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NOTE ABOUT THE MULTIPLE GROUP METHOD

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This note directs attention to the basic similarity between a factor analysis method described by Holzinger in 1944 and what Thurstone has called the multiple group method. With minor modifications and the application of Holzinger's method in several successive cycles until the residuals vanish, the methods are essentially the same.

The purpose of this note is to call attention to the fact that a paper by Holzinger on a new factoring method anticipated essential parts of the multiple group method of factoring which was described by the writer in this journal six months later (1, 2). Holzinger described his method as a factoring method that is applicable to a special kind of correlation matrix whereas, in fact, his method is entirely general with only slight modifications or extension. The statements that Holzinger made about his method and those which the writer has made about the multiple group method are entirely at variance so that it is natural for the reader to infer that the two methods are entirely different. Closer inspection shows that Holzinger's method is not restricted to the special case which he describes. Holzinger had a better idea than he thought and his method can be readily extended to cover any type of correlation matrix. It then becomes what the writer has called the multiple group method of factoring.

When my attention was called to the similarity of the two papers, it was difficult to see how they could describe essentially the same method because of our respective statements. Holzinger said: "The simple method here presented is applicable to the factoring of a correlation matrix in case the latter can be sectioned into portions of approximately rank unity"; (1, 257) and also: "There is no guarantee, of course, that any correlation matrix can be factored in the above manner, ..." (1, 261). In my description of the multiple group method in the *Psychometrika* paper (2) and in the text on multiple-factor analysis (3), I said: "The multiple group method of factoring is general in that it can be used on a correlation matrix of any rank, any order, and any configuration of test vectors, ..." (3, 170). Holzinger limited himself to the case in which the variables could be so arranged that each section of the correlation matrix was of unit rank. This restriction does not apply to the multiple group method. I said: "The method of selecting the groups of tests is not crucial for the multiple group method of factoring, since the only requirement is that the centroid vectors for the several groups shall be linearly independent"; (3, 171) and, further: "The computer will select, for each group, those tests which are nearly collinear, if such groups can be found in the correlation matrix. If such groups are not readily seen by inspection, then the grouping can be carried out by some other routine"; (3, 171) and "Further, it is not necessary that all of the tests in each group be collinear, or nearly so. All that is required is that the whole correlation matrix be divided into sections that are linearly independent." (2, 78). Holzinger said: "The adequacy of the solution may be tested in each section, . . . ," (1, 257) and he describes the procedure for "testing the solution by checking the rank of submatrices." (1, 261). The check consists in verifying that the correlations are proportional in each section. No such restriction applies to the multiple group method. On this subject I said: "The grouping was chosen quite arbitrarily in this example in order to illustrate in a numerical example that the multiple group method of factoring is independent of the method of grouping the variables. The only requirement is that the centroid vectors of the several groups must be linearly independent." (3, 175). On the treatment of residuals, I said: "If the first estimate of the number of clusters is too small, one merely repeats the process until the residuals vanish." (3, 175).

With only a few minor modifications, Holzinger's method becomes what I have called the multiple group method of factoring. These minor modifications are as follows:

1) Eliminate the restriction that the correlation matrix shall be divisible into sections of unit rank.

2) Eliminate the reservation that the method may not be applicable to any given correlation matrix. The method can easily be made general so that it is applicable to a correlation matrix of any order, any rank, and any configuration of tests.

3) Eliminate the check by which each section of the matrix is tested for unit rank because this check is not applicable to the multiple group method of factoring.

4) Holzinger's description implies that the number of groups is equal to the rank of the correlation matrix. This restriction is not necessary for the multiple group method, which implies only that the number of groups is equal to or less than the rank of the reduced correlation matrix. If the number of groups should exceed the rank

of the reduced correlation matrix, then this fact will be discovered in factoring the correlations between the centroid axes by the diagonal method. This is one step in the multiple group method.

5) Instead of avoiding the computation of an orthogonal factor matrix F, this matrix is the objective to be attained in the multiple group method (1, 261).

The actual computations can then proceed just as Holzinger has described. The residual correlations are then computed for the number of factors, which is equal to the number of groups that was used.

6) If the residuals do not vanish, then one or more groups of tests are selected from the residual matrix, and the procedure is repeated. The adequacy of the method is not determined by the residuals. If they do not vanish, the procedure is repeated until they do vanish. The additional columns so found are simply added to the columns of the factor matrix F that were obtained in previous cycles.

If these minor modifications are made in Holzinger's paper, and if the method which he describes is applied in several successive cycles until the residuals vanish, then his method becomes what the writer has called the multiple group method of factoring. Our failure to recognize the relation between the two papers was probably caused by the exposition in the earlier paper, which was limited explicitly to a special kind of correlation matrix, whereas the underlying idea could easily be extended to cover any correlation matrix. This circumstance leads to the inference that Holzinger evidently had a better idea than he realized when he limited his description to sections of unit rank.

REFERENCES

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3. Thurstone, L. L. Multiple-factor analysis. Chicago: University of Chicago Press, 1947.