Assignment -2

1. Find the chance that if a card is drawn at random from an ordinary pack, it is

(i) a black card = 26/52

(ii) one of the court cards(Court card is either King, Queen or Jack)

= 12/52

(iii) one of the aces.

= 4/52

2. What is the chance that a leap year selected at random will contain 53 Sundays?

Leap year contains two days as 53. So it may be

Sun,mon

Sat,sun

Fri,sat

Thu,fri

Wed,thu

Tue,wed

Mon,tue

Containing 53 Sundays = 2 / 7 = 0.2857

3. An urn contains 9 balls, two of which are red, three blue and four black. Three balls are drawn from the urn at random, that is every ball has an equal chance of being included in the three.

What is the chance that

1. three balls are of different colours?

This is possible if he picks

red,blue,black

red,black,blue

blue,red,black

blue,black,red

black,red,blue

black,blue,red

=(2/9)\*(3/8)\*(4/7) + (2/9)\*(4/8)\*(3/7) + (3/9)\*(2/8)\*(4/7) + (3/9) \* (4/8) \*(2/7)

+ (4/9) \* (2/8) \*(3/7) + (4/9) \* (3/8) \* (2/7)

= 0.2857

1. two balls are of the same colour and third of different?

This is possible if he picks

red,red,blue = (2/9)\*(1/8)\*(3/7) = 0.0119

red,red,black = (2/9)\*(2/8)\*(4/7) = 0.0317

blue,blue,red = (3/9)\*(2/8)\*(2/7) = 0.0238

blue,blue,black = (3/9)\*(2/8)\*(4/7) = 0.0476

black,black,red = (4/9)\*(3/8)\*(2/7) = 0.0476

black,black,blue = (4/9)\*(3/8)\*(3/7) = 0.0714

red,blue,red =(2/9)\*(3/8)\*(1/7) = 0.0119

blue,red,red = (3/9)\*(2/8)\*(1/7) = 0.0119

red,black,red = (2/9)\*(4/8)\*(1/7) = 0.0158

black,red,red = (4/9)\*(2/8)\*(1/7) = 0.0158

blue,red,blue = (3/9)\*(2/8)\*(2/7) = 0.0238

red,blue,blue = (2/9)\*(3/8)\*(2/7) = 0.0238

blue,black,blue = (3/9)\*(4/8)\*(2/7) = 0.0476

black,blue,blue = (4/9)\*(3/8)\*(3/7) = 0.0714

black,red,black = (4/9)\*(2/8)\*(3/7) = 0.0476

red,black,black = (2/9)\*(4/8)\*(3/7) = 0.0476

black,blue,black = (4/9)\*(3/8)\*(3/7) = 0.0714

blue,black,black = (3/9)\*(4/8)\*(3/7) = 0.0714

Total prob = 0.6944

1. the balls are of the same colour?

This is possible if he picks

Blue,blue,blue

Black,black,black

= (3/9)\*(2/8)\*(1/7) + (4/9)\*(3/8)\*(2/7)

= 0.0595

4. Three groups of children contain respectively 3 girls and 1 boy (1st group), 2 girls and 2 boys (2nd group) and 1 girl and 3 boys (3rd group). One child is selected at random from each group. What is the chance that the three selected consist of 1 girl and 2 boys is 13/32?

This is possible if he picks

Girl,boy,boy

Boy,boy,girl

Boy,girl,boy

= (3/4) \*(2/4)\*(3/4) + (1/4) \*(2/4)\*(1/4) + (1/4)\*(2/4)\*(3/4)

=0.4062

5. A random variable X has the following probability function: X = x: 1 2 3 4 5 6 P (x): 0.2 k 0.1 2k 0.3 3k

(i) Find k

1 = 0.2 + k + 0.1 + 2k + 0.3 + 3k

1 = 6k + 0.6

6k = 0.4

k = 0.066

(ii) Evaluate

P(X < 5) = 0.2 + 0.066 + 0.1 + (2 \* 0.066) = 0.4992

P(X ≤ 2) = 0.2 + 0.066 = 0.266,

P(X ≥ 2) = 1 – 0.2 = 0.8,

P(3<X<6) = 0.3

(iii) Find the minimum value of k, such that P(X ≤ 2)>0.25.

0.25 = 0.2 + k

k = 0.05

6. A bag contains 4 red and 3 blue balls. Two drawings of 2 balls (each time) are made. Find the chance that the first drawing gives 2 red balls and the second drawing 2 blue balls

(a) if the balls are returned to the bag after the first draw,

= (4/7 \* 3/6) \* (3/7\*2/6) = 0.0408

(b) if the balls are not returned

= (4/7 \* 3/6) \* (3/5\*2/4) = 0.0857

7. The probability that train will start from the source in time is 0.4 and the probability that it will reach its destination in time is 0.7. The probability that the train will start from source in time and will reach its destination in time is 0.3. What are the following probabilities?

(i) The probability that the train will reach its destination in time provided it starts in time

= 0.3/0.4 = 0.75

(ii) The probability that the train will start from source in time provided that it will reach in time

= 0.3/0.7 = 0.428

8. A man purchases a lottery ticket, in which he may win the first prize of Rs. 1,00000 with probability 0.0001 or the second prize of Rs. 4,0000 with probability 0.0004. Find his mathematical expectation.

E(x) = summation (x \* p(x))

= (100000 \* 0.0001) + (40000 \* 0.0004) = 10 + 16 = 26

9. Twelve coins are thrown simultaneously. Find the probability of getting at least six heads.

P =0.5

N=12

P(x>=6) = 1 – p(x<6)

= 1 – pbinom(5,12,0.5)

= 0.6127

10. Three coins are tossed. Find the probabilities of

P = 0.5

N = 3

1. 0 head

P(x=0) = dbinom(0,3,0.5) = 0.125

1. 1 head,

P(x=1) = dbinom(1,3,0.5) = 0.375

1. 2 heads,

P(x=2) = dbinom(2,3,0.5) = 0.375

1. 3 heads;

P(x=3) = dbinom(3,3,0.5) = 0.125

(ii) more than one head;

P(x>1) = 1 –p(x<=1) = 1 – pbinom(1,3,0.5) = 0.5

1. at least 1 head.

P(x>0) = 1 – p(x=0) = 1 – 0.125 = 0.975

11. Six unbiased coins are thrown 2000 times. Find the expected frequencies of the distribution of heads and tails, and tabulate the result. Calculate the mean and standard deviation of the number of heads.

N =2000

X P 2000\*p

(No. of heads)

0 1/64 31

1 6/64 188

2 15/64 469

3 20/64 625

4 15/64 468

5 6/64 188

6 1/64 31

Mean = 285.7142

s.d = 217.3842

12. The overall percentage of failures of students in a BBA examination is 1/10. What is the probability that out of a group of 5 BBA candidates at least 3 passed the examination?

Probability of passing = 1-0.1 = 0.9

N = 5

P(x>=3) = 1 – p(x <= 2) = 1 –pbinom(2,5,0.9) = 0.9914

13. Find the probability that at most 6 defective fuses will be found in a box of 300 fuses if experience shows that 3 per cent of such fuses are defective. (Given that as e-9 = 0000123)

Probability of fuse being defective = 0.03

N = 300

P(x<=6) = pbinom(6,300,0.03) = 0.2026

14. A random variable x follows Poisson distribution with parameter 4. Find the probabilities that the variable assumes the values

(i) 0, 1, 2, 3, 4;

P(x=0) = dpois(0,4) = 0.0183

P(x=1) = dpois(1,4) = 0.0732

P(x=2) = dpois(2,4) = 0.1465

P(x=3) = dpois(3,4) = 0.1953

P(x=4) = dpois(4,4) = 0.1953

(ii) less than 4;

P(x<4) = ppois(3,4) = 0.4334

(iii) at least 3. (Given that 𝑒−4 = 0.0183)

P(x>=3) = 1 – p(x<3) = 1- ppois(2,4) = 0.7618

15. In producing toys of a particular type through a manufacturing process in a factory, the average number of defectives is 6%. What is the probability of getting exactly 2 defectives in a sample of 10 toys chosen at random, by using the Poisson distribution as well as the binomial distribution?

average defective = 0.06

probability of toy being defective = p(x>0) = 1-p(x=0) = 1- dpois(0,0.06) = 0.05823

probability of 2 toys being defective among 10 = p(x=2) = dbinom(2,10,0.05823) = 0.0944

16. A business firm receives on an average 1.8 telephone calls per day during the period 9.55 -10.00 a.m. Find the probability that on a certain day, the firm receives

Average = 1.8 calls during 9.55 to 10

1. no call;

p(x=0) = dpois(0,1.8) = 0.16529

1. exactly 3 calls, during the same period. (Assume Poisson distribution; given 𝑒−1.8 = 0.1653).

p(x=3) = dpois(3,1.8) = 0.1606

17. In a factory 5% of the items made by a machine are defective. Find the probability that 4 or more items are defective in a sample of 100 items.

Probability of item being defective = 0.05

P(x>4) = 1 – p(x<=4) = 1 – pbinom(4,100,0.05) = 0.564

18. In a normal distribution whose mean is 10 and s.d. 3,

find the probability for the interval (area under the curve) from x = 8.5 to x = 12.4.

p(8.5<x<12.4) = pnorm(12.4,10,3) – pnorm(8.5,10,3) = 0.4796

(b) For a normal distribution with mean 2 and s.d. 3, find a value of the variate such that the probability of the variate from the mean to that value is 0.4115.

= qnorm(0.4115,2,3) = 1.3289

20. The height distribution of a group of 9000 men is Normal with mean height 66.5″ and s.d. 3.5″. Find the number of men whose height is

(a) less than 65″ but greater than 57″,

= p(57<x<65) \* 9000 = (pnorm(65,66.5,3.5) – pnorm(57,66.5,3.5)) \* 9000 = 2977

(b) less than 68.5″

=p(x<68.5) \* 9000 = pnorm(68.5,66.5,3.5) \* 9000 = 6445

(c) more than 68″.

=p(x>68) \* 9000 = (1 – p(x<68)) \* 9000 = (1 – pnorm(68,66.5,3.5)) \* 9000 =3007