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Statistical Techniques for Data Science

Correlation

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Objective

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After attending this session, you will be able to –

- **Explain what is Correlation**
- **Describe Correlation Co-efficient**
- **Explain effect of Outliers on Correlation**
- **Explain what is Multi-collinearity**

Correlation

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- **Correlation**: The degree of relationship between the variables under consideration is measured through the correlation analysis.
- The measure of correlation is called the correlation coefficient
- The degree of relationship is expressed by coefficient which range from correlation **(-1 ≤ r ≥ +1)**
- The direction of change is indicated by a sign
- The correlation analysis enable us to have an idea about the degree & direction of the relationship between the two variables under study.

Correlation

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- Correlation is a statistical tool that helps to measure and analyze the degree of relationship between two variables.
- Correlation analysis deals with the association between two or more variables.

Correlation & Causation

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- Causation means cause & effect relation.
- Correlation denotes the interdependency among the variables for correlating two phenomenon, it is essential that the two phenomenon should have cause-effect relationship,& if such relationship does not exist then the two phenomenon can not be correlated.
- If two variables vary in such a way that movement in one are accompanied by movement in other, these variables are called cause and effect relationship.
- Causation always implies correlation but correlation does not necessarily implies causation.

Methods of Studying Correlation

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- Karl Pearson's Coefficient of Correlation
- Scatter Diagram Method

Karl Pearson's Correlation Co-efficient

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- Covariance explains the relationship between 2 variables, but units of covariance is attached to the unit of covariance
- Expression of Covariance for Population –

$$\text{Covariance}(X, Y) = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{N}$$

- Expression of Covariance for Sample –

$$\text{Covariance}(X, Y) = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{n-1}$$

Karl Pearson's Correlation Co-efficient

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- Expression for Karl Pearson's Correlation Co-efficient – (it has no unit) –

$$r = \frac{\text{Covariance}(x, y)}{\sqrt{\text{Var}(x)}\sqrt{\text{Var}(Y)}}$$

Or,

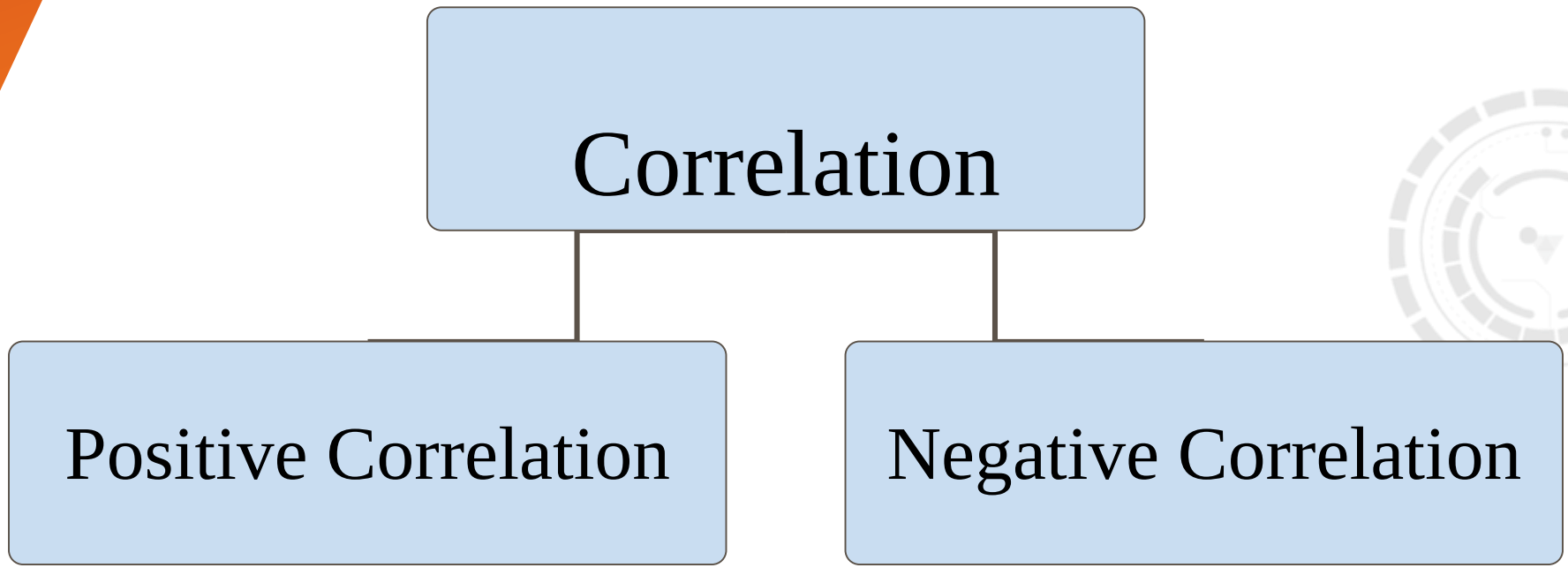
$$r = \frac{\frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{n-1}}{\sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}}\sqrt{\frac{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}{n-1}}}$$

$$= \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}}$$

- In general for computation purpose, the following formula is used -

$$r = \frac{n\sum_{i=1}^{n}X_iY_i - \left(\sum_{i=1}^{n}X_i\right)\left(\sum_{i=1}^{n}Y_i\right)}{\sqrt{\left(n\sum_{i=1}^{n}X_i^2 - \left(\sum_{i=1}^{n}X_i\right)^2\right)}\sqrt{\left(n\sum_{i=1}^{n}Y_i^2 - \left(\sum_{i=1}^{n}Y_i\right)^2\right)}}$$

Types of Correlation
Type I



Types of Correlation Type I

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- **Positive Correlation:** The correlation is said to be positive correlation if the values of two variables changing with same direction.

  Ex. Pub. Exp. & sales, Height & weight.

- **Negative Correlation:** The correlation is said to be negative correlation when the values of variables change with opposite direction.

  Ex. Price & qty. demanded.

Direction of the Correlation

- **Positive relationship** – Variables change in the same direction.
  - As X is increasing, Y is increasing
  - As X is decreasing, Y is decreasing
  - E.g., As height increases, so does weight.
- **Negative relationship** – Variables change in opposite directions.
  - As X is increasing, Y is decreasing
  - As X is decreasing, Y is increasing
  - E.g., As TV time increases, grades decrease

Indicated by sign; (+) or (-).

More examples

Positive relationships

water consumption and temperature.

study time and grades.

Negative relationships:

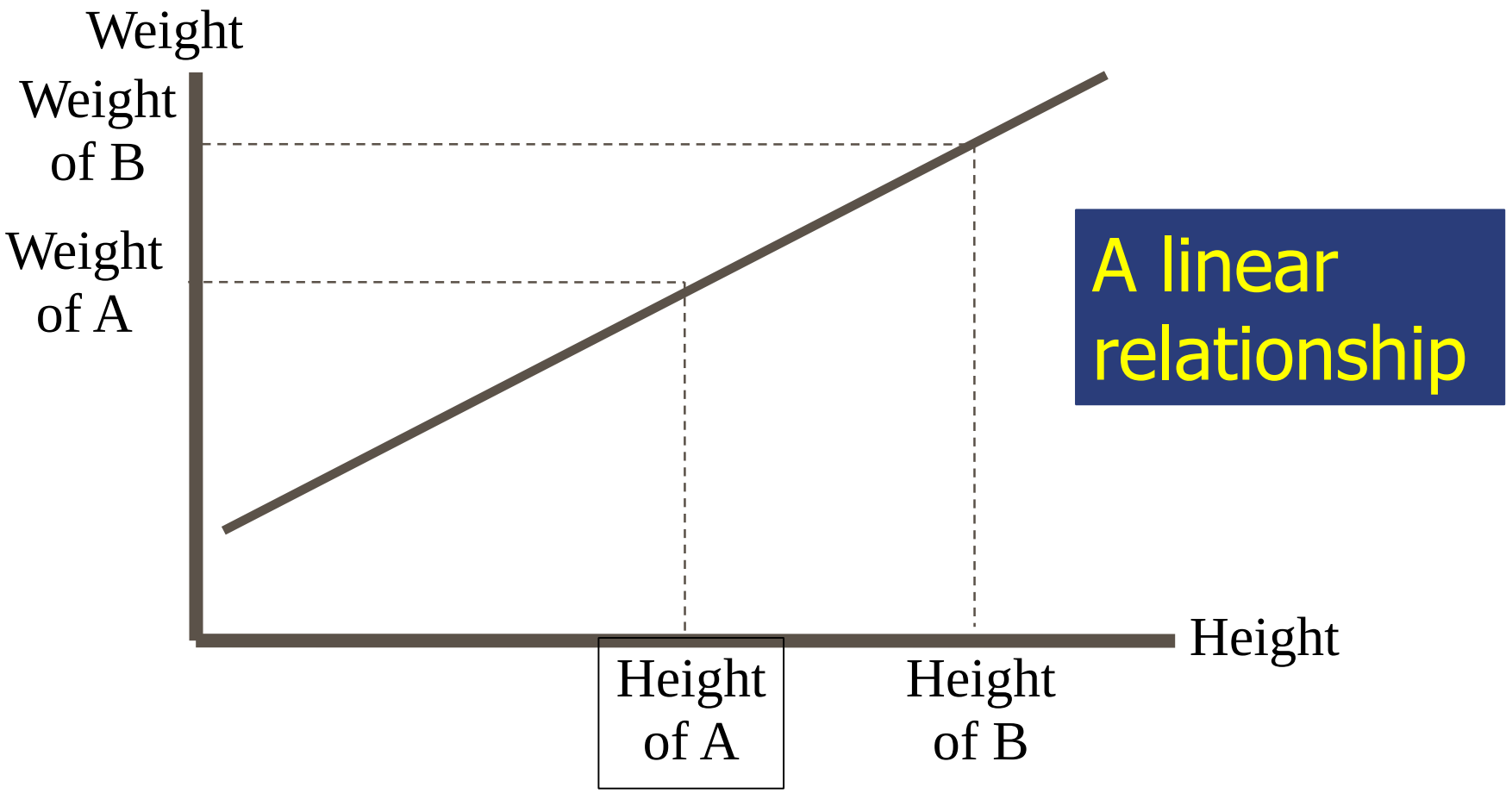
alcohol consumption and driving ability.

Price & quantity demanded

Scatter Diagram Method

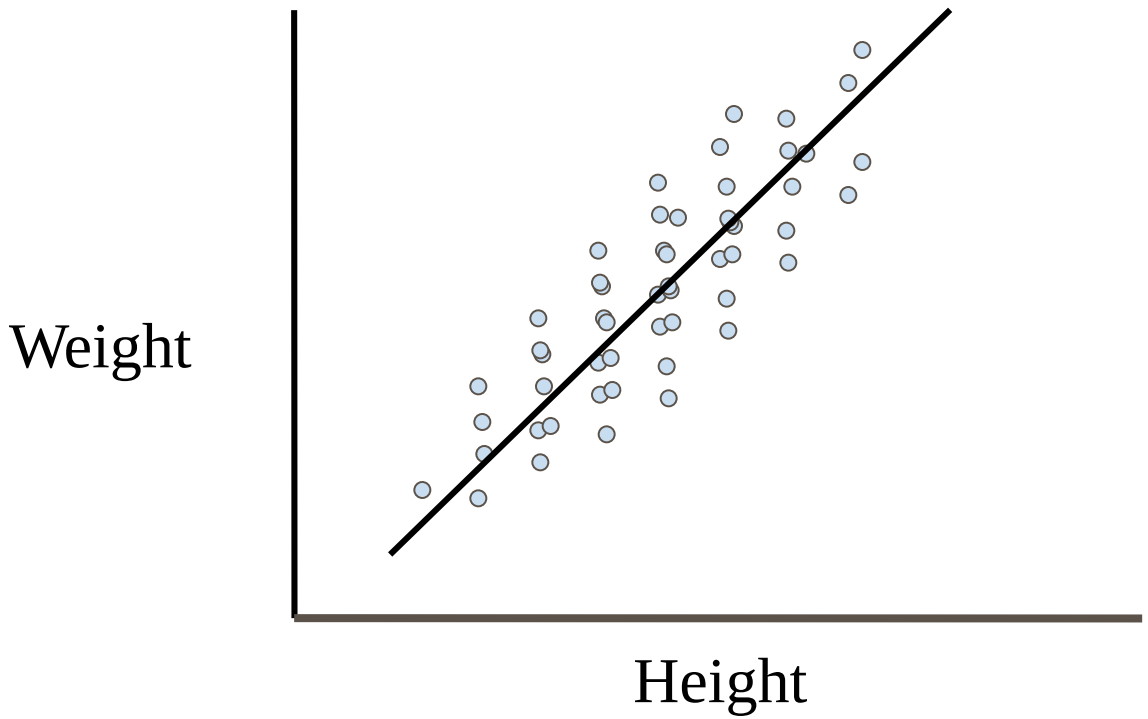
- Scatter Diagram is a graph of observed plotted points where each points represents the values of X & Y as a coordinate. It portrays the relationship between these two variables graphically.

A perfect positive correlation



High Degree of positive correlation

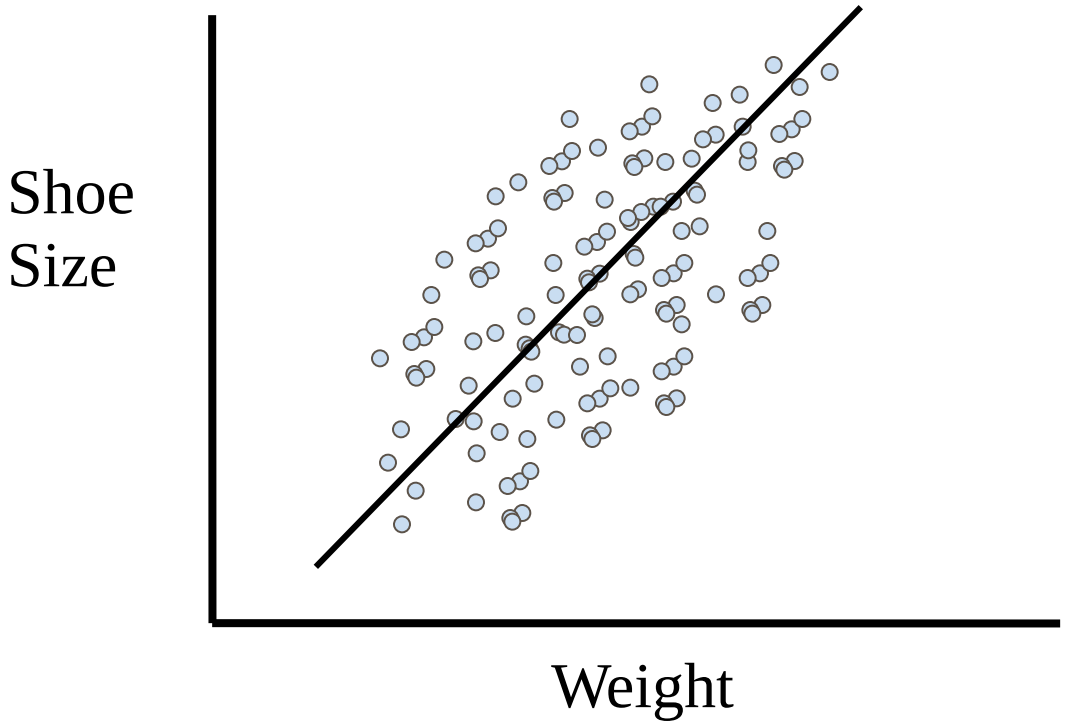
- Positive relationship



r = +.80

Degree of correlation

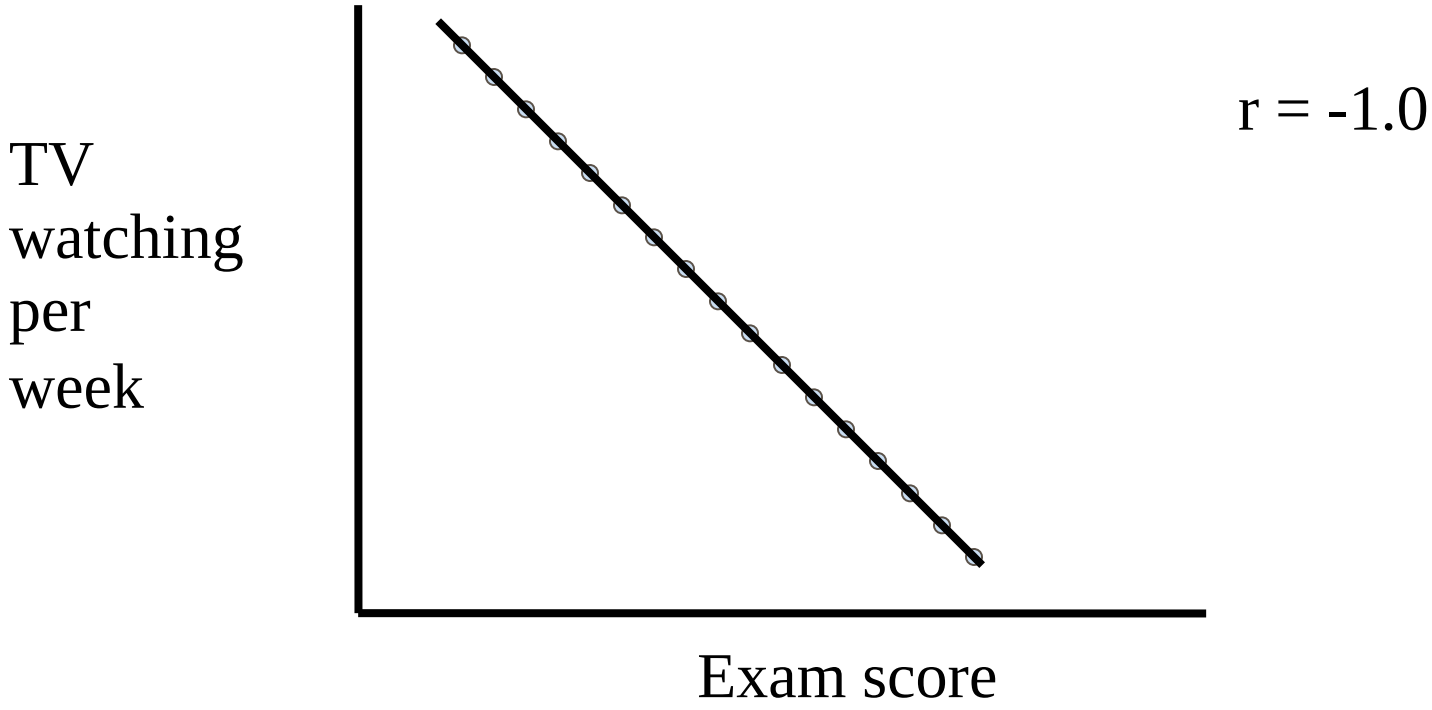
- **Moderate Positive Correlation**



r = + 0.4

Degree of correlation

- **Perfect Negative Correlation**

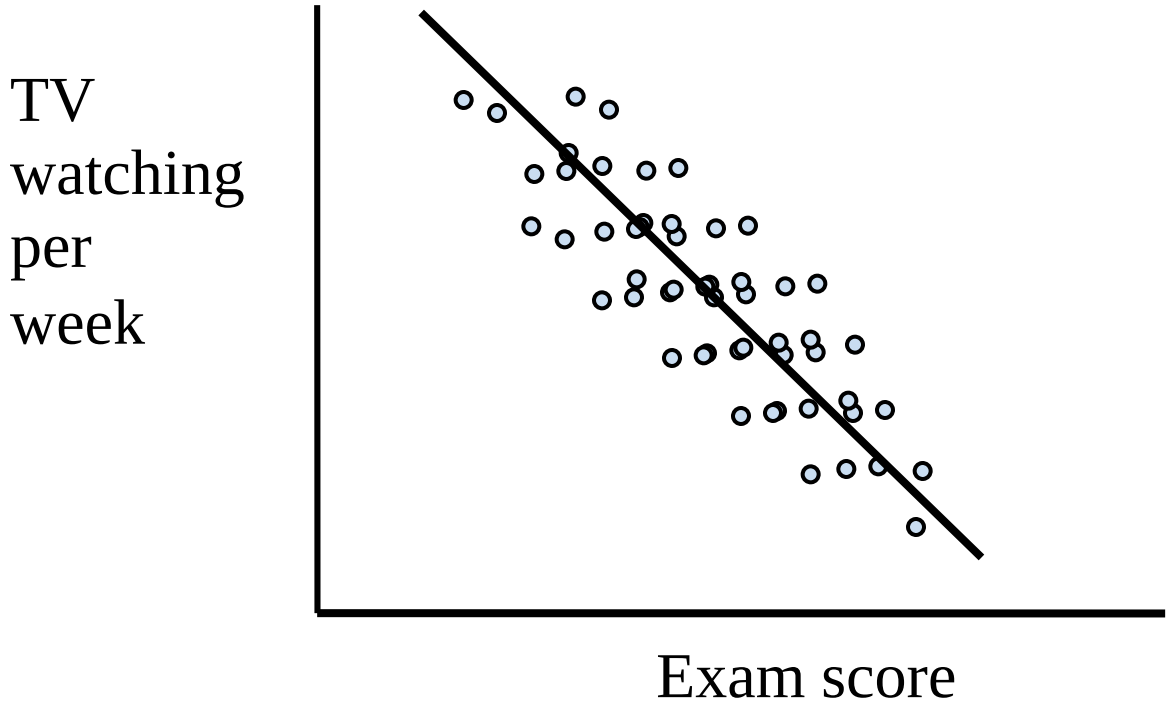


r = -1.0

Degree of correlation

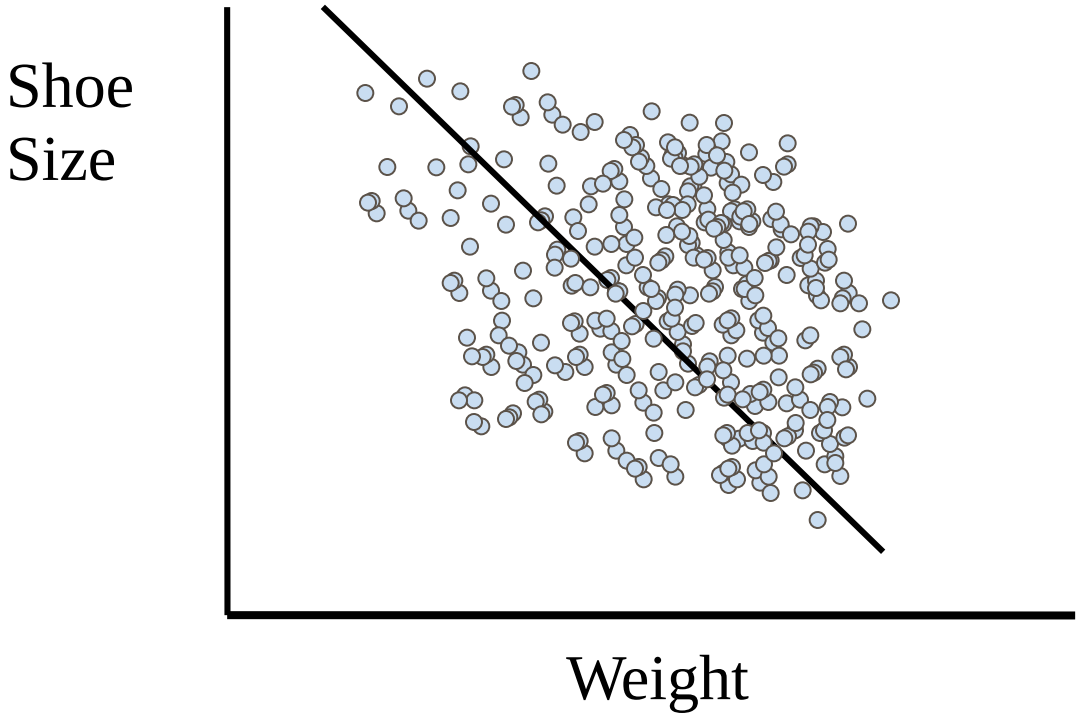
- **Moderate Negative Correlation**

r = -.80



Degree of correlation

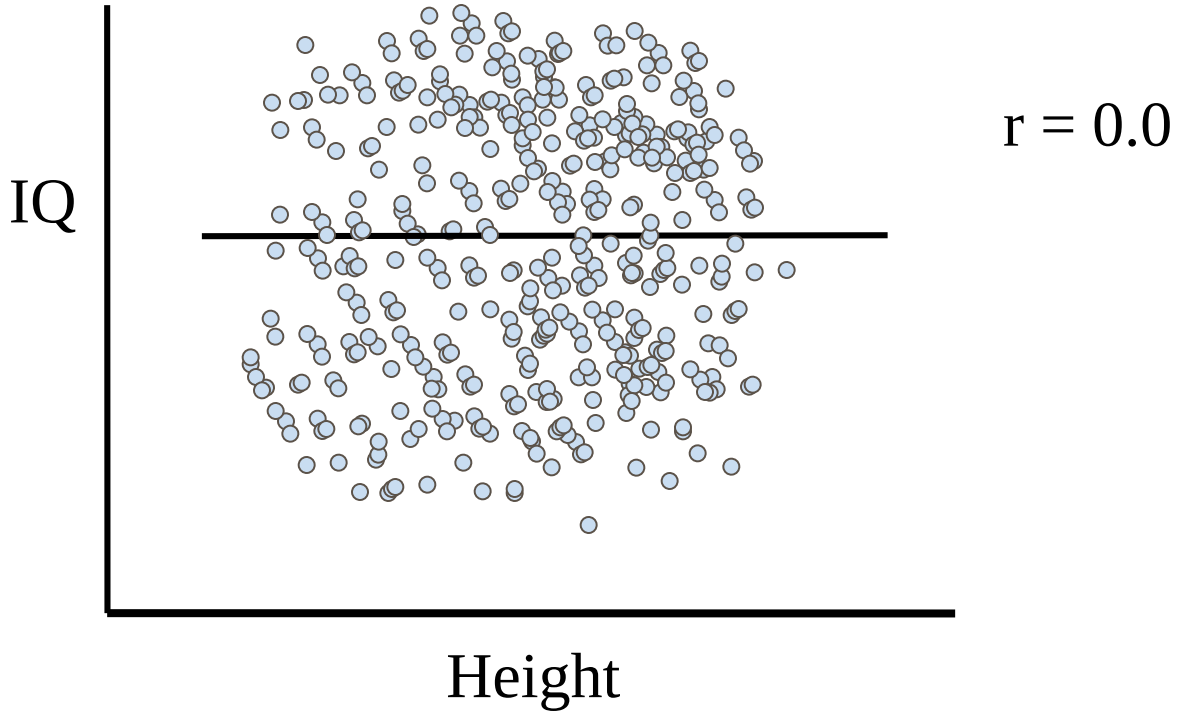
- **Weak negative Correlation**



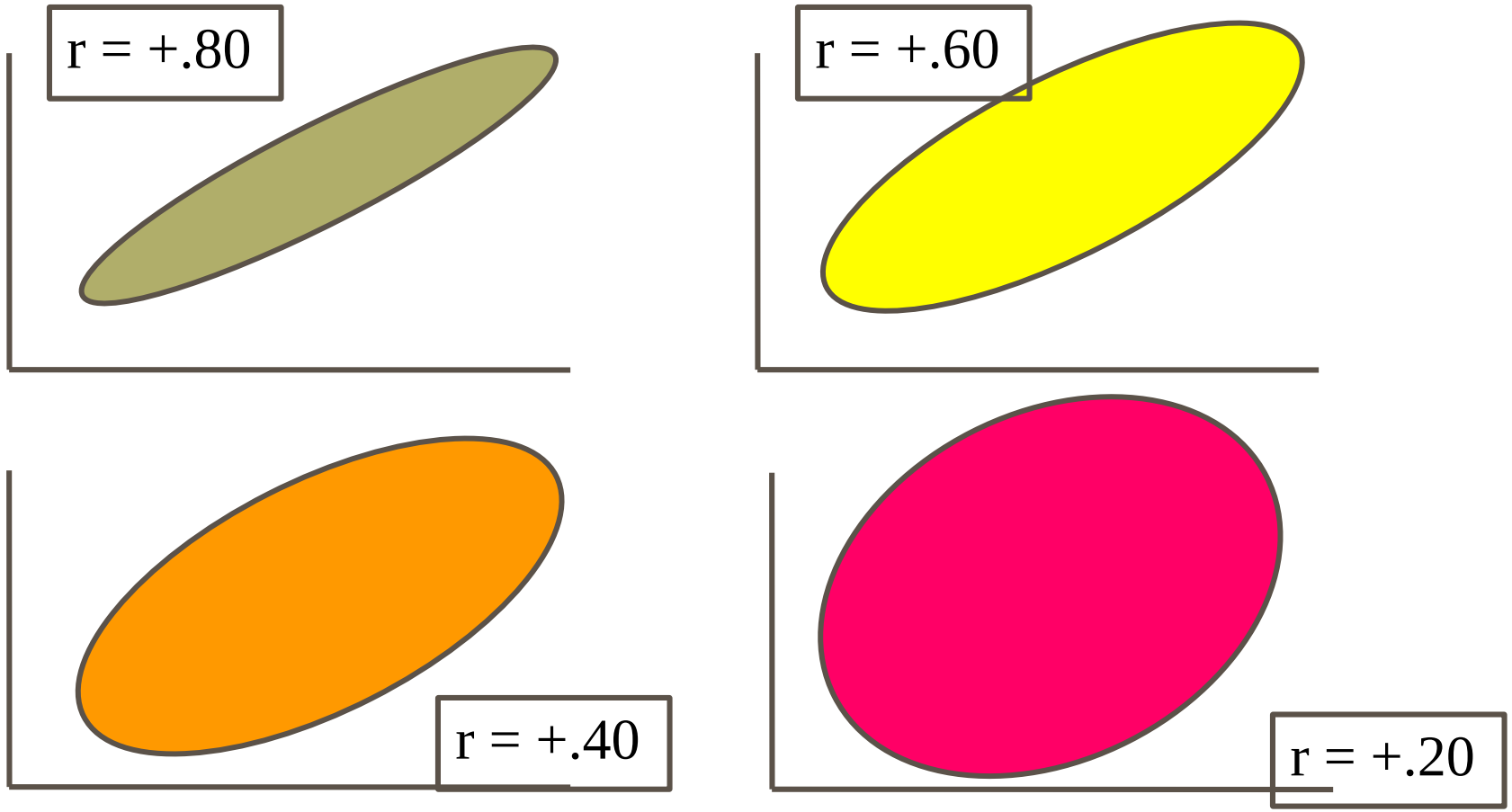
r = - 0.2

Degree of correlation

- **No Correlation (horizontal line)**



Degree of correlation (r)



Advantages of Scatter Diagram

- Simple & Non Mathematical method
- Not influenced by the size of extreme item
- First step in investing the relationship between two variables

Disadvantage of scatter diagram

Can not adopt the an exact degree of correlation

Assumptions of Pearson's Correlation Coefficient

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- There is linear relationship between two variables, i.e. when the two variables are plotted on a scatter diagram a straight line will be formed by the points.
- Cause and effect relation exists between different forces operating on the item of the two variable series.

Advantages of Pearson's Coefficient

- It summarizes in one value, the degree of correlation & direction of correlation also.

Limitation of Pearson's Coefficient

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- Always assume linear relationship
- Interpreting the value of r is difficult.
- Value of Correlation Coefficient is affected by the extreme values.
- Time consuming methods

Karl Pearson's correlation coefficient (r) Example

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Serial No	Age (yrs)	Weight (Kg)
1	7	12
2	6	8
3	8	12
4	5	10
5	6	11
6	9	13

Karl Pearson's correlation coefficient (r)

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Sl no.	Age (yrs) (X)	Wt (Kg) (Y)	XY	X^2	Y^2
1	7	12	84	49	144
2	6	8	48	36	64
3	8	12	96	64	144
4	5	10	50	25	100
5	6	11	66	36	121
6	9	13	117	81	169
Total	$\sum X=41$	$\sum Y=66$	$\sum XY= 461$	$\sum X^2= 291$	$\sum Y^2= 742$

Karl Pearson's correlation coefficient (r)

$$r = \frac{6 \text{ x } 461 - 41 \text{ x } 66}{\sqrt{\left[6 \text{ x } 291 - (41)^2\right]} . \sqrt{\left[6 \text{ x } 742 - (66)^2\right]}}$$

r = 0.759 (Positive (Direct) strong correlation)

Example: Relationship between anxiety and test Scores

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Anxiety (X)	Test score (Y)	X^2	Y^2	XY
10	2	100	4	20
8	3	64	9	24
2	9	4	81	18
1	7	1	49	7
5	6	25	36	30
6	5	36	25	30
$\sum X = 32$	$\sum Y = 32$	$\sum X^2 = 230$	$\sum Y^2 = 204$	$\sum XY = 129$

Karl Pearson's correlation coefficient (r)

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$$r = \frac{(6)(129) - (32)(32)}{\sqrt{\left(6(230) - 32^2\right)\left(6(204) - 32^2\right)}} = \frac{774 - 1024}{\sqrt{(356)(200)}} = -0.94$$

r = - 0.94 (Negative (Indirect) strong correlation)

Effect of Outlier on Correlation

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- In most practical circumstances an outlier decreases the value of a correlation coefficient
- If both the variables X and Y have outlier, then in certain case, the correlation coefficient may increase also

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