Number Theory And Applications

Assignment 3

1. Prove that $19 | 2^{2^{6k+2}} + 3$ for $k = 0, 1, 2, 3, \cdots$

2. Prove that for $F_n = 2^{2^n} + 1$ we have $F_n | 2^{F_n} - 2$ where $n = 1, 2, 3, \cdots$

3. Find all integers $n > 1$ such that $1^n + 2^n + \cdots + (n-1)^n$ is divisible by n.

4. Prove that for every odd prime p there exist infinitely many positive integers n such that $p | n \cdot 2^n + 1$.

5. Does there exist an integer n such that $\frac{n}{2}$ is a perfect square, $\frac{n}{3}$ is a cube and $\frac{n}{5}$ a fifth power?

6. (AIME-1989-9) One of the Euler's conjectures was disproved in the 1960s by three American mathematicians when they showed there was a positive integer such that

$$133^5 + 110^5 + 84^5 + 27^5 = n^5$$

Find the value of n.

7. (AIME-2006-II-14) Let S_n be the sum of the reciprocals of the non-zero digits of the integers from 1 to 10^n inclusive. Find the smallest positive integer n for which S_n is an integer.

8. Implement Euler's Totient function in code. Given a number n, find $\phi(n)$.

9. (Code) Given a number n, find ϕ for all number less than and equal to n.

10. (Code) Compute the remainder when $\binom{n}{r}$ is divided by p using fermat's little theorem. You are given n, r and p. Here p is a prime greater than n.

11. Implement Sieve of Eratosthenes in code. Given a number n, print all primes smaller than or equal to n.