

NUMBER THEORY AND APPLICATIONS

ASSIGNMENT 1

1. Prove that for positive integer n we have $169|3^{3n+3} - 26n - 27$.
2. Prove that for positive integer n we have $n^2|(n+1)^n - 1$.
3. Prove that if for integers a and b we have $7|a^2 + b^2$, then $7|a$ and $7|b$.
4. For numbers $2k - 1$ and $9k + 4$, find their greatest common divisor as a function of k .
5. Find the remainder when 2^{81} is divided by 17.
6. Prove that $2^n + 6 \cdot 9^n$ is always divisible by 7 for any positive integer n .
7. (1992 AHSME 17) The two-digit integers from 19 to 92 are written consecutively to form a large integer

$$N = 192021 \cdots 909192$$

Suppose that 3^k is the highest power of 3 that is a factor of N . What is k ?

8. Show that there are no integer solutions to $x^2 + y^2 = 10^z - 1$ for $z > 1$.
- 9*. Implement Euclidean Algorithm for computing the GCD of two given numbers a and b .
- 10*. Implement Extended Euclidean Algorithm to represent the GCD of two numbers a and b as their linear combination, i.e.,

$$GCD(a, b) = a \cdot x + b \cdot y$$

Given a and b , find corresponding x and y .

* These are implementation based programming questions and are optional.