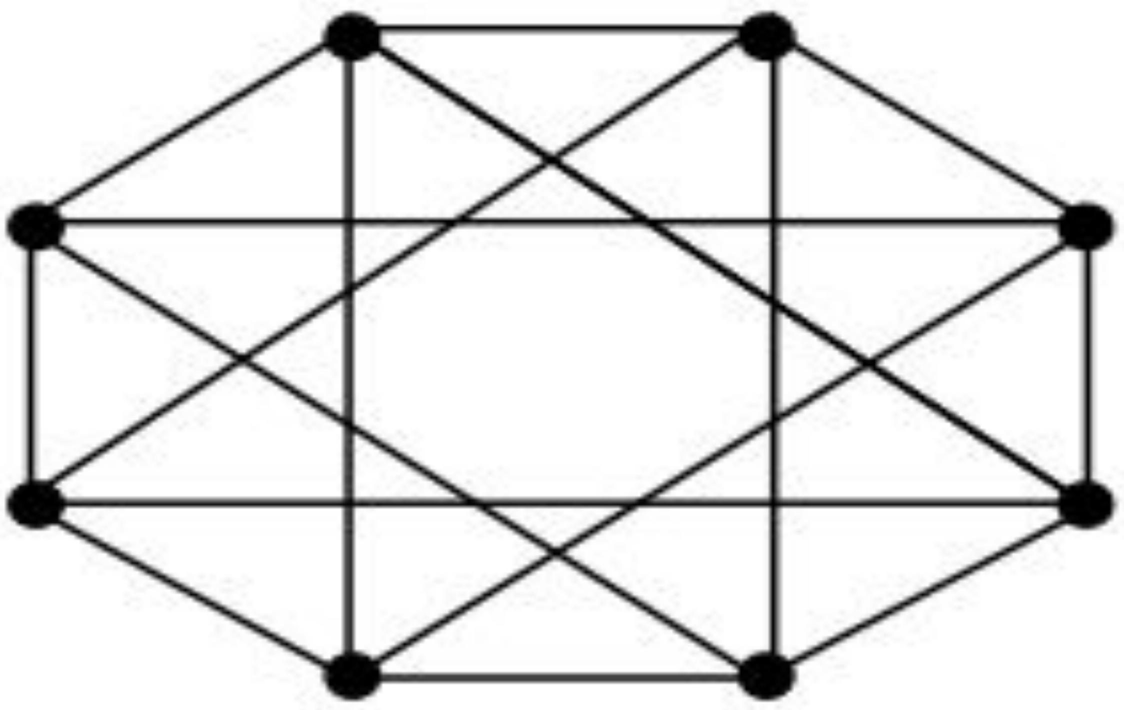
(a) G contains a complete subgraph with K vertices

(b) G contains an independent set of size at least n/K

(c) G contains at least K(K-1)/2 edges

(d) G contains a vertex of degree of at least K

Q The chromatic number of a graph is the minimum number of colours used in a proper colouring of the graph. The chromatic number of the following graph is __________**(Gate 2024 CS)(2 Marks)(NAT)**



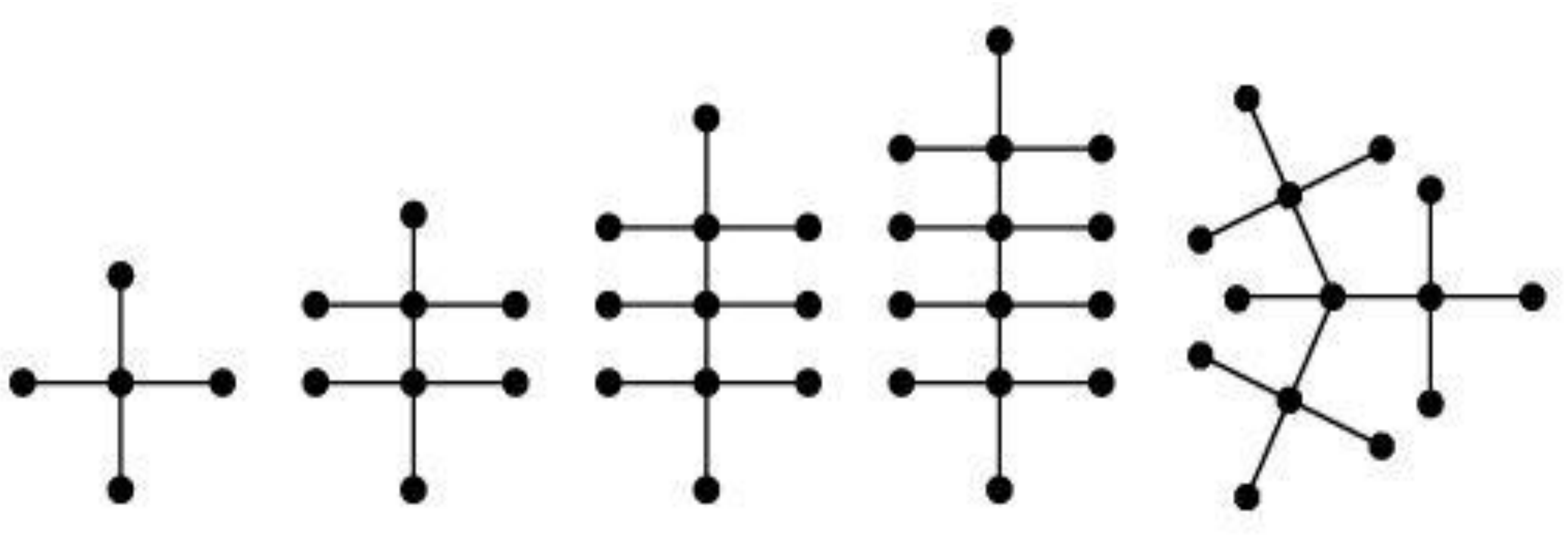
- Trivial graph is 1-chromatic
- A graph with 1 or more edge is at least 2-chromatic
- A complete graph K_n is n-chromatic

- Tree is always 2-chromatic
- Bi-partite graph is 2-chromatic
- C_n is 2-chromatic if n is even, C_n is 3-chromatic if n is odd

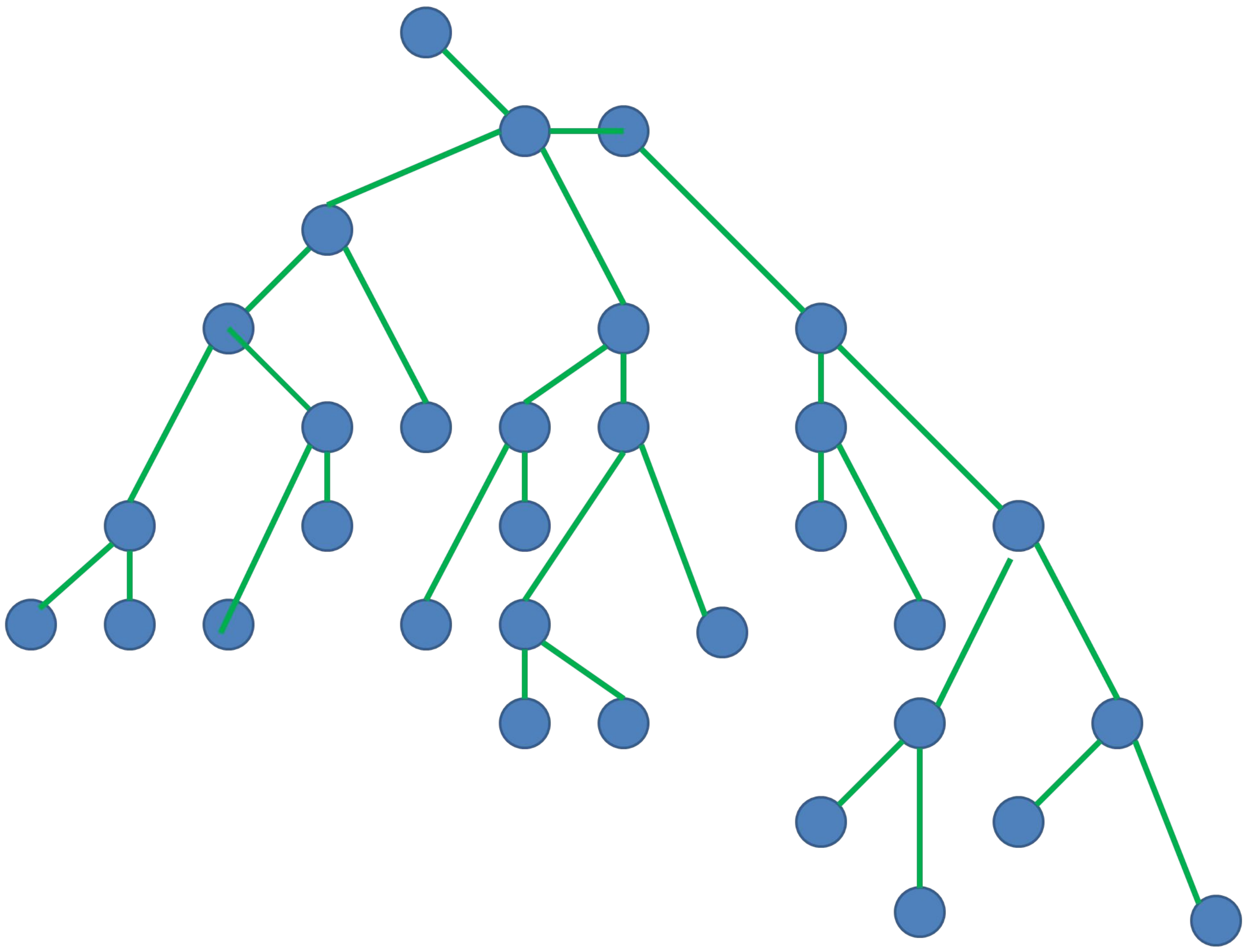
- 5-color theorem-any planer graph is at most 5-chromatic
- 4-colour theorem/hypothesis- any planer graph is 4-chromatic
- If $\Delta(G)$ is the maximum degree of any vertex in a graph then, $\chi(G) <= 1 + \Delta(G)$

Tree

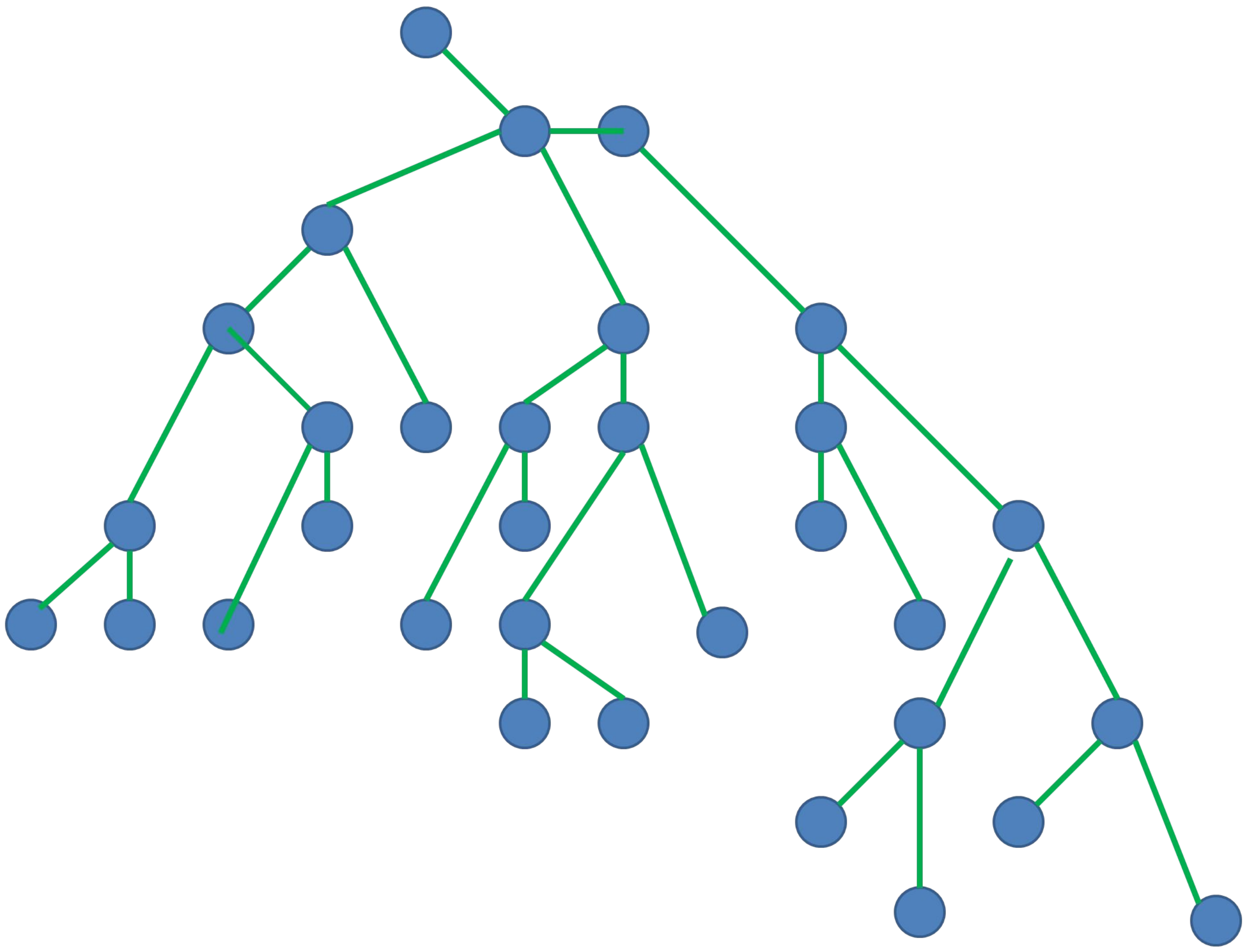
- A tree is a connected graph without any circuit.



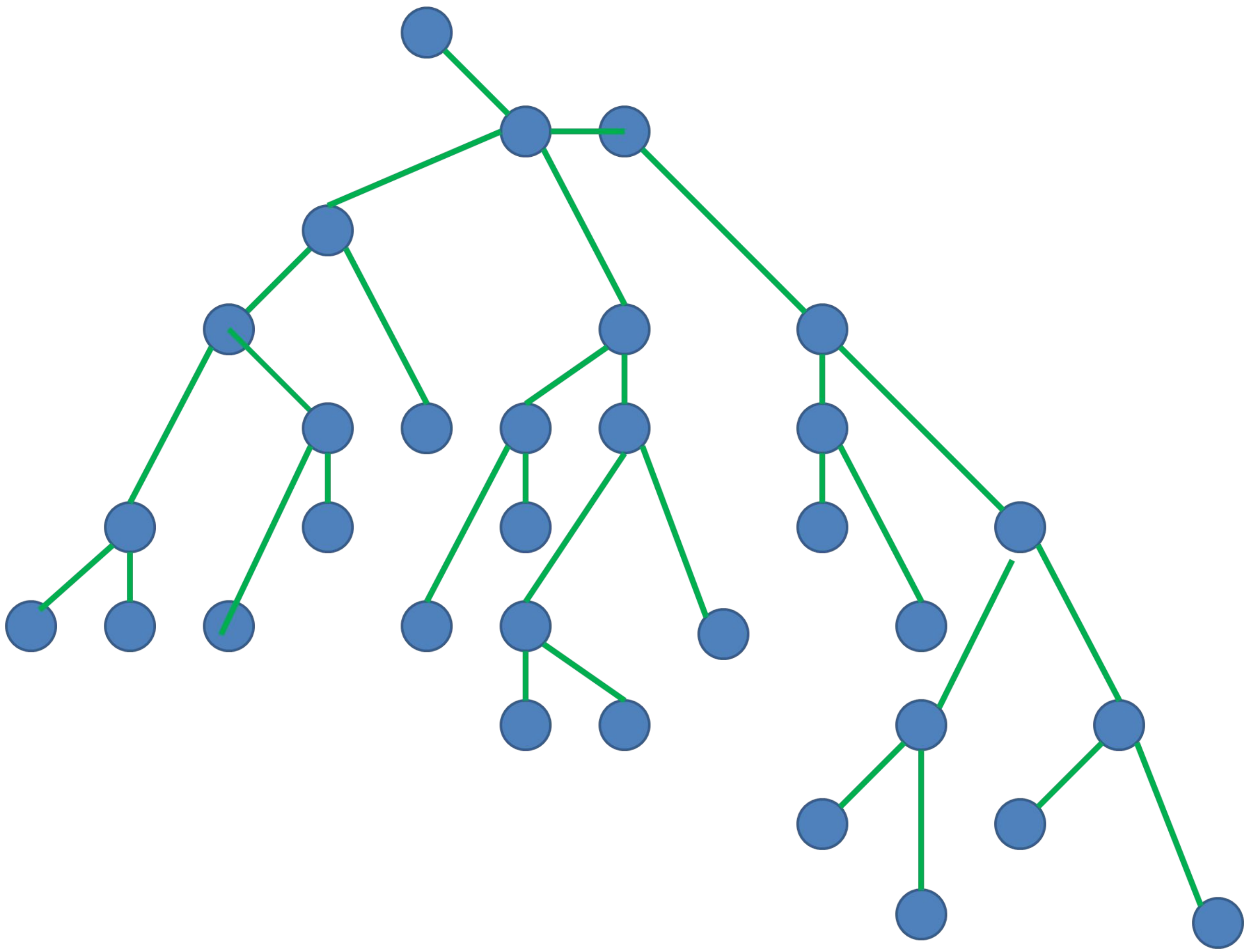
- There is one and only one path between every pair of vertices in a tree



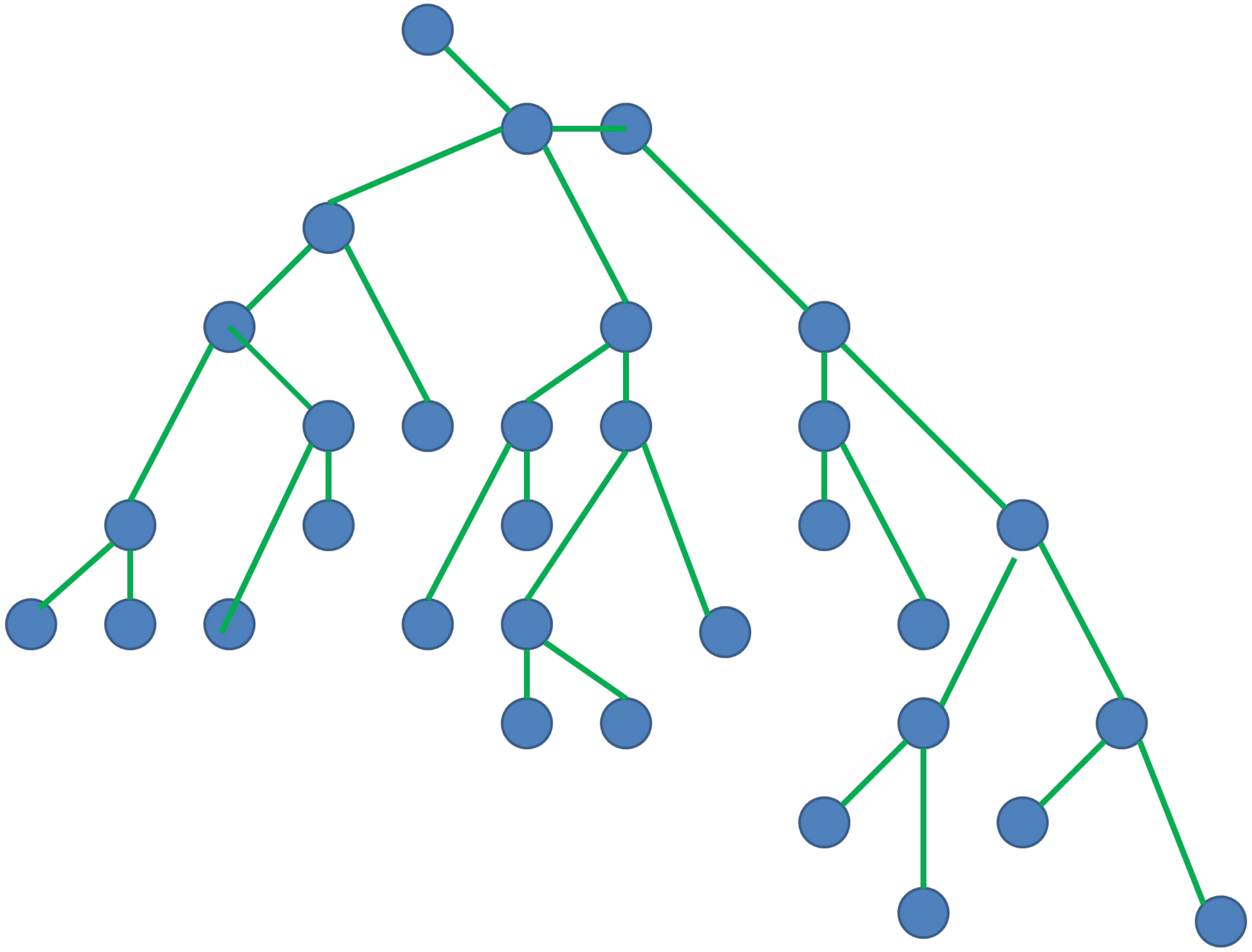
- If in a graph G, there is one and only one path between every pair of vertices then G is a tree



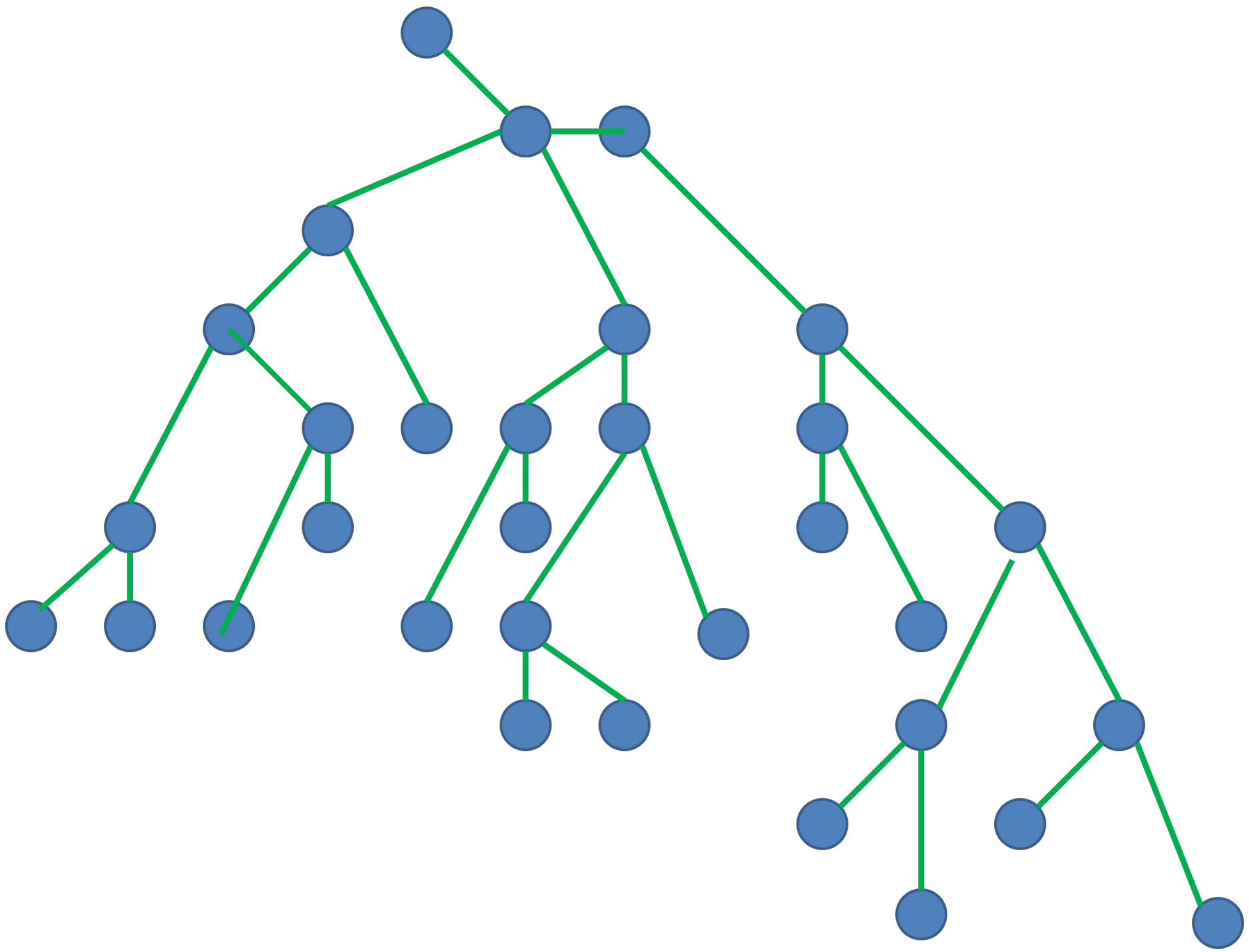
- A tree with n vertices has n-1 edges



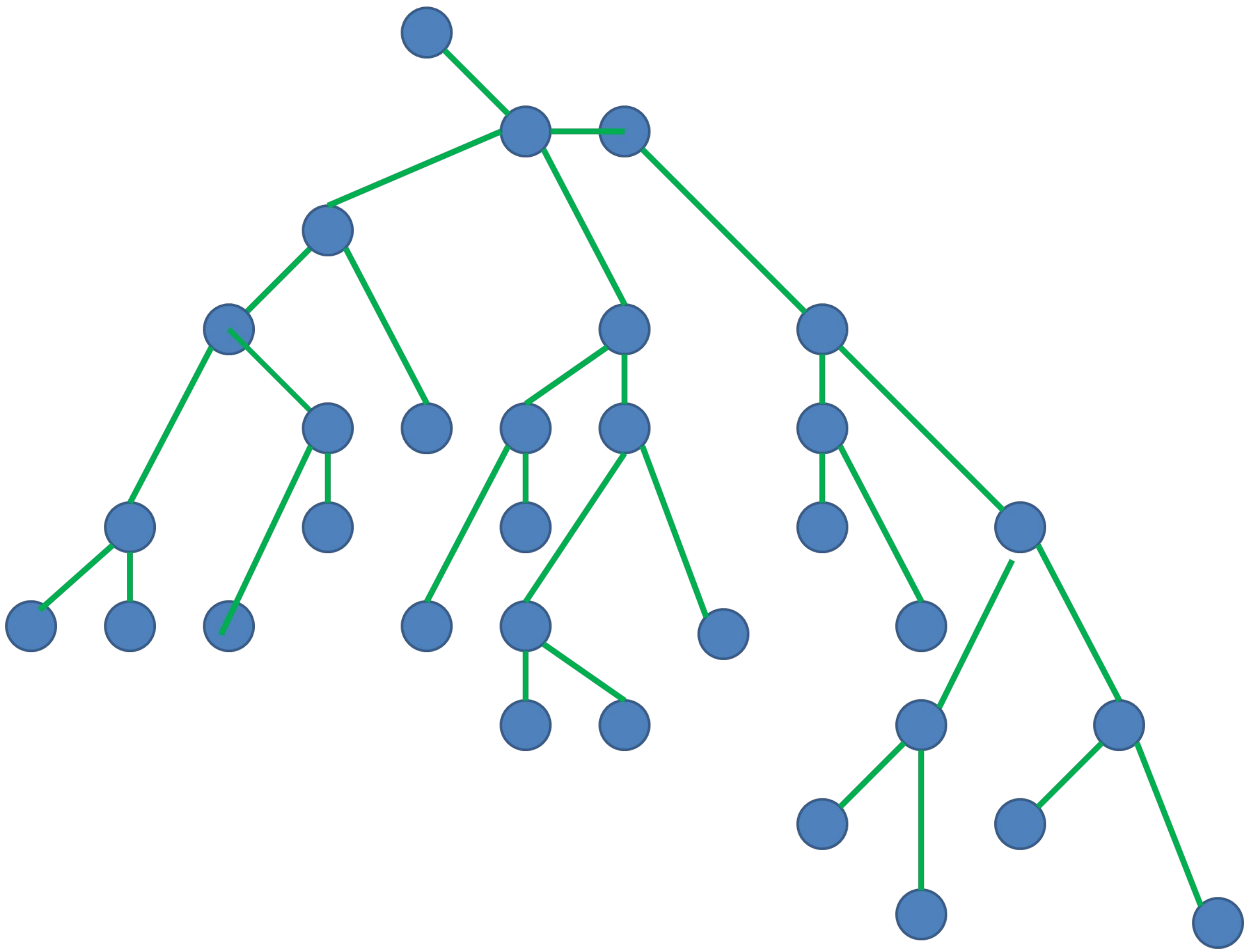
- Any connected graph with n vertices and n-1 edges in a tree



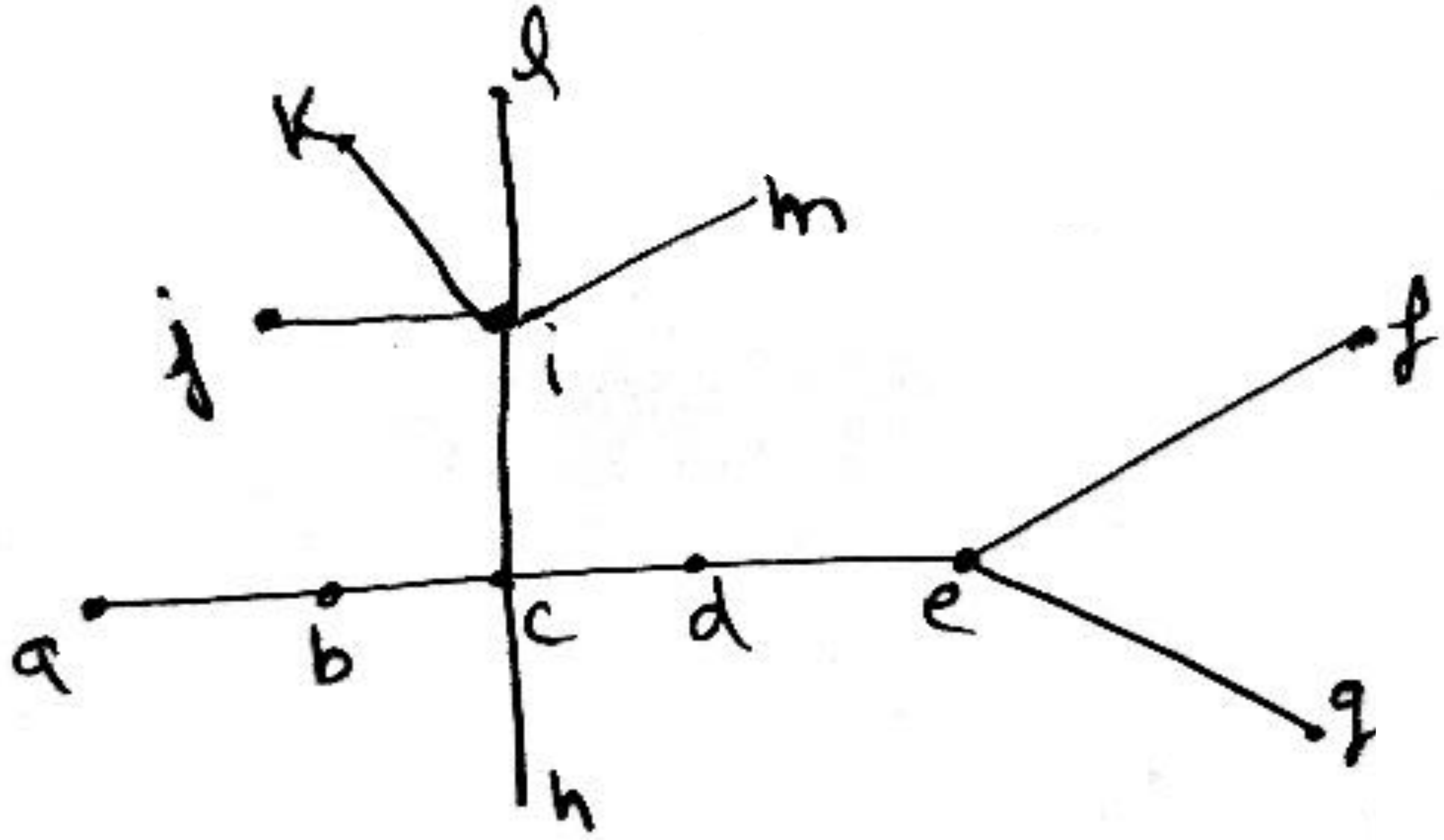
- A graph is a tree if and only if it is minimally connected



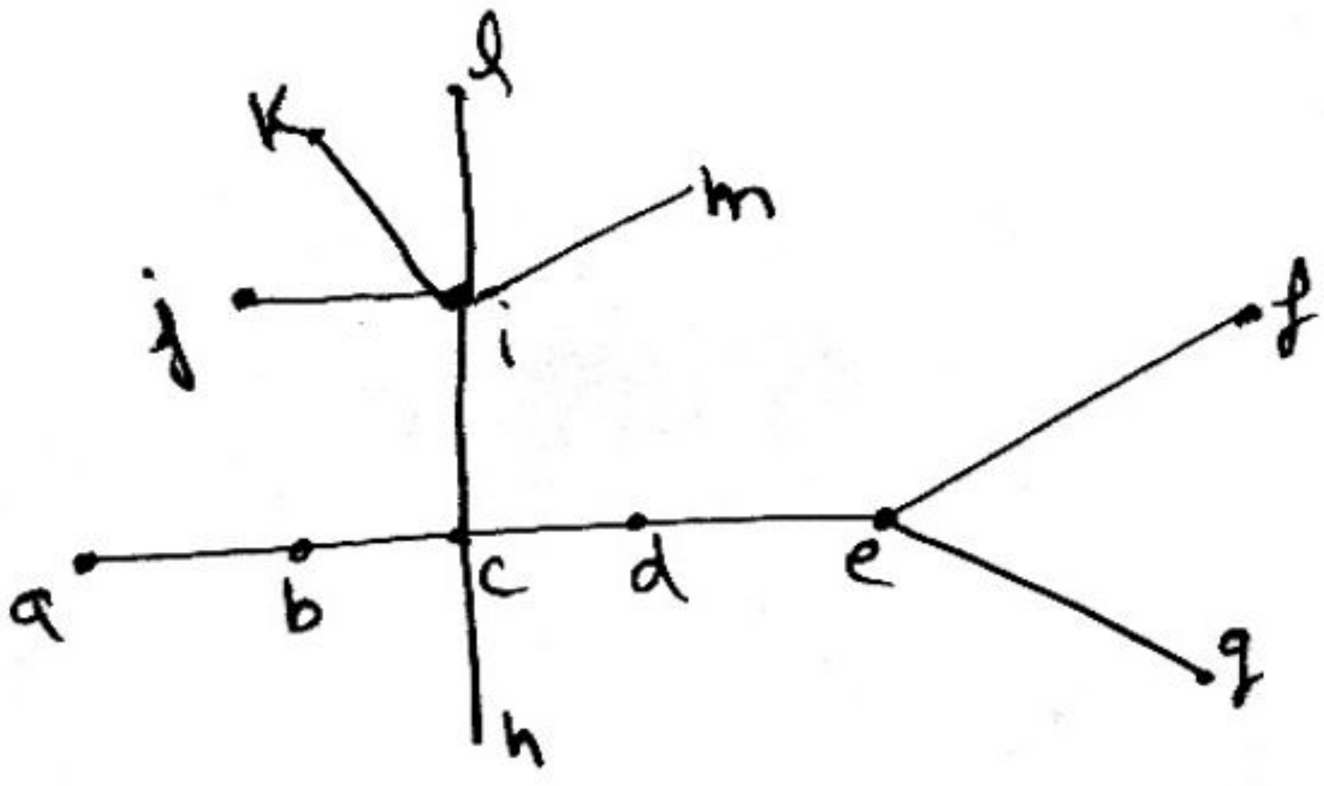
- A graph G with n vertices and n-1 edges and no circuit is connected

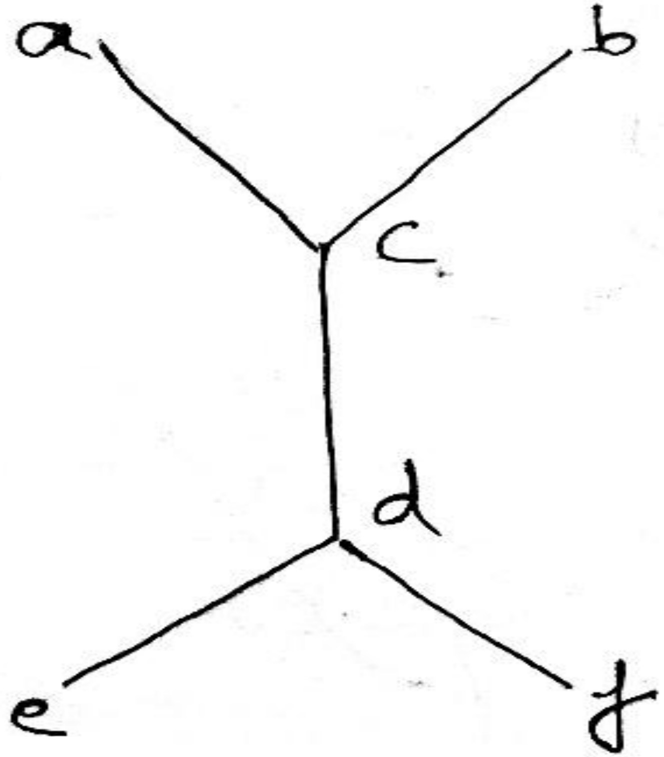


Eccentricity: - Eccentricity of a vertex is denoted by E(v) of a vertex v in a graph G, it is the distance from V to the vertex farthest from V in G. $E(v) = \max d(v, v_i)\ v_i \in G$

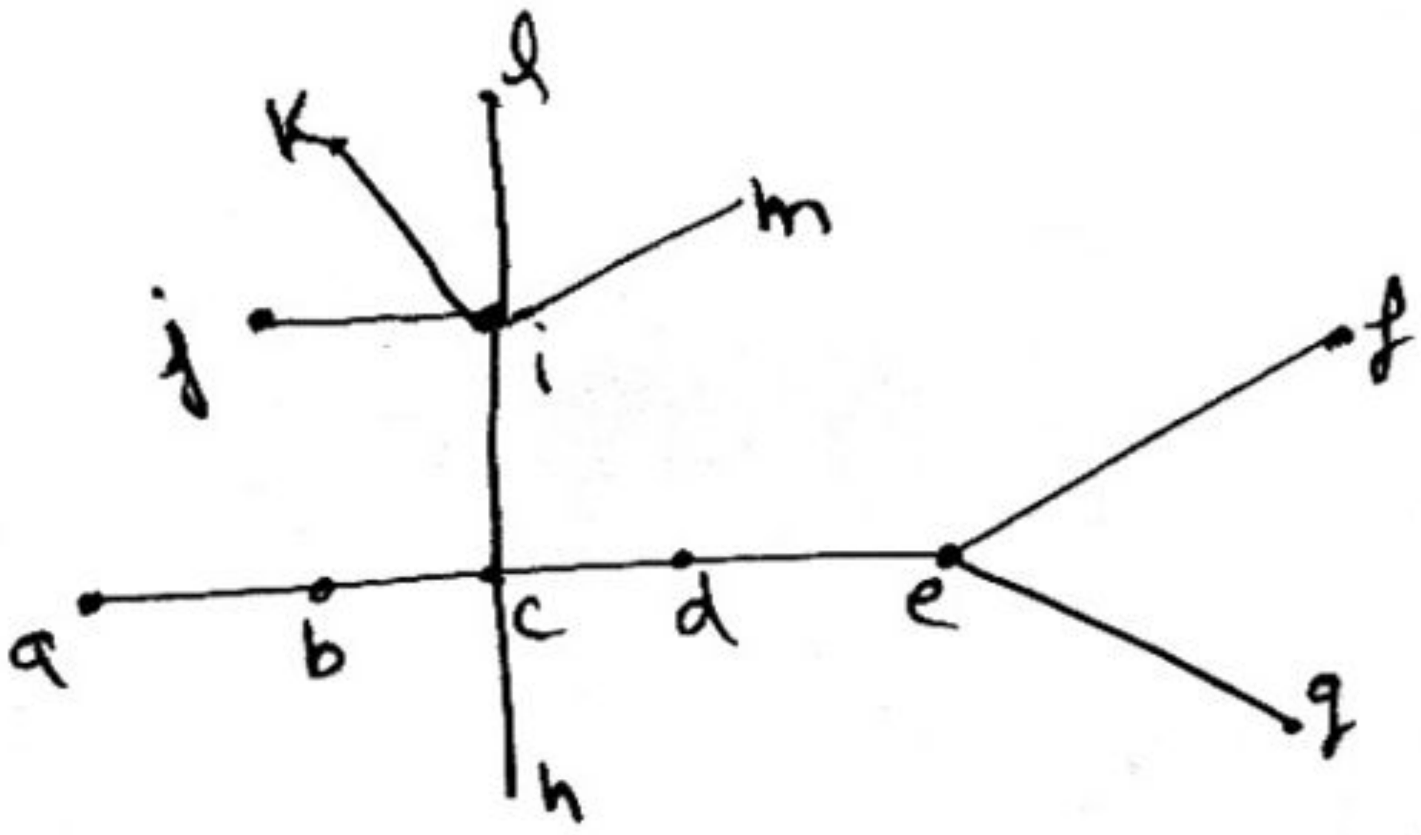


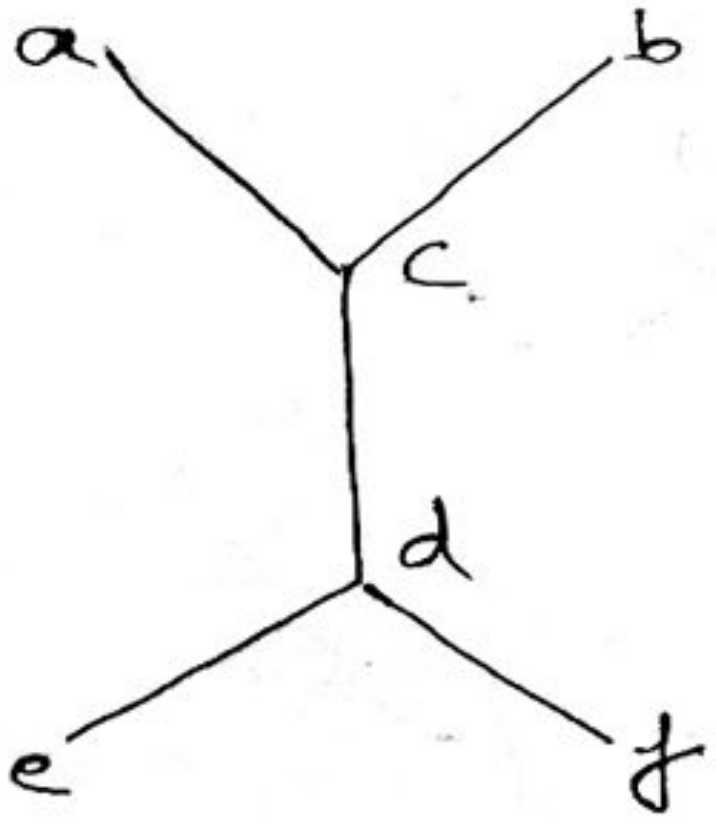
- A vertex with minimum eccentricity in a tree T is called center of T.
- Minimum eccentricity of any vertex in a tree T is called radius of tree. (eccentricity of center)
- Maximum eccentricity of any vertex in a tree T is called diameter of tree. (length of the longest path)





- Every tree has either one or two centers.





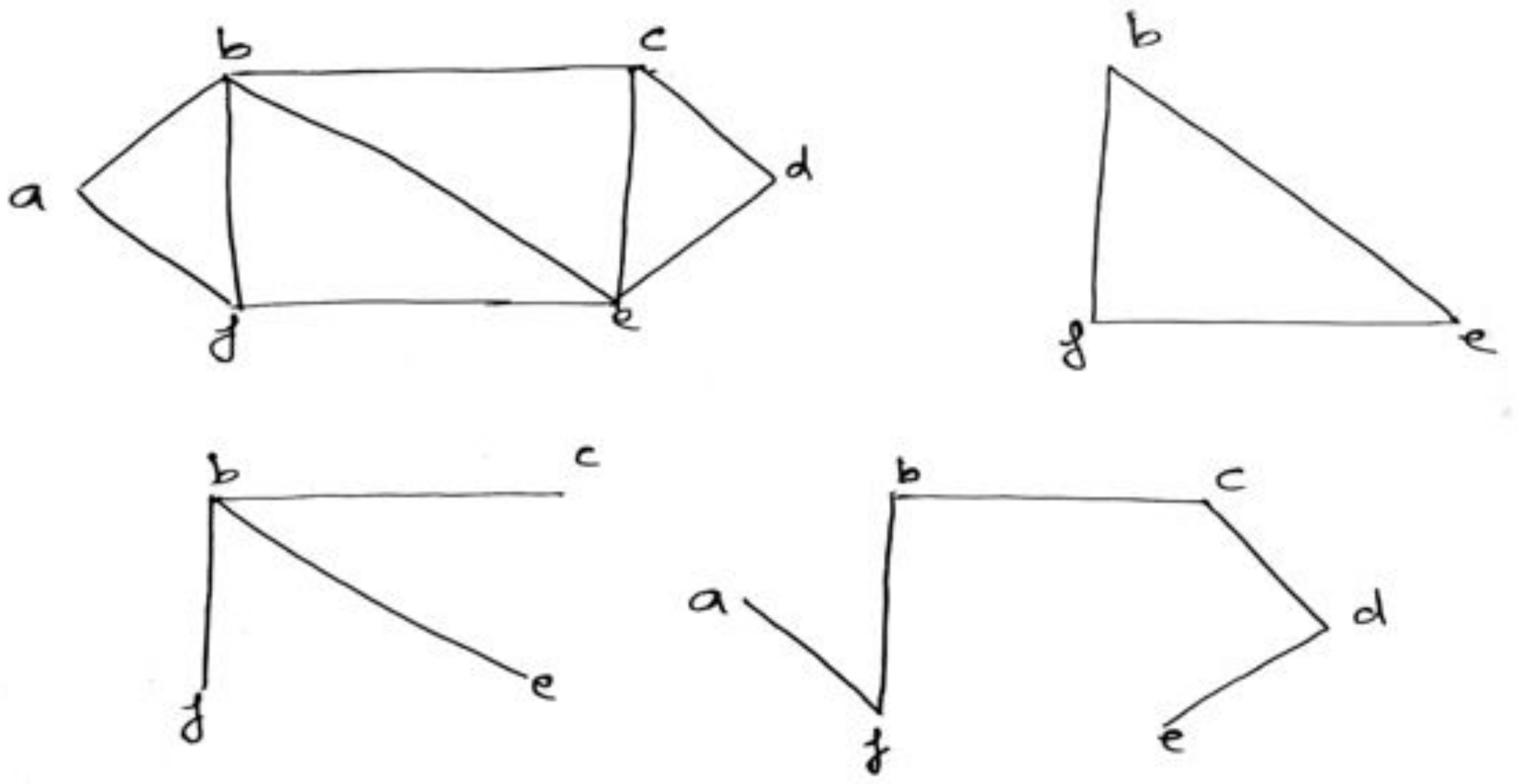
Q Let T be a tree with 10 vertices. The sum of the degrees of all the vertices in T is ______. **(GATE-2017) (1 Marks)**

Q What is the maximum number of edges in an acyclic undirected graph with n vertices? **(GATE-2004) (1 Marks)**

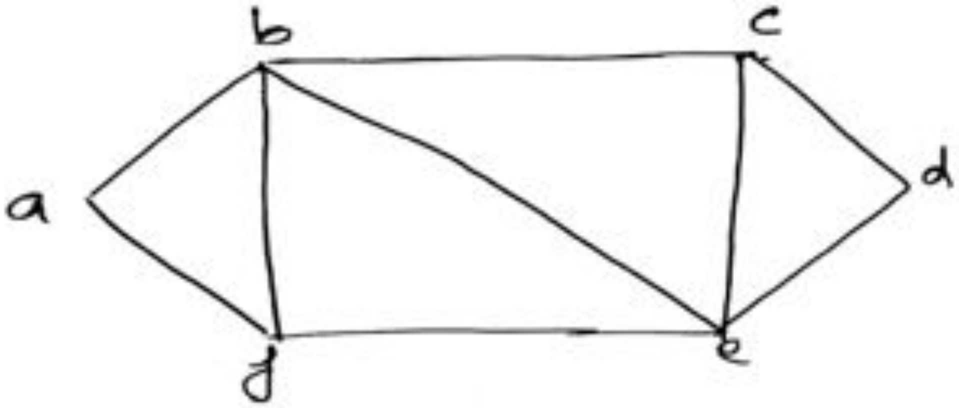
(A) n-1 **(B)** n **(C)** n + 1 **(D)** 2n-1

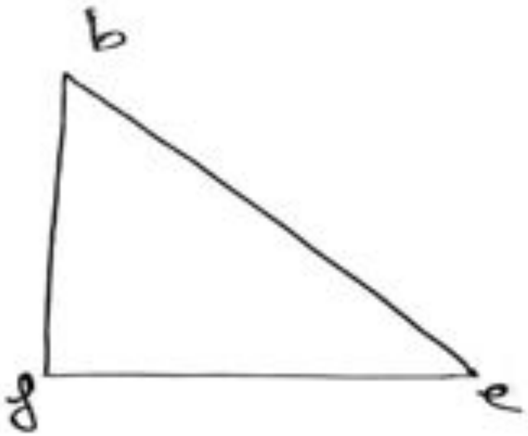
Spanning tree

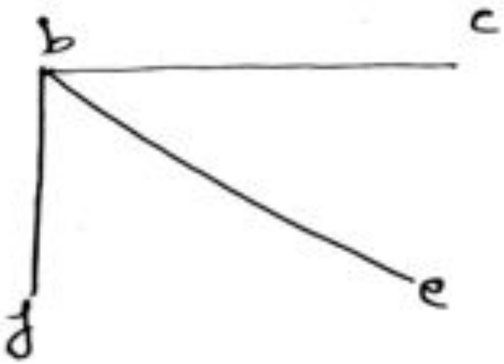
- A tree T is said to be spanning tree of a connected graph G, if T is a subgraph of G and T contains all vertices of G.

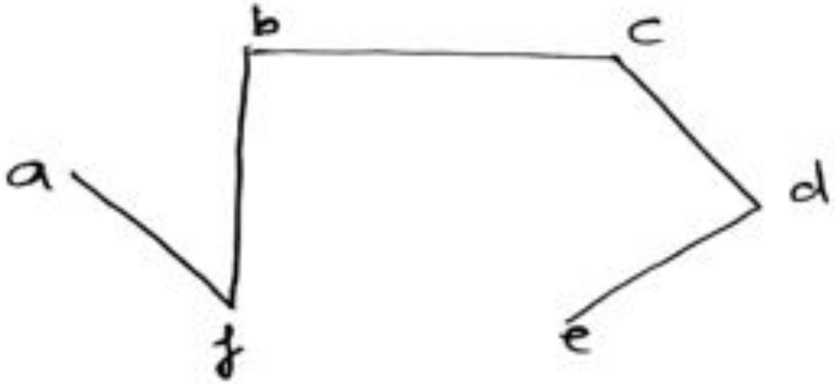


- An edge in a spanning tree T is called a branch of T
- An edge that is not in the given spanning tree T is called a chord.
- Branch and Chord are defined with respect to a given spanning tree.

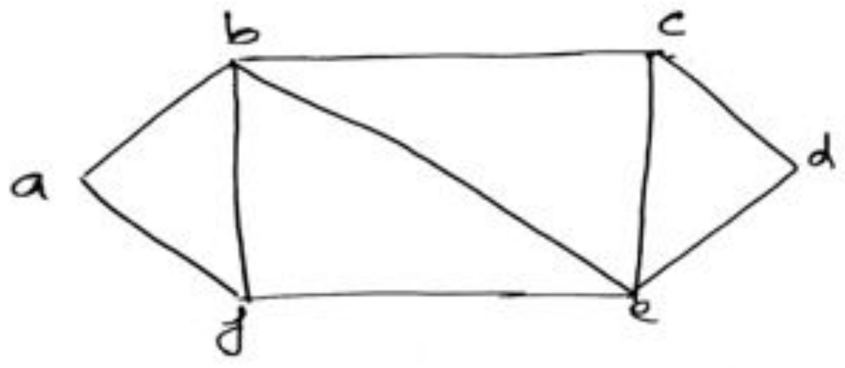


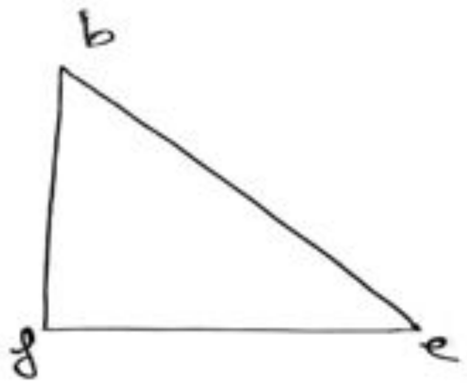


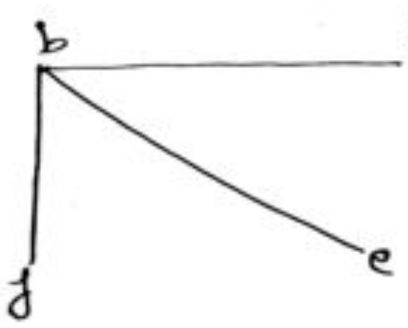


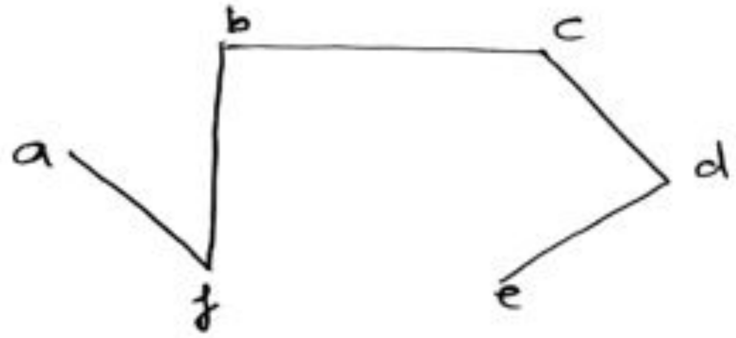


1. With respect to any of its spanning tree, a connected graph of n vertices and e edges has n-1 branches and e-n+1 chord
2. A connected graph G is a tree if and only if adding an edge between any two vertices in g creates exactly one cycle.
3. Rank(r) = n-1
4. Nullity(μ) = e – n + 1
5. Rank + nullity = number of edges in G





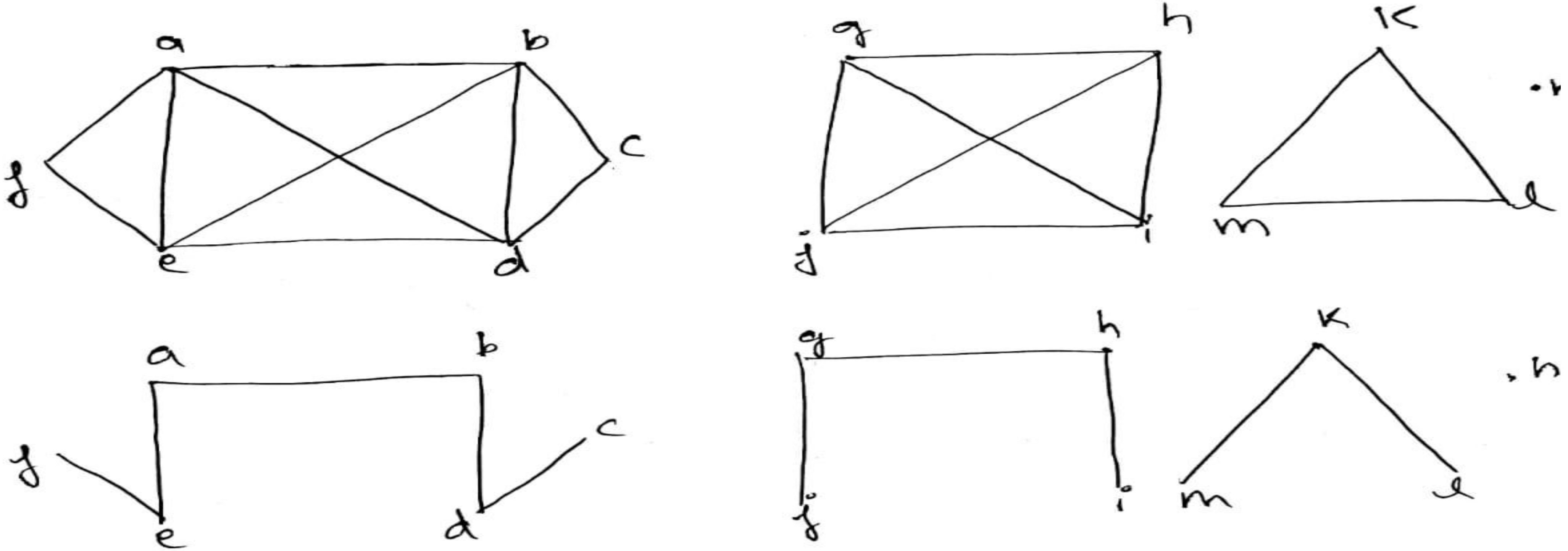




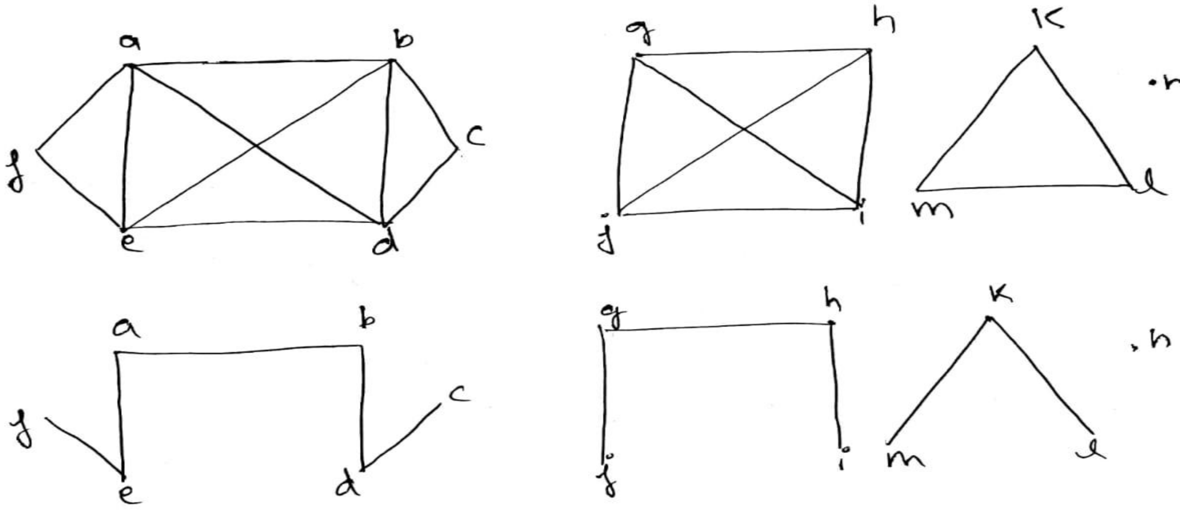
Q. The number of spanning trees in a complete graph of 4 vertices labelled A,B,C, and D is __________ **(Gate 2024,CS) (1 Marks)(NAT)**

<u>Spanning Forest:</u> - if a graph is not connected, then there is no possibility of finding a spanning tree, but we can find a spanning forest. If a graph is not connected then we can find connected components, finding a spanning tree in each component we can find spanning forest.

A disconnected graph with K components has a spanning forest consisting of K spanning tree.



1. Rank(r) = n-k
2. Nullity(μ) = e – n + k
3. Rank + nullity = number of edges in G



Fundamental circuit: - With respect to a spanning tree T in a connected graph G, adding any one chord to T will create exactly one circuit such a circuit formed by adding a chord to a spanning tree is called fundamental circuit.

Q if G is a forest with n vertices and k connected components, how many edges does G have? **(GATE-2014) (2 Marks)**

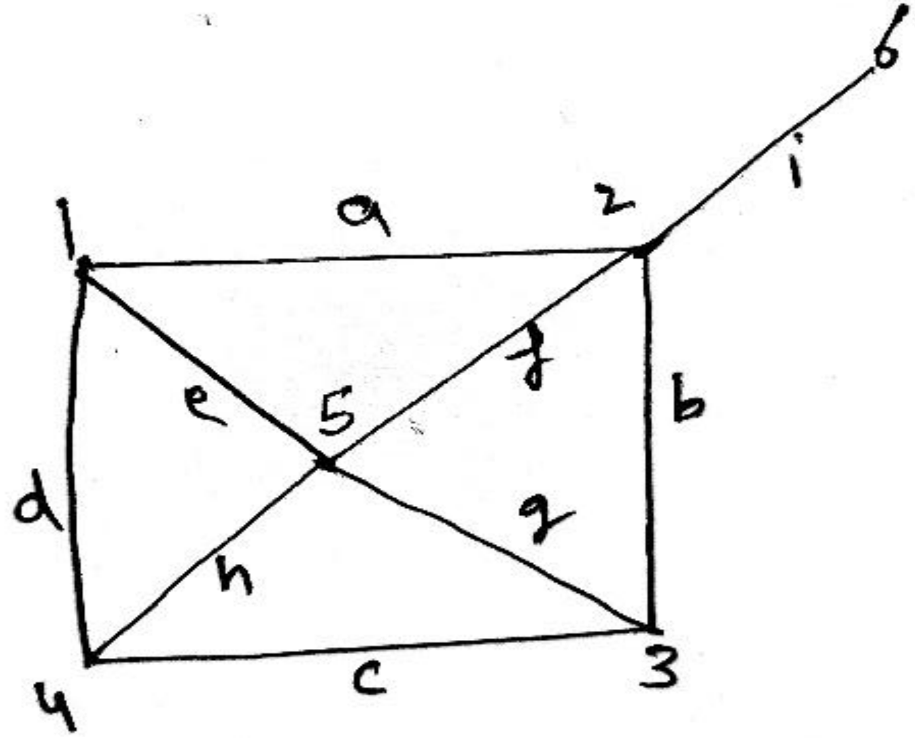
(A) floor(n/k) **(B)** ceil(n/k) **(C)** n-k **(D)** n-k+1

Cut-Set (edge and vertex connectivity)

Cut-Set (Edges)

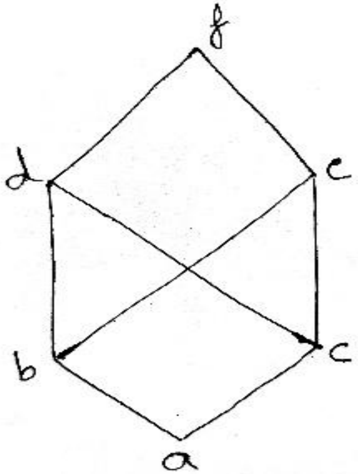
Cut Set: - In a connected graph G, a cut set is a set of edges whose removal from g leaves G disconnected, provided removal of no proper subset of these edges disconnects G.

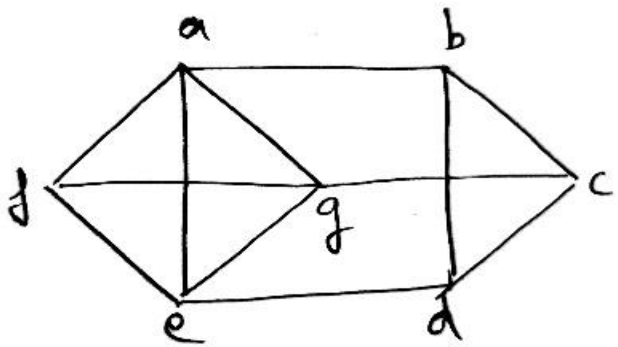
Cut Set	Validity
{a, f, g}	
{a, e, h, c}	
{a, i}	
{e, h, f, g}	
{d, h, c, g}	
{d, e, f}	

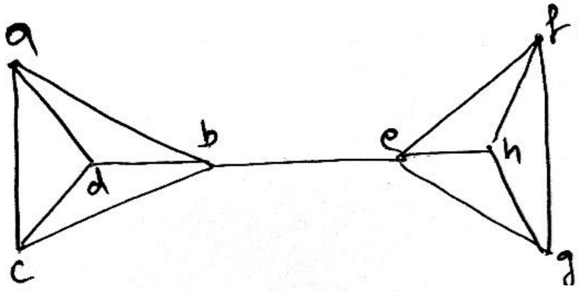


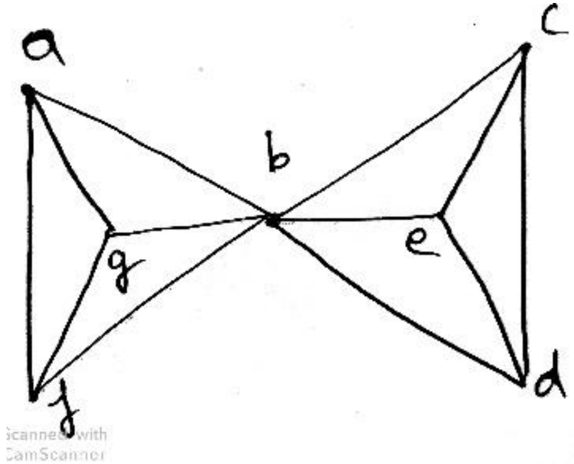
Connectivity: - each cut-set of a connected graph G consist of a certain number of edges. The number of edges in the smallest cut-set is defined as the edges connectivity of G. It is denoted by $\lambda(G)$.

- if the edge connectivity from a graph is one, then that edge how's removal disconnect the graph is called a bridge.









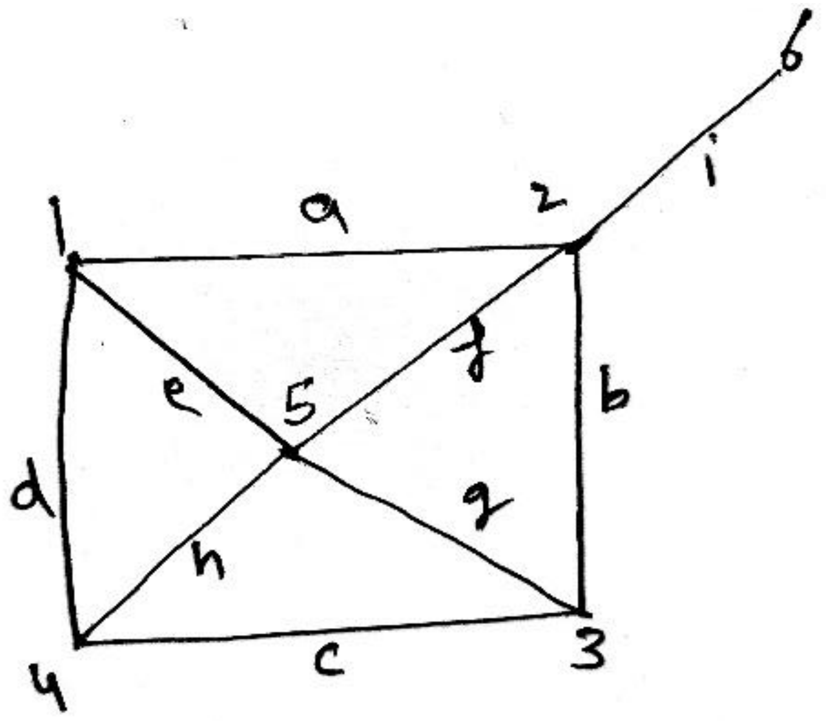
k(G)				
λ(G)				
δ(G)				

- Every Cut Set in a connected graph G must contain at least one branch of every spanning tree of G.
- Every circuit has an even number of edges in common with any Cut-Set.

Cut-Set (Vertex)

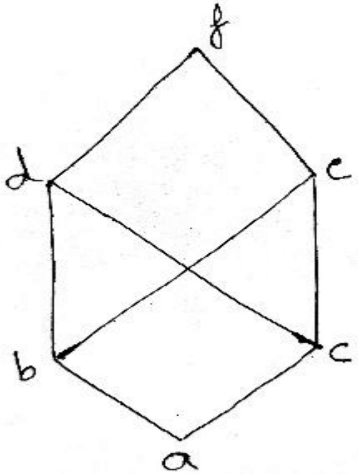
Cut Set: - In a connected graph G, a cut set is a set of vertices whose removal from g leaves G disconnected, provided removal of no proper subset of these vertices disconnects G.

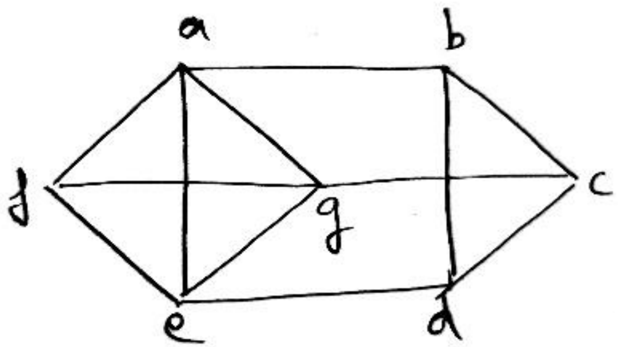
Cut Set	Validity
{5, 3}	
{6}	
{5, 2}	
{2}	
{1, 5, 3}	

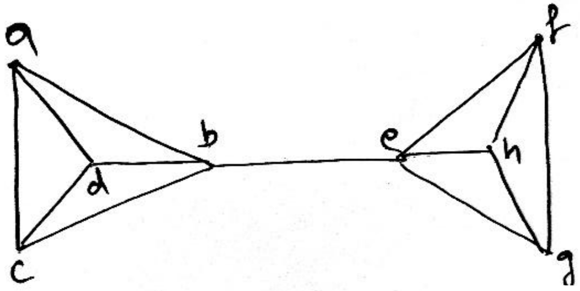


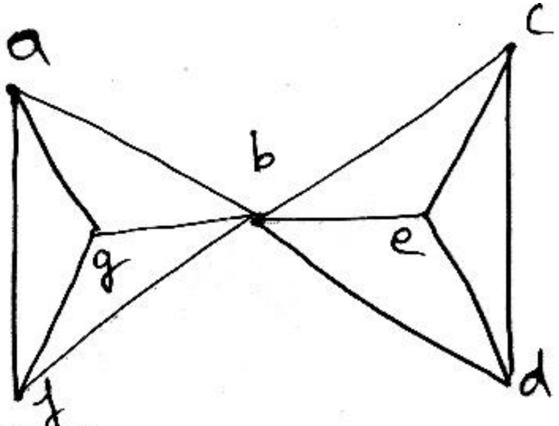
Vertex Connectivity: - Each cut-set of a connected graph G consist of a certain number of vertices. The number of vertices in the smallest cut-set is defined as the vertex connectivity of G. It is denoted by k(G).

- A connected graph is said to be separable of its vertex connectivity is one.
- If the vertex connectivity of a graph is one, then that vertex how removal disconnects a graph is called articulation point.









k(G)				
λ(G)				
δ(G)				

Q The maximum number of possible edges in an undirected graph with ‘a’ vertices and ‘k’ components is ______. **(GATE-1991) (2 Marks)**

Q G is a graph on n vertices and 2n – 2 edges. The edges of G can be partitioned into two edge-disjoint spanning trees. Which of the following is NOT true for G? **(GATE-2008) (2 Marks)**

(A) For every subset of k vertices, the induced subgraph has at most 2k-2 edges

(B) The minimum cut in G has at least two edges

(C) There are two edge-disjoint paths between every pair to vertices

(D) There are two vertex-disjoint paths between every pair of vertices

Q Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie between (k-1)(n-1) **(GATE-2003) (1 Marks)**

(A) k and n

(B) k – 1 and k + 1

(C) k – 1 and n – 1

(D) k + 1 and n – k

Q Let G be a graph with 100! vertices, with each vertex labelled by a distinct permutation of the numbers 1, 2, …, 100. There is an edge between vertices u and v if and only if the label of u can be obtained by swapping two adjacent numbers in the label of v. Let y denote the degree of a vertex in G, and z denote the number of connected components in G. Then y + 10z = ________. **(GATE-2018) (2 Marks)**

Q Let G = (V, E) be a directed graph where V is the set of vertices and E the set of edges. Then which one of the following graphs has the same strongly connected components as G? **(GATE-2014) (1 Marks)**

a) $G_1 = (V, E_1)$ where $E_1 = \{(u, v) \mid (u, v) \notin E\}$

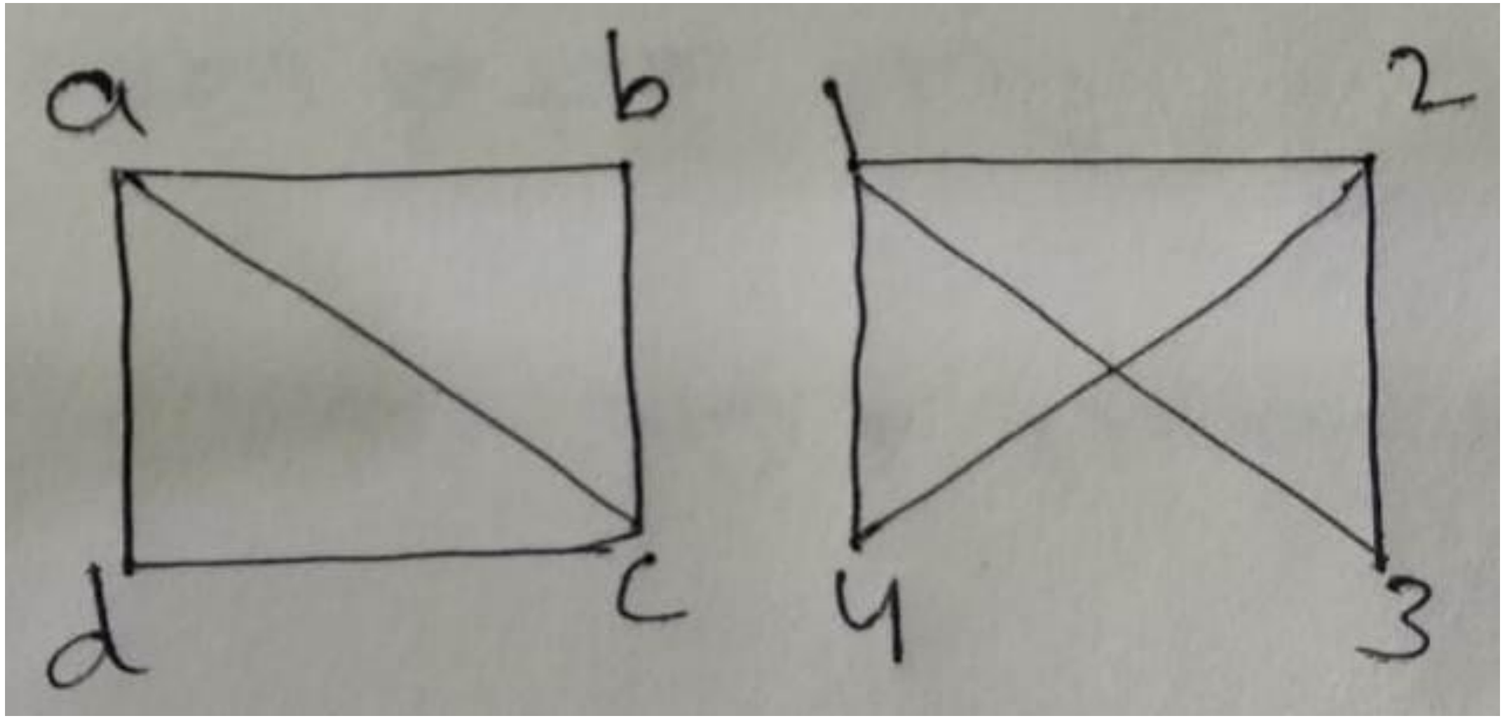
b) $G_2 = (V, E_2)$ where $E_2 = \{(u, v) \mid (v, u) \in E\}$

c) $G_3 = (V, E_3)$ where $E_3 = \{(u, v) \mid$ there is a path of length ≤ 2 from u to v in E$\}$

d) $G_4 = (V_4, E)$ where V_4 is the set of vertices in G which are not isolated

Isomorphism

1. In general, two graphs are said to be isomorphic if they are perhaps the same graphs, but just drawn differently with different names. i.e. two graphs are thought of as isomorphic if they have identical behavior in terms of graph-theoretic properties.
2. Formally speaking: - Two graphs G and G’ are said to be isomorphic, if there is a one to one correspondence between their vertices and between their edges such that the incidence relationship is preserved.



1. Determining if two graphs are isomorphic is thought to be neither an NP-complete problem nor a P-problem, although this has not been proved.

2. In fact, there is a famous complexity class called graph isomorphism complete which is thought to be entirely disjoint from both NP-complete and from P.

Q How many simple non isomorphic graphs are possible with 3 vertices ?

Q How many simple non isomorphic graphs are possible with 4 vertices and 2 edges ?

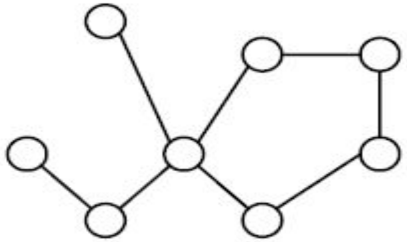
Q How many simple non isomorphic graphs are possible with 4 vertices and 3 edges ?

Q How many simple non isomorphic graphs are possible with 5 vertices and 3 edges ?

Q How many simple non isomorphic graphs are possible with 6 vertices and 6 edges, such that degree of every vertex must be same ?

Q How many simple non isomorphic graphs are possible with 8 vertices and 8 edges, such that degree of every vertex must be same ?

Q Which of the following graphs is isomorphic to **(GATE-2012) (2 Marks)**



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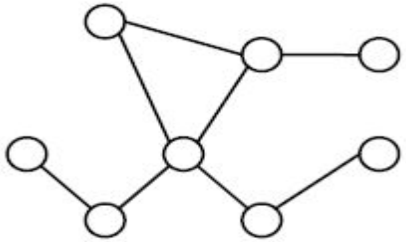

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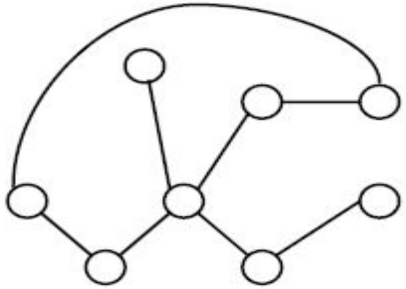
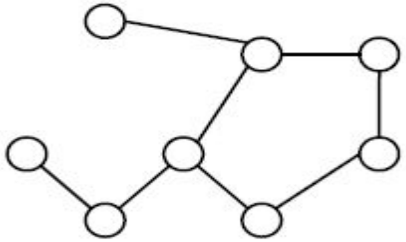


(C)



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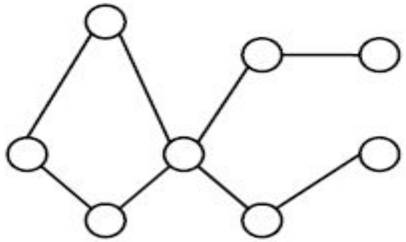


Q A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graphs on n vertices, n is **(GATE-2015) (2 Marks)**

(A) A multiple of 4

(B) Even

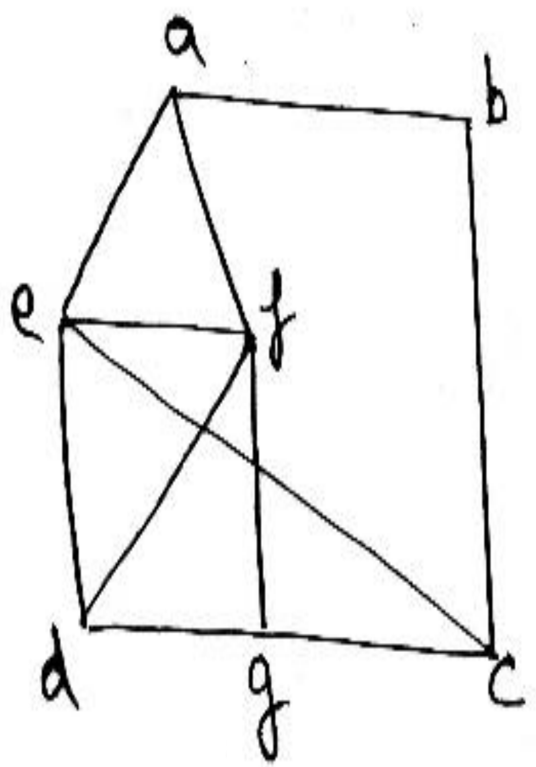
(C) Odd

(D) Congruent to 0 mod 4, or 1 mod 4

Q A cycle on n vertices is isomorphic to its complement. The value of n is ______. **(GATE-2014) (2 Marks)**

How to check weather two graphs are isomorphic or not

1. Number of vertices
2. Number of edges
3. Number of vertices with a given degree
4. Check degree property of vertices with their neighbor
5. Check minimum cycle length, maximum cycle length, or number of cycle with a specific length
6. Can check isomorphism for complement of the graph
7. Planer, non-planer
8. Connected disconnected
9. Chromatic number
0. Matching number, covering number
1. Edge connectivity, vertex connectivity
2. If it seems that graphs are isomorphic to each other then identify the similar vertex and delete both, and keep repeating the process until we are sure..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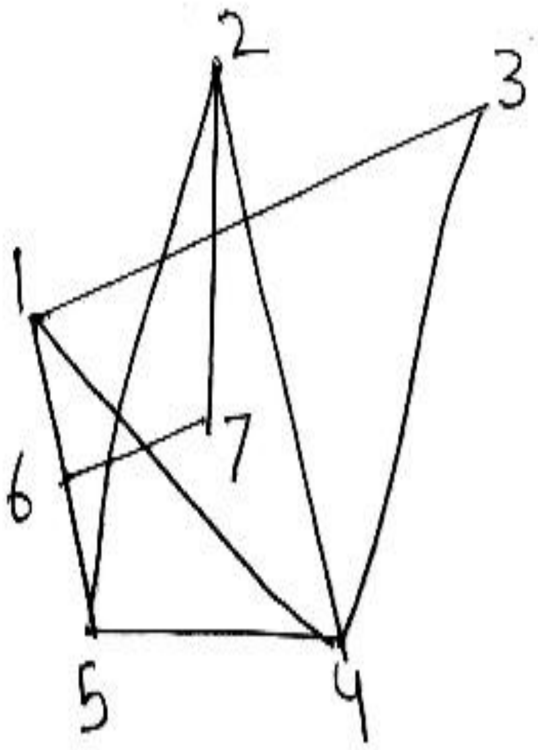
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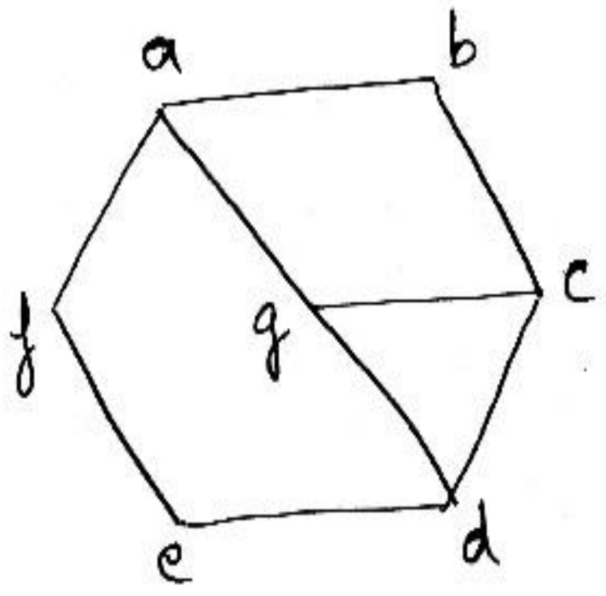
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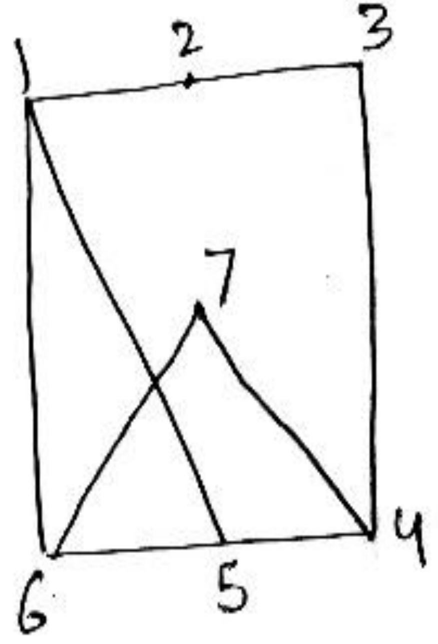
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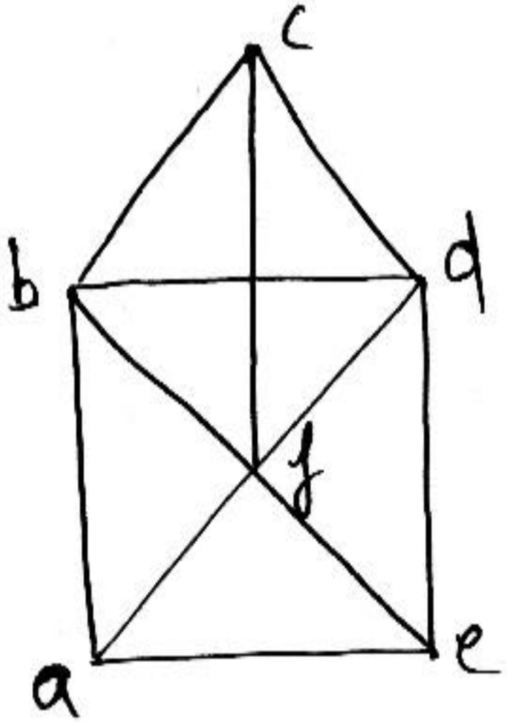
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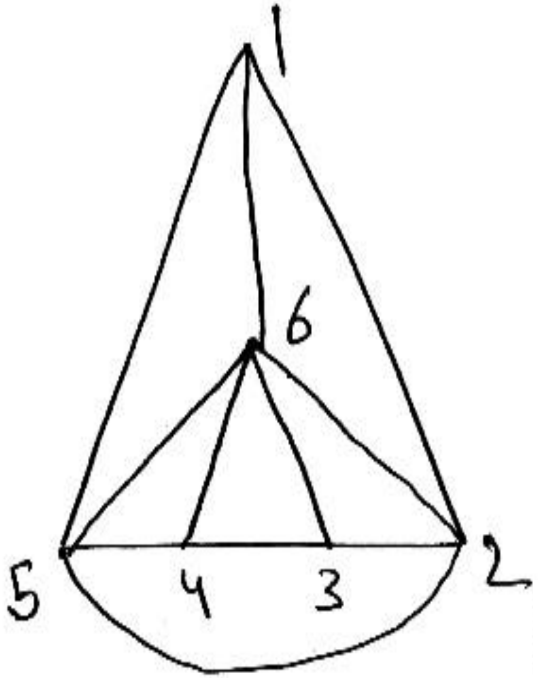
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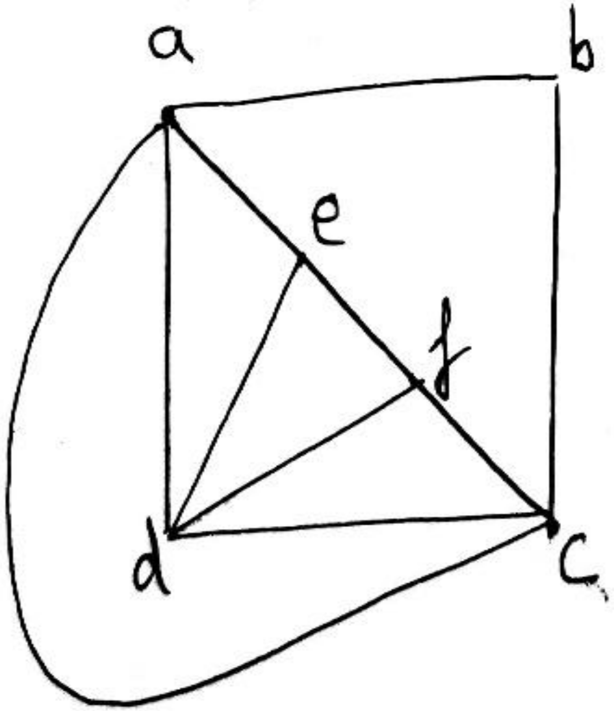
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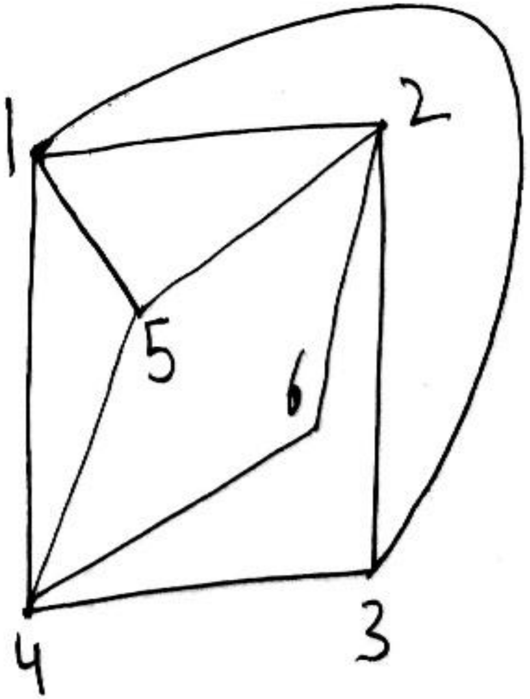
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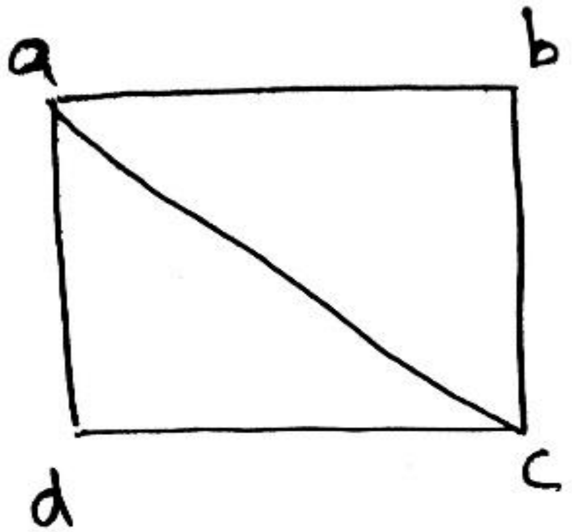
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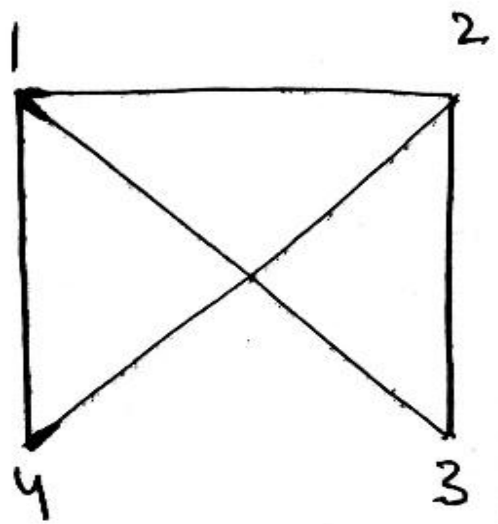
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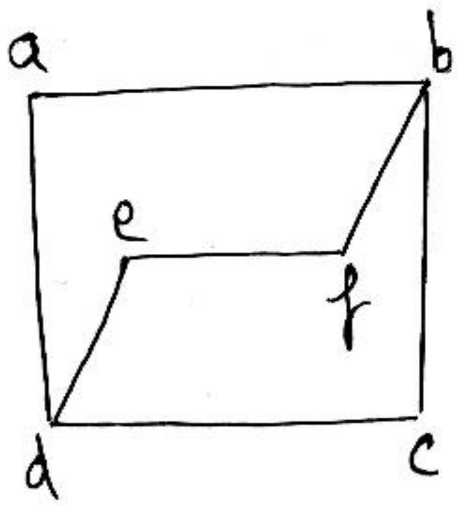
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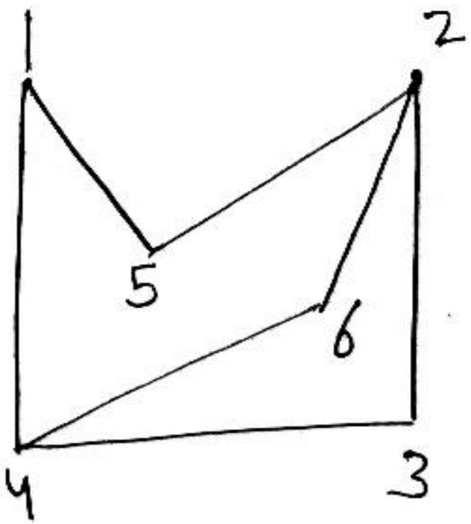
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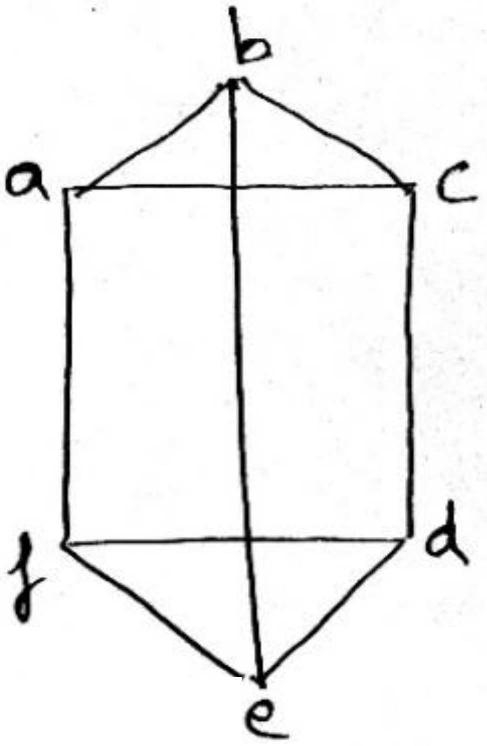

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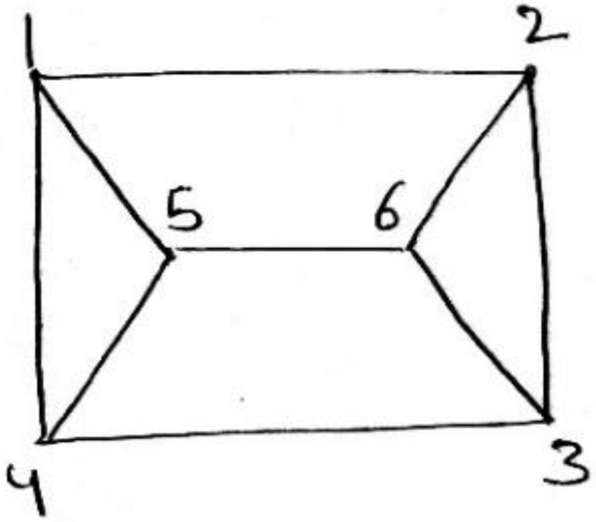

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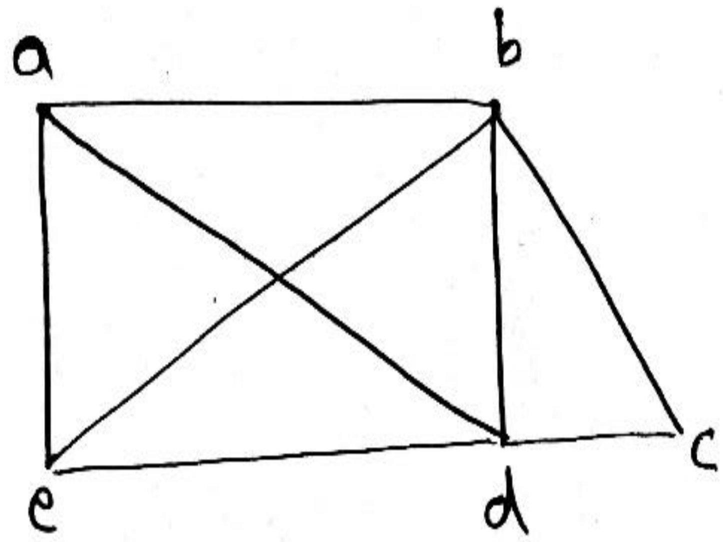
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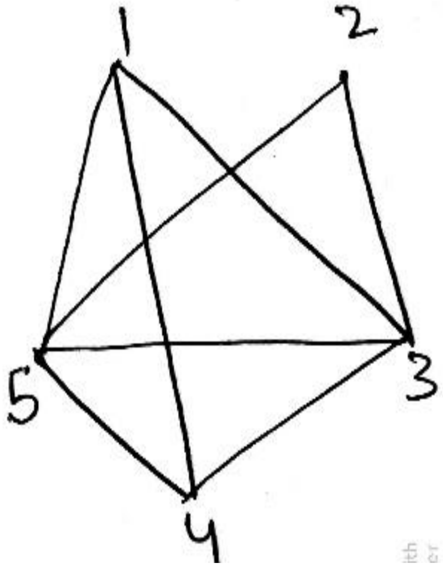
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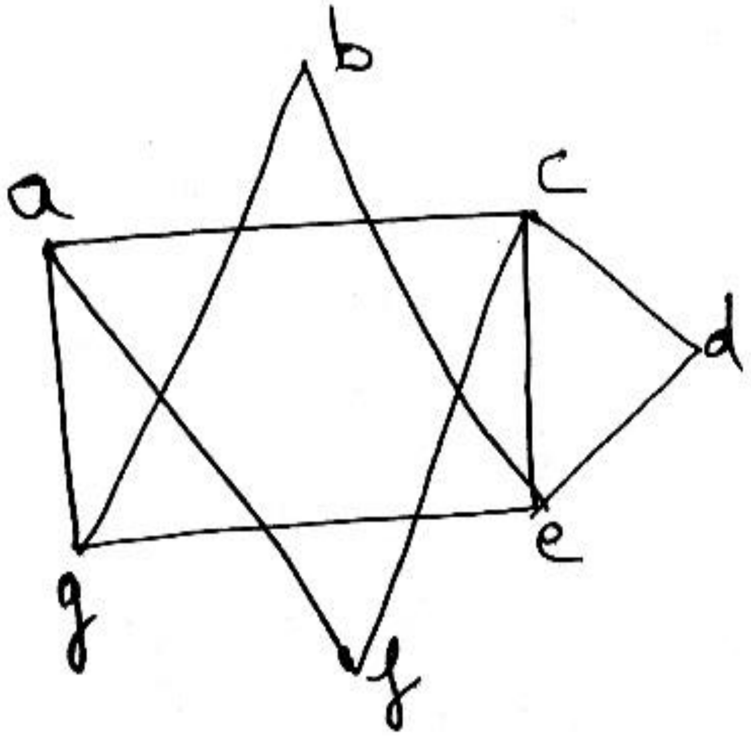
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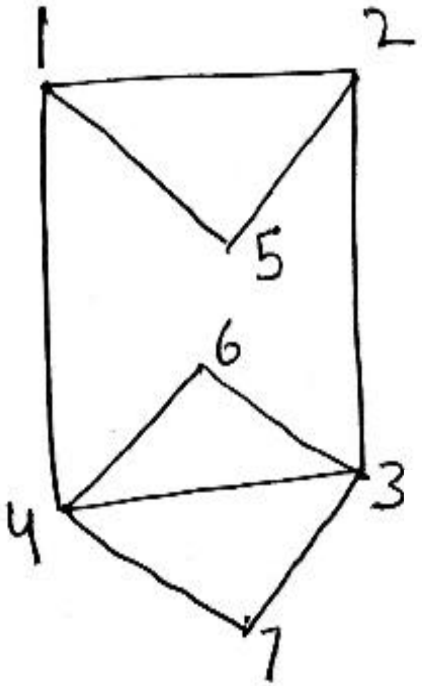
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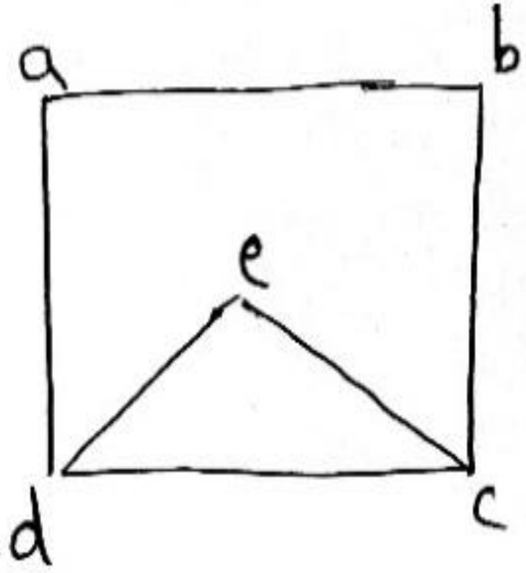
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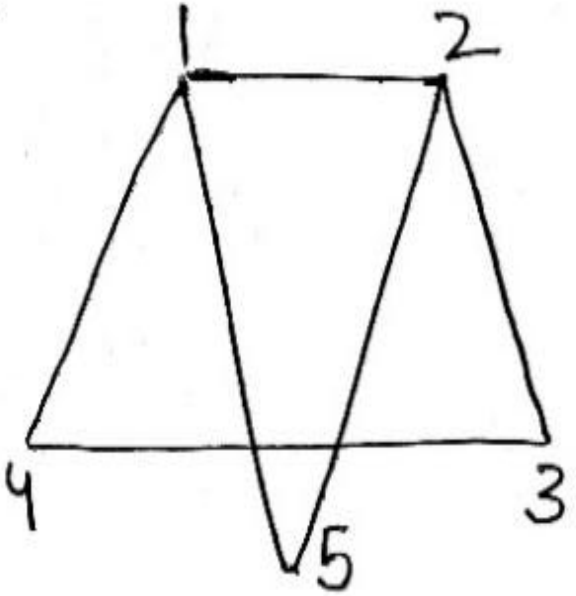
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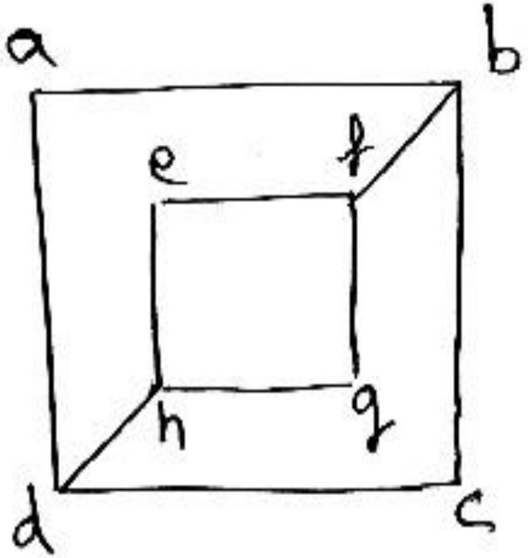
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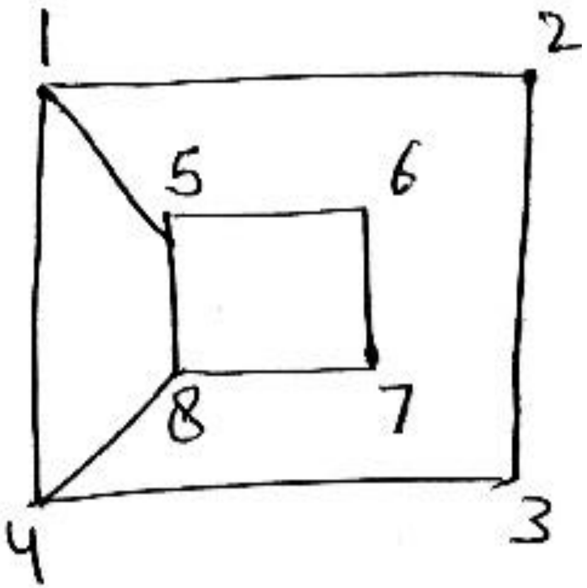
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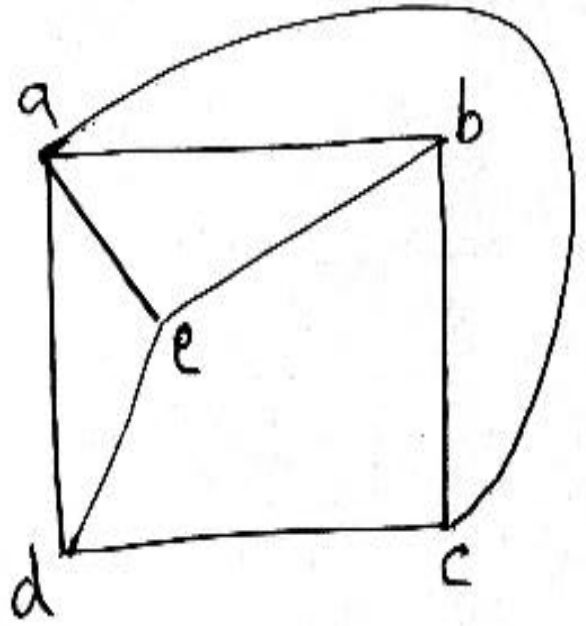

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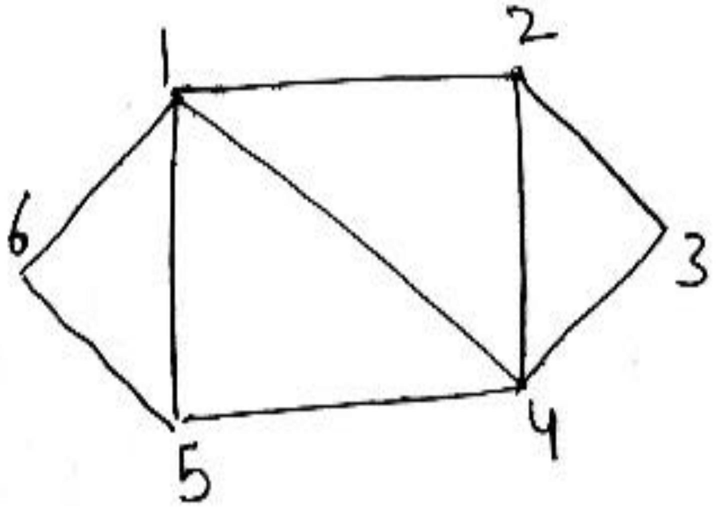

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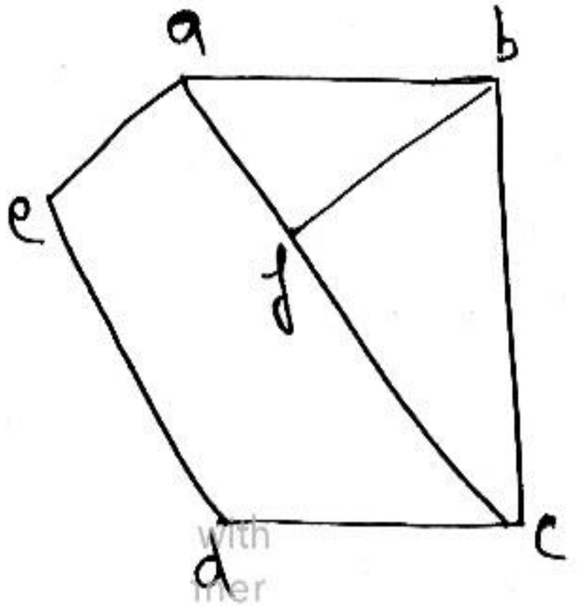
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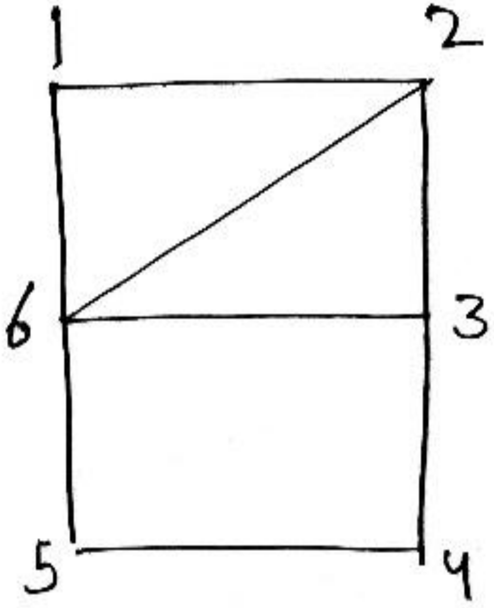
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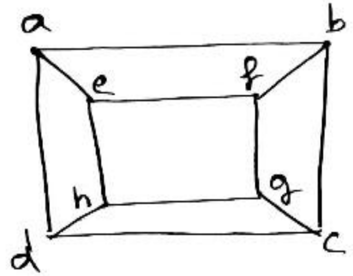
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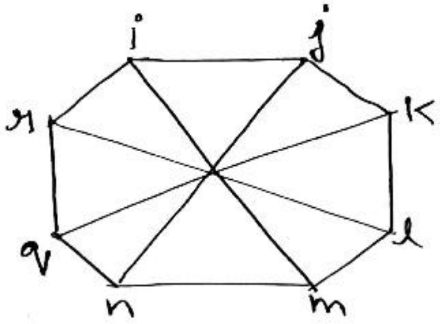
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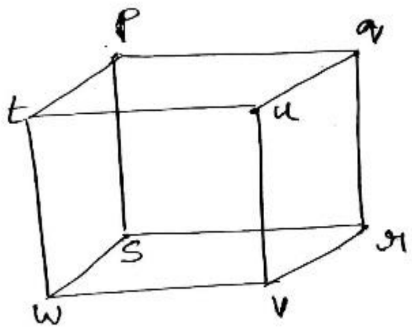
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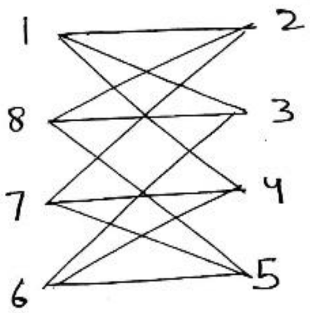
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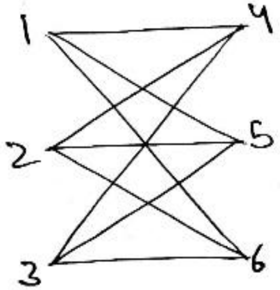
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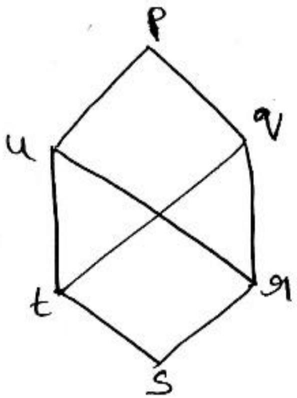
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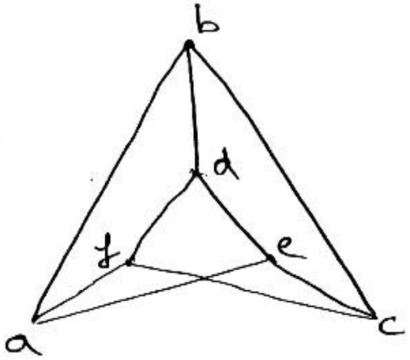
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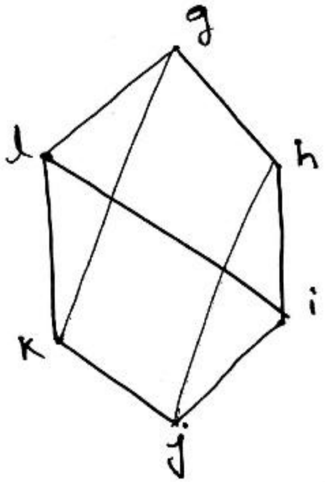
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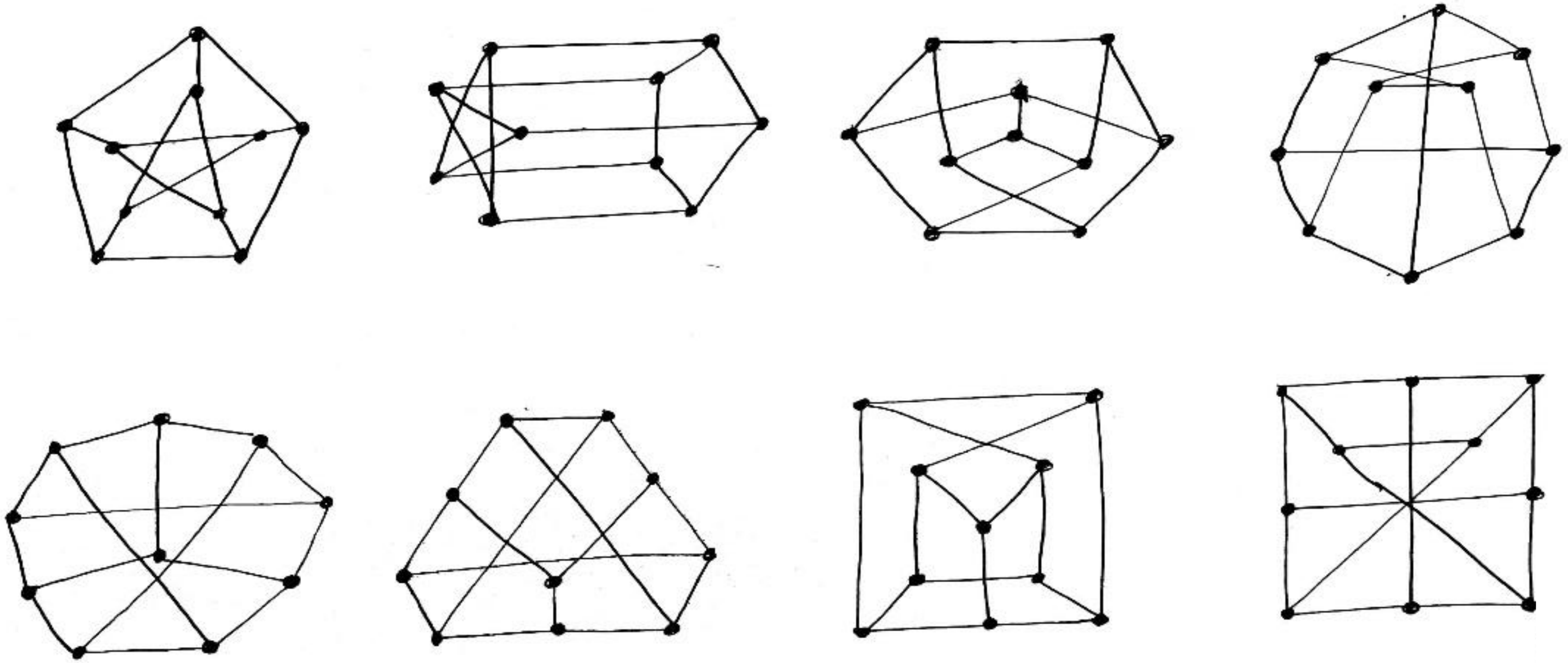
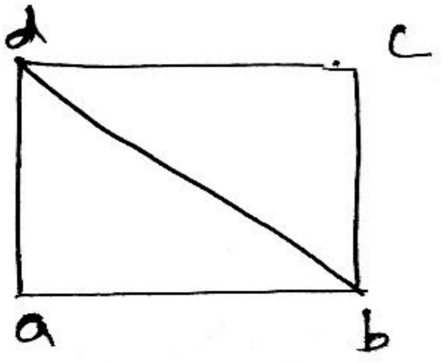
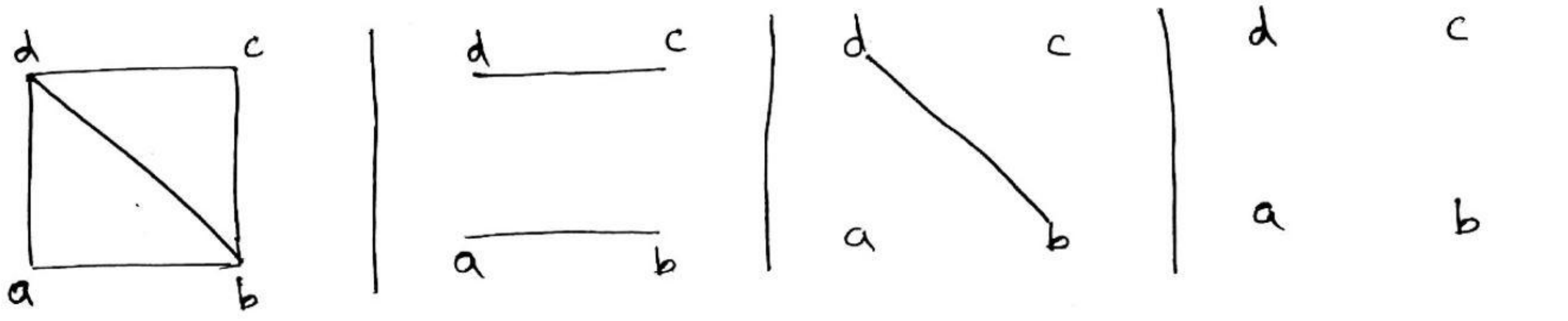


<u>Matching:</u> - Let G be a graph, a subgraph M of G is called a matching of G, if every vertex of G is incident with at most one edge in M.

$$\deg(v) <= 1, \ \forall_v \ V \in V(G)$$





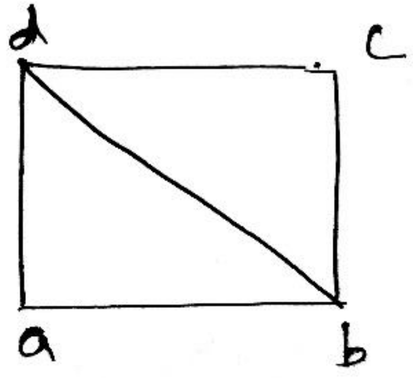
M_1	M_2	M_3	M_4

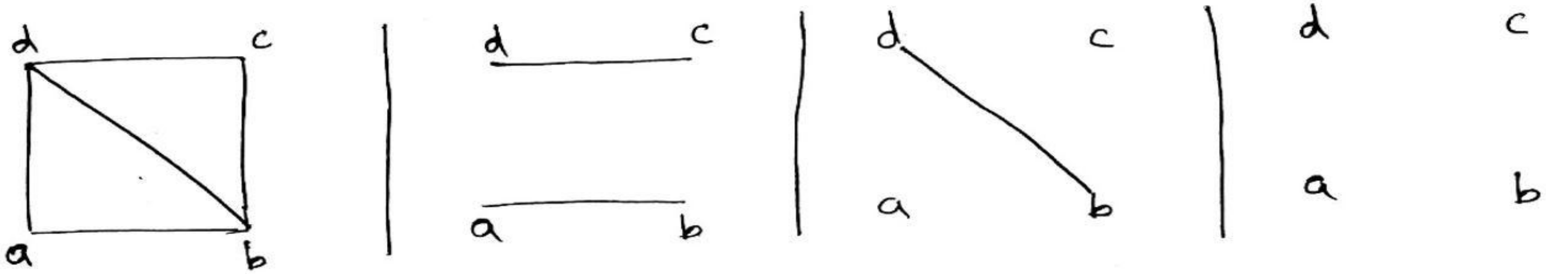
- In matching no two edges are adjacent

<u>Maximal Matching:</u> - A matching M of a graph G is said to be maximal, if no other edges of G can be added to M, without violating the deg condition.

<u>Maximum Matching:</u> - A matching of a graph with maximum no of edges is called a maximum matching of G.

- Number of edges in a maximum matching of G is called matching number.

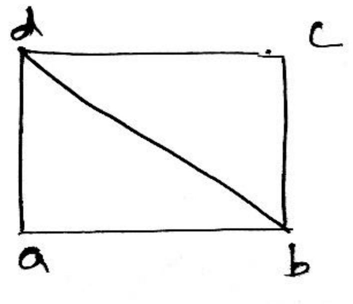


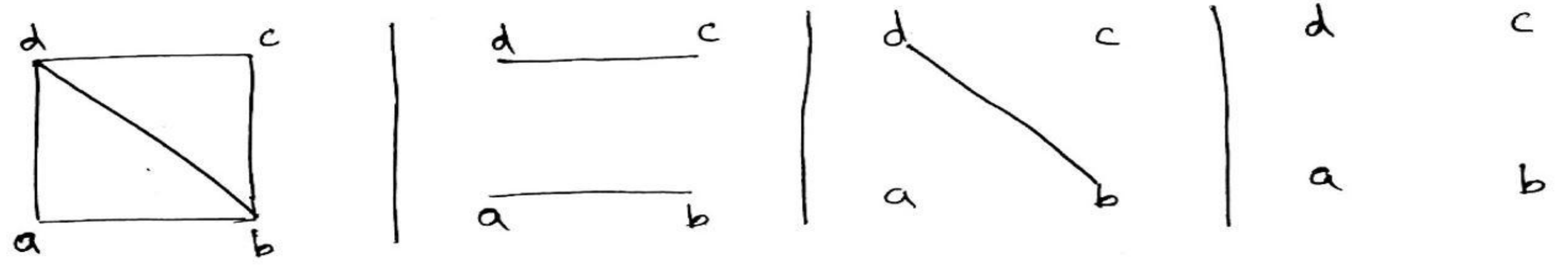


M_1	M_2	M_3	M_4

<u>Perfect Matching:</u> - A matching of a graph in which every vertex is matched is called perfect matching.

1. If a graph G has a perfect match then no of vertices in G is even.
2. If no of vertexes is even, it is not necessary to have a perfect match.
3. No of perfect matchings are there in a complete graph K_n is $[(2n)!]/n!2^n$





M_1	M_2	M_3	M_4

Q How many perfect matchings are there in a complete graph of 6 vertices?
(GATE-2003) (2 Marks)

(A) 15 **(B)** 24 **(C)** 30 **(D)** 60

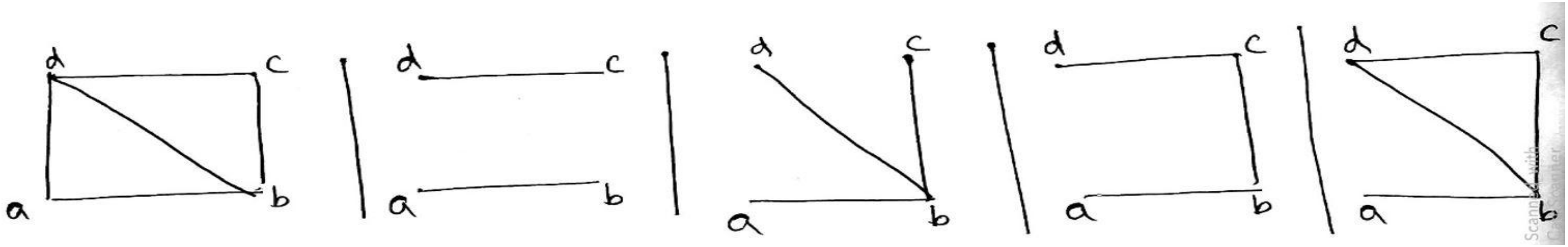
Q. Let A be the adjacency matrix of a simple undirected graph G. Suppose A is its own inverse. Which one of the following statements is always TRUE? **(Gate 2024 CS)(1Mark)(MCQ)**

(a) G is a cycle
(b) G is a perfect matching
(c) G is a complete graph
(d) There is no such graph G

Covering

<u>Line Covering: -</u> Let G (V, E) be a graph, a subset C of E is called a line covering of G, if every vertex of G is incident with at least one edge in C. (deg at least one)

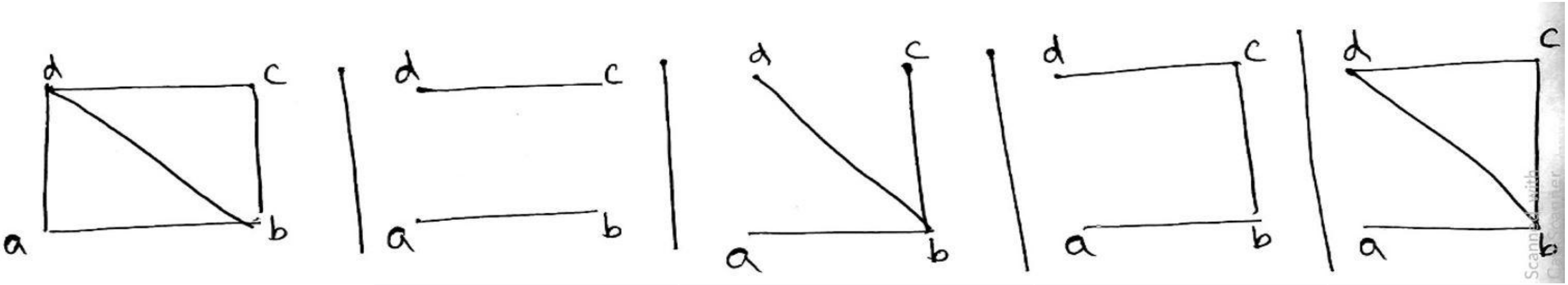
deg(v) >= 1



C_1	C_2	C_3	C_4

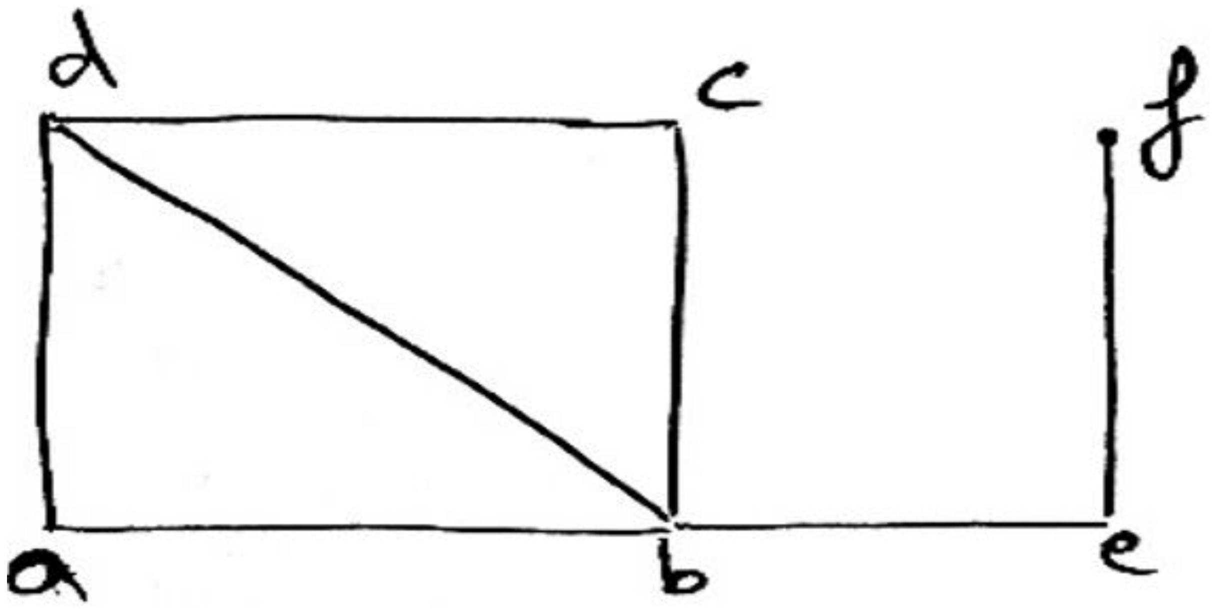
- Line covering of a graph G does not exist if G has an isolated vertex

- **Minimal Line covering:** - A line covering is said to be minimal if no edge can be deleted from the line covering, without destroying its ability to cover the graph.
- **Minimum line covering:** - A line covering with minimum no of edges is called a minimum line covering.



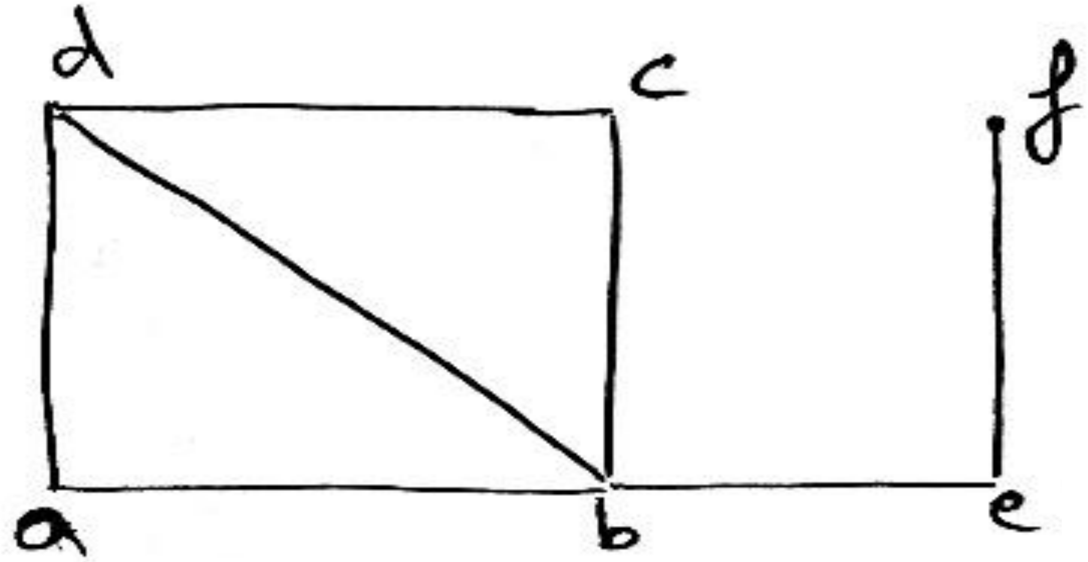
C_1	C_2	C_3	C_4

- No of edges in minimum line covering is called **line covering number** of a graph G, denoted by α_1
- line covering of a graph with n vertices contain at least upper bound(n/2) edges.
- no minimal line covering can contain a cycle.



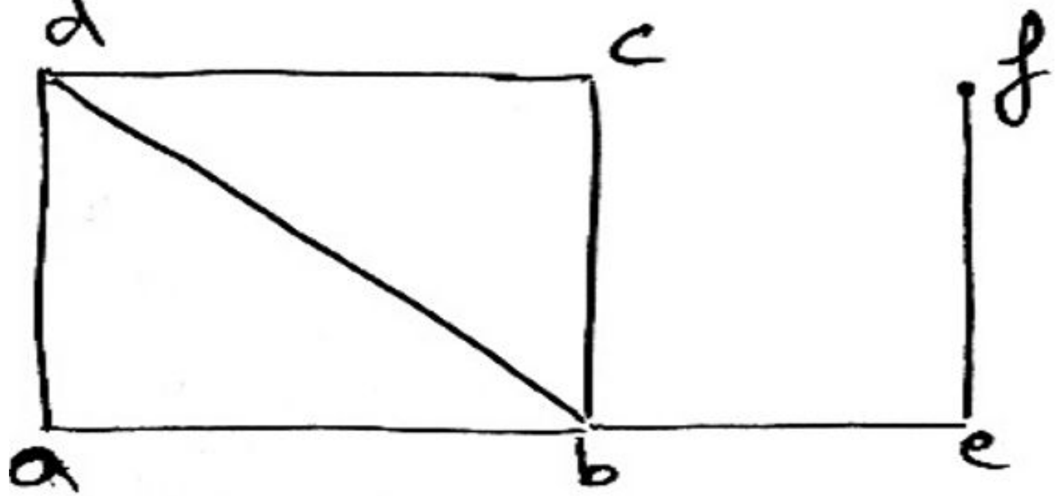
Independent Line set: - Let G (V, E) be a graph, a subset L of E is called independent line set of G, if no two edges are adjacent.

$L_1 = \{(b, d)\}$
$L_2 = \{(b, d), (e, f)\}$
$L_3 = \{(a, d), (b, c), (e, f)\}$
$L_4 = \{(a, b), (e, f)\}$
$L_5 = \{(a, b), (d, c), (e, f)\}$



- **Maximal independent Line set:** - An independent line set L of a graph G is said to be maximal if no other edges of G can be added to L.
- **Maximum independent line set:** - An independent line set L of a graph G, with maximum no of edges is called maximum independent line set.
- No of edges in maximum independent line set is called **Line independent number** of G denoted by $\beta_{1.}$
- line independent no = matching no of G

$$\alpha_1 + \beta_1 = |v|$$



$L_1 = \{(b, d)\}$

$L_2 = \{(b, d), (e, f)\}$

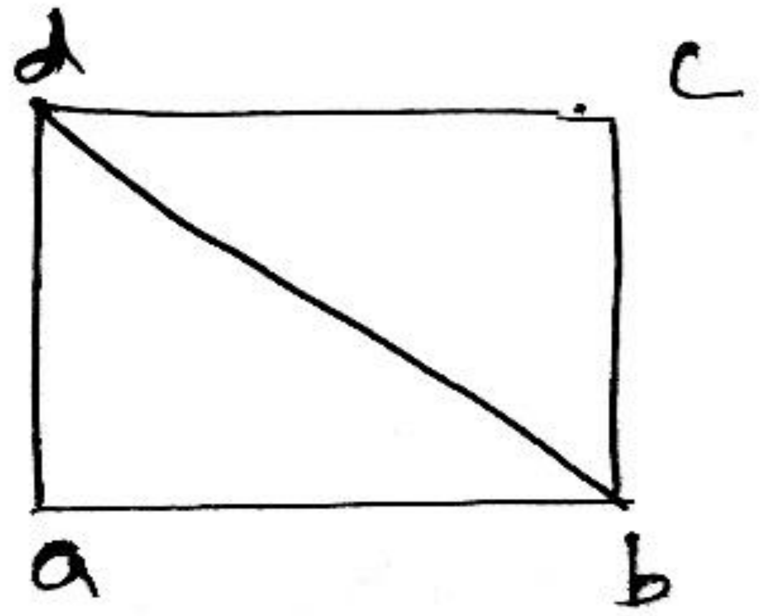
$L_3 = \{(a, d), (b, c), (e, f)\}$

$L_4 = \{(a, b), (e, f)\}$

$L_5 = \{(a, b), (d, c), (e, f)\}$

Vertex Covering: - Let G (V, E) be graph, a subset K of V is called a vertex coving of G. if every edge of G is incident with a vertex in K.

$K_1 = \{b, d\}$
$K_2 = \{a, b, c\}$
$K_3 = \{b, c, d\}$
$K_4 = \{a, b, c, d\}$



Minimal vertex cover: - Vertex covering K of a graph G is said to be minimal if no vertex can be deleted from K, without violating the condition.

Minimum vertex covering: - A vertex covering of a graph G with minimum number of vertices is called as minimum vertex covering.

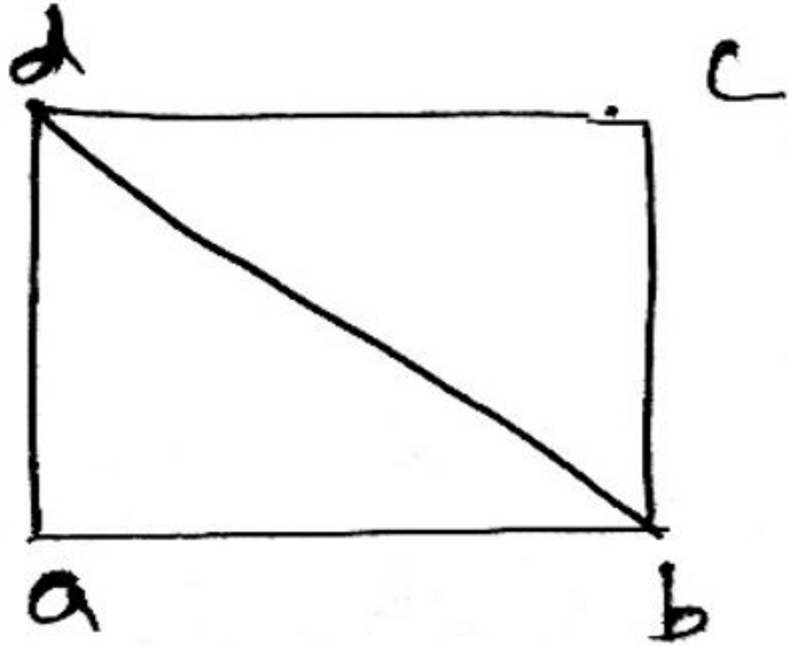
- No of vertices in a minimum vertex covering is called **vertex Covering no** of graph G denoted by α_2

$K_1 = \{b, d\}$

$K_2 = \{a, b, c\}$

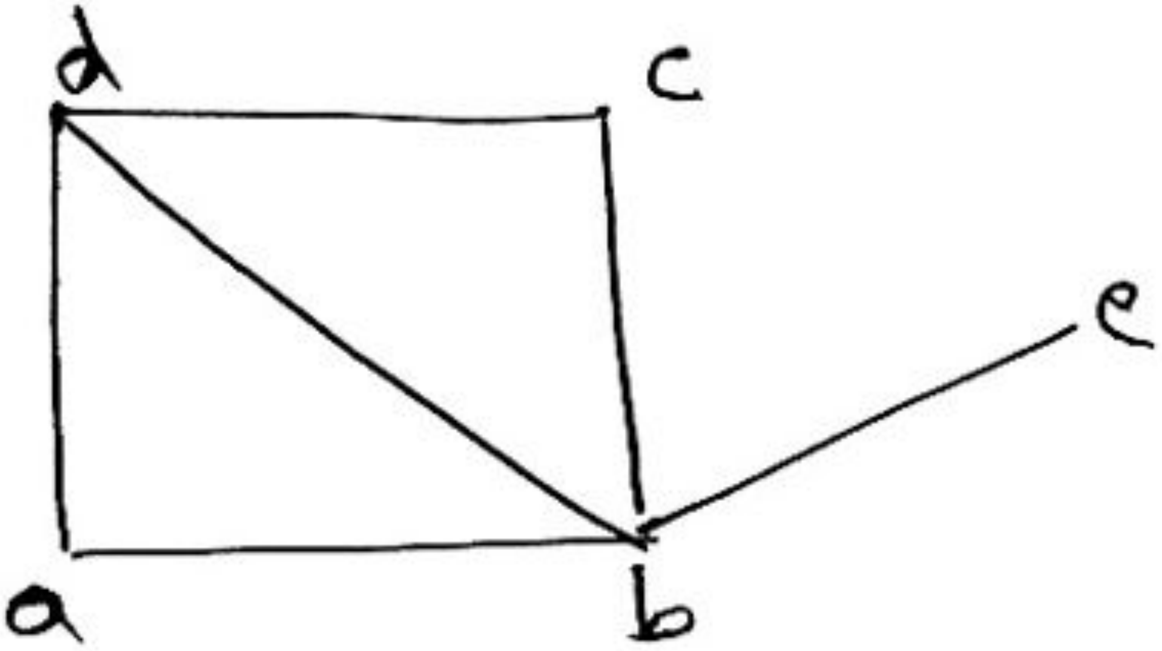
$K_3 = \{b, c, d\}$

$K_4 = \{a, b, c, d\}$



Independent vertex set: - let G (V, E) be a graph, a subset S of V is called an independent vertex set if no two vertices in S are adjacent.

$S_1 = \{b\}$
$S_2 = \{d, e\}$
$S_3 = \{a, c\}$
$S_4 = \{a, b, c\}$
$S_5 = \{a, c, e\}$



Maximum independent Vertex Set: - An independent vertex set is said to be maximal, if no other vertex of G can be added to the set.

Maximum independent vertex set: - An independent vertex set of graph G with maximum no of vertices is called maximum independent vertex set.

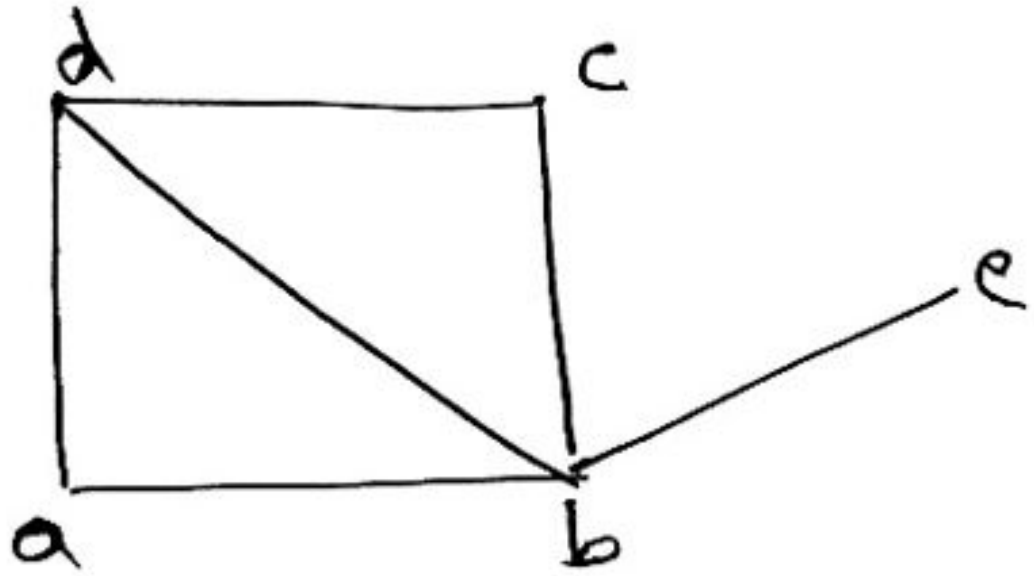
The number of vertices in maximum independent vertex set is called as **vertex independent number** of G donated by β_2

$\alpha_2 + \beta_2 = |V|$

$S_1 = \{b\}$

$S_2 = \{d, e\}$

$S_3 = \{a, c, e\}$



Q What is the size of the smallest MIS (Maximal Independent Set) of a chain of nine nodes? **(GATE-2008) (1 Marks)**

(A) 5 **(B)** 4 **(C)** 3 **(D)** 2

Q Let G be a simple graph with 20 vertices and 100 edges. The size of the minimum vertex cover of G is 8. Then, the size of the maximum independent set of G is **(GATE-2005) (1 Marks)**

(A) 12 **(B)** 8 **(C)** Less than 8 **(D)** More than 12

Q In a connected graph, a bridge is an edge whose removal disconnects a graph. Which one of the following statements is True? **(GATE-2015) (2 Marks)**

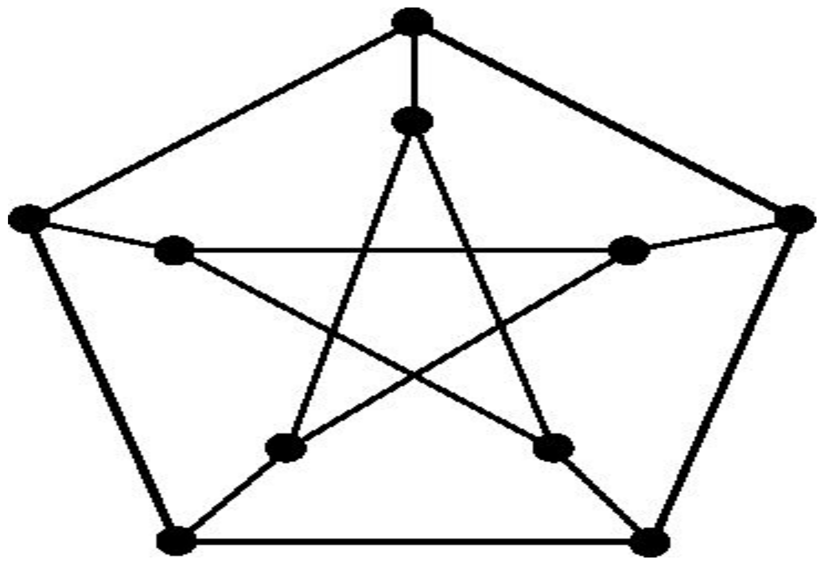
(A) A tree has no bridge

(B) A bridge cannot be part of a simple cycle

(C) Every edge of a clique with size ≥ 3 is a bridge (A clique is any complete subgraph of a graph)

(D) A graph with bridges cannot have a cycle

Q The following simple undirected graph is referred to as the Peterson graph. Which of the following statements is/are TRUE?**(GATE 2022) (2 MARKS)**



(A) The chromatic number of the graph is 3.

(B) The graph has a Hamiltonian path.

(C) The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)

(D) The following graph is isomorphic to the Peterson graph.