1 CHAPTER Introduction and Finite Automata

STRTING & LANGUAGES

1. How many substrings (of all lengths inclusive) can be formed from a character string of length n? Assume all characters to be distinct. Prove your answer. **[1989 : 2 Marks]**

2. The number of substrings (of all lengths inclusive) that can be formed from a characted string of length n is

   (a) n (b) n^2

   (c) $\frac{n(n-1)}{2}$ (d) $\frac{n(n+1)}{2}+1$

   [1994 : 2 Marks]

3. Which of the following definitions below generates the same language as L

   Where $L = \{x^n y^n \mid n \geq 1\}$

   (i) $E \rightarrow xEy \mid xy$

   (ii) $xy \mid (x^+xyy^+)$

   (iii) x^+y^+

   (a) (i) only (b) (i) and (ii)

   (c) (ii) and (iii) (d) (ii) only

   [1995 : 2 Marks]

4. Given $\Sigma = \{a, b\}$, which one of the following sets is not countable?

   (a) Set of the strings over Σ

   (b) Set of all language over Σ

   (c) Set of all regular languages over Σ

   (d) Set of all languages over Σ accepted by Turing Machines **[1997 : 1 Mark]**

5. The string 1101 does not belong to the set represented by

   (a) 110*(0+1) (b) 1(0+1)*0101

   (c) (10)*(01)*(00+11)* (d) (00+(11)*0)*

   [1998 : 1 Mark]

6. How many substrings of different lengths (non-zoro) can be formed from a character string of length n?

   (a) n (b) n^2

   (c) 2^n (d) $\frac{n(n+1)}{2}$

   [1998 : 1 Mark]

7. Consider the following languages:

   $L_1 = \{ww \mid w \in \{a,b\}^*\}$

   $L_2 = \{ww^R \mid w \in \{a, b\}^*$, w^R is the reverse of w$\}$

   $L_3 = \{0^{2i} \mid i$ is an interger$\}$

   $L_4 = \{0^{i^2} \mid i$ is an integer$\}$

   Which of the languages are regular?

   (a) Only L_1 and L_2 (b) Only L_2, L_3 and L_4

   (c) Only L_3 and L_4 (d) Only L_3

   [2001 : 2 Marks]

8. If s is a string over (0+1)*, then let $n_0(s)$ denote the number of 0's in s and $n_1(s)$ the number of 1's in s. Which one of the following languages is not regular?

   (a) $L = \{s \in (0+1)^* \mid n_0(s)$ is a 3-digit prime$\}$

   (b) $L = \{s \in (0+1)^* \mid$ for every prefix s' of s, $|n_0(s') - n_1(s')| \leq 2\}$

   (c) $L = \{s \in (0+1)^* \mid |n_0(s) - n_1(s)| \leq 4\}$

   (d) $L = \{s \in (0+1)^* \mid n_0(s) \bmod 7 = n_1(s) \bmod 5 = 0\}$

   [IT-2006 : 2 Marks]

9. Given the language L = {ab, aa, baa}, which of the following strings are in L*?

   1. abaabaaabaa 2. aaaabaaaa
   3. baaaaabaaaab 4. baaaaabaa

   (a) 1, 2 and 3 (b) 2, 3 and 4

   (c) 1, 2 and 4 (d) 1, 3 and 4

   [2012 : 1 Mark]

10. The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$ of the following regular expression is _____.

    a*b*(ba)*a*

    [2014 (Set-3) : 1 Mark]

11. The C language is

    (a) A context free language

    (b) A context sensitive language

    (c) A regular language

    (d) Parsable fully only by a Turing machines

    [2002 : 2 Marks]

ALPHABET

12. Let R_1 and R_2 be regular sets defined over the alphabet then

    (a) $R_1 \cap R_2$ is not regular (b) $R_1 \cup R_2$ is not regular

    (c) $\Sigma^* - R_1$ is regular (d) R_1^* is not regular

    [1990 : 2 Marks]

13. The two grammars given below generate a language over the alphabet {x, y, z}

G1: $S \to x \mid z \mid xS \mid zS \mid yB$

$B \to y \mid z \mid yB \mid zB$

G2: $S \to y \mid z \mid yS \mid zS \mid xB$

$B \to y \mid yS$

Which one of the following choice describes the properties satisfied by the strings in these languages?

(a) G1 : No y appears before any x
G2 : Every x is following by at least one y

(b) G1 : No y appears before any x
G2 : No x appears before any y

(c) G1 : No y appears after any x
G2 : Every x is following by at least one y

(d) G1 : No y appears after any x
G2 : Every y is following by at least one x

[IT-2007 : 2 Marks]

14. Let Σ be a finite non-empty alphabet and let 2^{Σ^*} be the power set of Σ^*. Which one of the following is TRUE ?

(a) Both 2^{Σ^*} and Σ^* are countable
(b) 2^{Σ^*} is countable and Σ^* is uncountable
(c) 2^{Σ^*} is uncountable and Σ^* is countable
(d) Both 2^{Σ^*} and Σ^* are uncountable

[2014 (Set-3) : 1 Mark]

DFA

15. Which of the following set can be recognized by a Deterministic Finite state Automaton?

(a) The numbers 1, 2, 4, 8, ... 2^n, ... written in binary
(b) The numbers 1, 2, 4, ... 2^n, ... written in unary
(c) The set of binary string in which the number of zeros is the same as the number of ones,
(d) The set {1, 101, 11011, 1110111, ...}

[1998 : 1 Mark]

16. Let L be the set of all binary strings whose last two symbols are the same. The number of states in the minimum state deterministic finite state automaton accepting L is

(a) 2 (b) 5
(c) 8 (d) 3 **[1998 : 2 Marks]**

17. Consider a DFA over Σ = {a, b} accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of state that the DFA will have?

(a) 8 (b) 14
(c) 15 (d) 48 **[2001 : 2 Marks]**

18. Consider the following deterministic finite state automaton M.

Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings is S that are accepted by M is

(a) 1 (b) 5
(c) 7 (d) 8

[2003 : 2 Mark]

19. Consider the regular language L = (111+11111)*. The minimum number of states in any DFA accepting this language is

(a) 3 (b) 5
(c) 8 (d) 9

[IT-2006 : 2 Marks]

20. A minimum state deterministic finite automation accepting the language L = {w | w ∈ (0, 1)*, number of 0s & 1s in w are divisible by 3 and 5, respectively} has

(a) 15 states (b) 11 states
(c) 10 states (d) 9 states

[2007 : 2 Marks]

21. Consider the following DFA in which s_0 is the start state and s_1, s_3 are the final states.

What language does this DFA recognize?

(a) All strings of x and y
(b) All strings of x and y which have either even number of x and even number of y or odd number or x and odd number of y
(c) All strings of x and y which have equal number of x and y
(d) All strings of x and y with either even number of x and odd number of y or odd number of x and even number of y

[IT-2007 : 2 Marks]

22. Consider the regular expression:

R = (a+b)* (aa+bb) (a+b)*

Which deterministic finite automaton accepts the language represented by the regular expression R?

(a) ⇒

(b) ⇒

(c) ⇒

(d) ⇒

[IT-2007 : 2 Marks]

23. If the final states and non-final states in the DFA below are interchanged, then which of the following languages over the alphabet {a, b} will be accepted the the new DFA?

(a) Set of all strings that do not end with ab
(b) Set of all strings that begin with either an a or ab
(c) Set of all strings that do not contain the substring ab,
(d) The set described by the regular expression b*aa*(ba)*b*

[IT-2008 : 2 Marks]

24. The following DFA accepts the set of all strings over {0, 1} that

(a) begin either with 0 or 1
(b) end with 0
(c) end with 00
(d) contain the substring 00

[2010 : 2 Marks]

25. Definition of a language L with albhabet {a} is given as follows:

L = {a^{nk} | K > 0, and n is a positive integer constant}.

What is the minimum number of states needed in a dfa to recognize L?

(a) k + 1 (b) n + 1
(c) 2^{n+1} (d) 2^{k+1}

[2011 : 2 Marks]

26. A deterministic finite automaton (DFA) D with alphabe Σ = {a, b} is given below:

Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?

(a)

(b)

(c)

(d)

[2011 : 2 Marks]

27. Consider the set of strings on {0, 1} in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially complete DFA that accepts this language is shown below.

The missing arcs in the DFA are

(a)

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

(b)

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

(c)

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

(d)

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

[2012 : 2 Marks]

28. Consider the DFA A is given below:

Which of the following are False?

1. Complement of L(A) is context-free.
2. L(A) = L(11 *0+0) (0 + 1)*0*1*)
3. For the language accepted by A, A is the minimal DFA.
4. A accepts all strings over {0, 1} of length at least 2.

(a) 1 and 3 only (b) 2 and 4 only
(c) 2 and 3 only (d) 3 and 4 only

[2013 : 2 Marks]

29. Which of the regular expression given below represent the following DFA?

I. 0*1 (1 + 00* 1)* II. 0*1*1 + 11* 0* 1
III. (0 + 1)* 1

(a) I and II only
(b) I and III only
(c) II and III only
(d) I, II and III

[2014 (Set-1) : 2 Marks]

30.

Consider the DFAs M and N given above. The number of states in minimal DFA that accepts the language L(M) ∩ L(N) is ____.

[2015 (Set-1 : 2 Marks]

31. The number of states in the minimal deterministic finite automaton corresponding to the regular expression (0+1)*(10) is ____.

[2015 (Set-2) : 2 Marks]

32. The number of states in the minimum sized DFA that accepts the language defined by the regular expression (0+1)* (0+1) (0+1)* is___.

[2016 (Set-2) : 1 Mark]

33. Consider the language L given by the regular expression (a + b)* b(a + b) over the alphabet {a, b}. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is _____.

[2017 (Set-1) : 1 Mark]

34. The minimum possible number of states of a deterministic finite automaton that accepts the regular language $L = \{w_1 a w_2 \mid w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is ____.

[2017 (Set-2) : 1 Mark]

35. For $s \in (0 + 1)^*$ let d(s) denote the decimal value of s (e.g. d (101) = 5).

Let $L = \{s \in (0 + 1)^* \mid d(s) \bmod 5 = 2 \text{ and } d(s) \bmod 7 \neq 4\}$

Which one of the following statements is true ?

(a) L is recursively enumerable, but not recursive
(b) L is recursive, but not context-free
(c) L is context-free, but not regular
(d) L is regular

[2006 : 2 Marks]

36. Which one of the following pairs have DIFFERENT expressive power

(a) Deterministic finite automata (DFA) and Non-deterministic finite automata (NFA).
(b) Determinstic push down automata (DPDA) and Non-deterministic push down automata (NPDA)
(c) Deterministic single-tape Turing machine and Non-deterministic single-tape Turing machine
(d) Single-tape Turing machine and multi-tape Turing machine

[2011 : 1 Mark]

37. Which of the following problems are decidable?

1. Does a given program ever produce an output?
2. If L is a context-free language, then, is $\bar{L}$ also context-free?
3. If L is a regular language, thenm is $\bar{L}$ also regular
4. If L is a recursive language, then, is $\bar{L}$ also recursive?

(a) 1, 2, 3, 4 (b) 1, 2
(c) 2, 3, 4 (d) 3, 4

[2012 : 1 Mark]

38. Let Σ be the set of all bijections from {1, ..., 5} to {1, ..., 5}, where id denotes the identity function, i.e. id(j) = j, ∀ j. Let ∘ denote composition on functions.

For a string $x = x_1 x_2 \ldots x_n \in \Sigma^n, n \geq 0$,

let $\pi(x) = x_1 \circ x_2 \circ \ldots \circ x_n$.

Consider the language $L = \{\in \Sigma^* \mid \pi(x) = id\}$. The minimum number of states in any DFA accepting L is _____.

[2019: 2 Marks]

39. Consider the following language.

L = {x ∈ {a, b}* | number of a's in x is divisible by 2 but not divisible by 3}

The minimum number of states in a DFA that accepts L is _____.

[2020: 2 Marks]

40. Let (M) denote an encoding of an automaton M.

Suppose that $\sum = \{0, 1\}$. Which of the following languages is/are NOT recursive ?

(a) L = {⟨M⟩ | M is a DFA such that L (M) = θ}
(b) L = {⟨M⟩ | M is a DFA such that L (M) = Σ*}
(c) L = {⟨M⟩ | M is a PDA such that L (M) = θ}
(d) L = {⟨M⟩ | M is a PDA such that L (M) = Σ*}

[2021 (Set-1) : 1 Mark]

41. Consider the following language.

$L = \{w \in \{0, 1\}^* \mid w \text{ ends with the substring } 011\}$

Which one of the following deterministic finite automata accepts L ?

[2021 (Set-1) : 2 Marks]

42. Consider the following deterministic finite automaton (DFA).

The number of strings of length 8 accepted by the above automaton is ____.

[2021 (Set-2) : 1 Mark]

43. Consider the language L over the alphabet {0, 1}, given below:

$L = \{w \in \{0, 1\}^* \mid w \text{ does not contain three or more consecutive 1's}\}$.

The minimum number of states in a Deterministic Finite-State Automaton (DFA) for L is_____.

[2023 : 2 Marks]

NFA

44. Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in a equivalent minimized DFA is at least

(a) N^2 (b) 2^N

(c) 2N (d) N! [2001 : 1 Mark]

45. Consider the NFAM shown below.

Let the language accepted by M be L. Let L_1 be the language accepted by the $NFAM_1$, obtained by changing the accepting state of M to a non-accepting state and by changing the non-accepting state of M to accepting states. Which of the following statemets is true?

(a) $L_1 = \{0, 1\}^* \cdot L$ (b) $L_1 = \{0, 1\}^*$

(c) $L_1 \subseteq L$ (d) $L_1 = L$

[2003 : 2 Marks]

46. Consider the non-deterministic finite automaton (NFA) shown in the figure.

State X is the starting state of the automaton. Let the language accepted by the NFA with Y as the only accepting state be L1. Similarly, let the language accepted by the NFA with Z as the only accepting state be L2. Which of the following statements about L1 and L2 is TRUE?

(a) l1 = L2 (b) L1 ⊂ L2

(c) L2 ⊂ L1 (d) None of these

[IT-2005 : 2 Marks]

47. Consider the regular expression:

R = (a+b)* (aa+bb) (a+b)*

Which of the following non-deterministic finite automata recognizes the language defined by the regular expression R? Edges labeled λ denote transitions on the empty strings

[IT-2007 : 2 Marks]

48. Let N be an NFA with n states and let M be the minimized DFA with m states recognizing the same language. Which of the following in necessarily true?

(a) $m \le 2^n$

(b) $n \le m$

(c) M has one accept state

(d) $m = 2^n$

[IT-2008 : 1 Mark]

49. Let ω be any string of length n in {0, 1}*, Let L be the set of all substrings of ω. What is the minimum number of states in a non-deterministic finite automata that accepts L?

(a) n - 1 (b) n

(c) n + 1 (d) 2^{n+1}

[2010 : 2 Marks]

50. What is the complement of the language accepted by the NFA shown below? Assume Σ = {a} and ε is the empty string.

(a) ∅ (b) {ε}

(c) a* (d) {a, ε} [2012 : 1 Mark]

51. Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the ε-NFA whose transition table is given below:

δ	∈	a	b
→ q_0	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	ϕ	ϕ
q_3	ϕ	ϕ	$\{q_2\}$

The $\hat{\delta}$ (q_2, aba) is

(a) ϕ (b) $\{q_0, q_1, q_3\}$

(c) $\{q_0, q_1, q_2\}$ (d) $\{q_0, q_2, q_3\}$

[2017 (Set-2) : 2 Marks]

52. Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true?

(a) $k \ge 2^n$ (b) $k \ge n$

(c) $k \le n^2$ (d) $k \le 2^n$

[2018 : 1 Mark]

53. Consider the following definition of a lexical token id for an identifier in a programming language, using extended regular expressions:

letter → [A – Za – a]

digit → [0 – 9]

id → letter (letter | digit)*

Which one of the following Non-deterministic Finite-state Automata with ∈-transitions accepts the set of valid identifiers? (A double-circle denotes a final state)

[2023 : 1 Mark]

AUTOMATA & GRAMMAR

54. Let L denotes the languate the language generated by the grammar S →0S0 | 00. Which of the following is true?

(a) L = 0^*

(b) L is regular but not 0^*

(c) L is context free but not regular

(d) L is not context free

[2000 : 1 Mark]

55. Consider the regular grammar:

S → Xa | Ya

X → Za

Z → Sa | ε

Y → Wa

W → Sa

where S is the starting symbol, the set of terminals is {a} and the set of non-terminals is {S, W, X, Y, Z}. We wish to construct a deterministic finite automaton (DFA) to recognize the same language. What is the minimum number of states required for the DFA?

(a) 2 (b) 3

(c) 4 (d) 5

[IT-2005 : 2 Marks]

56. Conside the regular grammar below:

S → bS | aA | ε

A → aS | bA

The Myhill-Nerode equivalence classes for the language generated by the grammar are

(a) {w ∈ (a+b)* | $\#_a$(w) is even} and {w ∈ (a+b)* | $\#_a$(w) is odd}

(b) {w ∈ (a+b)* | $\#_b$(w) is even} and {w ∈ (a+b)* | $\#_b$(w) is odd}

(c) {w∈(a+b)* | $\#_a$(w)=$\#_b$(w)} and {w∈(a+b)* | $\#_a$(w) ≠ $\#_b$(w)}

(d) {ε}, {wa | w ∈ (a+b)*} and {wb | w∈(a+b)*}

[IT-2006 : 2 Marks]

57. Consider the grammar given below:

S → xB | yA

A → x | xS | yAA

B → y | yS | yBB |

Consider the following strings.

(i) xxyyx (ii) xxyyxy

(iii) xyxy (iv) yxxy

(v) yxx (vi) xyx

Which of the above strings are generated by the grammar?

(a) (i), (ii), and (iii) (b) (ii), (v), and (vi)

(c) (iii) and (iv) (d) (i), (ii), and (iv)

[IT-2007 : 2 Marks]

58. Consider the following finite automata P and Q over the alphabet {a, b, c}. The start states are indicated by a double arrow and final states are indicated by a double circle. Let the languages recognized by them be denoted by L(P) and (Q) respectively.

(e) None of the above

[IT-2007 : 2 Marks]

59. The lexical analysis for a modern computer language such as Java needs the power of which one of the following machine models in a necessary and sufficient sense?

(a) Finite state automata

(b) Deterministic pushdown automata

(c) Non-deterministic pushdown automata

(d) Turing machine

[2011 : 1 Mark]

60. Consider alphabe Σ = {0, 1}, the null/empty string λ and the sets of strings X_0, X_1 and X_2 generated by the corresponding non-terminals of a regular grammar. X_0, X_1 and X_2 are related as follows:

$X_0 = 1X_1$

$X_1 = 0X_1 + 1X_2$

$X_2 = 0X_1 + \{\lambda\}$

Which one of the following choices precisely represents the strings in X_2?

(a) 10(0*+(10)*)1

(b) 10(0*+(10)*)*1

(c) 1(0+10)*1

(d) 10(0+10)*1+110(0+10)*1

[2015 (Set-2) : 2 Marks]

61. Which of the following language is generated by the given grammar?

S → aS | bS | ε

(a) {$a^n b^m$ | n, m ≥ 0}

(b) {w∈{a,b}* | w has equal number of a's and b's}

(c) {a^n | n ≥ 0} ∪ {b^n | n ≥ 0} ∪ {$a^n b^n$ | n ≥ 0}

(d) {a, b}*

[2016 (Set-1) : 1 Mark]

62. If G is a grammar with productions

$S \to SaS \mid aSb \mid bSa \mid SS \mid \epsilon$

where S is the start variable, then which one of the following strings is not generated by G?

(a) *abab* (b) *aaab*

(c) *abbaa* (d) *babba*

[2017 (Set-1) : 2 Marks]

63. S → aSa | b Sb | a | b

The language generated by the above grammar over the alphabet {a, b} is the set of

(a) all palindromes

(b) all odd length palndromes

(c) strings that begin and end with the same symbol

(d) all even length palindromes

[2009 : 1 Mark]

ANSWERS

1. (*)	2. (d)	3. (a)	4. (b)	5. (c)	6. (d)	7. (d)	8. (c)	9. (c)	10. (3)
11. (b)	12. (c)	13. (a)	14. (c)	15. (a)	16. (b)	17. (d)	18. (c)	19. (d)	20. (a)
21. (d)	22. (a)	23. (a)	24. (c)	25. (b)	26. (a)	27. (d)	28. (d)	29. (b)	30. (1)
31. (3)	32. (2)	33. (4)	34. (8)	35. (d)	36. (b)	37. (d)	38. (120)	39. (6)	40. (d)
41. (d)	42. (256 to 256)		43. (4)	44. (b)	45. (b)	46. (d)	47. (a)	48. (a)	49. (c)
50. (b)	51. (c)	52. (d)	53. (c)	54. (b)	55. (b)	56. (a)	57. (c)	58. (e)	59. (a)
60. (c)	61. (d)	62. (a)	63. (b)						

EXPLANATIONS

1. Number of substrings (All lengths inclusive) can be formed from a character string of length n is determine by using the following formula.

$$\frac{n(n+1)}{2}+1$$

Example : If n = 3 and string is → abc

No. of substrings of '0' length is 1 → λ (null)

No. of substring of 1 length is 3 → (a, b, c)

No. of subtring of 2 length is 2 → (ab, bc)

No. of subtring of 3 length is 1 → (abc)

So total No. of substrings is

$$1+3+2+1=7$$

So $\frac{n(n+1)}{2}+1 \Rightarrow \frac{3\times 4}{2}+1=7$

2. $\frac{n(n+1)}{2}+1$

By using this formula we can calculate the number of substrings (of all length inclusive)

for n = 3 total 7 strings are possible.

string = abc

substrings = {λ, a, b, c, ab, bc, abc}

3. $E \Rightarrow XEY \mid XY$

$E \Rightarrow XEY$

$\Rightarrow XXEYY$

$\Rightarrow XXXYYY$

$\Rightarrow X^n\overline{Y}^n$

4. Set of all languages over Σ is incountable infinite set.

5. (a) $110^*(0+1) = 1100, 1101$

(b) $1(0+1)^*101 = 10^*1^*101$

$= 101101$

(c) $(10)^*(01)^*(00+01)^*$

$= (10)^*(01)^*(00)^*(01)^*$

$= 10\,01\,0001$

(d) $(00+(11)^*0)^*$

$= (00)^*(11)^*0)^*$

$= (00)^*(110)^*$

$= 00\,110110$

From above calculation (c) does not belong to 1101.

6. Total non-zero substring of different lengths from string of length n is

$$\frac{n(n+1)}{2}$$

Example n = 3, "abc"

substrings ⇒ {a, b, c, ab, bc, abc}

7. * 0^{i^2} will regular when $i \geq 1$ [integer → 0 – n]

* 0^{i^2} will not regular when $i \geq 1$

* {WWR(a,b)*, WR is the reverse of W} not regular

* $(a,b)^*(b,a)^*$ not is regular.

$[0^{i^2}, 0^{i^3}, 0^{2^n}$, is not regular]

8. Choice (a) is regular since it is finite.

Choice (b) is regular since although comparison is made between 0's and 1's, it is for all prefixes and this can be done by dfa.

Note : $|n_0(s') - n_1(s')| \leq 2$ is same as $n_0(s') - n_1(s') \geq 2$ or $n_1(s') - n_0(s') \geq 2$.

Choice (c) involves comparison of number of 0's and 1's, but for the string as a whole, and this cannot be done by a dfa, since it has finite memory and has no stack for counting upto infinity. Therefore, choice (c) is not regular.

Choice (d) is regular since $n_0(s) \bmod 7 = n_1(s) \bmod 5 = 0$ means number of 0's is divisible by 7 and number of 1's is divisible by 5 and this can be accepted by a dfa with 7 × 5 = 35 states.

A dfa that will accept the language of choice (b) is shown below :

9. String 1, 2 and 4 are generate by language l^*, L = {ab, aa, baa}.

10. R .E = a*b* (ba) *a*

Length 0 is present as it accepts ∈ all length 1 strings are present (a,b) also aa, ab, ba, bb are present, But 'bab' is not present. So it is 3.

11. All morden languages are CSL.

12. Regular language (Regular sets) are closed under intersection (∩), union (∪), Kleene star (*) so option a, b, and d are wrong.

In option (c)

$\Sigma^* - R_1$ is Regular

$\Rightarrow \Sigma^* \cap \overline{R}_1$ (We can change "–" to intersection)

$\Rightarrow \Sigma^* \cap$ Regular {complement of regular is also regular)

⇒ Regular {inter section of two regular is also regular).

13. G1 : S → x | z | xS | yB

B → y | z | yB | zB

(In which No 'y' is appear before any x)

G2 : S → y | z | yS | zS | xB

B → y | yS

(In which every x is following by at least one y).

14. 2^{ε^*} is the power set of ε*

ε* is countabily infinite

The power set of countabily infinite set is uncountable.

So 2^{ε^*} is uncountable, and ε* is countable.

15.

Deterministic Automata for the series

16.

A to D → 00, 100, 1100, 01100

A to E → 11, 011, 0011, 10011

From above example we see A to D end with 00 and A to E end with 11.

So, number of state for deterministic finite state automation accepting L is **A, B, C, D, E** means 5.

17. For DFA over S {a, b}, If No. of a is divisible by "m" and no. of b is divisible by "n" then total minimum number of state is:-

(m × n)

so for given question

6 * 8 = 48

18. Total 7 strings are possible

1 ------1 ----- 1

1001001

1001011

1001101

1001111

1011001

1101001

1111001

19. L(111 + 11111)*

DFA is

Total 9 state.

20. The minimum state deterministic finite automata accepting specified language will have 15 states.

21.

Here S_1 accept all string of x & y where No. of x is odd and y is even and S_2 accept all string where no. of x is even and y is odd.

22. R = (a + b)* (aa + bb) (a + b)*

DFA for R is

23. DFA

(Accept all strings end with ab)

Complemented DFA

(It accepts all strings that do not end with ab)

24.

Accepts all strings end with 00.

25. Let $n = 3$ and $k = 1$

26. Options (b) and (c) will accept the string b
Option (d) will accept the string "bba"
Both are invalid strings.
So minimized DFA is option a

27. The complete DFA is

28.

Minimal DFA for given.
DFA

So statement 3 is false

* String 11 is not accept by the given DFA

So statement 4 is false.

29. Given DFA will accept all the strings over $\Sigma = \{0,1\}$ which are ending with 1.
$0^*1(1+00^*1)^*$ and $(0+1)^*1$, are the regular expressions for ending with 1.

30. $L(M) \cap L(N) = \Sigma^* - (\Sigma^* - L(M)) \cup (\Sigma^* - L(N))$

$\Sigma^* - L(M) \Rightarrow$

$\Sigma^* - L(N) \Rightarrow$

M accept all string end with a and N accept all string end with b. The intersection of both will accept empty language.

Hence

Then no. of state is 1.

31.

Ans = 3

32.

33. The Minimal DFA is

34. Given the regular language is :

Min DFA for L

$$= \{w_1 a w_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1| = 2, |w_2| \geq 3\}$$

Hence the regular expression for L is

$$(a+b)(a+b)\,a\,(a+b)\,(a+b)(a+b)\,(a+b)^*$$

The minimal DFA is

Hence the minimum possible number of states of DFA is 8.

35. $L_1 = d(s) \bmod 5 = 2$ is regular with 5 states.
$L_2 = d(s) \bmod 7 = 4$ is regular with 7 states.
$L_1 \cap L_2$ is also regular with 35 states.

36. NPDA is more powerful than DPDA.
Hence answer is (b)

37. Regular and recuesive language are closed under complement operation. Hence they both are decidable.

38. The DFA for accepting L will have 5! = 120 states, since we need one state for every possible permutation function on 5 elements. The starting state will be "id" state, name as $\begin{pmatrix}12345\\12345\end{pmatrix}$ and from there n! arrows will go the n! states each named with a distinct permutation of the set {1, 2, 3, 4, 5}. Since composition of permutation function is closed every arrow has to go to some permutation and hence some state.
Since the language only has those strings where $\pi(x)$ = id only the starting ("id" state) will be the final state. Sample machine with only 2 states is shown below

39. $L = \{w \in \{a, b\}^* \mid n_a(w) = \text{multiple of 2 but not 3}\}$
Let $L_1 = \{w \in \{a, b\}^* \mid n_a(w) = \text{multiple of 2}\}$
$L_2 = \{w \in \{a, b\}^* \mid n_a(w) = \text{multiple of 3}\}$
Given language is $L_1 - L_2 = L_1 \cap L_2^c$
We can make a DFA for L_1 with 2 states.
We can make a DFA for L_2^c with 3 states.
Then use product automata construction to get a DFA for $L_1 - L_2 = L_1 \cap L_2^c$ with 6 sates.
Since neither of the L_1 or L_2^c has a trap state, so resulting product automata also has no trap state.
Hence, min DFA has 6 states.
Example:-
If numbers that are divisible by 2 but not by 3 = {2, 4, 8, 10, 14, 16, ...}
= {aaa, aaaa, aaaaaaaa, ...}

40. (a) is L = { M | M is a DFA such that L(M) is Σ^* } is a trivial problem for DFA and DECIDABLE and **Recursive.**
(b) is **DECIDABLE** and **Recursive**; The language accepted we can check for usefulness of start symbol. If the start symbol is useful then L(G) non empty otherwise L(G) is empty
(c) is **UNDECIDABLE** and **NOT Recursive**; we can't check whether L(G) = Σ^* or not but rather

we can check complemented of L(G) is $\varnothing$. Since context free language are not closed under complement operation $\overline{L(G)}$ may be language accepted by Turing Machine and we can't check emptiness for Turing machine.
Option (d) is **DECIDABLE** and **recursive**; emptiness of the finite state machine can be checked by reachability of the final state from initial state in transition graph. If the final state is reachable then non empty otherwise empty.
Hence, option (d) is the correct answer.

41.

Ending pattern required n+1 states where n is length of the string, in this case 3 and at least on input goes out of the final state.
Here input 1 from the final state goes to initial state which remember all 1's
And input 0 goes to second state which remember last alphabet as 0. And other than option (d) are Accepting 01111. Hence, option (d) is correct.

42. The regular expression for the given DFA is
$= (0(0+1)(0+1)+1(0+1)(0+1))(0+1)^*$
It can be written as:
$= (0+1)(0+1)(0+1)(0+1)^*$
Which can be defined as all string over {0, 1} of length greater than 3
As per the given condition in the string length it should be exactly 8.
We can write the expression for length 8 string as:
$\Rightarrow (0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1) = [0+1]^8$
So, all possible combination
For length 8, all $2^8 = 256$ string will be accepted.

43. For ω, that does not contain 3 or more consecutive is, For this constraint, final state will not be reached after transition on, third '1',
L is the language of the strings over {0, 1}* that does not contains '111' as substring.
The DFA for L is

2 CHAPTER

Regular Language & Regular Expression

REGULAR EXPRESSION

1. Let r =(1+ 0)*, s = 11*0 and t = 1*0 be three regular expressions. Which one of the following is true?
   (a) L(s)⊆L(r) and L(s)⊆L(t)
   (b) L(r)⊆L(s) and L(s)⊆L(t)
   (c) L(s)⊆L(t) and L(s)⊆L(r)
   (d) L(t)⊆L(s) and L(s)⊆L(r) **[1991 : 2 Marks]**

2. Which of the following regular expression identifies are true?
   (a) $r(*) = r^*$ (b) $(r^* s^*)^* = (r + s)^*$
   (c) $(r + s)^* = r^* + s^*$ (d) $r^* s^* = r^* + s^*$
   [1992 : 2 Marks]

3. Which two of the following four regular expressions are equivalent?
   (i) $(00)^* (\varepsilon+0)$ (ii) $(00)^*$
   (iii) 0^* (iv) $0(00)^*$
   (a) (i) and (ii) (b) (ii) and (iii)
   (c) (i) and (iii) (d) (iii) and (iv)
   [1996 : 1 Mark]

4. Let $L \subseteq \Sigma^*$ where $\Sigma = \{a, b\}$, which of the following is true?
   (a) L = {x | x has an equal number of a's and b's is regular
   (b) $L = \{a^n b^n \mid n \geq 1\}$ is regular
   (c) L = {x | x has more a's is regular
   (d) $L = \{a^m b^n \mid m \geq, n \geq 1\}$ is regular
   [1996 : 1 Mark]

5. Which one of the following regular expressions over {0, 1} denotes the set of all strings not containg 100 as substrings?
   (a) 0*(1+0)* (b) 0*1010*
   (c) 0*1*01* (d) 0*(10+1)*
   [1997 : 2 Marks]

6. If the regular set A represented by A = (01+1)* and the regular set 'B' is represented by B = (((01)*1*)* which of the following is true?
   (a) A ⊂ B
   (b) B ⊂ A
   (c) A and B are incomparable
   (d) A = B **[1998 : 1 Mark]**

7. Let S and T be language over Σ = {a, b} represented by the regular expressions (a+b*)* and (a+b)*, respectively. Which of the following is true?
   (a) S ⊂ T (b) T ⊂ S
   (c) S = T (d) S ∩ T = ϕ
   [2000 : 1 Mark]

8. Which one of the following regular expressions is NOT equivalent to the regular expression (a+b+c)*?
   (a) (a* + b* + c*)* (b) (a*b*c*)*
   (c) ((ab)* + c*)* (d) (a*b* + c*)*
   [IT-2004 : 1 Mark]

9. Which of the following statement is TRUE about the regular expression 01*0?
   (a) It represents a finite set of finite strings.
   (b) It represents an infinite set of finite strings.
   (c) It represent a finite set of infinite strings.
   (d) It represents an infinite set of infinite strings.
   [IT-2005 : 1 Mark]

10. Which regular expression best describes the language accepted by the non-deterministic automaton below?

   (a) (a+b)* a(a+b)b
   (b) (abb)*
   (c) (a+b)* a(a+b) *b(a+b)*
   (d) (a+b)* **[IT-2006 : 1 Mark]**

11. Which of the following statements about regular languages is NOT true?
   (a) Every language has a regular superset
   (b) Every language has a regular subset
   (c) Every subset of a regular language is regular
   (d) Every subset of a finite language is regular
   [IT-2006 : 2 Marks]

Directions for Q.12 to Q.13:

Let L be a regular language. Consider the constructions on L below:

I. repeat (L) = {ww | w ∈ L}
II. prefix (L) = {u | ∃v : uv ∈ L}
III. suffix (L) = {v | ∃u : uv ∈ L}
IV. half (L) = {u | v : |∃v| = |u| and uv ∈ L}

12. Which of the constructions could lead to a non-regular language?
   (a) Both I and IV (b) Only I
   (c) Only IV (d) Both II and II
   [IT-2006 : 2 Marks]

13. Which choice of L is best suited to support your answer above?

(a) (a+b)* (b) {ε, a, ab, bab}

(c) (ab)* (d) $\{a_n b_n \mid n \geq 0\}$

[IT-2006 : 2 Marks]

14. Which of the following is TRUE?

(a) Every subset of the regular set is regular

(b) Every finite subset of a non-regular set is regular

(c) The union of two non-regular sets is not regular

(d) Infinite union of finite sets is regular

[IT-2007 : 1 Mark]

15. Consider the regular expression:

R = (a+b)* (aa+bb) (a+b)*

Which one of the regular expressions given below defines the same language as defined by the regular expression R?

(a) (a(ba)* + b(ab)*) (a+b)*

(b) (a(ba)* + b(ab)*)*(a+b)*

(c) (a(ba)* (a+bb) + b(ab)* (b+aa))(a+b)*

(d) (a(ba)* (a+bb) + b(ab*(b+aa))(a+b)*

[IT-2007 : 2 Marks]

16. Which of the following regular expressions describes the language over {0, 1} consisting of strings that contain exactly two 1's?

(a) (0 + 1)* 11(0 + 1)*

(b) 0* 110*

(c) 0* 10* 10*

(d) (0 + 1)* 1(0 + 1)* 1(0 + 1)*

[IT-2008 : 1 Mark]

17. Math List-1 with List-II and select the correct answer using the codes given below the lists:

List-I

A.

B.

C.

D.

List-II

1. ε + 0(01*1+00)*01*
2. ε + 0(10*1+00)*0
3. ε + 0(10*1+10)*1
4. ε + 0(10*1+10)*10*

Codes:

	A	B	C	D
(a)	2	1	3	4
(b)	1	3	2	4
(c)	1	2	3	4
(d)	3	2	1	4

[2008 : 2 Marks]

18. Which one of the following language over the alphabet {0, 1} is described by the regular expression: (0 + 1)*0(0 + 1)*(0 + 1)*?

(a) The set of all strings containing the substring 00

(b) The set of all strings containing at most two 0's

(c) The set of all strings containing at least two 0's

(d) The set of strings that begin and end with either 0 or 1

[2009 : 1 Mark]

19. Let L = {ω∈ (0+1)* | ω has even number of 1s}, i.e. L is the set of all bit strings with even number of 1s. Which one of the regular expression below represents L?

(a) (0*10*1)*

(b) 0*(10*10*)*

(c) 0*(10*1)*0*

(d) 0*1(10*1)*10*

[2010 : 2 Marks]

20. Let L be the language represented by the regular expression Σ*0011Σ* where Σ = {0, 1}. What is the minimum number of states in a DFA that recognizes $\overline{L}$ (complement of L)?

(a) 4 (b) 5

(c) 6 (d) 8

[2015 (Set-3) : 1 Mark]

21. Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0s and two consecutive 1s?

(a) (0+1)*0011(0+1)* + (0+1)*1100(0+1)*

(b) (0+1)*(00(0+1)*11+11(0+1)*00)(0+1)*

(c) (0+1)*00(0+1)*+(0+1)*11(0+1)*

(d) 00(0+1)*11+11(0+1)*00

[2016 (Set-1) : 1 Mark]

22. Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I

A. Checking that identifies are declared before their use

B. Number of formal parameters in the declaration of a function agrees with the number of actual parameters in a use of that function

C. Arithmetic expression with matched pairs of parentheses

D. Palindromes

List-II

1. $L = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$
2. X → X b X | X c X | d X f | g
3. L = {wcw | w ∈ (a | b)*}
4. X → b X b | c X c | ε

Codes:

	A	B	C	D
(a)	1	3	2	4
(b)	3	1	4	2
(c)	3	1	2	4
(d)	1	3	4	2

[2008 : 2 Marks]

23. Which one of the following regular expressions represents the set of all binary strings with an odd number of 1's?

(a) ((0 + 1)* 1(0 + 1)*1)*10*

(b) (0*10*10*)*0*1

(c) 10* (0*10*10*)*

(d) (0*10*10*)*10*

[2020 : 1 Marks]

24. Which of the following regular expressions represent(s) the set of all binary numbers that are divisible by three? Assume that the string ∈ is divisible by three.

(a) (0 + 1(01*0)*1)*

(b) (0 + 11 + 10 (1 + 00)*01)*

(c) (0*(1(01*0)*1)*)*

(d) (0 + 11 + 11 (1 + 00)*00)*

[2021 (Set-2) : 2 Marks]

25. Which one of the following regular expressions correctly represents the language of the finite automaton given below?

(a) ab*bab* + ba*aba*

(b) (ab*b)* ab* + (ba*a)*ba*

(c) (ab*b + ba*a)* (a* + b*)

(d) (ba*a + ab*b)* (ab* + ba*)

[2022 : 1 Mark]

FINITE AUTOMATON

26. State True of False with one line explanation. A FSM (Finite State Machine) can be designed to add two integers of any arbitrary length (arbitrary number of digits).

[1994 : 1 Mark]

27. A finite state machine with the follows state table has a sngle input X and a single output Z.

present state	next state, z	next state, z
	x = 1	x = 0
A	D, 0	B, 0
B	B, 1	C, 1
C	B, 0	D, 1
D	B, 1	C, 0

It the initial state is unknown, then the shortest input sequence to reach the final state C is

(a) 01 (b) 10

(c) 101 (d) 110

[1995 : 2 Marks]

28. The finite state machine described by the following state diagram with A as starting state, where an arc lable is x/y and x stands for 1-bit input and y stands for 2-bit output

(a) Output the sum of the present and the previous bits of the input
(b) Outputs 01 whenever the input sequence contains 11
(c) Output 00 whenever the input sequence contains 10
(d) None of the above **[2002 : 2 Marks]**

29. The smallest finite automation which accepts the language L = {x | length of x is divisible by 3} has
(a) 2 states (b) 3 states
(c) 4 states (d) 5 states **[2002 : 2 Marks]**

30. Let M = (K, Σ, δ, s, F} b a finite state automaton, where
K = {A, B}, Σ = {a, b}, s = A, F = {B},
δ(A, a)=A, δ(A, b)=B, δ(B, a)=B and δ(B, b)=A
A grammar to generate the language accepted by M can be specified as G = (V, Σ, R, S),Where V = K ∪ Σ, and S = A
Which one of the following set of rules will make L(G) = L(M)?
(a) {A→aB, A→bA, B→bA, B→aA, B→ε}
(b) {A→aA, A→bB, B→aB, B→bA, B→ε}
(c) {A→bB, A→aB, B→aA, B→bA, B→ε}
(d) {A→aA, A→bA, B→aB, B→bA, A→ε}
[IT-2004 : 2 Marks]

31. The following finite state machine acepts all those binary strings in which the number of 1's and 0's are respectively

(a) divisible by 3 & 2
(b) odd and even
(c) even and odd
(d) divisible by 2 & 3
[2004 : 2 Marks]

32. Consider the machine M

The language recognized by M is
(a) {w∈{a, b}* | every a in w is following by exactly two b's}
(b) {w∈{a, b}* every a in w is following by at least two b's}
(c) {w∈{a, b}* w containts the substring 'abb'
(d) {w∈{a, b}* w does not contain 'aa' as a substring}
[2005 : 2 Marks]

33. The following diagram represents a finite state machine which takes as input a binary number from the least significant bit

Which one the following is TRUE?
(a) It computes 1's complement fo the input number
(b) It computes 2's complement of the input nujmber
(c) It increments the input number
(d) It decrements the input number
[2005 : 2 Marks]

34. In the automaton below, s is the start state and t is the only final state.

Consider the strings u = abbaba, v = bab, and w = aabb. Which of the following statements is true?
(a) The automaton accepts u and v but not w
(b) The automaton accepts each of u, v, and w
(c) The automaton rejects each of u, v, and w
(d) The automaton accepts u but rejects v and w
[IT-2006 : 1 Mark]

Common Data for Q. 35 & 36

Consider the following Finite State Automation

35. The language accepted by this automaton is given by the regular expression
(a) b * ab* ab* ab*
(b) (a + b)*
(c) b* a(a + b)*
(d) b* ab* ab*
[2007 : 2 Marks]

36. The minimum state automaton equivalent to the above FSA has the following number of states
(a) 1 (b) 2
(c) 3 (d) 4 **[2007 : 2 Marks]**

37. Consider the following two finite automata. M_1 accepts L_1 and M_2 accepts L_2. Which one of the following is TRUE?

(a) $L_1 = L_2$ (b) $L_1 \cap L_2 = \phi$
(c) $L_1 \cap \overline{L_2} = \phi$ (d) $L_1 \cup L_2 \neq L_1$
[IT-2008 : 2 Marks]

38. Given below are two finite state automata (→ indicates the start and F indicates a final state)

Y:

	a	b
→1	1	2
2(F)	2	1

Z:

	a	b
→1	2	2
2(F)	1	1

Which of the following represents the product automaton Z × Y?

(a)

	a	b
→P	S	R
Q	R	S
R(F)	Q	P
S	P	Q

(b)

	a	b
→P	Q	S
Q	R	S
R(F)	Q	P
S	Q	P

(c)

	a	b
→P	S	Q
Q	R	S
R(F)	Q	P
S	P	Q

(d)

	a	b
→P	S	Q
Q	S	R
R(F)	Q	P
S	Q	P

[2008 : 2 Marks]

39. Consider the finite automaton in the following figure.

What is the set of reachable states for the input string 0011?
(a) $\{q_0, q_1, q_2\}$ (b) $\{q_0, q_1\}$
(c) $\{q_0, q_1, q_2, q_3\}$ (d) $\{q_3\}$
[2014 (Set-1 : 1 Mark]

40. Given the following state table of an FSM with two states A and B, one input and one output :

Present State A	Present State B	Input	Next State A	Next State B	Output
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	0	1	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	0	0	1

If the initial state is A = 0, B = 0, what is the minimum length of an input string which will take the machine to the state A = 0, B = 1 with Output = 1?
(a) 3 (b) 4
(c) 5 (d) 6
[2009 : 2 Marks]

41. Suppose we want to design a synchronous circuit that processes a string of 0's and 1's. Given a string, it produces another string by replacing the first 1 in any subsequence of consecutive 1's by a 0. Consider the following example.
Input sequence : 00100011000011100
Output sequence : 00000001000001100
A Mealy Machine is a state machine where both the next state and the output are functions of the present state and the current input.
The above mentioned circuit can be designed as a two-state Mealy machine. The states in the Mealy machine can be represented using Boolean values 0 and 1. We denote the current state, the next state, the next incoming bit, and the output bit of the Mealy machine by the variables s, t, b and y respectively. Assume the initial state of the Mealy machine is 0.

What are the Boolean expressions corresponding to t and y in terms of s and b ?

(a) t = s + b
y = s b

(b) t = b
y = s b

(c) t = b
y = s $\bar{b}$

(d) t = s + b
y = s $\bar{b}$

[2021 (Set-2) : 2 Marks]

RE WITH FA

42. The regular expression for the language recognized by the finite state automaton

[1994 : 2 Marks]

43. Consider the regular expression (0+1)(0+1)...n times. The minimum state finite automation that recognizes the language represented by this regular expression contains

(a) n states (b) n+1 states
(c) n+2 states (d) None of these

[1999 : 1 Mark]

44. What can be said about a regular language L over {a} whose minimal finite state automation has two states?

(a) Can be $\{a^n \mid n \text{ is odd}\}$
(b) Can be $\{a^n \mid n \text{ is even}\}$
(c) Can be $\{a^n \mid \geq 0\}$
(d) Either L can be $\{a^n \mid n \text{ is odd}\}$ or L can be $\{a^n \mid n \text{ is even}\}$

[2000 : 2 Marks]

45. Let $L \subseteq \{0, 1\}^*$ be an arbitrary regular language accepted by a minimal DFA with k states. Which one of the following languages must necessarily be accepted by a minimal DFA with k states ?

(a) L − {01} (b) L ∪ {01}
(c) $\{0, 1\}^* - L$ (d) L . L

[2021 (Set-2) : 1 Mark]

46. Consider the Deterministic Finite-state Automaton (DFA) A shown below. The DFA runs on the alphabet {0, 1}, and has the set of states {s, p, q, r}, with s being the start state and p being the only final state.

Which one of the following regular expressions correctly describes the language accepted by A?

(a) 1 (0*11)* (b) 0(0 + 1)*
(c) 1(0 + 11)* (d) 1(110*)*

[2023 : 1 Mark]

REGULAR LANGUAGE

47. Consider the following statements:

S1: $\{0^{2n} \mid n \geq 1\}$ is a regular language

S2: $\{0^m 1^n 0^{m+n} \mid m \geq 1 \text{ and } n \geq 1\}$ is a regular language

Which of the following is true about S_1 and S_2?

(a) Only S_1 is correct
(b) Only S_2 is correct
(c) Both S_1 and S_2 are correct
(d) None of S_1 and S_2 is correct [2001 : 1 Mark]

48. Which of the following language is regular?

(a) $\{ww^R \mid w \in \{0, 1\}^*\}$ (b) $\{ww^R x \mid x, w \in (0, 1)^*\}$
(c) $\{wxw^R \mid x, w \in (0, 1)^*\}$ (d) $\{xww^R \mid x, w \in \{0, 1)^*\}$

[2007 : 2 Marks]

49. Which of the following languages is (are) non-regular?

$L_1 = \{0^m 1^n \mid 0 \leq m \leq n \leq 10000\}$

$L_2 = \{w \mid w \text{ reads the same forward and backward}\}$

$L_3 = \{w \in \{0, 1\}^* \mid w \text{ contains an even number of 0's and an even number of 1's}\}$

(a) L_2 and L_3 only (b) L_1 and L_2 only
(c) L_3 only (d) L_2 only

[IT-2008 : 2 Marks]

50. Which of the following are regular sets?

1. $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$ 2. $\{a^n b^m \mid n = 2m\}$
3. $\{a^n b^m \mid n \neq m\}$ 4. $\{xcy \mid x, y \in \{a, b\}^*\}$

(a) 1 and 4 only (b) 1 and 3 only
(c) 1 only (d) 4 only

[2008 : 2 Marks]

51. Consider the language $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$?

(a) $\{\varepsilon\}$ (b) ϕ
(c) a^* (d) $\{\varepsilon, a\}$

[2013 : 1 Mark]

52. Which one of the following is TRUE?

(a) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.
(b) The language $L = \{a^n \mid n \text{ is prime}\}$ is regular.
(c) The language $L = \{w \mid w \text{ has } 3k + 1 \text{ b's for some } k \in N \text{ with } \Sigma = \{a, b\}\}$
(d) The language $L = \{ww \mid w \in \Sigma^* \text{ with } \Sigma = \{0, 1\}\}$ is regular.

[2014 (Set-1) : 1 Mark]

53. If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

I. $L_1 . L_2$ is a regular language
II. $L_1 . L_2 \{a^n b^n \mid n \geq 0\}$

Which one of the following is CORRECT?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor Ii

[2014 (Set-2) : 1 Mark]

54. Let $L_1 = \{w \in \{0,1\}^* \mid w \text{ has at least as many occurrences of (110)'s as (011)'s}\}$. Let $L_2 = \{w \in \{0,1\}^* \mid w \text{ has at least as many occurrence of (000)'s as (111)'s}\}$. Which one of the following is TRUE?

(a) L_1 is regular but not L_2
(b) L_2 is regular but not L_1
(c) Both L_1 and L_2 are regular
(d) Neither L_1 not L_2 are regular

[2014 (Set-2) : 2 Marks]

55. Which of the following language is/are regular?

L_1: $\{wxw^R \mid w, x \in \{a,b\}^* \text{ and } |w|, |x| > 0\} w^R$ is the reverse of string w

L_2: $\{a^n b^m \mid m \neq n \text{ and } m, n \geq 0\}$

L_3: $\{a^p b^q c^r \mid p, q, r \geq 0$

(a) L_1 and L_3 only
(b) L_2 only
(c) L_2 and L_3 only
(d) L_3 only

[2015 (Set-2) : 2 Marks]

56. Consider the following two statements:

I. If all states of an NFA are accepting states then the language accepted by the NFA is Σ^*.
II. There exists a regular language A such that for all languages B, $A \cap B$ is regular.

Which one of the following is CORRECT?

(a) Only I is true
(b) Only II is true
(c) Both I and II are true
(d) Both I and II are false

[2016 (Set-2) : 2 Marks]

57. Given a language L, define L^i as follows:

$L^0 = \{\epsilon\}$

$L^i = L^{i-1}. L$ fore $i > 0$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$.

Consider the language L_1 (over alphabet 0) accepted by the following automaton.

The order of L1 is _____. [2018 : 2 Marks]

58. Fortran is

(a) regular language
(b) context free language
(c) context sensitive language
(d) None of the above [1987 : 2 Marks]

59. Which of the following statement is false?

(a) Every finite subset of a non-regular set is regular
(b) Every subset of a regular set is regular
(c) Every finite subset of a regular set is regular
(d) The intersection of two regular sets is regular

[1998 : 2 Marks]

60. Let L be a regular language and M be a context-free language, both over the alphabet . Let L^c and M^c denote the complements of L and M respectively.

Which of the following statements about the language $L^c \cup M^c$ is TRUE ?

(a) It is necessarily regular but not necessarily context-free.
(b) It is necessarily context-free.
(c) It is necessarily non-regular.
(d) None of the above

[IT-2005 : 1 Mark]

61. Which of the following statements is false?

(a) Every NFA can be converted to an equivalent DFA.
(b) Every o-deterministic Turing machine can be converted to an equivalent deterministic Turing machine
(c) Every regular language is also a context-free language
(d) Every subset of a recursively enumerable set is recursive

[2008 : 2 Marks]

62. Match all items in Group I with correct options from those given in Group 2

Group 1	Group 2
P. Regular expression	1. Syntax analysis
Q. Pushdown automata	2. Code generation
R. Dataflow analysis	3. Lexical analysis
S. Register allocation	4. Code Optimization

(a) P-4, Q-1, R-2, S-3
(b) P-3, Q-1, R-4, S-2
(c) P-3, Q-4, R-1, S-2
(d) P-2, Q-1, R-4, S-2 **[2009 : 1 Mark]**

63. Which one of the following is the strongest correct statement about a finite language over some fiite alphabet Σ?

(a) It could be undecidable
(b) It is turing-machine recognizable
(c) It is a regular language
(d) It is a context-sensitive language

[1991 : 2 Marks]

64. Let L(R) be the language represented by regular expression r. Let L(G) be the language generated by a context free grammar G. Let L(M) be the language accepted by a Turing machine M.

Which of the following decision problems are undecidable?

I. Given a regular expression R and a string w, is $\omega \in L(R)$?
II. Given a context-free grammar G, is $L(G) = \phi$?
III. Given a context-free grammar G, is $L(G) = \Sigma^*$ for some alphabet Σ?
IV. Given a Turing machine M and a string w, is $\omega \in L(M)$?

(a) I and IV only (b) II and III only
(c) II, III and IV only (d) III and IV only

[2017 (Set-2) : 2 Marks]

65. If L is a regular language over $\Sigma = \{a, b\}$, which one of the following language is NOT regular?

(a) $L \cdot L^R = \{xy \mid x \in L, y^R \in L\}$
(b) $\{ww^R \mid \omega \in L\}$
(c) Suffix (L) = $\{y \in \Sigma^* \mid \exists x \in \Sigma^*$ such that $xy \in L\}$
(d) Prefix (L) = $\{x \in \Sigma^* \mid \exists y \in \Sigma^*$ such that $xy \in L\}$

[2019 : 1 Mark]

66. For $\Sigma = \{a, b\}$, let us consider the regular language $L = \{x \mid x = a^{2+3k}$ or $x = b^{10+12k}, k \geq 0\}$. Which one of the following can be a pumping length (the constant guaranted by the pumping lemma) for L?

(a) 9 (b) 5
(c) 3 (d) 24

[2019 : 1 Mark]

67. Consider the following statements.

I. If $L_1 \cup L_2$ is regular, then both L_1 and L_2 must be regular.
II. The class of regular languages is closed under infinite union.

Which of the above statements is/are TRUE?

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

[2020 : 1 Mark]

68. Consider the following two statements about regular languages :

S1 : Every infinite regular language contains an undecidable language as a subset.

S2 : Every finite language is regular.

Which one of the following choices is correct?

(a) Only S_1 is true.
(b) Only S_2 is true.
(c) Both S_1 and S_2 are true.
(d) Neither S_1 nor S_2 is true.

[2021 (Set-2) : 2 Marks]

69. Consider the following languages:

$L_1 = \{a^n w a^n \mid w \in \{a, b\}^*\}$

$L_2 \{wxw^R \mid w, x \in \{a, b\}^*, |w|, |x| > 0\}$

Note that w^R is the reversal of the string w. Which of the following is/are TRUE?

(a) L_1 and L_2 are regular.
(b) L_1 and L_2 are context-free.
(c) L_1 is regular and L_2 is context-free.
(d) L_1 and L_2 are context-free but not regular.

[2022 : 2 Marks]

ANSWERS

1. (a,c)	2. (b)	3. (c)	4. (d)	5. (d)	6. (d)	7. (c)	8. (*)	9. (b)	10. (a)
11. (c)	12. (b)	13. (a)	14. (b)	15. (c)	16. (c)	17. (c)	18. (c)	19. (b)	20. (b)
21. (b)	22. (a)	23. (d)	24. (a, b, c)	25. (d)	26. (*)	27. (b)	28. (a)	29. (b)	30. (b)
31. (a)	32. (b)	33. (b)	34. (d)	35. (c)	36. (b)	37. (a)	38. (a)	39. (a)	40. (a)
41. (b)	42. (*)	43. (b)	44. (d)	45. (c)	46. (c)	47. (a)	48. (c)	49. (d)	50. (a)
51. (a)	52. (c)	53. (a)	54. (a)	55. (a)	56. (b)	57. (2)	58. (b)	59. (b)	60. (d)
61. (d)	62. (b)	63. (c)	64. (d)	65. (b)	66. (d)	67. (d)	68. (c)	69. (a,b,c) 7	

EXPLANATIONS

1. Here $r = (1 + 10)^*$

$\Rightarrow$ (set of All strings of 0 & 1)

$$S = (11^* 0) \Rightarrow 1^* 0$$

$$t = 1^* 0 \Rightarrow 1^* 0$$

Set r are include all strings of s and t and 't' is superset of "s".

So $L(s) \leq L(r)$

$L(s) \leq L(t)$

2. According to regular expression rule

$$(r^* s^*) = (r + s)^*$$

3. From options given, we see that $(00)^* (\epsilon + 0)$ and 0^* is equivalent.

4. $L \leq \Sigma^*$ where $\Sigma = \{a, b\}$

It is function of regular function,

$$\Sigma^* = (a, b)^*$$

or we write $a^m b^n$, $m > 1$, $n > 1$

5. Option 'a' and 'b' generate 100 string as substring directly

Option 'c' and 'd' both are not generate 100 substring directly

But option c does not accept λ(null).

But null is a string which does not contain 100 as a substring.

So option 'd' is correct.

6. $A = (01 + 1)^*$

$B = ((01^*)1^*)^*$

So Both are equal according to identity rules.

$$(R1 + R2)^* = (R1^* R2^*)^*$$

Option (d) is correct.

7. $(a + b^*)$ and $(a + b)^*$ Both are equal according to identity rules.

$$(R_1 + R_2)^* = (R_1 + R_2^*)^* = (R_1^* + R_2)^*$$

8. In option C $\Rightarrow ((ab)^* + c^*)^*$ we can not generate string a and b separately. But in given expression $(a + b + c)^*$ we can generate string a and b separately.

9. The Regular expression represents infinite set of finite strings.

(string is always finite in length).

10.

option 'b' and 'd' generate λ(null) string which is not accepted by given automata option 'C' generate minimum length string "ab" which is not accepted given Automata.

So option "a" is correct which generate all strings accepted by given automata.

11. Every subset of a Regular language is not always regular

Example $L = a^* b^*$ is a Regular.

But $L1 = a^n b^n$ is not Regular but $\boxed{L_1 \leq L}$ Not Regular.

12. Prefix; suffix and half of any regular language is also regular, according to properties of regular.

But repeat (L) is not regular language.

13. $L = (a + b)^*$ then Repeat of L is

Repeat (L) = $\{ww \mid w \in \{a, b\}^*\}$ is

Clearly not Regular.

14. Every finite subset of a non – regular set is regular.

66. $L = \{a^{2+3k} \text{ or } b^{10+12k}\}$ for $k \geq 0$

$= a^2(a^3)^* \text{ or } b^{10}(b^{12})^*$

$= \{a^2, a^5, a^7, \ldots, b^{10}, b^{22}, b^{34} \ldots\}$

The smallest string which can be pumped is a^5

$$a^5 = \underset{x}{a^2}\underset{y}{(a^3)}\underset{z}{\epsilon}$$

So minimum pumping length = 5

67. If $L_1 \cup L_2$ is Regular, then L_1 and L_2 are also regular languages: false, since $\{a^n b^n\} \cup \{a^n b^n\} = \Sigma^*$ which is regular, but $\{a^n b^n\}$ not regular.

Infinite union of regular languages are also regular: false, since may infinite language can be broken into invidual strings which will be infinite union of finite languages. So if regular is closed under infinite union, then all infinite language will become regular, which is untrue. Hence, II is also false.

68. S1 is TRUE, With the particular focus on regular languages, the intended solution might have been something like using the pumping lemma to find x,y,z such that xy^nz is in your language for every n, and then consider the subset $\{xy^nz \mid n \in A\}$, where A is your favorite undecidable set of natural numbers.

S2 is also TRUE, Any finite language is regular.

Hence, option (c) is the correct answer.

69. As given languages;

$L_1 = \{a^n wa^n \mid w \in \{a, b\}^*\}$

$L_2 = \{wxw^R \mid w, x \in \{a, b\}^*, |w|, |x| > 0$

- L_1 is regular because if we putting n = 0, we create a subset $\{w \mid w \in \{a, b\}^*\}$ which is the set of all possible strings (which is regular, so, it is context free as well). So if subset of L_1 $(a + b)^*$, then $L_1 = (a + b)^*$.
- L_2 is regular because by putting w as "a" and "b" we get a regular expression $a(a + b)^*a + b(a+ b)^*b$, which covers all other string which can be obtained by putting w as "aa", "ab", "ba", "bb", etc

So, L_2 : $a(a + b)^*a + b(a + b)^*b$, is regular expression for L_2 (L_2 is regular, so it is context free as well).

Therefore, L_1 and L_2 both are regular and context-free.

Hence, option (a), (b), (c) are all true and option (d) is false

3 CHAPTER

Context Free Language and Push Down Automata

CFG

1. The language $\{0^n 1^n 2^n \mid 1 \leq n \leq 10^6\}$ is
   (a) Regular
   (b) Context-free but not regular.
   (c) Context-free but its complement is not context-free.
   (d) Not context-free. **[IT-2005 : 1 Mark]**

2. A language L satisfies the Pumping Lemma for regular languages, and also the Pumping Lemma for contex-free language. Which of the following statements about L is TRUE?
   (a) L is necessarily a regular language.
   (b) L is necessarily a context-free language, but not necessarily a regular language.
   (c) L is necessarily a non-regular language.
   (d) None of the above. **[IT-2005 : 2 Marks]**

3. Consider the context-free grammars over the alphabet {a, b, c,} given below, S and T are non-terminals.
   $G_1 : S \rightarrow aSb \mid T,\ T \rightarrow cT \mid \epsilon$
   $G_2 : S \rightarrow bSa \mid T,\ T \rightarrow cT \mid \epsilon$
   The language $L(G_2) \cap L(G_2)$ is
   (a) Finite
   (b) Not finite but regualr
   (c) Context-free but not regular
   (d) Recursive but not context-free
   [2017 (Set-1) : 1 Mark]

4. A context-free grammar is ambiguous if
   (a) the grammar contains useless non-terminals
   (b) it produces more than one parse tree for some sentence
   (c) some production has two non-terminals side by side on the right-hand side
   (d) None of the above **[1987 : 2 Marks]**

5. If G is a context-free grammar and w is a string of length n in L(G), how long is a derivation of w in G, if G is Chomsky normal form ?
   (a) 2n (b) 2n + 1
   (c) 2n – 1 (d) n **[1992 : 2 Marks]**

6. Which of the following features cannot be captured by context-free grammar ?
   (a) Syntax of if-then-else statements
   (b) Syntax of recursive procedures
   (c) Whether a variable has been declared before its use
   (d) Variable names of arbitrary length.
   [1994 : 2 Marks]

7. Consider the grammar with the following productions.
   $S \rightarrow a\alpha b \mid b\alpha c \mid ab$
   $S \rightarrow \alpha S \mid b$
   $S \rightarrow \alpha bb \mid ab$
   $S\alpha \rightarrow bdb \mid bd$
   the above grammar is
   (a) context free grammar
   (b) regular grammar
   (c) context sensitive grammar
   (d) LR(k) **[1995 : 1 Mark]**

8. Let G be a context free grammar with productions given below.
   $S \rightarrow ABAC$
   $A \rightarrow aA \mid \varepsilon$
   $B \rightarrow bB \mid \varepsilon$
   $C \rightarrow d$
   Transform the grammar G to an equivalent form G' that has no ε productions and no unit productions. **[1996 : 2 Marks]**

9. If L_1 is a context free language and L_2 is a regular language which of the following is are false ?
   (a) $L_1 - L_2$ is not context free
   (b) $L_1 \cap L_2$ is not context free
   (c) $\sim L_1$ is context free
   (d) $\sim L_2$ is regular **[1999 : 2 Marks]**

10. Let G = ({S}, {a, b}, R, S) ne a context free grammar where the rule set R is
    $S \rightarrow aSb \mid SS \mid \varepsilon$
    Which of the following statements is true ?
    (a) G is not ambiguous
    (b) There exist x, y $\in$ L(G) such that xy $\notin$ L(G)
    (c) There is a deterministic pushdown automation that accepts L(G)
    (d) We can find a determintic finite state automation that accepts L(G) **[2003 : 2 Marks]**

11. Consider the following grammar C
    $S \rightarrow bS \mid aA \mid b$
    $A \rightarrow bA \mid aB$
    $B \rightarrow bB \mid aS \mid a$
    Let $N_a(w)$ and $N_b(w)$ denote the number of a's and b's in a string w respectively. The language L(G) $\subseteq \{a, b\}^*$ generated by G is
    (a) $\{w \mid N_a(w) > 3N_b(w)\}$
    (b) $\{w \mid N_b(w) > 3N_a(w)\}$
    (c) $\{w \mid N_a(w) = 3k,\ k \in \{0, 1, 2, \ldots\}\}$
    (d) $\{w \mid N_b(w) = 3k,\ k \in \{0, 1, 2, \ldots\}\}$
    [2004 : 2 Marks]

12. In the context-free grammar below, S is the start symbol, a and b are terminals, and denotes the empty string

S aSa | bSb | a | b|

Which of the following strings is NOT generated by the grammar ?

(a) aaaa (b) baba
(c) abba (d) babaaabab

[IT-2006 : 1 Mark]

13. In the context free grammar below, S is the start symbol, a and b are terminals, and denotes the empty string.

$S \to aSAb \mid \varepsilon$

$A \to bA \mid \varepsilon$

The grammar generates the language

(a) $((a + b)^* b)^*$ (b) $\{a^m b^n \mid m \le n\}$
(c) $\{a^m b^n \mid m = n\}$ (d) $a^* b^*$

[IT-2006 : 2 Marks]

14. Consider the following statements about the context-free grammar,

$G = \{S \to SS, S \to ab, S \to ba, S \to \varepsilon\}$

1. G is ambiguous.
2. G produces all strings with equal number of a's and b's.
3. G can be accepted by a deterministic PDA.

Which combination below expresses all the true statements about G?

(a) 1 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

[2006 : 2 Marks]

15. Consider an ambiguous grammar G and its disambiguated version D. Let the language recognized by the two grammars be denoted by L(G) and L(D) respectively. Which one of the following is true ?

(a) $L(D) \subset L(G)$ (b) $L(D) \supset L(G)$
(c) $L(D) = L(G)$ (d) L(D) is empty

[2007 : 1 Mark]

16. Consider the following gramamrs. Names representing terminals have been specified in capital letters.

G1 : stmnt → WHILE (expr) stmnt
stmnt → OTHER
expr → ID

G2 : stmnt → WHILE (expr) stmnt
stmnt → OTHER
expr → expr + expr
expr → expr * expr
expr → ID

Which one of the following statements is true?

(a) G1 is context-free but not regular and G2 is regular
(b) G2 is context-free but not regular and G1 is regular
(c) Both G1 and G2 are regular
(d) Both G1 and G2 are context-free but neither of them is regular **[IT-2007 : 2 Marks]**

17. Which of the following statements are true ?

1. Every left-recursive grammar can be converted to a right-recursive grammar and vice-versa
2. All ε-productions can be removed from any context-free grammar by suitable tranformations
3. The language generated by a context-free grammar all of whose productions are of the form X → w or X → wY (where, w is a string of terminals and Y is a non-terminal), is always regular
4. The derivation trees of strings generated by a context-free grammar in Chomsky Normal Form are always binary trees

(a) 1, 2, 3 and 4 (b) 2, 3 and 4 only
(c) 1, 3 and 4 only (d) 1, 2 and 4 only

[2008 : 2 Marks]

18. Consider a CFG with the following productions.

S → AA|B

A → 0A | A0 | 1

B → 0B00|1

S is the start symbol, A and B are non-terminals and 0 and 1 are the terminals. The language generated by this grammar is

(a) $\{0^n 10^{2n} \mid n \ge 1\}$
(b) $\{0^i 10^j 10^k \mid i, j, k \ge 0\} \cup \{0^n 10^{2n} \mid n \ge 0\}$
(c) $\{0^i 10^j \mid i, j \ge 0\} \cup \{0^n 10^{2n} \mid n \ge 1\}$
(d) The set of all strings over {0, 1} containing at least two 0's

[IT-2008 : 2 Marks]

Directions for Question 19 and 20 :

A CFG G is given with the following productions where S is the start symbol, A is a non-terminal and a and b are terminals.

S → aS | A

A → aAb | bAa | ε

19. Which of the following strings is generated by the grammar above?

(a) aabbaba (b) aabaaba
(c) abababb (d) aabbaab

[IT-2008 : 2 Marks]

20. For the correct answer in above Question, how many steps are required to derive the string and how many parse trees are there?

(a) 6 and 1 (b) 6 and 2
(c) 7 and 2 (d) 4 and 2

[IT-2008 : 2 Marks]

21. Consider the following context-free grammars :

$G_1 : S \to aS \mid B, B \to b \mid bB$

$G_2 : S \to aA \mid bB, A \to aA \mid B \mid \varepsilon, B \to bB \mid \varepsilon$

Which of the following pairs of languages is generated by G1 and G2, respectively?

(a) $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0$
(b) $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n \ge 0$
(c) $\{a^m b^n \mid m \ge 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0$
(d) $\{a^m b^n \mid m \ge 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0$

[2016 (Set-3) : 2 Marks]

22. Language L_1 is defined by the grammar :

$S_1 \to aS_1 b \mid \varepsilon$

Language L_2 is defined by the grammar :

$S_2 \to abS_2/\epsilon$.

Consider the following statements :

P : L_1 is regular

Q : L_2 is regular

Which one of the following is TRUE ?

(a) Both P and Q are true
(b) P is true and Q is false
(c) P is false and Q is true
(d) Both P and Q are false

[2016 (Set-2) : 2 Marks]

23. Which one of the following grammars is free from left recursion ?

S → AB S → Ab | Bb | c

(a) A → Aa | b, B → c, S → Aa | B
(b) A → Bd | ε, B → e, S → Aa | Bb | c
(c) A → Bb | Sc | ε, B → d
(d) A → Bd | ε, B → Ae | ε

[2016 (Set-2) : 2 Marks]

24. Consider the following context-free grammar over the alphabet = {a, b, c} with S as the start symbol:

S → abScT | abcT

T → bT | b

Which one of the following represents the language generated by the above grammar?

(a) $\{(ab)^n (cb)^n \mid n \ge 1\}$
(b) $\{(ab)^n cb^{m_1} cb^{m_2} ... cb^{m_n} \mid n, m_1, m_2, ..., m_n \ge 1\}$
(c) $\{(ab)^n (cb^m)^n \mid m, n \ge 1\}$
(d) $\{(ab)^n (cb^n)^m \mid m, n \ge 1\}$ **[2017 (Set-1) : 1 Mark]**

25. Identify the language generated by the following grammar, where S is the start variable.

S → XY

X → aX | a

Y → aYb | ∈

(a) $\{a^m b^n \mid m \ge n, n > 0\}$ (b) $\{a^m b^n \mid m \ge n, n \ge 0\}$
(c) $\{a^m b^n \mid m > n, n \ge 0\}$ (d) $\{a^m b^n \mid m > n, n > 0\}$

[2017 (Set-1) : 1 Mark]

CFL

26. Let P be a regular language and Q be a context-free language such that $Q \subseteq P$. (For example, let P be the language represented by the regular expression $p^* q^*$ and Q be $\{p^n q^n \mid n \in N\}$. Then which of the following is ALWAYS regular?

(a) $P \cap Q$ (b) P - Q
(c) $\Sigma^* - P$ (d) $\Sigma^* - Q$

[2011 : 1 Mark]

27. Context tree languages and regular languages both are closed under the operation (s) of

(a) union (b) interecection
(c) concatenation (d) complement

[1989 : 2 Marks]

28. **Context free languages are**

(a) closed under union
(b) closed under complementation
(c) closed under intersection
(d) closed under Kleene closure

[1992 : 2 Marks]

29. If L1 and L2 are context free languages and R a regular set, one of the languages below is not necessarily a context free languages. Which one?

(a) $L_1 L_2$ (b) $L_1 \cap L_2$
(c) $L_1 \cap R$ (d) $L_1 \cup L_2$

[1996 : 2 Marks]

30. Context free languages are closed under

(a) union, intersection
(b) union, kleene closure
(c) intersection, complement
(d) complement, kleene clousre **[1999 : 1 Mark]**

31. Which of the following statement is true ?

(a) if a language is context free is can always be accepted by a deterministic push-down automation
(b) the union of two context free languages is context free
(c) the intersection of two context free languages is context free
(d) the complement of a context free languages is context free **[2001 : 1 Mark]**

32. Which of the following statements is FALSE?
(a) There exist context free languages such that all the context free grammars generating them are ambiguous
(b) An unambiguous context-free grammar always has a unique parse tree for each string of the language generated by it
(c) Both deterministic and non-deterministic pushdown automata always accept the same set of languages
(d) A finite set of strings from some alphabet is always a regular language **[IT-2004 : 1 Mark]**

33. The language $\{a^m b^n c^{m+n} \mid m, n \geq 1\}$ is
(a) regular
(b) context-free but not regular
(c) context sensitive but not context free
(d) type-0 but not context sensitive
[2004 : 2 Marks]

34. Consider the languages
$L_1 = \{a^n b^n c^m \mid n, m > 0\}$ and $L_2 = \{a^n b^m c^m \mid n, m > 0\}$
Which one of the following statements is FALSE?
(a) $L_1 \cap L_2$ is a context-free language
(b) $L_1 \cup L_2$ is a context-free language
(c) L_1 and L_2 are context-free language
(d) $L_1 \cap L_2$ is a context sensitive language
[2005 : 2 Marks]

35. Consider the languages
$L_1 = \{ww^R \mid w \in \{0, 1\}^*\}$
$L_2 = \{w \# w^R \mid w \in \{0, 1)^*\}$, where # is a special symbol
$L_3 = \{ww \mid w \in \{0, 1\}^*\}$
Which one of the following is TRUE ?
(a) L_1 is a deterministic CFL
(b) L_2 is a deterministic CFL
(c) L_3 is a CFL, but not a determistic CFL
(d) L_3 is a deterministic CFL **[2005 : 2 Marks]**

36. Let $L_1 = \{0^{n+m} 1^n 0^m \mid n, m \geq 0\}$,
$L_2 = \{0^{n+m} 1^{n+m} 0^m \mid n, m \geq 0\}$, and
$L_3 = \{0^{n+m} 1^{n+m} 0^{n+m} \mid n, m \geq 0\}$.
Which of these languages are NOT context free ?
(a) L_1 only (b) L_3 only
(c) L_1 and L_2 (d) L_2 and L_3
[2006 : 1 Mark]

37. Let L be a context-free language and M a regular language. Then the language $L \cap M$ is
(a) always regular
(b) never regular
(c) always a deteministic context-free language
(d) always a context-free language
[IT-2006 : 2 Marks]

38. The language $L = \{0^i 2 1^i \mid i \geq 0\}$ over the alphabet $\{0, 1, 2)$ is
(a) not recursive
(b) is recursive and is a deterministic CFL
(c) is a regular language
(d) is not a deterministic CFL but a CFL
[IT-2007 : 2 Marks]

39. Let $L = L_1 \cap L_2$, where L_1 and L_2 are language as defined below :
$L_1 = \{a^m b^m c a^n b^n \mid m, n \geq 0\}$
$L_2 = \{a^i b^j c^k \mid i, j, k \geq 0\}$
Then L is
(a) not recursive
(b) regular
(c) context-free but not regular
(d) recursively enumerable but not context-free
[2009 : 2 Marks]

40. Consider the language $L1 = \{0^i 1^j \mid i = j\}$, $L2 = \{0^i 1^j \mid i = j\}$, $L3 = \{0^i 1^j \mid i = 2j + 1\}$, $L4 = \{0^i 1^j \mid i \neq 2j\}$.
Which one of the following statements is true?
(a) Only L2 is context free
(b) Only L2 and L3 are context free
(c) Only L1 and L2 are context free
(d) All are context free **[2010 : 2 Marks]**

41. Which of the following languages are context-free?
$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$
$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$
$L_3 = \{a^m b^n \mid m = 2n + 1\}$
(a) L_1 and L_2 only (b) L_1 and L_3 only
(c) L_2 and L_3 only (d) L_3 only
[2015 (Set-3) : 2 Marks]

42. Consider the transition diagram of a PDA given below with input alphabet = {a, b} and stack alphabet l = {X, Z}. Z is the initial stack symbol. Let L denote the language accepted by the PDA.

Which one of the following is TRUE ?
(a) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata
(b) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is not accepted by any deterministic PDA
(c) L is not accepted by any Turning machine that halts on every input
(d) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is deterministic context-free
[2016 (Set-1) : 2 Marks]

43. Consider the following language are the alphabet = {a,b,c}
Let $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ and $L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$
Which of the following are context free languages?
I. $L_1 \cup L_2$ II. $L_1 \cap L_2$
(a) I only (b) II only
(c) Both (d) None of these.
[2017 (Set-1) : 2 Marks]

44. Let L1, L2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?
I. $L_1 \cup L_2$ is context-free. II. L_1 is context-free.
III. $L_1 - R$ is context-free. IV. $L_1 \cap L_2$ is context-free.
(a) I, II and IV only (b) I and III only
(c) II and IV only (d) I only
[2017 (Set-2) : 1 Mark]

45. Consider the following languages :
$L_1 = \{a^p \mid p$ is a prime number$\}$
$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$
$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$
$L_4 = \{a^n b^n \mid n \geq 1\}$
Which of the following are CORRECT?
I. L_1 is context-free but not regular.
II. L_2 is not context-free.
III. L_3 is not context-free but recursive.
IV. L_4 is deterministic context-free.
(a) I, II and IV only (b) II and III only
(c) I and IV only (d) III and IV only
[2017 (Set-2) : 2 Marks]

46. Consider the following languages :
I. $\{a^m b^n c^p d^q \mid m + p = n + q$, where m, n, p, $q \geq 0\}$
II. $\{a^m b^n c^p d^q \mid m = n$ and $p = q$, where m, n, p, $q \geq 0\}$
III. $\{a^m b^n c^p d^q \mid m = n = p$ and $p = q$, where m, n, p, $q \geq 0\}$
IV. $\{a^m b^n c^p d^q \mid mn = p + q$, where m, n, p, $q \geq 0\}$
Which of the language above are context-free?
(a) I and IV only (b) I and II only
(c) II and III only (d) II and IV only
[2018 : 2 Marks]

47. Let L be a language and $\overline{L}$ be its complement. Which one of the following is NOT a viable possibility?
(a) Neither L nor is recursively enumerable (r.e.).
(b) One of L and $\overline{L}$ is r.e. but not recursive, the other is not r.e.
(c) Both L and $\overline{L}$ are r.e. but not recursive.
(d) Both L and $\overline{L}$ are recursive.
[2014 (Set-1) : 2 Marks]

48. Which one of the following languages over $\Sigma = \{a, b\}$ is NOT context-free?
(a) $\{wa^n w^R b^n \mid w \in \{a,b\}^*, n \geq 0\}$
(b) $\{a^n b^i \mid i \in \{n, 3n, 5n\}, n \geq 0\}$
(c) $\{ww^R \mid w \in \{a, b\}^*\}$
(d) $\{wa^n b^n w^R \mid w \in \{a, b\}^*, n \geq 0\}$ **[2019 : 2 Marks]**

49. Consider the language $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and the following statements.
I. L is deterministic context-free.
II. L is context-free but not deterministic context-free.
III. L is not LL(k) for any k.
Which of the above statements is/are TRUE?
(a) I only (b) II only
(c) I and III only (d) III only
[2020 : 1 Marks]

50. Consider the following languages.
$L_1 = \{w\,x\,yx \mid w, x, y \in (0 + 1)^+\}$
$L_2 = \{xy \mid x, y \in (a + b)^*, |x| = |y|, x \neq y\}$
Which one of the following is TRUE?
(a) L_1 is regular and L_2 is context-free.
(b) L_1 is context-free but not regular and L_2 is context-free.
(c) Neither L_1 nor L_2 is context-free.
(d) L_1 is context-free but L_2 is not context-free.
[2020 : 2 Marks]

51. Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free ?
(a) $L_1 \cap L_2$ (b) $L_1 . L_2$
(c) $L_1 - L_2$ (d) $L_1 \cup L_2$
[2021 (Set-1): 1 Mark]

52. Let L1 be a regular language and L2 be a context-free language. Which of the following languages is/are context-free ?
(a) $L_1 \cap \overline{L_2}$
(b) $\overline{\overline{L_1} \cup \overline{L_2}}$
(c) $L_1 \cup (L_2 \cup \overline{L_2})$
(d) $(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2)$
[2021 (Set-2) : 1 Mark]

53. For a string w, we define w^R to be the reverse of w. For example, if w = 01101 then w^R = 10110.

Which of the following languages is/are context-free ?

(a) $\{wxw^Rx^R \mid w, x \in \{0, 1\}^*\}$

(b) $\{ww^Rxx^R \mid w, x \in \{0, 1\}^*\}$

(c) $\{wxw^R \mid w, x \in \{0, 1\}^*\}$

(d) $\{wxx^Rw^R \mid w, x \in \{0, 1\}^*\}$

[2021 (Set-2) : 2 Marks]

54. Consider the following languages:

$L_1 = \{ww \mid w \in \{a, b\}^*\}$

$L_2 = \{a^nb^nc^m \mid m, n \geq 0\}$

$L_3 = \{a^mb^nc^n \mid m, n \geq 0\}$

Which of the following statements is/are FALSE?

(a) L_1 is not context-free but L_2 and L_3 are deterministic context-free.

(b) Neither L_1 nor L_2 is context-free.

(c) L_2, L_3 and $L_2 \cap L_3$ all are context-free

(d) Neither L_1 nor its complement is context-free.

[2022 : 2 Marks]

55. Consider the context-free grammar G below:

$S \rightarrow aSb \mid X$

$X \rightarrow aX \mid Xb \mid a \mid b$

where S and X are non-terminals, and a and b are terminal symbols. The starting non-terminal is S.

Which one of the following statements is CORRECT?

(a) The language generated by G is (a + b)*

(b) The language generated by G is a*(a + b)b*

(c) The language generated by G is a*b*(a + b)

(d) The language generated by G is not a regular language

[2023 : 1 Mark]

PDA

56. Which of the following language over {a, b, c} is accepted by a deterministic push down automata?

(a) $\{wcw^R \mid w \in \{a, b\}^*\}$

(b) $\{ww^R \mid w \in \{a, b, c\}^*\}$

(c) $\{a^n b^n c^n \mid n \geq 0\}$

(d) {W | W is a palindrome over {a, b, c}}

[1997 : 2 Marks]

57. Let L_D be the set of all language accepted by a PDA by final state and L_E the set of all languages accepted by empty stack. Which of the following is true ?

(a) $L_D = L_E$ (b) $L_D \supset L_E$

(c) $L_D \subset L_E$ (d) None of the above

[1999 : 1 Mark]

58. The language accepted by a Pushdown Automation in which the stack is limited to 10 items is best described as

(a) context free

(b) regular

(c) deterministic context free

(d) recursive

[2002 : 1 Mark]

59. Let $M = (K, \Sigma, \Gamma, \Delta, s, F)$ be a pushdown automaton, where

$K = (s, f)$, $F = \{f\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a\}$ and

$\Delta = \{((s, a, \varepsilon), (s, a)), ((s, b, \varepsilon), (s, a)), ((s, a, \varepsilon), (f, \varepsilon)), ((f, a, a), (f, \varepsilon)), ((f, b, a), (f, \varepsilon))\}$

Which one of the following strings is not a member of L(M)?

(a) aaa (b) aabab

(c) baaba (d) bab

[IT-2004 : 2 Marks]

60. Consider the pushdown automaton (PDA) below which runs over the input alphabet (a, b, c). It has the stack alphabet $\{Z_0, X\}$ where Z_0 is the bottom of stack marker. The set of states of the PDA is {s, t, u, f} where is the start state and f is the final state. The PDA accepts by final state. The transitions of the PDA given below are depicted in a standard manner. For example, the transition $(s, b, X) \rightarrow (t, XZ_0)$ means that if the PDA is in state s and the symbol on the top of the stack is X, then it can read b from the input and move to state t after popping the top of stack and pushing the symbols Z_0 and X_0 and X (in that order) on the stack.

$(s, a, Z_0) \rightarrow (s, XYZ_0)$

$(s, \varepsilon, Z_0) \rightarrow (f, \varepsilon)$

$(s, a, X) \rightarrow (s, XXX)$

$(s, b, X) \rightarrow (t, \varepsilon)$

$(t, b, X) \rightarrow (t, \varepsilon)$

$(t, c, X) \rightarrow (f, \varepsilon)$

$(u, c, X) \rightarrow (t, \varepsilon)$

$(u, \varepsilon, Z_0) \rightarrow (f, \varepsilon)$

The language accepted by the PDA is

(a) $\{a^l b^m c^n \mid l = m = n\}$

(b) $\{a^l b^m c^n \mid l = m\}$

(c) $\{a^l b^m c^n \mid 2l = m + n\}$

(d) $\{a^l b^m c^n \mid m = n\}$

[IT-2006 : 2 Marks]

61. Which one of the following is FALSE ?

(a) There is a unique minimal DFA for every regular language

(b) Every NFA can be converted to an equivalent PDA

(c) Complement of every context-free language is recursive

(d) Every nondeterministic PDA can be converted to an equivalent deterministic PDA

[2009 : 1 Mark]

62. Consider the language L1, L2 and L3 are given below :

$L1 = \{0^p1^q \mid p, q \in N\}$,

$L2 = \{0^p1^q \mid p, q \in N \text{ and } p = q\}$ and

$L3 = \{0^p1^q0^r \mid p, q, r \in N \text{ and } p = q = r\}$

Which of the following statements is NOT TRUE?

(a) Push Down Automata (PDA) can be used to recognize L1 and L2

(b) L1 is a regular language

(c) All the three languages are context free

(d) Turing machines can be used to recognize all the languages

[2011 : 2 Marks]

63. Consider the following languages

$L1 = \{0^p1^q0^r \mid p, q, r \geq 0\}$

$L2 = \{0^p1^q0^r \mid p, q, r \geq 0, p = r\}$

Which one of the following statements is FALSE?

(a) L_2 is context free

(b) $L_1 \cap L_2$ is context free

(c) Complement of L_2 is recursive

(d) Complement of L_1 is context-free but not regular

[2013 : 2 Marks]

64. Consider the following languages over the alphabet = {0, 1, c} :

$L_1 = \{0^n1^n \mid n 0\}$

$L_2 = \{wcw^r \mid w \in \{0, 1\}^*\}$

$L_3 = \{ww^r \mid w \in \{0, 1\}^*\}$

Here, w^r is the reverse of the string w. Which of these languages are determinstic Context-free languages?

(a) None of the languages

(b) Only L_1

(c) Only L_1 and L_2

(d) All the three languages

[2014 (Set-3) : 2 Marks]

65. In a pushdown automaton $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$, a transition of the form,

where $p, q \in Q$, $a \in \Sigma \cup \{\epsilon\}$, and $X, Y \in \Gamma \cup \{\epsilon\}$, represents $(q, Y) \in \delta(p, a, X)$.

Consider the following pushdown automaton over the input alphabet Σ = {a, b} and stack alphabet Γ = {#, A}.

The number of strings of length 100 accepted by the above pushdown automaton is _____ .

[2021 (Set-1) : 2 Marks]

66. Consider the pushdown automaton (PDA) P below, which runs on the input alphabet { a, b}, has stack alphabet {⊥, A}, and has three states {s, p, q}, with s being the start state.

A transition from state u to state v, labelled c |X| γ, where c is an input symbol or ϵ, X is a stack symbol, and γ is a string of stack symbols, represents the fact that in state u, the PDA can read c from the input, with X on the top of its stack, pop X from the stack, push in the string γ on the stack, and go to state v. In the initial configuration, the stack has only the symbol ⊥ in it. The PDA accepts by empty stack.

Which one of the following options correctly describes the language accepted by P?

(a) $\{a^m b^n \mid 1 \le m \text{ and } n < m\}$

(b) $\{a^m b^n \mid 0 \le n \le m\}$

(c) $\{a^m b^n \mid 0 \le m \text{ and } 0 \le n\}$

(d) $\{a^m \mid 0 \le m\} \cup \{b^n \mid 0 \le n\}$

[2023 : 1 Mark]

NPDA

67. Let P be a non-deterministic push-down automaton (NPDA) with exactly one state, q, and exactly one symbol, Z, in its stack alphabet. State q is both the starting as well as the accepting state of the PDA. The stack is intialized with one Z before the start of the operation of the PDA. Let the input alphabet of the PDA be Σ.

Let L(P) be the language accepted by the PDA by reading a string and reaching its accepting state. Let N(P) be the language accepted by the PDA by reading a string and emptying its stack. Which of the following statements is TRUE ?

(a) L(P) is necessarily Σ^* but N(P) is not necessarily Σ^*.

(b) N(P) is necessarily Σ^* but N(P) is not necessarily Σ^*.

(c) Both L(P) and N(P) is necessarily Σ^*.

(d) Neither L(P) nor N(P) are necessarily Σ^*.

[IT-2005 : 1 Mark]

68. Let N_f and N_p denote the classes of languages accepted by non-deterministic finite automata and non-deterministic push-down automata, respectively. Let D_f and D_p denote the classes of languages accepted by deterministic finite automata and deterministic push-down automata, respectively. Which one of the following is TRUE?

(a) $D_f \subset N_f$ and $D_p \subset N_p$

(b) $D_f \subset N_f$ and $D_p = N_p$

(c) $D_f = N_f$ and $D_p = N_p$

(d) $D_f = N_f$ and $D_p \subset N_p$

[2005 : 2 Marks]

69. Consider the NPDA $\langle Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}$ $\Gamma = \{0, 1, \perp\}$ $\delta, q_0, \perp, F = \{q_2\}$, where (as per usual convention) Q is the set of states, Σ is the input alphabet, is stack alphabet, δ is the state transition function, q_0 is the initial state, $\perp$ is the initial stack symbol, and F is the set of accepting states. The state transition is as follows :

$0/1/\varepsilon, Z \to Z\,\varepsilon\perp\to\varepsilon$

Which one of the following sequences must follow the string 101100 so that the overall string is accepted by the automation?

(a) 10110 (b) 10010

(c) 01010 (d) 01001

[2015 (Set-1) : 2 Marks]

70. In which of the cases stated below is the following statement true ?

"For every non-deterministic machine M_1 there exists an equivalent deterministic machine M_2 recognizing the same language".

(a) M_1 is non-deterministic finite automaton

(b) M_1 is a non-deterministic PDA

(c) M_1 is a non-deterministic Turing machine

(d) For no machine M1 use the above statement true **[1992 : 2 Marks]**

71. Which of the following conversions is not possible (algorithmically) ?

(a) Regular grammar to context free grammar

(b) Non-deterministic FSA to deterministic FSA

(c) Non-determinstic PDA to deterministic PDA

(d) Non-deterministic Turing machine to deterministic Turing machine

[1994 : 2 Marks]

72. Regarding the power of recognition of languages, which of the following statements is false?

(a) The non-deterministic finite-state automata are equivalent to deterministic finite-state automata

(b) Non-deterministic push-down automata are equivalent to deterministic push-down automata

(c) Non-deterministic Turning machines are equivalent to deterministic Turning machines

(d) Multi-tape Turning machines are equivalent to Single-tape Turning machines.

[1998 : 1 Mark]

DPDA

73. Which of the following languages is accepted by a non-deterministic push down automaton (PDA) but NOT by a deterministic PDA?

(a) $\{a^n b^n c^n \mid n \ge 0\}$

(b) $\{a^l b^m c^n \mid l \ne m \text{ or } m \ne n\}$

(c) $\{a^n b^n \mid n \ge 0\}$

(d) $\{a^m b^n \mid m, n \ge 0\}$

[IT-2006 : 2 Marks]

74. Consider the following languages.

$L_1 = \{a^i b^j c^k \mid i = j, k \ge 1\}$

$L_2 = \{a^i b^j \mid j = 2i, i \ge 0\}$

Which of the following is true ?

(a) L_1 is not a CFL but L_2 is

(b) $L_1 \cap L_2$ = and ϕ L_1 is non-regular

(c) $L_1 \cup L_2$ is not a CFL but L_2 is

(d) There is a 4 state PDA that accept L_1, but there is no DPDA that accepts L_2

[IT-2008 : 2 Marks]

ANSWERS

1. (a)	2. (d)	3. (b)	4. (b)	5. (c)	6. (c)	7. (c)	8. (*)	9. (a, c)	10. (c)
11. (c)	12. (b)	13. (b)	14. (b)	15. (c)	16. (d)	17. (c)	18. (*)	19. (d)	20. (a)
21. (d)	22. (c)	23. (b)	24. (b)	25. (c)	26. (c)	27. (a,c)	28. (a,c)	29. (b)	30. (b)
31. (b)	32. (c)	33. (b)	34. (a)	35. (b)	36. (d)	37. (d)	38. (b)	39. (c)	40. (d)
41. (b)	42. (d)	43. (a)	44. (b)	45. (d)	46. (b)	47. (c)	48. (a)	49. (c)	50. (a)
51. (c)	52. (b, c, d)	53. (b, c, d)	54. (b,c,d)	55. (b)	56. (a)	57. (a)	58. (b)	59. (c)	60. (c)
61. (d)	62. (c)	63. (d)	64. (c)	65. (50 to 50)		66. (a)	67. (d)	68. (d)	69. (b)
70. (a, c)	71. (c)	72. (b)	73. (b)	74. (b)					

4 CHAPTER

Turing Machine and Recursive Functions

UNDECIDABILITY

1. Which of the following decision problems are undecidable?
   I. Given NFAs N_1 & N_2 is $L(N_1) \cap L(N_2) = \Phi$?
   II. Given a CFG $G = (N, \Sigma, P, S)$ and a string $x \in \Sigma^*$, does $x \in L(G)$?
   III. Given CFGs G_1 and G_2, is $L(G_1) = L(G_2)$?
   IV. Given a TM M, is $L(M) = \Phi$?
   (a) I and IV only (b) II and III only
   (c) III and IV only (d) II and IV only
   [2016 (Set-1) : 1 Mark]
2. Which of the following problems are undecidable?
   (a) Membership problem in context-free languages
   (b) Whether a given context-free language is regular
   (c) Whether a finite state automation halts on all inputs
   (d) Membership problem for type 0 languages
   [1989 : 2 Marks]
3. It is undecidable whether
   (a) an arbitrary Turing machine halts within 10 steps
   (b) a Turing machine prints a specific letter
   (c) A Turing machine computes the product of two numbers
   (d) None of the above **[1990 : 2 Marks]**
4. Which of the following statements is false?
   (a) The halting problem for Turing machine is undecidable
   (b) Determining whether a context free grammar is ambiguous is undecidable
   (c) Given two arbitrary context free grammars G_1 and G_2, it is undecidable whether $L(G_1) = L(G_2)$
   (d) Given two regular grammars G_1 and G_2, it is undecidable whether $L(G_1) = L(G_2)$
   [1996 : 1 Mark]
5. Which one of the following is not decidable ?
   (a) Given a Turing machine M, a strings s and an integer k, M accepts a within k steps
   (b) Equivalence of two Turing machines
   (c) Language accepted by a given finite state machine is non empty
   (d) Language accepted by a context free grammar is non empty **[1997 : 2 Marks]**
6. Consider the following decision problems :
   P_1 : Does a given finite state machine accept a given string
   P_2 : Does a given context free grammar generate an infinite number of strings.
   Which of the following statement is true ?
   (a) P_1 and P_2 are decidable
   (b) Neither P_1 nor P_2 are decidable
   (c) Only P_1 is decidable
   (d) Only P_2 is decidable **[2002 : 2 Marks]**
7. Consider the following Problem X.
   "Given a Turing machine M over the input alphabet Σ, any state q of M and a word Σ^*, does the computation of M on w visit the state q"
   Which of the following statements about X is correct?
   (a) X is decidable
   (b) X is undecidable but partially decidabe
   (c) X is undecidable and not even partially decidable
   (d) X is not a decision problem **[2002 : 2 Marks]**
8. Consider three decision problems P_1, P_2 and P_3. It is known that P_1 is decidable and P_2 is undecidable. Which one of the following is TRUE?
   (a) P_3 is decidable if P_1 is reducible to P_3
   (b) P_3 is undecidable if P_3 is reducible to P_2
   (c) P_3 is undecidable if P_2 is reducible to P_3
   (d) P_3 is decidable if P_3 is reducible to P_2's complement
   [2005 : 2 Marks]
9. Which one of the following problems is undecidable?
   (a) Membership problem for CFGs
   (b) Ambiguity problem for CFGs
   (c) Finiteness problem for FSAs
   (d) Equivalence problem for FSAs
   [2007 : 1 Mark]
10. Which one of the following are decidable?
    1. Whether the intersection of two regular languages is infinite
    2. Whether a given context-free language is regular
    3. Whether two push-down automata accept the same language
    4. Whether a given grammar is context-free
    (a) 1 and 2 (b) 1 and 4
    (c) 2 and 3 (d) 2 and 4
    [2008 : 1 Mark]

11. Which of the following is/are undecidable?
1. G is CFG. Is $L(G) = \phi$?
2. G is a CFG. Is $L(G) = \Sigma^*$?
3. M is a Turing machine. Is L(M) regular?
4. A is a DFA and N is an NFA. Is L(A) = L(N)?

(a) 3 only (b) 3 and 4 only
(c) 1, 2 and 3 only (d) 2 and 3 only

[2013 : 2 Marks]

12. Which one of the following problems is undeciable?
(a) Deciding if a given context-free grammar is ambiguous.
(b) Deciding if a given string is generated by a given context-free grammar.
(c) Deciding if the language generated by a given context-free grammar is empty.
(d) Deciding if the language generated by a given context-free grammar is finite.

[2014 (Set-3) : 2 Marks]

13. Consider the following statements :
1. The complement of every Turning decidable language is Turning decidable.
2. There exists some language which is in NP but is not Turing decidable.
3. If L is a language in NP, L is Turing decidable.

Which of the above statements is/are True?
(a) Only 2 (b) Only 3
(c) Only 1 and 2 (d) Only 1 and 3

[2015 (Set-2) : 1 Mark]

14. Consider the following problems L(G) denotes the language generated by a grammar G. L(M) denotes the language accepted by a machine M.
I. For an unrestricted grammar and G and a string w, whether $\omega \in L(G)$.
II. Given a Turing Machine M, whether L(M) is regular.
III. Given two grammars G1 and G2, whether L(G1) = L(G2).
IV. Given an NFA N, whether there is a determinstic PDA P such that N and P accept the same language.

Which one of the following statements is correct?
(a) Only I and II are undecidable
(b) Only III is undecidable
(c) Only II and IV are undecidable
(d) Only I, II and III are undeciable.

[2018 (Set-2) : 1 Marks]

15. For a Turing machine M, ⟨M⟩ denotes an encoding of M. Consider the following two languages.

L_1 = {⟨M⟩ | M takes more than 2021 steps on all inputs}

L_2 = {⟨M⟩ | M takes more than 2021 steps on some input}

Which one of the following options is correct?
(a) Both L_1 and L_2 are decidable.
(b) L_1 is decidable and L_2 is undecidable.
(c) L_1 is undecidable and L_2 is decidable.
(d) Both L_1 and L_2 are undecidable.

[2021 (Set-1) : 2 Marks]

16. Which of the following is/are undecidable?
(a) Given two Turing machines M_1 and M_2, decide if $L(M_1) = L(M_2)$.
(b) Given a Turing machine M, decide if L(M) is regular.
(c) Given a Turing machine M, decide if M accepts all strings.
(d) Given a Turing machine M, decide if M takes more than 1073 steps on every string.

[2022 : 2 Marks]

RECURSIVE & RECURSIVE ENUMERABLE FUNCTION

17. Which of the following is true?
(a) The complement of a recursive language is recursive
(b) The complement of recursively enumerable language is recursively enumerable
(c) The complement of a recursively enumerable language is either recursive or recursively enumerable
(d) The complement of a context-free language is context-free **[2002 : 2 Marks]**

18. Nobody knows yet if P = NP. Consider the language L defined as follows

$$L = \begin{cases} (0+1)^* \text{ if } P = NP \\ \phi \text{ otherwise} \end{cases}$$

Which of the following statements is true?
(a) L is recursive
(b) L is recursively enumerable but not recursive
(c) L is not recursively enumerable
(d) Whether L is recursive or not will be known after we find out if P = NP **[2003 : 1 Mark]**

19. If the strings of a language L can be effectively enumerated in lexicographic (i.e., alphabetic) order, which of the following statements is true?
(a) L is necessarily finite
(b) L is regular but not necessarily finite
(c) L is context free but not necessarily regular
(d) L is recursive but not necessarily context free

[2003 : 1 Mark]

20. Let L_1 be a recursive language, and let L_2 be a recursively enumerable but not a recursive language. Which one of the following is TRUE?
(a) $\overline{L_1}$ is recursive and $\overline{L_2}$ is recursively enumerable.
(b) $\overline{L_1}$ is recursive and $\overline{L_2}$ is not recursively enumerable.
(c) $\overline{L_1}$ and $\overline{L_2}$ are recursively enumerable.
(d) $\overline{L_1}$ is recursively enumerable and $\overline{L_2}$ is recursive.

[2005 : 2 Marks]

21. Let L_1 be regular language, L_2 be a deterministic context-free language and L_3 a recursively enumerable, but not recursive, language. Which one of the following statements is false?
(a) $L_1 \cap L_2$ is a deterministic CFL
(b) $L_3 \cap L_1$ is recursive
(c) $L_1 \cup L_2$ is context-free.
(d) $L_1 \cap L_2 \cap L_3$ is recursively enumerable

[2006 : 2 Marks]

22. If L and $\overline{L}$ are recursively enumerable then L is
(a) regular (b) context-free
(c) context-sensitive (d) recursive

[2008 : 1 Mark]

23. Let L_1 be a recursive language. Let L_2 and L_3 be language that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?
(a) $L_2 - L_1$ is recursive enumerable
(b) $L_1 - L_3$ is recursive enumerable
(c) $L_2 \cap L_3$ is recursive enumerable
(d) $L_2 \cup L_1$ is recursive enumerable

[2010 : 1 Mark]

24. Let $A \leq_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B. Which one of the following is FALSE?

(a) If $A \leq_m B$ and B is recursive then A is recursive.
(b) If $A \leq_m B$ and a is undecidable then B is undecidable.
(c) If $A \leq_m B$ and B is recursively enumerable then A is recursively enumerable.
(d) If $A \leq_m B$ and B is not recursively enumerable then A is not recursively enumerable.

[2014 (Set-2) : 1 Marks]

25. For any two languages L_1 and L_2 such that L_1 is context free and L_2 is recursively enumerable but not recursive, which of the following is/are necessarily true?
1. $\overline{L_1}$ (complement of L_1) is recursive
2. $\overline{L_2}$ (complement of L_2) is recursive
3. $\overline{L_1}$ is context-free
4. $L_1 \cap L_2$ is recursively enumerable

(a) 1 only (b) 3 only
(c) 3 and 4 only (d) 1 and 4 only

[2015 (Set-1) : 1 Mark]

26. Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that $\overline{Y}$ reduces to W, and Z reduces to $\overline{X}$ (reduction means the standard many-one reduction). Which one of the following statements is TRUE ?
(a) W can be recursively enumerable and Z is recursive
(b) W can be recursive and Z is recursively enumerable.
(c) W is not recursively enumerable and Z is recursive.
(d) W is not recursively enumerable and Z is not recursive. **[2016 (Set-1) : 1 Mark]**

27. Consider the following types of languages L_1 : Regular L_2 : Context-free, L_3 : Recursive, L_4 : Recursively enumerable.
Which of the following is/are TRUE?
I. $\overline{L_3} \cup L_4$ is recursively enumerable.
II. $\overline{L_2} \cup L_3$ is recursive.
III. $\overline{L_1^*} \cap L_2$ is context-free.
IV. $L_1 \cup \overline{L_2}$ is is context-free.
(a) I only (b) I and III only
(c) I and IV only (d) I, II, and III only

[2016 (Set-2) : 1 Mark]

28. Consider the following languages :

- $L_1 = \{\langle M \rangle \mid M$ takes at least 2016 steps on some input$\}$
- $L_2 = \{\langle M \rangle \mid M$ takes at least 2016 steps on all inputs$\}$ and
- $L_3 = \{\langle M \rangle \mid M$ accepts $\epsilon\}$

where for each turning machine M, $\langle M \rangle$ denotes a specific encoding of M. Which one of the following is TRUE ?

(a) L_1 is recursive and L_2, L_3 are not recursive

(b) L_2 is recursive and L_1, L_3 are not recursive

(c) L_1, L_2 are recursive and L_3 is not recursive

(d) L_1, L_2, L_3 are recursive

[2016 (Set-2) : 2 Marks]

29. Let A and B be finite alphabets and let # be a symbol outside both A and B. Let f be a total function from A^* to B^*. We say f is computable if there exists a Turing machine M which given an input x in A^*, always halts with f(x) on its tape. Let L_f denote the language $\{x\#f(x) \mid x \in A^*\}$. Which of the following statements is true :

(a) f is computable if and only if L_f is recursive.

(b) f is computable if and only if L_f is recursively enumerable

(c) If f is computable then L_f is recursive, but not convesely.

(d) If f is computable then L_f is recursively enumerable , but not conversely.

[2017 (Set-1) : 1 Mark]

30. The set of all recursively enumerable languages is

(a) closed under complementation

(b) closed under intersection

(c) a subset of the set of all recursive languages

(d) an uncountable set

[2018 (Set-1) : 1 Mark]

31. Consider the following sets:

S1. Set of all recursively enumerable languages over the alphabet {0, 1}

S2. Set of all syntactically valid C programs

S3. Set of all languages over the alphabet {0, 1}

S4. Set of all non-regular languages over the alphabet {0, 1}

Which of the above sets are uncountable?

(a) S2 and S3 (b) S1 and S2

(c) S1 and S4 (d) S3 and s4

[2019 : 2 Marks]

32. Which of the following statements is/are TRUE?

(a) Every subset of a recursively enumerable language is recursive.

(b) If a language L and its complement $\overline{L}$ are both recursively enumerable, then L must be recursive.

(c) Complement of a context-free language must be recursive.

(d) If L_1 and L_2 are regular, then $L_1 \cap L_2$ must be deterministic context-free.

[2022 : 1 Mark]

33. Which of the following statements is/are CORRECT?

(a) The intersection of two regular languages is regular.

(b) The intersection of two context-free languages is context-free.

(c) The intersection of two recursive languages is recursive.

(d) The intersection of two recursively enumerable languages is recursively enumerable.

[2023 : 1 Mark]

TM

34. A single tape Turing Machine M has two states q_0 and q_1, of which q_0 is the starting state. The tape alphabet of M is {0, 1, B} and its input alphabet is {0, 1}. The symbol B is the blank symbol used to indicate end of an inptu string. The transition function of M is described in the following table:

	0	0	B
q_0	q_1,1,R	q_1,1,R	Halt
q_1	q_1,1,R	q_0,1,L	q_0,B,L

The table is interpreted as illustrated below. The entry (q_1, 1, R) in row q_0 and column 1 signifies that if M is in state q_0 and reads 1 on the current tape square, then it writes 1 on the same tape tape square, then it writes 1 on the same tape square, moves its tape head one position to the right and transitions to state q_1. Which of the following statements is true about M?

(a) M does not halt on any string in $(0 + 1)^*$

(b) M does not halt on any string in $(00 + 1)^*$

(c) M halts on all string ending in a 0

(d) M halts on all string ending in a 1

[2003 : 2 Marks]

35. Which one of the following is true for the language $\{a^p \mid p$ is a prime$\}$?

(a) It is not accepted by a Turning Machine

(b) It is regular but not context free

(c) It is context-free but not regular

(d) It is neither regular nor context-free, but accepted by a Turing machine

[2008 : 1 Mark]

36. Which of the following statements is/are FALSE?

1. For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine.
2. Turing recognizable languages are closed under union and complementation.
3. Turing decidable languages are closed under intersection and complementation.
4. Turing recognizable languages are closed under union and intersection.

(a) 1 and 4 only (b) 1 and 3 only

(c) 2 only (d) 3 only

[2013 : 1 Mark]

37. Let <M> be the encoding of a Turing machine as a string over = {0, 1}. Let L = {<m> | M is a Turing machine that accepts a string of length 2014}. Then, L is

(a) decidable and recursively enumerable

(b) undecidable and recursively enumerable

(c) undecidable and not recursively enumerable

(d) decidable but not recursively enumerable

[2014 (Set-2) : 2 Marks]

38. Which of the following languages are undecidable? Note that $\langle M \rangle$ indicates encoding of the Turing machine M.

$L_1 = \{\langle M \rangle \mid L(M) = \phi\}$

$L_2 = \{\langle M \rangle\ w, q) \mid M$ on input w reaches state q in exactly 100 steps$\}$

$L_3 = \{\langle M \rangle \mid L(M)$ is not recursive$\}$

$L_4 = \{\langle M \rangle \mid L(M)$ contains at least 21 members$\}$

(a) L_1, L_3 and L_4 only

(b) L_1 and L_3 only

(c) L_2 and L_3 only

(d) L_1, L_3 and L_4 only

[2020 : 2 Marks]

ANSWERS

1.(c)	2.(b,d)	3.(b)	4.(d)	5.(a,b)	6.(a)	7.(b)	8.(c)	9.(b)	10.(b)
11.(d)	12.(a)	13.(d)	14.(d)	15.(a)	16.(a,b,c)	17.(a)	18.(a)	19.(d)	20.(b)
21.(b)	22.(d)	23.(b)	24.(d)	25.(d)	26.(c)	27.(d)	28.(c)	29.(a)	30.(b)
31.(d)	32.(b,c,d)	33.(a, c, d)	34.(a)	35.(d)	36.(c)	37.(b)	38.(a)		