Number Theory And Applications

Assignment 2

1. Solve the following system of congruences:

$$x \equiv 2 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$
$$x \equiv 2 \pmod{7}$$

2. Solve the following system of congruences:

$$x \equiv 11 \pmod{36}$$
$$x \equiv 7 \pmod{40}$$
$$x \equiv 32 \pmod{75}$$

3. Solve the following system of congruences:

$$x^2 \equiv 1 \pmod{3}$$
$$x \equiv 2 \pmod{4}$$

4. (2012 AIME II) For a positive integer p, define the positive integer n to be p-safe if n differs in absolute value by more than 2 from all multiples of p. For example, the set of 10-safe numbers is $\{3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23, ...\}$. Find the number of positive integers less than or equal to 10,000 which are simultaneously 7-safe, 11-safe, and 13-safe.

5. (2011 AIME II) There are N permutations $(a_1, a_2, ..., a_{30})$ of 1, 2, . . . , 30 such that for $m \in \{2, 3, 5\}$, m divides $a_{n+m} - a_n$ for all integers n with $1 \leq n < n + m \leq 30$. Find the remainder when N is divided by 1000.

6. Implement Chinese Remainder Theorem in code: You are given n numbers which are pairwise co-prime and corresponding remainders when these numbers are divided by some number x. You need to find minimum possible value of x that produces given remainders.