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Blind Deconvolution of EEG Signals Using the Stochastic Calculus

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***Abstract*—a new tool, in the blind deconvolution, for the estimation of both the source signals and the unknown channel dynamics has been developed. The framework for this methodology is based on a multi-channel blind deconvolution technique that has been reformulated to use Stochastic Calculus. The convolution processes is modeled as Finite Impulse Response (FIR) filters with unknown coefficients. Assuming that one of the FIR filter coefficients is time-varying, we have been able to get accurate estimation results for the source signals, even though the filter order is unknown. The time-varying filter coefficient was assumed to be a stochastic process. A stochastic differential equation (SDE), with some unknown parameters, was developed that described its evolution over time. The SDE parameters have been estimated using methods in stochastic calculus. The method was applied to the problem of two chatting persons and the problem of EEG contaminated by EOG. Comparisons to existing methods are also reported.**

***Index Term*—EEG, EOG, Blind Deconvolution, Convolutive BSS, Ornstein-Uhlenbeck process, Ito Calculus**

I. INTRODUCTION:

Blind deconvolution (BD) is very active research domains of the signal processing community. The application range of BD includes, but not limited to, EEG analysis [1], EKG analysis [2], [3], EMG analysis [4] and many more. In BD one is interested in the estimation of n unknown signals, given just a set of filtered mixtures observed at m sensors (in this study m>n). The term "blind" refers to our incomplete knowledge of the mixing operator. The BD has been extensively studied and a number of efficient algorithms have been developed [5], [6]-[8]. One of the most popular and effective techniques is to assume a finite impulse response filter (FIR) model for the modulating channels/paths and to estimate the coefficients of this model. Through the inversion of the FIR filter we get the original source signals. A major drawback of this model and the others is that if the filter order is incorrect, the estimated source signals are far from the true values [9].

This report is an extension to our previous work presented in [10]. The sources are now more than one (specifically and for the sake of simplicity we present the case of two sources and three measurements). The proposed method does not assume that the sources are independent. This is a big advantage over existing methods. It is also assumed that FIR filters are adequate to describe the channels though their orders and values are unknown.

In this report, we suggest making one of the FIR filter parameters changing over time. This way, the ambiguity in the filter order is compensated by the time variations of one of the parameters. The time-varying parameter is estimated through the stochastic calculus [11]. In Sec. 2, we describe the blind deconvolution problem, and we propose a method based on regression analysis. In Sec. 3, we modify this method by assuming that one of the filter parameters is following what is known as Ornstein-Uhlenbeck (OU) process. In Sec. 4, the estimated sources are given, using the proposed method and conventional methods, for the case of two mixed voices and EEG corrupted signal. The same approach could be used in other signals as well. Finally in Sec. 5, we provide summary and conclusions.

II. PROBLEM FORMULATION:

For exposition purposes assume that we have two unknown sources $u_1(t), u_2(t)$ and three measurements $y_1(t), y_2(t), y_3(t)$. The measurements are the convolution of the sources with unknown linear time invariant finite impulse response (FIR) filters. This could be represented by the equation:

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \\ h_{31}(t) & h_{32}(t) \end{bmatrix} * \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \varepsilon_3(t) \end{bmatrix} \tag{1}$$

Where "*" is the convolution operation, $\varepsilon_i(t)$ is the ith error or noise that is assumed to be white and Gaussian. The objective is to find an estimate of the source signals $u_1(t)$ and $u_2(t)$. Equation (1) could be written in the Z domain as:

$$\begin{bmatrix} Y_1(z) \\ Y_2(z) \\ Y_3(z) \end{bmatrix} = \begin{bmatrix} h_{11}(z) & h_{12}(z) \\ h_{21}(z) & h_{22}(z) \\ h_{31}(z) & h_{32}(z) \end{bmatrix} \begin{bmatrix} U_1(z) \\ U_2(z) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(z) \\ \varepsilon_2(z) \\ \varepsilon_3(z) \end{bmatrix} \tag{2}$$

After some manipulations we get:

$$\frac{[h_{21}(z)Y_1(z) - h_{11}(z)Y_2(z)]}{[h_{21}(z)h_{12}(z) - h_{11}(z)h_{22}(z)]} \approx U_2(z) \approx \frac{[h_{31}(z)Y_1(z) - h_{11}(z)Y_3(z)]}{[h_{31}(z)h_{12}(z) - h_{11}(z)h_{32}(z]}$$

i.e

$$\begin{aligned} &[h_{31}(z)h_{12}(z) - h_{11}(z)h_{32}(z][h_{21}(z)Y_1(z) - h_{11}(z)Y_2(z)] \\ &\approx [h_{21}(z)h_{12}(z) - h_{11}(z)h_{22}(z)][h_{31}(z)Y_1(z) - h_{11}(z)Y_3(z)] \end{aligned} \tag{3}$$

Similar equations could be obtained for the source signals

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$U_1(z)$ and $U_2(z)$.

Through regression analysis or other methods and using (3), one is able to find estimated for the filters $h_{ij}(z)$ and consequently an estimate for $U_2(z)$ is obtained using for example:

$$[h_{21}(z)y_1(z)-h_{11}(z)y_2(z)]\approx[h_{21}(z)h_{12}(z)-h_{11}(z)h_{22}(z)]U_2(z) \quad (4)$$

The above steps could be repeated for $U_1(z)$. Thus, the two sources are obtained. The following example explains the steps in great details.

Example: To further simplify the analysis, assume that the filters are first order i.e.

$$h_{ij}(z)=1+h_{ij1}z^{-1} \qquad \text{i=1, 2, 3} \quad (5)$$

Using (3) we get:

$$(\alpha_1 z^{-1}+\alpha_2 z^{-2})Y_2(z)+(\alpha_3 z^{-1}+\alpha_4 z^{-2})Y_1(z)\approx (\alpha_5 z^{-1}+\alpha_6 z^{-2})Y_3(z) \quad (6)$$

Define:

$$\alpha_1=(-(h_{121}+h_{311})+(h_{111}+h_{321})),$$
$$\alpha_2=(-(h_{311}h_{121})+h_{111}h_{321})$$
$$\alpha_3=(-(h_{321}+h_{211})+(h_{221}+h_{311})),$$
$$\alpha_4=(-h_{211}h_{321}+h_{221}h_{311})$$
$$\alpha_5=(-(h_{211}+h_{121})+(h_{111}+h_{221})),$$
$$\alpha_6=(-h_{211}h_{121}+h_{111}h_{221})$$

Thus (6) could be written compactly as:

$$[\alpha_1 y_2(k-1)+\alpha_2 y_2(k-2)]+[\alpha_3 y_1(k-1)+\alpha_4 y_1(k-2)] \approx[\alpha_5 y_3(k-1)+\alpha_6 y_3(k-2)] \quad (7)$$

Through regression analysis or other methods and using (3), one is able to find an estimate for only five of the six unknown coefficients that represent the transfer function. The sixth coefficient $\alpha_6=(-h_{211}h_{121}+h_{111}h_{221})$ will be taken as the numeraire. Another two similar equations to (7) could be obtained and from which we could get estimates for all the six parameters. The filter coefficients h_{ij1} are then estimated from the estimated α_i. Once the h_{ij1} are estimated, we use these filter coefficients to find an estimate for the unknown sources using:

$$u_2(k) \approx \frac{-\alpha_6}{\alpha_5}u_2(k-1) + \frac{[-y_1(k+1)+y_2(k+1)]-h_{211}y_1(k)+h_{111}y_2(k)}{\alpha_5} \quad (8)$$

In matrix format and for N data points we have for equation (8):

$$\underline{Y}_{12}=H_{12}\underline{U}_2+\underline{v}_1 \quad (9)$$

Where

$$\underline{Y}_{12}=\begin{bmatrix} y_1(2)-y_2(2)+h_{211}y_1(1)-h_{111}y_2(1)\\ y_1(3)-y_2(3)+h_{211}y_1(2)-h_{111}y_2(2)\\ \cdot\\ \cdot\\ \cdot\\ y_1(N-1)-y_2(N-1)+h_{211}y_1(N-2)-h_{111}y_2(N-2)\end{bmatrix},$$

$\underline{U}_2^T=[u_2(0)\quad u_2(1)\quad\cdots\quad\cdots\quad u_2(N-1)]$, $\underline{v}_1^T=[v_1(0)\quad v_1(1)\quad\cdots\quad\cdots\quad v_1(N-1)]$,

And $$H_{12}=\begin{bmatrix}-\alpha_6 & -\alpha_5 & 0 & \ldots\\ 0 & -\alpha_6 & -\alpha_5 & 0\\ \ldots & \ldots & \ldots & ..\\ 0 & \ldots & -\alpha_6 & -\alpha_5\end{bmatrix}$$

Once the coefficients of the FIR filters are estimated, we use inverse filtering to find and estimate $\underline{\hat{U}_2}$ for the source signal $\underline{U_2}$ as follows:

$$\underline{\hat{U}_2}=\hat{H}_{12}^T(\hat{H}_{12}\hat{H}_{12}^T)^{-1}\underline{Y}_{12} \quad (10)$$

Where "T" stands for transpose, and"^" on top of the variable symbol means the estimate of that variable.

III. STOCHASTIC CALCULUS FOR THE ESTIMATION OF THE UNKNOWN SOURCES AND THE UNKNOWN TIME VARYING COEFFICIENT:

The linear filter assumption, as a description to the communication/transmission/measurement channel, is just an approximation to reality. Sometimes the media is nonlinear, time varying, random, or all of the above. Moreover, the measured signals are usually noisy. The orders of the filters are usually unknown. All these factors suggest that the FIR filter model is an approximation. To compensate for these assumptions, we suggest to make one or all of the unknown filter coefficients varying with time, i.e. $\alpha_1(t), \alpha_2(t)$, $\alpha_3(t), \alpha_4(t)$, $\alpha_5(t)$, or $\alpha_6(t)$. In this paper, however, we restrict ourselves to only one time-varying parameter/coefficient $\alpha_5(t)$. Any of the other parameters could also be used. Now the problem becomes that of the estimation of the unknown time-varying coefficients. The details of the estimation procedure are given in this section.

It is proposed to model the unknown time-varying coefficients as an Ornstein-Uhlenbeck (OU) processes. Other models could also be used. The OU model, however, is used when the trend in the time-varying parameter is known. The OU model represents a signal that is bouncing around its trend [12]. In our case we assume that all the coefficients are constants and only $\alpha_5(t)=(-(h_{211}+h_{121}(t))+(h_{111}+h_{221}(t)))$ has an SDE of the OU form:

$$d\alpha_5(t)=c[\alpha_5(0)-\alpha_5(t)]dt+e\,dW(t) \quad (11)$$

Where c and e are unknown constants to be estimated, W(t) is a Wiener process, and $\alpha_5(0)=\hat{\alpha}_5$ is the estimated value through the constant coefficient model of the regression method. Using the model of Sec. II, we get:

$$\alpha_5(k) = \frac{[\hat{\alpha}_1 y_2(k-1) + \hat{\alpha}_2 y_2(k-2)]}{y_3(k-1)} + \frac{[\hat{\alpha}_3 y_1(k-1) + \hat{\alpha}_4 y_2(k-2)] - \hat{\alpha}_6 y_3(k-2)}{y_3(k-1)} \tag{12}$$

Where $\hat{\alpha}_i$ is the estimate, through regression, of the coefficient α_i. $\alpha_5(k)$ of (11) is treated as discrete noisy measurements of the true time-varying coefficient $\alpha_5(t)$. Estimating the coefficients of $\alpha_5(t)$, c and e, will yield an estimate for $\alpha_5(t)$.We use the maximum likelihood method to achieve this objective. Other methods could be used as well [1].

A. Estimation of the Diffusion Parameter of Equation (11)

For an observation period [0, T], squaring both sides of (11) we get:

$$d[\alpha_5(t)]^2 = e\, dt \tag{13}$$

Where we use the properties of the Ito calculus, mainly:

$dt\, dt = 0$, $dt\, dW(t) = 0$, $dt\, dW_2(t) = 0$

and $dW(t)\, dW(t) = dt$. (14)

$$e = \frac{1}{T}\int_0^T d[\alpha_5(t)]^2 \tag{15}$$

Thus,

Equation (15) yields an estimate of the diffusion coefficient e.

Equation (12) Estimation of the drift parameter of (11)

Following [1], the maximum likelihood estimate of the drift parameter, c, is given as:

$$c = \frac{\int_0^T \alpha_5(t) d\alpha_5(t)}{\int_0^T \alpha_5^2(t)dt - \alpha_5(0)\int_0^T \alpha_5(t)dt} \tag{16}$$

Thus, an estimate $\hat{\alpha}_5(t)$ for $\alpha_5(t)$ is obtained by substituting (15) and (16) in (11) and we use inverse filtering to find $u_2(t)$ (one of the source signals) as follow:

$$\underline{\hat{U}_2} = \hat{H}_{12}^T(k)(\hat{H}_{12}(k)\hat{H}_{12}^T(k))^{-1}\underline{Y}_{12} \tag{17}$$

Where now

$$\hat{H}_{12}(k) = \begin{bmatrix} -\hat{\alpha}_6 & -\hat{\alpha}_5(1) & \dots & 0 \\ 0 & -\alpha_6 & -\hat{\alpha}_5(2) & \dots \\ \dots & \dots & \dots & \dots \\ 0 & \dots & -\alpha_6 & -\hat{\alpha}_5(N-2) \end{bmatrix} \tag{18}$$

B. Summary of the Proposed Algorithm and its Assumptions

Our blind convolutive BSS technique is based on a set of assumptions:

1. The mixtures of the common inputs, not necessarily independent, are obtained at the output of more sensors than the number of input sources. The channels relating the common inputs to reach distinct outputs are linear time invariant.
2. The channels are each represented by an FIR filter of some unknown order.
3. Since the description of the channels is approximate, one of the parameters is assumed to be varying with time $\alpha_5(t)$. The rest of the parameters are constants.
4. An OU model is assumed for the time varying parameter $\alpha_5(t)$.
5. $\alpha_5(t)$ has an SDE as shown in (11). The parameters, in this equation c and e, are estimated by the maximum likelihood method.
6. Equation (17) is used with the estimate of the time-varying parameter $\hat{\alpha}_5(t)$ to find an estimate for the inputs $u_1(t)$ and $u_2(t)$.

The input source signals are thus reconstructed to within an arbitrary scale factor by deconvolving the measured output signals.

IV. Results:

Two cases are presented. In the first case we recorded two persons having a chat $u_1(t)$, and $u_2(t)$. We simulated a convolutive mixture according to (1). The FIR filters were taken to be first order except $h_{11}(t)$ and $h_{12}(t)$ were taken to be second order. Other experiments were performed for higher order FIR models for the channels. The results were similar.

Thus, $h_{11}(z) = 1 + h_{111}z^{-1} + h_{112}z^{-2}$, $h_{12}(z) = 1 + h_{121}z^{-1} + h_{122}z^{-2}$ and the filter coefficients are shown in the following table I. The proposed BD algorithms assume only first order filters as was shown in Sec. II.

TABLE I, Parameters of the FIR filters representing the convolutive mixing or the communication channels (i and j stand for the ijth filter).

	h11	h12	h21	h22	h31	h32
hij1	-0.9	-0.8	0.4	-0.7	0.4	0.6
hij2	-0.5	0.1				

As we observe in Table I, $h_{111} = -0.9$, $h_{111} = -0.5$. White noises were added, $\varepsilon_1(t)$, $\varepsilon_1(t)$, and $\varepsilon_1(t)$, such that the signal to noise ratio (SNR) of the observations is around 10 db. The signal to noise ratio of the estimates (SNRE) was taken as the measure of performance for the evaluation of the proposed algorithms. It is defined as:

$$SNRE = 10\log_{10}\frac{\sum_k u_i^2(k)}{\sum_k [u_i(k) - \hat{u}_i(k)]^2} \tag{19}$$

Where $\hat{u}_i(k)$ is the estimate (using regression alone or regression followed by OU model) of the source signal $u_i(k)$ at instant k. Following the steps of Sec. III, the source signal $u_2(k)$, the estimated source signal using the method in [8], the estimated source signal using only regression, and the estimated source signal using regression followed by OU model for $\alpha_5(k)$ are all shown in Figure 1. The SNRE using [8] is 1.97 db., using only regression is 10.28 db., and using regression followed by OU models is 11.90 db. In the second case, one channel of EEG signal and the vertical EOG signal were mixed with exactly the same filters as in the first case. the estimated EEG signal, using all methods, are shown in Fig. 2. The SNRE using [8] is 0.34 db., using only regression is 7.40 db., and using regression followed by OU models is 8.90 db.

V. SUMMARY AND CONCLUSIONS

In this report we presented a novel technique to deconvolve the source signals from multiple waveform measurements, using multi-channel blind deconvolution. We applied the technique to the case of two chatting persons and to the case of EEG contaminated by EOG signal. We assumed that one of the FIR filter coefficients is time varying. Its values were estimated using methods based on the stochastic calculus. By this assumption, time-varying parameter, we were able to compensate for the wrong FIR filter order and the possible time variations of the channels. The results showed superior performance for our proposed approach compared to conventional methods.

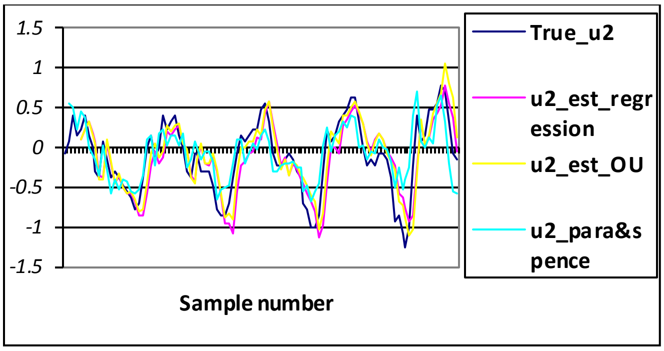

Fig. 1, True u2 and estimated u2 using regression(u2_est_regression), using OU model(u2_est_OU), and using Para&Spence (u2_para&spance)

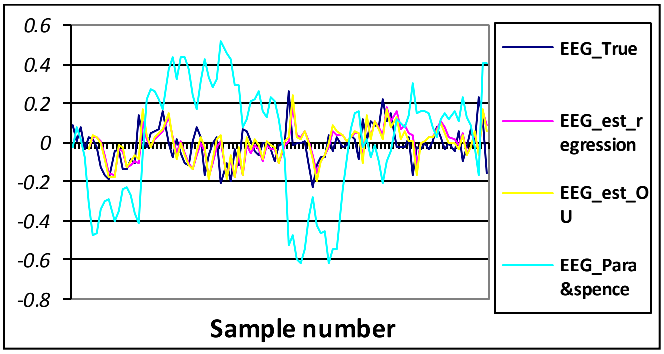

Fig. 2, True EEG and estimated EEG using regression(EEG_est_regression), using OU model(EEG_est_OU), and using Para&spence

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