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A Theory of Marriage: Part I

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I present in this paper the skeleton of a theory of marriage. The two basic assumptions are that each person tries to do as well as possible and that the "marriage market" is in equilibrium. With the aid of several additional simplifying assumptions, I derive a number of significant implications about behavior in this market. For example, the gain to a man and woman from marrying compared to remaining single is shown to depend positively on their incomes, human capital, and relative difference in wage rates. The theory also implies that men differing in physical capital, education or intelligence (aside from their effects on wage rates), height, race, or many other traits will tend to marry women with like values of these traits, whereas the correlation between mates for wage rates or for traits of men and women that are close substitutes in household production will tend to be negative. The theory does not take the division of output between mates as given, but rather derives it from the nature of the marriage market equilibrium. The division is determined here, as in other markets, by marginal productivities, and these are affected by the human and physical capital of different persons, sex ratios (that is, the relative numbers of men and women), and some other variables.

1. Introduction

In recent years, economists have used economic theory more boldly to explain behavior outside the monetary market sector, and increasing numbers of noneconomists have been following their examples. As a result, racial discrimination, fertility, politics, crime, education, statistical

decision making, adversary situations, labor-force participation, the uses of "leisure" time, and other behavior are much better understood. Indeed, economic theory may well be on its way to providing a unified framework for *all* behavior involving scarce resources, nonmarket as well as market, nonmonetary as well as monetary, small group as well as competitive.

Yet, one type of behavior has been almost completely ignored by economists,[1] although scarce resources are used and it has been followed in some form by practically all adults in every recorded society. I refer to marriage. Marital patterns have major implications for, among other things, the number of births and population growth, labor-force participation of women, inequality in income, ability, and other characteristics among families, genetical natural selection of different characteristics over time, and the allocation of leisure and other household resources. Therefore, the neglect of marriage by economists is either a major oversight or persuasive evidence of the limited scope of economic analysis.

In this essay, it is argued that marriage is no exception and can be successfully analyzed within the framework provided by modern economics. If correct, this is compelling additional evidence on the unifying power of economic analysis.

Two simple principles form the heart of the analysis. The first is that, since marriage is practically always voluntary, either by the persons marrying or their parents, the theory of preferences can be readily applied, and persons marrying (or their parents) can be assumed to expect to raise their utility level above what it would be were they to remain single. The second is that, since many men and women compete as they seek mates, a *market* in marriages can be presumed to exist. Each person tries to find the best mate, subject to the restrictions imposed by market conditions.

These two principles easily explain why most adults are married and why sorting of mates by wealth, education, and other characteristics is similar under apparently quite different conditions. Yet marital patterns differ among societies and change over time in a variety of ways that challenge any single theory. In some societies divorce is relatively common, in others, virtually impossible, and in Western countries it has grown rapidly during the last half-century. Some societies adjust to legal difficulties in receiving divorces by delaying marriage, whereas others adjust by developing more flexible "consensual," "common-law," or "trial" marriages. In many the bride brings a dowry, in others the groom pays a bride-price, and in still others couples marry for "love" and disdain any financial bargaining. In some the newly married usually set up their own household, in others they live with one set of parents.

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[1] To the best of my knowledge, the only exception prior to my own work is an unpublished paper by Gronau (1970). His paper helped stimulate my interest in the subject.

I do not pretend to have developed the analysis sufficiently to explain all the similarities and differences in marital patterns across cultures or over time. But the "economic" approach does quite well, certainly far better than any available alternative.[2] It is hoped that the present essay will stimulate others to carry the analysis into these uncharted areas.

Section 2 considers the determinants of the gain from marriage compared to remaining single for one man and one woman. The gain is shown to be related to the "compatibility" or "complementarity" of their time, goods, and other inputs used in household production.

Section 3 considers how a group of men and women sort themselves by market and nonmarket characteristics. Positive assortive mating—a positive correlation between the values of the traits of husbands and wives—is generally optimal, one main exception being the sorting by the earning power of men and women, where a negative correlation is indicated. Empirically, positive assortive mating is the most common and applies to IQ, education, height, attractiveness, skin color, ethnic origin, and other characteristics.

Section 4 considers how the total output of a household gets divided between the husband and wife. The division is not usually fixed, say at 50-50, or determined mechanically, but changes as the supply of and demand for different kinds of mates changes.

Part II, to be published in a later issue of this *Journal*, develops various extensions and modifications of the relatively simple analysis in this part. "Caring" is defined, and some of its effects on optimal sorting and the gain from marriage are treated. The factors determining the incidence of polygamous marital arrangements are considered. The assumption that the characteristics of potential mates are known with certainty is dropped, and the resulting "search" for mates, delays in marriage, trial marriage, and divorce are analyzed. Divorce and the duration of marriage are also related to specific investments made during marriage in the form of children, attachments, and other ways. We also briefly explore the implications of different marital patterns for fertility, genetical natural selection, and the inequality in family incomes and home environments.

2. The Gain from Marriage

This section considers two persons, M and F, who must decide whether to marry each other or remain single. For the present, "marriage" simply means that they share the same household. We assume that

[2] Some of the best work has been done by Goode (1963), but there is no systematic theory in any of his fine work.

marriage occurs if, and only if, both of them are made better off—that is, increase their utility.[3]

Following recent developments in the theory of household behavior, we assume that utility depends directly not on the goods and services purchased in the market place, but on the commodities produced "by" each household.[4] They are produced partly with market goods and services and partly with the own time of different household members. Most important for present purposes, commodities are not marketable or transferable among households, although they may be transferable among members of the same household.

Household-produced commodities are numerous and include the quality of meals, the quality and quantity of children, prestige, recreation, companionship, love, and health status. Consequently, they cannot be identified with consumption or output as usually measured: they cover a much broader range of human activities and aims. We assume, however, that all commodities can be combined into a single aggregate, denoted by Z. A sufficient condition to justify aggregation with fixed weights is that all commodities have constant returns to scale, use factors in the same proportion, and are affected in the same way by productivity-augmenting variables, such as education. Then different commodities could be converted into their equivalent in terms of any single commodity by using the fixed relative commodity prices as weights.[5] These weights would be independent of the scale of commodity outputs, the prices of goods and the time of different members, and the level of productivity.

Maximizing utility thus becomes equivalent for each person to maximizing the amount of Z that he or she receives. Moreover, our concentration on the output and distribution of Z does not presuppose transferable utilities, the same preference function for different members of the same household, or other special assumptions about preferences.

Each household has a production function that relates its total output of Z to different inputs:

$$Z = f \quad (x_1, \ldots, x_m; t_1, \ldots, t_k; E), \tag{1}$$

where the x_i are various market goods and services, the t_j are the time

[3] More precisely, if they *expect* to increase their utility, since the latter is not known with certainty. Part II discusses some consequences of this uncertainty, especially for the time spent searching for an appropriate mate and the incidence of divorce and other marital separations.

[4] An exposition of this approach is given in Michael and Becker (1972).

[5] One serious limitation of these assumptions is that they exclude the output of commodities from entering the production functions of other commodities. With such "joint production," the relative price of a commodity would depend partly on the outputs of other commodities (Grossman 1971). Joint production can result in complementarity in consumption, and thereby affect the gain from marriage and the sorting of mates. See the brief discussion in section 3.

inputs of different household members, and E represents "environmental" variables. The budget constraint for the x_i can be written as:

$$\sum^m p_i x_i = \sum^k w_j l_j + v, \tag{2}$$

where w_j is the wage rate of the jth member, l_j the time he spends working in the market sector, and v property income. The l_j and t_j are related by the basic time constraint

$$l_j + t_j = T \qquad \text{all } j, \tag{3}$$

where T is the total time of each member. By substituting equation (3) into (2), the goods and time constraints can be combined into a single "full" income constraint:

$$\sum^m p_i x_i + \sum^k w_j t_j = \sum^k w_j T + v = S, \tag{4}$$

where S stands for full income, the maximum money income achievable, if the w_j are constants.

We assume that a reduction in the household's total output of Z makes no member better off and some worse off.[6] Consequently, each member would be willing to cooperate in the allocation of his time and goods to help maximize the total output of Z. Necessary conditions to maximize Z include

$$\frac{MP_{t_i} \equiv (\partial Z / \partial t_i)}{MP_{t_j} \equiv (\partial Z / \partial t_j)} = \frac{w_i}{w_j}, \qquad \text{for all } 0 < t < T. \tag{5}$$

If the household time of the kth member $= T$, then

$$\frac{MP_{t_k}}{MP_{t_j}} = \frac{\mu_k}{w_j}, \tag{6}$$

where $\mu_k \geq w_k$ is the "shadow" price of the time of k. Also

$$\frac{MP_{x_i}}{MP_{t_j}} = \frac{p_i}{w_j} \qquad \text{for all } x_i > 0 \text{ and } 0 < t_j < T. \tag{7}$$

Each member must cooperate and allocate his time between the market and nonmarket sectors in the appropriate proportions.

If M and F are married, their household is assumed to contain only the two time inputs t_m and t_f; for simplicity, the time of children and others living in the same household is ignored. As long as they remain married, $T_m = T_f = 24$ hours per day, 168 hours per week, and so forth, and conditions (5) to (7) determine the allocation of the time of M and F between the market and nonmarket sectors. More time would be allocated to the market sector by M than by F (less to the nonmarket sector) if

[6] This assumption is modified in section 3 and in Part II.

$w_m > w_f$ and if $MP_{t_f} \geq MP_{t_m}$ when $t_f = t_m$. Indeed, F would specialize in the nonmarket sector $(l_f = 0)$ if either w_m/w_f or MP_{t_f}/MP_{t_m} were sufficiently large.

A singles household is taken to be exactly the same as a married one except that $T_f = 0$ when M is single and $T_m = 0$ when F is single. A singles household allocates only its own time between the market and nonmarket sectors to satisfy equation (7). Single persons generally allocate their time differently than married persons because the former do not have time and goods supplied by a mate. These differences depend partly on the elasticities of substitution among the x_i, t_f, and t_m, and partly on the differences between the market wage rates w_m and w_f. For example, single F are more likely to "work" more than married F and single M less than married M, the greater the percentage excess of w_m over w_f. Empirically, single women clearly "work" more than married women and single men less than married men.[7]

If Z_{m0} and Z_{0f} represent the maximum outputs of single M and F, and m_{mf} and f_{mf} their incomes when married, a necessary condition for M and F to marry is that

$$m_{mf} \geq Z_{m0}$$
$$f_{mf} \geq Z_{0f}. \tag{8}$$

If $m_{mf} + f_{mf}$, the total income produced by the marriage, is identified with the output of the marriage,[8] a necessary condition for marriage is then that

$$m_{mf} + f_{mf} \equiv Z_{mf} \geq Z_{m0} + Z_{0f}. \tag{9}$$

Since most men and women over age 20 are married in all societies, equation (9) must generally hold because of fundamental reasons that are not unique to time or place. We have a useful framework for discovering these reasons.

The obvious explanation for marriages between men and women lies in the desire to raise own children and the physical and emotional attraction between sexes. Nothing distinguishes married households more from singles households or from those with several members of the same sex than the presence, even indirectly, of children. Sexual gratification, cleaning, feeding, and other services can be purchased, but not *own* children:[9] both the man and woman are required to produce their own children and perhaps to raise them. The physical and emotional involvement called "love" is also primarily between persons of the opposite sex.

[7] See, e.g., *Employment Status and Work Experience* (U.S., Bureau of the Census 1963), tables 4 and 12.

[8] Income and output can differ, however, because some output may be jointly consumed. See the discussion in section 3 and Part II.

[9] The market in adoptions is used primarily by couples experiencing difficulties in having their own children and by couples paid to raise other persons' children.

Moreover, persons in love can reduce the cost of frequent contact and of resource transfers[10] between each other by sharing the same household.

Economies of scale may be secured by joining households, but two or more males or females could equally well take advantage of these economies and do so when they share an apartment and cooking. Consequently, the explanation of why men and women live together must go beyond economies of scale.

The importance of own children and love implies that, even with constant returns to scale, M (now standing for a man) and F (now standing for a woman) gain from marriage because t_m and t_f are not perfect substitutes for each other or for goods and services supplied by market firms or households. When substitution is imperfect, single persons cannot produce small-scale equivalents of the optimal combination of inputs achieved by married couples.

Consequently, the "shadow" price of an hour of t_f to a single M—the price he would be willing to pay for t_f—would exceed w_f, and the "shadow" price of t_m to a single F—the price she would be willing to pay for t_m—would exceed w_m. Both gain from marriage because M then, in effect, can buy an hour of t_f at w_f and F can buy an hour of t_m at w_m, lower prices they then would be willing to pay. Of course, this is also why married households use positive amounts of t_f and t_m.

Our explanation of the gain from marriage focuses on the complementarity between M and F. The gain from complementarity can be illustrated in much-exaggerated measure by assuming that the production function relating Z to t_m, t_f, and x has the Cobb-Douglas form

$$Z = kx^a t_m{}^b t_f{}^c. \tag{10}$$

Clearly, $Z_{m0} = Z_{0f} = 0$ since both t_m and t_f are needed to produce Z ($Z = 0$ if t_m or $t_f = 0$), whereas Z_{mf} can take any value. Other functions have less extreme "complementarity" and permit positive production when some inputs are absent but less "efficiently" than when all are present.

Some sociological literature also suggests that complementarity between men and women is the major source of the gain from marriage (Winch 1958, 1967; Goode 1963), but the meaning of "complementarity" is left rather vague and ill defined. By building on the substantial economic literature that analyzes complementarity and substitution in production, we have shown how "complementarity" determines the gain from marriage.

Can this analysis also explain why one man is typically married to one woman, rather than one man to several women, several men to one woman, or several men to several women? The importance of own children is sufficient to explain why marriages of several men to one or several

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[10] The relation between love and such transfers is discussed in Part II.

women are uncommon since it would be difficult to identify the father of a child if many men had access to the same woman, whereas the identity of the mother is always known. The marriage of several women to one man does not suffer from this defect, and, indeed, such marriages have been more common. However, if the sex ratio equalled about unity, each household having several women and one man would have to be balanced by households having only men. If we assume that all men and all women are identical, and if we make the rather plausible assumption of "diminishing returns" from adding persons to a household having one man and one woman, the total output from say two single male households and one household with three women and one man would be smaller than the total output from three households each having one man and one woman.[11] Consequently, monagamous unions—one man married to one woman—predominate because it is the most efficient marital form. Polygamy is encouraged when the sex ratio is significantly different from unity and when men or women differ greatly in wealth, ability, or other attributes.[12]

Our definition of marriage in terms of whether a man and a woman share the same household differs from the legal definition because our definition includes persons in "consensual" and casual unions and excludes legally married persons who are separated. However, our analysis does have useful implications about the choice between legally recognized and other unions (Kogut 1972), as well as about the decisions to remain married, divorce, remarry legally, remarry "consensually," remain single, and so forth, that must be made in the course of a lifetime (see Part II).

The gain from marriage has to be balanced against the costs, including legal fees and the cost of searching for a mate, to determine whether marriage is worthwhile. The larger the gain is relative to costs, the larger the net gain from marriage; presumably, therefore, the larger too is the fraction of persons who marry. We now consider the more important determinants of this net gain.

The gain is greater the more complementary are the inputs: the time of spouses and market goods. Since we have argued that these inputs are complementary in good part because of the desire to raise own children, the gain would be positively related to the importance of children. Hence, persons desiring relatively few or low-"quality" children either marry later, end their marriages earlier, or do both.[13]

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[11] For example, assume that singles households have an output of 5 units of Z, one man and one woman 13 units, one man and two women 20 units, and one man and three women 26 units. Three households each with one man and one woman would produce 39 units, whereas two single male households and one household having three women and one man would produce only 36 units.

[12] See the more extensive discussion of polygamy in Part II.

[13] A further discussion can be found in Keeley (1972).

The gain from marriage also depends on market opportunities. The effect of a change in opportunities can be analyzed most easily by equating the maximum output of any household to its full income deflated by the average cost of producing a unit of output. For example, with constant returns to scale, the output of a married household with both members participating in the labor force can be written as

$$Z_{mf} = \frac{\text{full income}}{\text{average cost of production}} \equiv \frac{S_{mf}}{C_{mf}(w_m, w_f, p)} \equiv \frac{S_m + S_f}{C_{mf}},$$

$$(11)$$

where C_{mf} depends on the wage rates of t_m and t_f and the price of x.[14] The output of a singles household can be written in the same form except that only one price of time enters the average cost functions C_m and C_f.[15]

What is the effect of an increase in income on the incentive to marry? If only the property incomes of M and F, v_m and v_f, rose exogenously by the same percentage, and if $v_m/S_m = v_f/S_f$, then S_m, S_f, and S_{mf} would all rise by the same percentage. With constant returns to scale, Z_{m0}, Z_{0f}, and Z_{mf}, and thus the absolute gain from marriage, would also rise by the same percentage as full income since neither C_{mf}, C_m, nor C_f would be affected by the rise in property incomes, as long as both M and F continue to participate in the labor force,[16] and assuming that property income is unaffected by the allocation of time.[17] Since a rise in property income should not greatly affect the cost of getting married, the incentive to marry would also rise.

The effect of a rise in wage rates alone[18] on the incentive to marry is less clear-cut. A rise in the wage rates of M and F by the same percentage would increase outputs by smaller percentages than full incomes, even with constant returns to scale, because costs of production also rise.[19] Moreover, the cost of getting married rises to the extent that the own time of M and F enters into search and other marital costs. Consequently, the

[14] Duality theory shows that C is the dual of the production function.

[15] Or, alternatively, the shadow price of F to M enters C_m, and the shadow price of M to F enters C_f.

[16] Even if married F did not participate in the labor force, the percentage rise in Z_{mf} would still equal the share of property income in full income (see section 2 of the Appendix).

[17] The gain from marriage would increase even more if the income from nonhuman capital, i.e., property income, was positively related to the time allocated to "portfolio management" (see the discussion in section 3).

[18] By alone is meant in particular that the productivity of time in household production or marital search is unchanged.

[19] The percentage rise in output equals the percentage rise in wage rates multiplied by the ratio of total earnings to full income. Although this relation holds whether or not married F is in the labor force (see section 2 of the Appendix), the ratio of total earnings to full income can depend—positively or negatively—on her participation.

effect on the net gain from marriage is not clear a priori and depends on the relative importance of own time in marriage costs and in the production of output in single and married households.

Consequently, our analysis predicts that a rise in property income, necessarily, and a rise in wage rates, possibly, increase the incentive to marry. This implication runs counter to the popular opinion that poor persons marry earlier and divorce less than rich persons but is consistent with the empirical evidence. In the United States, at least, the probability of separation and divorce is negatively related to income (U.S., Bureau of the Census 1971). Keeley (1972) finds too that when years of schooling and a few other variables are held constant, higher-wage persons appear to marry earlier than others.

Our analysis implies that a rise in w_f relative to w_m, F's wage rate relative to M's, with the productivity of time in the nonmarket sector held constant, would decrease the gain from marriage if w_f were less than w_m: the gain from substituting M's time in the market for F's time (and F's time in the household for M's time) is greater the lower w_f is relative to w_m. As a proof, consider an increase in w_f "compensated" by a sufficient decrease in w_m to maintain constant the combined output of the two singles households. The increase in w_f would not increase married output as much as the decrease in w_m would decrease it if married F worked sufficiently fewer hours in the market sector than single F, and married M worked at least as much as single M. Since married women do work much less than single women and married men work more than single men, an increase in the wage rate of women relative to men would decrease the incentive to marry.[20] As supporting evidence, note that American states that have higher wage rates of women relative to men also have smaller fractions of men and women who are married (Santos 1970; Freiden 1972).

The gain from marriage also depends on traits, such as beauty, intelligence, and education, that affect nonmarket productivity as well, perhaps, as market opportunities. The analysis of sorting in section 3b implies that an increase in the value of traits that have a positive effect on nonmarket productivity, market productivity held constant, would generally increase the gain from marriage. Presumably this helps explain why, for example, less attractive or less intelligent persons are less likely to marry than are more attractive or more intelligent persons.[21]

[20] A fortiori, if married women were not in the labor force, a compensated increase in their wage rate would decrease the incentive to marry since an increase in their wage rate would not affect married output, whereas a decrease in the male wage rate would decrease output. This footnote as well as the text assumes that compensated changes in w_f and w_m do not much affect the cost of getting married.

[21] Evidence on marriage rates by intelligence can be found in Higgins, Reed, and Reed (1962) and Bajema (1963). The statement on marriage rates by attractiveness is not based on any statistical evidence.

3.   The Marriage Market and Sorting of Mates

a) Optimal Sorting

We now consider not one M and F who must decide whether to marry or remain single, but many M's and F's who must decide whom to marry among numerous potential candidates, as well as whether to marry. If there are n M's and n F's (unequal numbers of M and F are discussed in section 4), each is assumed to know all the relevant[22] entries in an $n + 1 \times n + 1$ payoff matrix showing the maximum household commodity output that can be produced by any combination of M and F:

$$
\begin{array}{c|ccc}
 & F_1 & \cdots & F_n \\
\hline
M_1 & Z_{11} & \cdots & Z_{1n}\ Z_{10} \\
 & & Z_{ij} & \\
M_n & Z_{n1} & \cdots & Z_{nn}\ Z_{n0} \\
 & Z_{01} & \cdots & Z_{0n}
\end{array}
\tag{12}
$$

The last row and column give the output of single M and F. Each person has $n + 1$ possibilities and the $2n$ persons together have $n^2 + 2n$ possibilities. We assume that each person gains from marriage, so that the singles row and column of the payoff matrix can be ignored.

There are $n!$ different combinations that permit each M to marry one F and vice versa; that is, there are $n!$ ways to select one entry in each married row and column. The total output over all marriages produced by any one sorting can be written as

$$
Z^k = \sum_{i \in M,\ j \in F} Z_{ij}, \qquad k = 1, \ldots, n!.
\tag{13}
$$

Number one of the sortings that maximizes total output so that its entries lie along the diagonal and write

$$
Z^* = \sum_{i=1}^{n} Z_{ii} = \max_k Z^k \geq Z^k \qquad \text{all } k.
\tag{14}
$$

If the total output of any marriage is divided between the mates,

$$
m_{ij} + f_{ij} = Z_{ij},
\tag{15}
$$

where m_{ij} is the income of the ith M from marriage to the jth F, and similarly for f_{ij}. If each chooses the mate who maximizes his or her "income," the optimal sorting must have the property that persons not married to each other could not marry and make one better off without making the other worse off. In game theoretic language, the optimal

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[22] That is, all the entries relevant to their decisions. This strong assumption of sufficient information is relaxed in Part II, where "search" for a mate is analyzed.

sorting is in the "core" since no "coalition" outside the core could make any of its members better off without making some worse off.

Persons entering noncore marriages could not produce more together than the sum of their incomes in the core. For, if they could, and if any division of output between mates were feasible, they could find a division of their output that would make each better off, a contradiction of the definition of the core. If the sorting along the diagonal were in the core, this condition states that

$$m_{ii} + f_{jj} \geq Z_{ij} \qquad \text{all } i \text{ and } j. \tag{16}$$

Conditions (15) and (16) immediately rule out any sorting that does not maximize the total output of commodities over all marriages, for at least one M and one F would then be better off with each other than with their mates.[23] Moreover, the theory of optimal assignments, which has the same mathematical structure as the sorting of persons by marriage, implies the existence of a set of incomes that satisfy conditions (15) and (16) for sortings that maximize total output.[24]

The solution can be illustrated with the following 2×2 matrix of payoffs:

$$\begin{array}{c} \\ M_1 \\ M_2 \end{array} \begin{array}{cc} F_1 & F_2 \\ \begin{bmatrix} 8 & 4 \\ 9 & 7 \end{bmatrix} \end{array}. \tag{17}$$

Although the maximum output in any marriage is between M_2 and F_1, the optimal sorting is M_1 to F_1 and M_2 to F_2. For, if $m_{11} = 3$, $f_{11} = 5$, $m_{22} = 5$, and $f_{22} = 2$, M_2 and F_1 have no incentive to marry since $m_{22} + f_{11} = 10 > 9$, and neither do M_1 and F_2 since $m_{11} + f_{22} = 5 > 4$. In other words, the marriage market chooses not the maximum household commodity output of any single marriage but the maximum sum of the outputs over all marriages, just as competitive product markets maximize the sum of the outputs over all firms. Let us stress again that the commodity output maximized by all households is not to be identified with national output as usually measured, but includes conversation, the quantity and quality of children, and other outputs that never enter or enter only imperfectly into the usual measures. Put still differently, the marriage market acts as if it maximizes not the

[23] If M_i married F_j and F_i married M_p in an optimal sorting that did not maximize total output, condition (16) requires that $m_{ij} + f_{pi} \geq Z_{ii}$, all ij, pi, or, by summation,

$$Z_p = \sum_{\text{all } ij,\, pi}^{n} m_{ij} + f_{pi} \geq \sum_i Z_{ii} = Z^*.$$

Since Z^* is the maximum total output, it must exceed Z_p, by assumption less than the maximum. Hence, a contradiction, and a proof that the optimal sorting cannot produce less than the maximum total output.

[24] For a proof, see Koopmans and Beckman (1957).

gain from marriage compared to remaining single for any particular marriage, but the average gain over all marriages.[25]

Each marriage can be considered a two-person firm with either member being the "entrepreneur" who "hires" the other at the "salary" m_{ij} or f_{ij} and receives residual "profits" of $Z_{ij} - m_{ij}$ or $Z_{ij} - f_{ij}$. Another interpretation of the optimal sorting is that only it enables each "entrepreneur" to maximize "profits" for given "salaries" of mates because only the optimal sorting satisfies condition (16). With all other sortings, some "entrepreneurs" could do better by "hiring" different mates than those assigned to them.

b) Assortive Mating

We now consider the optimal sorting when M and F differ in a trait, or set of traits, such as intelligence, race, religion, education, wage rate, height, aggressiveness, tendency to nurture, or age. Psychologists and sociologists have frequently discussed whether likes or unlikes mate, and geneticists have occasionally assumed positive or negative assortive mating instead of random mating. But no systematic analysis has developed that predicts for different kinds of traits when likes or unlikes are motivated to mate.[26] Our analysis implies that likes or unlikes mate when that maximizes total household commodity output[27] over all marriages, regardless of whether the trait is financial (like wage rates and property income), or genetical (like height and intelligence), or psychological (like aggressiveness and passiveness).

Assume that M differs only in the quantitative trait A_m, and F only in

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[25] Clearly,

$$\left[\sum_{i}^{n} Z_{ii} - \sum_{j=1}^{n} (Z_{0j} + Z_{j0}) \right] \bigg/ n = \left\{ \sum_{i} [Z_{ii} - (Z_{0j} + Z_{j0})] \right\} \bigg/ n$$

is maximized if

$$\sum Z_{ii}$$

is, since Z_{0j} and Z_{j0} are given and independent of the marital sorting.

[26] Winch (1958) essentially assumes that each person tries to maximize utility ("In mate selection each individual seeks within his or her field of eligibles for that person who gives the greatest promise of providing him or her with maximum need gratification" [pp. 88–89]) and stresses complementary needs as a prerequisite for mating (especially in chap. 4), but he only considers psychological traits, brings in "eligibles" as a deus ex machina, and nowhere shows how mating by complementary needs brings equilibrium into the marriage market.

[27] Let me emphasize again that commodity output is not the same as national product as usually measured, but includes children, companionship, health, and a variety of other commodities.

A_f, that each trait has a monotonic effect on the output of any marriage, and that higher values have the larger effect:

$$\frac{\partial Z_{ij}(A_m, A_f)}{\partial A_m} > 0, \qquad \frac{\partial Z_{ij}}{\partial A_f}(A_m, A_f) > 0. \tag{18}$$

If increasing both A_m and A_f adds the same amount to output as the sum of the additions when each is increased separately, all sortings of M and F would give the same total output. On the other hand, if increasing both adds more to output than the sum of the separate additions, a sorting of large A_m with large A_f and small A_m with small A_f would give the greatest total output since an increase in A_m reinforces the effect of an increase in A_f. The converse holds if increasing both adds less to output than the sum of the separate additions. Mathematically, this states that positive or negative assortative mating—mating of likes or unlikes—is optimal as

$$\frac{\partial^2 Z(A_m, A_f)}{\partial A_m \, \partial A_f} \gtrless 0 \tag{19}$$

(proofs in Appendix, section 1).

Consider, as an example, a matrix of outputs when $n = 2$:

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} A_1 \qquad A_2 \\ \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \end{array}, \qquad \text{with } A_2 > A_1. \tag{20}$$

If $Z_{22} - Z_{12} > Z_{21} - Z_{11}$, if equality (19) is positive, then obviously $Z_{11} + Z_{22} > Z_{12} + Z_{22}$, and a positive correlation between A_m and A_f maximizes total output, as predicted from (19).

One tradition in production theory distinguishes substitution from complementarity by the sign of the cross-derivative of output with respect to different inputs into a production function. Although condition (19) is not defined in terms of household production functions, duality theory implies that the same condition holds when A_m and A_f are treated as inputs into these production functions.[28] Condition (19) says, therefore, that the association of likes is optimal when traits are complements and the association of unlikes is optimal when they are substitutes, a plausible conclusion since high values of different traits reinforce each other when they are complements, and offset each other when they are substitutes.

Economists have generally considered the sorting of different *quantities* of different traits, such as labor and capital, not different *qualities* of the same trait. Although sorting by quantity and quality are related analytically, many applications of sorting by quality are also directly available in economics, such as the optimal sorting of more able workers and more

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[28] Wage rates or other monetary variables, however, cannot be treated as productive inputs.

able firms,[29] more "modern" farms and more able farmers, or more informed customers and more honest shopkeepers. As already mentioned (n. 26 above), some sociologists have considered "complementarity" to be an important determinant of sorting, but have not given a rigorous analysis of the effects of "complementarity" or embedded their discussions in the context of a functioning marriage market.

Mating of likes—positive assortive mating—is extremely common, whether measured by intelligence, height, skin color, age, education, family background, or religion, although unlikes sometimes also mate, as measured, say, by an inclination to nurture or succor, to dominate or be deferential. This suggests that traits are typically but not always complements.

The determinants of complementarity and substitutability are best discovered by going explicitly to the household production function and the maximization process. All households are assumed to have the same production *function*; that is, if the inputs of time, goods, and *all* traits were exactly the same, the output of commodities would be exactly the same. Different families can, of course, produce different outputs from the same input of goods and time if their education, ability, and so forth, differ.

We consider a number of determinants in turn. First, if M and F differ *only* in their market wage rates—each M and each F are identical in all other market and in nonmarket traits—according to equation (11), the optimal output between M and F who are both participating in the labor force can be written as

$$Z = \frac{S}{C(w_m, w_f, p)} , \tag{21}$$

where the subscripts on Z, S, and C have been omitted and constant returns to scale assumed. Then, by differentiation and by using equation (4),

$$Z^m = \frac{T}{C} - \frac{S}{C^2} C^m,$$

where

$$Z^m = \frac{\partial Z}{\partial w_m} \quad \text{and} \quad C^m \equiv \frac{\partial C}{\partial w_m} \tag{22}$$

Since

$$C^m = t_m Z^{-1}, \tag{23}$$

where t_m is the time spent by M in the household,

$$Z^m = l_m C^{-1} > 0 \tag{24}$$

[29] This sorting is discussed for Japanese firms by Kurantani (1972). Hicks (1948, chap. 2, sec. 3) asserts that more able workers work for more able firms without offering any proof. Black (1926) discusses the sorting of workers and firms with a few numerical examples.

if l_m, the time spent at work, is greater than zero. Similarly,

$$Z^f = \frac{T}{C} - \frac{S}{C^2} \, C^f = l_f C^{-1} > 0. \tag{25}$$

Positive or negative assortive mating by wage rates is optimal as

$$\frac{\partial^2 Z}{\partial w_m \, \partial w_f} \equiv Z^{mf} \equiv Z^{fm} \gtrless 0. \tag{26}$$

Differentiate Z^f with respect to w_m to get

$$Z^{fm} = -C^{-2}C^m l_f + C^{-1} \frac{\partial l_f}{\partial w_m}. \tag{27}$$

The first term on the right is clearly negative, so Z^{fm} will be negative if the second term, $\partial l_f / \partial w_m \le 0$, is nonpositive, that is, if t_m and t_f are not gross complements, as these terms are usually defined.[30] Consequently, a perfectly negative rank correlation between w_m and w_f would maximize total commodity output if the time of M and F were not such gross complements as to swamp the first term in (27). Considerable empirical evidence supports the conclusion that t_m and t_f are not gross complements (Ofek 1970; Smith 1972).

A negative correlation between w_m and w_f maximizes total output because the gain from the division of labor is maximized. Low-wage F should spend more time in household production than high-wage F because the foregone value of the time of low-wage F is lower; similarly, low-wage M should spend more time in household production than high-wage M. By mating low-wage F with high-wage M and low-wage M with high-wage F, the cheaper time of both M and F is used more extensively in household production, and the expensive time of both is used more extensively in market production.

All persons have been assumed to participate in the labor force. During any year, however, most married women in the United States do not participate, and a significant number never really participate throughout their married life. Our analysis does predict that many women would have only a weak attachment to the labor force since low-wage women would be discouraged from participation both by their low wage and by the high wage of their husbands.[31]

[30] This definition is different from the one given earlier in terms of the sign of the cross-derivative of profit or production functions. The definition in equation (28) is preferable, at least as a predictor of responses to changes in input prices. By "gross" rather than "net" complements is meant in the usual way that the income effect is included along with the substitution effect. Even if t_m and t_f were net complements they could still be gross substitutes since the income effect of an increase in w_m would tend to increase t_f.

[31] Low-wage men also would be encouraged to work less both because of their low wage and the relatively high wage of their wives. They would not leave the labor force in large numbers, however, partly because average wage rates of men are so much higher than those of women and partly because the nonmarket productivity of women is higher than that of men.

If some women are not in the labor force, however, the wage rates of men and women need not be perfectly negatively correlated to maximize total output. For assume that all women with wage rates below a certain level would not participate in the labor force with a perfectly negative correlation between the wage rates of men and women. These women have $\partial Z/\partial w_f = 0$,[32] and, thus, $Z^{fm} = 0$; therefore, up to a point, they could switch mates without lowering total output. Consequently, other sortings having weaker negative, and conceivably even positive, correlations would also maximize total output; that is, many sortings would be equally good, and wage rates would not be a decisive determinant of the optimal sorting.

If M and F differ only in their stock of nonhuman capital, K_m and K_f, and if everyone participates in the labor force, $\partial C/\partial K_m = \partial C/\partial K_f = 0$ since the value of time is measured by the market wage rates. If the rate of return on K, denoted by r, depended positively on the amount of time allocated to "portfolio management," r would be positively related to K.[33] It then follows that

and

$$
\left.
\begin{aligned}
\frac{\partial Z}{\partial K_m} &= \frac{\partial Z}{\partial K_f} = rC^{-1} > 0 \\[2em]
\frac{\partial^2 Z}{\partial K_m\, \partial K_f} &= \frac{dr}{dK}\, C^{-1} > 0
\end{aligned}
\right\}^{34}
\tag{28}
$$

A perfectly positive correlation between the nonhuman capital of M and F would be optimal, an implication that is consistent with evidence on sorting by, say, parental wealth.

If some F did not participate in the labor force, the value of their time would be measured by a "shadow" price that exceeded their wage rate and was not constant but positively related to the sum of their nonhuman capital.[35] Moreover, a perfectly positive correlation of this capital is no longer necessarily optimal because of diminishing returns to an increase in the time of M and goods for a given amount of the time of F (for proof, see Appendix, section 2).

All differences in the output of commodities, by assumption the only determinant of behavior, not related to differences in wage rates or non-human capital are, by definition, related to differences in nonmarket

[32] As long as they are not indifferent at the margin to working in the market sector.

[33] For this result and a more complete analysis of the allocation of time to portfolio management, see Ben-Zion and Ehrlich (1972).

[34] If time is allocated to portfolio management, $S = wT + Kr(\ell_p) - w\ell_p$, where ℓ_p is the time so allocated. Then $\partial S/\partial K = r + (K\, dr/d\ell_p)(d\ell_p/dK) - w(d\ell_p/dK) = r + d\ell_p/dK\,[(K\, dr/d\ell_p) - w]$. Since, however, $K\, dr/d\ell_p = w$ is one of the first-order maximization conditions, then $\partial S/\partial K = r$.

[35] See the discussion in section 2 of the Appendix.

productivity.[36] The widespread differences between men and women in nonmarket productivity are caused by differences in intelligence, education, health, strength, height, personality, religion, and other traits. We now consider the optimal sorting of traits that affect nonmarket productivity, while assuming that wage rates and nonhuman capital are the same for all M and for all F.

To demonstrate the tendency toward complementarity of nonmarket traits in the context of household commodity outputs, rewrite the optimal output equation given by (21) as

$$Z = \frac{S}{C(w_m, w_f, p, A_m, A_f)} , \qquad (29)$$

where A_m and A_f are the traits of M and F. Then using the assumption that w_m, w_f, and the rate of return on nonhuman capital are independent of A_m and A_f,

$$\left. \begin{array}{l} \dfrac{\partial C}{\partial A_m} \equiv C_{a_m} \\[3mm] \dfrac{\partial C}{\partial A_f} \equiv C_{a_f} \end{array} \right\} < 0 \quad \text{and} \quad \frac{\partial S}{\partial A_f} = \frac{\partial S}{\partial A_m} = 0. \qquad (30)$$

Then,

$$\left. \begin{array}{l} \dfrac{\partial Z}{\partial A_m} = -SC^{-2}C_{a_m} \\[3mm] \dfrac{\partial Z}{\partial A_f} = -SC^{-2}C_{a_f} \end{array} \right\} > 0, \qquad (31)$$

and

$$\frac{\partial^2 Z}{\partial A_m \, \partial A_f} > 0 \quad \text{if } 2C^{-1}C_{a_m}C_{a_f} > C_{a_m,a_f}. \qquad (32)$$

Since the term on the left is positive, equation (32) necessarily holds if A_m and A_f have either independent or reinforcing effects on productivity, for then $C_{a_m,a_f} \leq 0$; moreover, (32) might hold even if they had offsetting effects. Therefore, perfectly positive assortive mating is definitely optimal if the traits have reinforcing effects; less obvious and more impressive, however, is the conclusion that positive assortive mating is also optimal

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[36] Differences in the earning power of children are assumed to be derived from differences in either the nonmarket productivity or incomes of their parents, and are not considered separately.

if they have independent effects because C enters inversely in the equation for Z, or even if they have offsetting effects if these are weaker than a multiple of the direct ones.[37]

The reasons for the prevalence of a complementary relation between traits that raise nonmarket productivity can be seen more transparently by considering a couple of special cases. If the percentage effect on output of a trait were independent of the quantities of goods and time, the optimal output equation could be written as

$$Z = \frac{S}{b(A_m, A_f)K(w_m, w_f, p)}, \qquad (33)$$

where $\partial b/\partial A_m \equiv b_{a_m} < 0$, and $\partial b/\partial A_f \equiv b_{a_f} < 0$. Hence,

$$\frac{\partial^2 Z}{\partial A_m \, \partial A_f} > 0 \qquad \text{as } 2b^{-1}b_{a_m}b_{a_f} > b_{a_m,a_f}, \qquad (34)$$

which must hold if $b_{a_m,a_f} \leq 0$ and can easily hold even if $b_{a_m,a_f} > 0$. Positive assortive mating is optimal even when these productivity effects are independent because productivity is raised multiplicatively: higher A_m (or A_f) have bigger *absolute* effects when combined with higher A_f (or A_m). A fortiori, this multiplicative relation encourages the mating of likes when the effects are reinforcing and can do so even when they are offsetting.[38]

The effect of most traits on nonmarket output is not independent of goods and time, but generally operates through the time supplied to the household; for example, if the time supplied became zero, so would the effect. A simple way to incorporate this interaction is to assume that each trait affects outputs only by augmenting the effective amount of own household time. It is shown in section 3 of the Appendix that positive assortive mating would still be optimal as long as the elasticity of substitution between the household time of M and F was not very high.[39] Negative assortive mating can be expected for own-time-augmenting traits only if they augment dimensions that are easily substitutable between M and F. Dominant and deferential persons tend to marry each other (Winch 1958), perhaps, therefore, because the dominant person's time can be used when households encounter situations calling for dominance and the deferential person's time can be used when they call for deference.

[37] Equation (32) can be written as

$$2|\varepsilon_{c_{a_m}}| > \varepsilon_{c_{a_f},a_m},$$

where $\varepsilon_{c_{a_m}} = (C_{a_m} \cdot A_m)/C < 0$, and $\varepsilon_{c_{a_m},a_f} = C_{a_f,a_m} \cdot A_m/C_{a_f} > 0$ if the effects are offsetting. The cross-elasticity must be smaller than twice the absolute value of the direct elasticity.

[38] Section 3 of the Appendix shows that positive assortive mating of A_m and A_f is still optimal even when F do not participate in the labor force.

[39] The elasticity estimates of Ofek (1970) and Smith (1972) are only of modest size.

Note that it is shown in section 2 that the gain from marriage is also greater when substitution between the time of M and F is more difficult. Therefore, the mating of likes should be more common when marriage is more attractive, an important and subtle implication of the analysis.

How do the nonmarket traits of one sex combine with the market traits of the other? In particular, does our analysis justify the popular belief that more beautiful, charming, and talented women tend to marry wealthier and more successful men? Section 4 of the Appendix shows that a positive sorting of nonmarket traits with nonhuman wealth always, and with earning power usually,[40] maximizes commodity output over all marriages. The economic interpretation is basically that nonmarket productivity and money income tend to combine multiplicatively, so that higher values of a trait have larger absolute effects when combined with higher income.

Scattered references have been made to the empirical evidence on sorting, and this evidence is now considered a little more systematically. The simple correlations between the intelligence, education, age, race, nonhuman wealth, religion, ethnic origin, height, and geographical propinquity of spouses are positive and strong.[41] A small amount of evidence suggests that the correlations between certain psychological traits, such as a propensity to dominate, nurture, or be hostile, are negative.[42] The correlation by intelligence is especially interesting since, although intelligence is highly inheritable, the correlation between mates is about as high as that between siblings (Alstrom 1961). Apparently, the marriage market, aided by coeducational schools, admissions tests, and the like, is more efficient than is commonly believed.

This evidence of positive simple correlations for a variety of traits, and of negative correlations for some, is certainly consistent with our theory of sorting. A more powerful test of the theory, however, requires evidence on partial correlations, when various other traits are held constant. For example, how strong is the correlation by intelligence, when years of schooling and family background are held constant? We do not yet have results on partial correlations by intelligence, but do have some on years of schooling, wage rates, and age, for samples of white and black families.[43] Even when age and wage rates are held constant, the correlation between years of schooling is high, $+.53$ for whites and virtually the same $(+.56)$

[40] By "usually" is meant that a positive sorting with earnings always maximizes total output when an increase in a trait does not decrease the spouses' hours worked in the market sector and *could* maximize output even when they do decrease.

[41] Many of the relevant studies are listed in Winch (1958, chap. 1).

[42] See Winch (1958, chap. 5). Deference is treated as negative values of dominance, succorance as negative values of nurturance, and abasement as negative values of hostility.

[43] A 20 percent random sample of the approximately 18,000 married persons in the 1967 Survey of Economic Opportunity was taken. Families were included only if the husband and wife both were less than age 65 and were employed, the wife for at least 20 hours in the survey week.

for blacks. Although the partial correlations between wage rates are much lower, they are also positive, +.32 for whites and a bit lower (+.24) for blacks.

The strong positive partial correlation between years of schooling is predicted by the theory, but the positive correlation between wage rates is troublesome since the theory predicts a negative correlation when nonmarket productivity is held constant. Note, however, that the sample is biased because it is restricted to women in the labor force in a particular year. Since the higher the husband's wage rate the higher must be his wife's wage rate to induce her to enter the labor force, a negative correlation across all mates is consistent with a positive one for those having wives in the labor force.[44] Indeed, Gregg Lewis has shown[45] that a correlation of about +.3 for mates who are participating almost certainly implies a negative one (about −.25) for all mates, given the relatively small fraction of married women who participate. If his calculations hold up, this would be striking confirmation of our theory since it is counter to common impressions and is one of the few examples (and a predicted one!) of negative associative mating.

Other evidence, probably less affected by unobserved differences in nonmarket productivity, does suggest that the gain from marriage is greater when differentials between male and female wage rates are greater. For example, a larger percentage of persons are married in American states that have higher wages of males and lower wages of females, even when age, years of schooling, the sex ratio, the fraction Catholic, and other variables are held constant (Santos 1970; Frieden 1972). Or a larger fraction of black households are headed by women in metropolitan areas with higher earnings of black women relative to black men (Reischauer 1970).

Quantitative evidence on the association of traits that affect nonmarket productivity with earnings and other income is scarce. The evidence we put together and referred to earlier indicates that husband's wage rate and wife's education are significantly positively correlated, even when husband's education and wife's wage rate are held constant.[46] One interpretation, stressed in Benham's (1972) paper, is that a wife's education contributes to her husband's earnings, just as a mother's education is said to contribute to her children's earnings (Leibowitz 1972). An

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[44] Also, nonmarket productivity varies even when years of schooling and age are held constant. If investments that raise nonmarket productivity also raise, somewhat, market earning power (Heckman [1972] finds that the education of women raises their nonmarket productivity almost as much as their market earning power), the positive correlation between wage rates may really be picking up the predicted positive correlation between husband's wage rate and wife's nonmarket productivity.

[45] Via an unpublished memorandum extending some work of Gronau (1972).

[46] In a more detailed analysis, Benham (1972) finds similar results, after several additional variables are also held constant. Note, however, that the husband's wage rate is much more strongly related to his own than to his wife's education.

alternative suggested by our theory of sorting is that a wife's education is a proxy for traits affecting her nonmarket productivity, especially when her wage rate is held constant[47] and that women with higher nonmarket productivity marry men with higher earning power. Although the relative importance of these alternative interpretations has not been determined, Benham (1972) does find that hours worked by husbands are positively related to wife's education, a sufficient condition for positive sorting (see n. 40 above).

Our analysis of mating and sorting has assumed perfect certainty in the production of household commodities. Uncertainty surrounds the production of many commodities, but our concern here is only with uncertainty about the "quality" of own children since children are a major source of the gain from marriage. An important result in population genetics is that positive assortive mating of inheritable traits, like race, intelligence, or height, increases the correlation of these traits among siblings; the increase would be greater the more inheritable the trait is and the greater the degree of assortive mating (Cavalli-Sforza and Bodmer 1971, chap. 9, sec. 7). Therefore, inheritable traits of M and F can be said to be complements in reducing the uncertainty about one's children. Positive assortive mating of inheritable traits would increase the utility of total output if more certainty about the "quality" of children is desirable—perhaps because friction between siblings or the cost of raising them is increased by uncertainty.

Our analysis of sorting is based on several other simplifying assumptions that ought to be modified in a fuller treatment. For example, the conclusion in section 2, that the gain from marriage is independent of preferences, assumes, among other things, no joint production and constant returns to scale in households. With beneficial joint production[48] or increasing returns, mating of persons with similar preferences would be optimal and conversely with detrimental production or decreasing returns. Similarly, the conclusion in section 2, that a monagamous union is always optimal, which is taken for granted in the discussion of sorting, should be modified to consider polygamy (we do this in Part II) and remaining single (see the discussion of search in Part II). Further, we have considered only one trait at a time, holding all other traits constant. But since people differ in many interdependent traits, optimal sortings should be determined for a set of traits, perhaps using the canonical correlation coefficient or related statistics as the measure of association.

Probably the assumption that would be most questioned is that any division of output between mates is feasible. Some of the output may not

[47] We argued earlier that her wage rate also is a proxy for such traits, when her education is held constant.

[48] Grossman (1971) distinguishes beneficial from detrimental production by the effect of an increase in output of one commodity on the cost of producing others.

be divisible at all and may constitute a "public," or better still, a "family" commodity. Children might be said to be largely a family commodity, and, as shown in Part II, "caring" can convert the whole output into family commodities. Or some divisions may not be feasible because they are not enforceable. For example, even though the marriage market might dictate a 2/5 share for a particular husband, he may receive a 3/5 share because his wife cannot "police" the division perfectly.

Although the rigidities resulting from family commodities and enforcement problems can often be overcome (through dowries and other capital transfers), it is instructive to consider a model of sorting that incorporates these rigidities in an extreme fashion. How robust are the conclusions about optimal sorting when complete rigidity in the division of output replaces the assumption of complete negotiability?

Rigidity is introduced by assuming that M_i would receive a constant fraction e_i of commodity output in *all* marriages, and F_j receive d_j. Note that e_i and e_k $(k \neq i)$ or d_j and d_k $(k \neq j)$ need not be equal, and that

$$e_i + d_j \gtrless 1, \qquad (35)$$

as family commodities or enforcement costs were dominant. The matrix showing the incomes for all combinations of M and F would then be

$$
\begin{array}{c|ccc|}
 & F_1 & F_j & F_n \\
\hline
M_1 & e_1 Z_{11},\, d_1 Z_{11} & \ldots\ldots\ldots & e_1 Z_{1n},\, d_n Z_{nn} \\
M_i & & e_i Z_{ij},\, d_j Z_{ij} & \\
M_n & e_n Z_{nj},\, d_1 Z_{n1} & \ldots\ldots\ldots & e_n Z_{nn},\, d_n Z_{nn} \\
\hline
 & & &
\end{array}
\qquad (36)
$$

If

$$\hat{Z}_1 \equiv Z_{st} > Z_{ij}, \qquad \text{all } i \neq s, \text{ all } j \neq t, \qquad (37)$$

were the maximum output in any possible marriage and if each person tried to maximize his commodity income, M_s would marry F_t since they could not do as well in any other marriage.[49] Now exclude M_s and F_t from consideration, and if

$$\hat{Z}_2 = Z_{uv} > Z_{ij}, \qquad \text{all } i \neq u \text{ or } s, \text{ all } j \neq v \text{ or } t, \qquad (38)$$

were the maximum output in all other marriages, M_u would marry F_v. This process can be continued through the $\hat{Z}_3, \ldots, \hat{Z}_n$ until all the M and F are sorted.

How does this sorting, which combines the various maxima, compare with that obtained earlier, which maximizes total output? As the example in (17) indicates, they are not necessarily the same: combining the maxima in that example sorts M_2 with F_1 and M_1 with F_2, whereas maximizing total output sorts M_1 with F_1 and M_2 with F_2. Yet, in perhaps the most

[49] Clearly, $e_s Z_{st} > e_s Z_{sj}$, all $j \neq t$, and $d_t Z_{st} > d_t Z_{lt}$, all $i \neq s$ by condition (37).

realistic cases, they are the same, which means that the sum of the maxima would equal the maximum of the sums.

Assume that an increase in trait A_m or A_f always increases output and that M and F are numbered from lower to higher values of these traits. Then, $\hat{Z}_1$ is the output of M_n with F_n, $\hat{Z}_2$ is that of M_{n-1} with F_{n-1}, and $\hat{Z}_n$ that of M_1 with F_1. Consequently, when traits have monotonic effects on output, the most common situation, combining the various maxima implies perfectly positive assortive mating.

We showed earlier that, in a wide variety of situations, namely, where traits are "complementary," maximizing total output also implies perfectly positive assortive mating. In these situations, permitting the market to determine the division of output and imposing the division a priori gives exactly the same sorting. Therefore, the implication of the theory about the importance of positive assortive mating is not weakened, but rather strengthened, by a radical change in assumptions about the determinants of the division of output.

When maximizing total output implies negative assortive mating, as it does between wage rates (with nonmarket productivity held constant), and between own-time augmenting traits that are close substitutes, these assumptions about the division of output have different implications. The empirical evidence on sortings cannot yet clearly choose between these assumptions, however, because positive sortings are so common: perhaps the positive correlation between observed wage rates is evidence of rigidities in the division, but several alternative interpretations of this correlation have been suggested that are consistent with a negative "true" correlation, and some psychological traits are apparently negatively correlated. Moreover, dowries and other capital transfers (discussed in Part II) provide more effective fluidity in the division than may appear to the casual observer.

4.   The Division of Output between Mates

With complete negotiability the division of output is given by condition (15) and (16). The m_{ii} and f_{ii} are determined by their marginal productivity in the sense that if $Z_{ki} > Z_{kk}$, necessarily $f_{ii} > f_{kk}$,[50] and similarly for the m_{ii}. Also, if $f_{ii} > f_{kk}$, necessarily $Z_{ii} > Z_{ik}$.[51] The following limits are easily derived:

$$\left. \begin{array}{c} Z_{ii} - \mathrm{Max}_k \, (Z_{ki} - Z_{kk}) \geq m_{ii} \geq \mathrm{Max}_k \, (Z_{ik} - Z_{kk}) \\ Z_{ii} - \mathrm{Max}_k \, (Z_{ik} - Z_{kk}) \geq f_{ii} \geq \mathrm{Max}_k \, (Z_{ki} - Z_{kk}) \end{array} \right\} \text{.}[52] \qquad (39)$$

[50] Since $f_{kk} + m_{kk} = z_{kk}$, all k, and $f_{ii} + m_{kk} \geq z_{ki}$, all i and k, then $f_{ii} - f_{kk} \geq z_{ki} - z_{kk} > 0$ by assumption.

[51] That is, if $f_{ii} > f_{kk}$, then $Z_{ii} = m_{ii} + f_{ii} > m_{ii} + f_{kk} \geq Z_{ik}$.

[52] Given conditions (15) and (16), $m_{ii} - m_{kk} \geq Z_{ik} - Z_{kk}$, all k, or, since $m_{kk} \geq 0$, $m_{ii} \geq Z_{ik} - Z_{kk}$, all k. The other conditions in (39) can be proved in a similar way.

The division of output resulting from conditions (15) and (16) is not unique, however. For if a set of m_{ii} and f_{ii} satisfies these conditions with all $0 < m_{ii} < Z_{ii}$, a positive quantity λ exists, such that $m_{ii} + \lambda$ and $f_{ii} - \lambda$ also satisfy these conditions. The range of indeterminacy in the division would narrow as the sum of $\text{Max}_k \, (Z_{ik} - Z_{kk})$ and $\text{Max}_k \, (Z_{ki} - Z_{kk})$ approached closer to Z_{ii}.

Clearly, the indeterminacy would vanish if the distribution of Z_{ik} became continuous. It could also vanish in a second case to which we turn. Assume v_i identical M_i and u_i identical F_i; by identical is meant that they would produce the same output with any mate or while single, so that they would receive the same income in market equilibrium. If the number of v_i were sufficiently large for a competitive equilibrium, there would be a supply curve of M_i to the marriage market: it would be horizontal at the singles income Z_{io} until all $v_i{}^o$ were married, and then would rise vertically (see S_o in fig. 1). Similarly, if the number of u_i were sufficiently large, there would be a market supply curve of F_i: it would be horizontal at Z_{oi} until all $u_i{}^o$ were married, and then would rise vertically. If initially we assume, for simplicity, that the M_i and F_i either marry each other or remain single, the supply curve of F_i would also be a derived demand curve for M_i that would be horizontal at $Z_{ii} - Z_{oi}$ until all $u_i{}^o$ were married, and then would fall vertically (D_o in fig. 1); moreover, the supply curve of M_i to the market would be its supply curve to F_i.
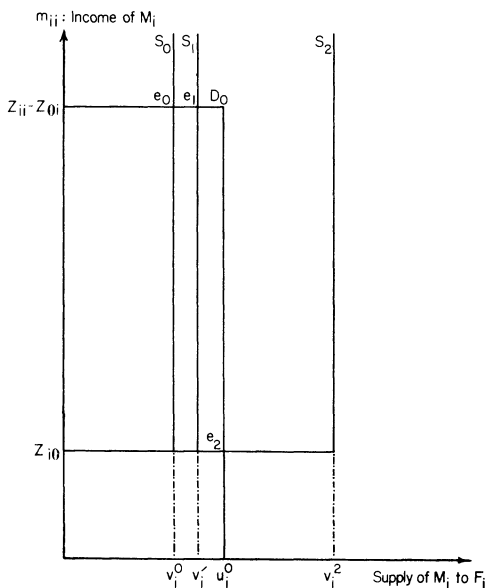

FIG. 1

The equilibrium income to each M_i is given by point e_o, the intersection of S_o and D_o. If the sex ratio (v_i^o/u_i^o) were less than unity, the equilibrium position is necessarily on the horizontal section of the derived demand curve, as is e_o. All the M_i would marry and receive the whole difference between their married output and the singles output of F_i. All the F_i would receive their singles output and, therefore, would be indifferent between marrying and remaining single, although market forces would encourage v_i^o of them to marry.

An increase in the sex ratio due to an increase in the number of M_i would lengthen the horizontal section of the supply curve and shift the equilibrium position to the right, say, to e_1. All the M_i would continue to marry and a larger fraction of the F_i also would. If the sex ratio rose above unity, equilibrium would be on the horizontal section of the supply rather than the derived demand curve (see e_2). Now all the F_i would marry and receive the whole difference between their married output and the singles output of M_i; market forces would induce u_i^o of the M_i to marry, and $v_i^2 - u_i^o$ to remain single.

The importance of sex ratios in determining the fraction of men and women who marry has been verified by numerous episodes and in several studies. An aftermath of a destructive war is many unmarried young women pursuing the relatively few men available, and men usually either marry late or not at all in rural areas that have lost many young women to cities. Statistical studies indicate that the fraction of women currently married at different ages is positively related to the appropriate sex ratio.[53]

I know of only highly impressionistic evidence on the effects of the sex ratio, or for that matter any other variable, on the division of output between mates. This division usually has not been assumed to be responsive to market forces, so that no effort has been put into collecting relevant evidence. Admittedly, it is difficult to separate expenditures of goods and time into those that benefit the husband, the wife, or both, but with enough will something useful could be done. For example, the information giving the separate expenditures on husband's and wife's clothing in some consumer surveys, or on the "leisure" time of husbands and wives in some time budget studies could be related to sex ratios, wage rates, education levels, and other relevant determinants of the division of output.

If we drop the assumption that all the M_i and F_i must either marry each other or remain single, M_i's supply curve to F_i would differ from its market supply curve because marriage to other persons would be sub-

[53] See the studies essentially of whites by Santos (1970) and Freiden (1972), of blacks by Reischauer (1970), of Puerto Rico by Nerlove and Schultz (1970), and of Ireland by Walsh (1972). By "appropriate" is meant that a group of women must be matched with the men they are most likely to marry, e.g., college-educated women with college-educated men, or women aged 20–24 with men aged 25–29.

stituted for marriage to F_i; similarly, F_i's supply curve to M_i would differ from its market supply curve. To demonstrate this, suppose that, at point e_o in figure 1, M_i does better by marrying F_i than by marrying anyone else; that is, condition (16) is a strict inequality for M_i. If M_i's income from marrying F_i were less than at e_o, the difference between the sum of M_i's income and that of other $F_j \neq F_i$, and what they could produce together would be reduced. At some income, this difference might be eliminated for an F, say, F_k: then all the M_i would be indifferent between marrying F_i and F_k.

At lower values of M_i's income from marrying F_i, some of the M_i would try to marry F_k. The increase in the supply of mates to F_k would raise M_i's income and reduce that of M_i's mates. In equilibrium, just enough M_i would marry F_k to maintain equality between the income M_i receives with F_i and F_k. The important point is that if some M_i marry F_k, the number marrying F_i would be less than the number supplied to the marriage market (v_i). Moreover, the number marrying F_i might fall still further as M_i's income with F_i fell further because some might marry, say, F_p, if they could then do as well with F_p as with F_i or F_k.

The net effect of these substitutions toward other F is a rising supply curve of M_i to F_i, shown by S_o in figure 2, with an elasticity determined both by the distribution of substitute F and by the effect on the income of
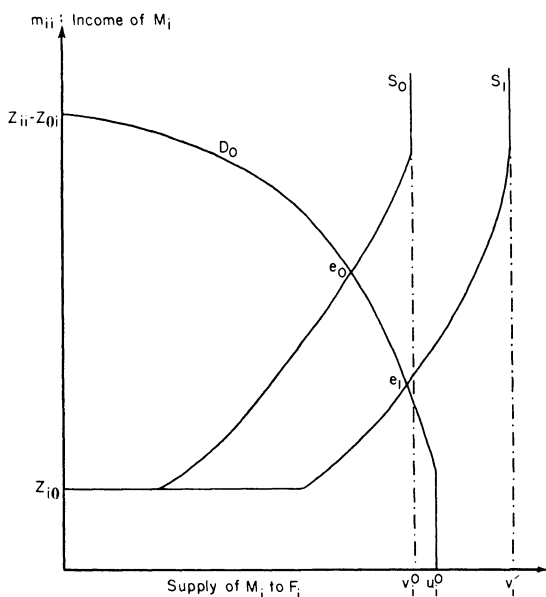

FIG. 2

these F of a given increase in the number of M_i available to marry them. Since F_i would also substitute toward other M, its derived demand curve for M_i would also fall, as D_o does in figure 2. The equilibrium position e_o determines both the division of output between M_i and F_i and the number marrying each other. The difference between the total number of M_i, v_i^o, and the number marrying F_i no longer measures the number of M_i remaining single, since at e_o all M_i marry, but rather it measures the number marrying other F and receiving the same income as the M_i marrying F_i; similarly, for the F_i.

An increase in the number of M_i from v_i^o to v'_i would shift their supply curve to F_i to the right and lower the equilibrium position to e_1 in figure 2. The reduction in M_i's income (equal to the increase in F_i's income) is negatively related to the elasticities of the demand and supply curves, which are determined by the availability of substitute M and F. The additional M_i all marry, some to F_i and some to other F; a larger fraction of the F_i are induced to marry M_i by the increase in F_i's income.

An increase in the sex ratio between M_i and F_i would not necessarily increase the fraction of F_i or decrease the fraction of M_i who marry since all can marry if some marry other F or M. However, if all F_i and M_i married, an increase in their sex ratio would tend to decrease the number of other M or increase the number of other F who marry, if the quantity of other M and F were fixed. For an increase in the ratio of M_i to F_i not only lowers M_i's and raises F_i's income, but also lowers the incomes of substitute M and raises those of substitute F. Some of these M would thereby be induced not to marry because their gain from marriage would be eliminated, and some F would be induced to marry because a gain from marriage would be created. Consequently, an increase in the ratio of M_i to F_i would still decrease the fraction of M and increase the fraction of F marrying, if substitute M and F as well as M_i and F_i were considered.

To illustrate these effects, assume an autonomous increase (perhaps due to selective immigration) in the size of a group of identical men, aged 24, who initially were indifferent between marrying women aged 22 and those slightly older or younger, although most married 22-year-olds. The increase in their numbers would decrease their income and the proportion marrying women aged 22. For if the percentage increase in the number marrying women aged 22 were as large as the increase in the number marrying other women, the income of those marrying 22-year-olds would fall by more than others, since men aged 24 are a larger fraction of all men marrying women aged 22 than of all men marrying women of other ages. Moreover, the income of women aged 22 would increase and more of them would marry men aged 24; the income of older or younger men marrying women aged 22 would fall and they would be encouraged to marry women of other ages; the income of

women somewhat older or younger than 22 would increase too, and so on.[54]

5.   Summary and Concluding Remarks

I have presented in this paper only the skeleton of a theory of marriage. The two basic assumptions are that each person tries to do as well as possible and that the "marriage market" is in equilibrium. With the aid of several additional simplifying assumptions, I am able to derive a number of significant implications about behavior in this market.

   For example, the gain to a man and woman from marrying compared to remaining single is shown to depend positively on their incomes, human capital, and relative difference in wage rates.

   The theory also implies that men differing in physical capital, education or intelligence (aside from their effects on wage rates), height, race, or many other traits will tend to marry women with like values of these traits, whereas the correlation between mates for wage rates or for traits of men and women that are close substitutes in household production will tend to be negative.

   My theory does not take the division of output between mates as given, but rather derives it from the nature of the marriage market equilibrium. The division is determined here, as in other markets, by marginal productivities, and these are affected by the human and physical capital of different persons, sex ratios, that is, the relative numbers of men and women, and some other variables.

   In Part II I put some flesh on the skeleton by incorporating into the analysis love in marriage, the incidence and viability of polygamy, and separations, divorce, remarriage, and other life-cycle marital decisions. If my present plans materialize, subsequent papers will deal more quantitatively with the marriage market, including an empirical analysis of separation and divorce, and of married households as producers of market and nonmarket skills.

Mathematical Appendix

1. *Optimal Sorting*[55]

Given a function $f(x, y)$, we first show that if $\partial^2 f / \partial x \partial y < 0$,

$$\frac{\partial [f(x_2, y) - f(x_1, y)]}{\partial y} \equiv \frac{\partial Q(x_2, x_1, y)}{\partial y} < 0 \qquad \text{for } x_1 < x_2. \qquad \text{(A1)}$$

Since $\partial Q / \partial y = (\partial f / \partial y)(x_2, y) - \partial f / \partial y (x_1, y)$, $\partial Q / \partial y = 0$ for $x_2 = x_1$.   By

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[54] The permanence of these effects depends on whether the immigration continues or is once and for all.

[55] I owe the proofs in this section to William Brock.

assumption, $(\partial/\partial x_2)(\partial Q/\partial y) = (\partial^2 f/\partial x \partial y)(x_2, y) < 0$. Since $\partial Q/\partial y = 0$ for $x_2 = x_1$ and $\partial Q/\partial y$ decreases in x_2, $\partial Q/\partial y < 0$ for $x_2 > x_1$; hence (A1) is proved. It follows immediately from (A1) that if $y_2 > y_1$,

$$f(x_2, y_1) - f(x_1, y_1) > f(x_2, y_2) - f(x_1, y_2). \tag{A2}$$

A similar proof shows that if $\partial^2 f/\partial x \partial y > 0$,

$$f(x_2, y_1) - f(x_1, y_1) < f(x_2, y_2) - f(x_1, y_2). \tag{A3}$$

We now are prepared to prove the following theorem: Let $f(x, y)$ satisfy $\partial^2 f/\partial x \partial y > 0$. Suppose $x_1 < x_2 < \cdots < x_n$ and $y_1 < y_2 < \cdots < y_n$. Then,

$$\left. \begin{array}{c} \displaystyle\sum_{j=1}^{n} f(x_j, y_{i_j}) < \sum_{i=1}^{n} f(x_i, y_i) \\[2em] \text{for all permutations} \\[1em] (i_1, i_2, \ldots i_n) \neq (1, 2, \ldots n) \end{array} \right\} . \tag{A4}$$

Assume the contrary; namely, that the maximizing sum is for a permutation $i_1 \cdots i_n$, not satisfying $i_1 < i_2 < \cdots < i_n$. Then there is (at least) one j_o with the property $i_{j_o} > i_{j_o+1}$. Therefore,

$$f(x_{j_o}, y_{i_{j_o}}) + f(x_{y_o+1}, y_{i_{j_o}+1}) < f(x_{j_o}, y_{i_{j_o}+1}) + f(x_{j_o+1}, y_{i_{j_o}}), \tag{A5}$$

by (A3) since $y_{i_{j_o}+1} < y_{i_{j_o}}$. But this contradicts the optimality of $i_1, \ldots i_n \cdot QED$.

A similar proof shows that if $\partial^2 f/\partial x \partial y < 0$, then

$$\left. \begin{array}{c} \displaystyle\sum_{j=1}^{n} f(x_j, y_{i_j}) < \sum_{i=1}^{n} f(x_i, y_{n+1-i}) \\[2em] \text{for all permutations} \\[1em] (i_1, i_2, \ldots i_n) \neq (n, n-1, \ldots, 1) \end{array} \right\} . \tag{A6}$$

2. Women Not in the Labor Force

If F did not participate in the labor force,

$$S = Tw_m + T\hat{w}_f + r(l_{pm}, l_{pf})(K_m + K_f) - l_{pm}w_m - l_{pf}\hat{w}_f, \tag{A7}$$

where $\hat{w}_f$, the "shadow" price of F, is greater than w_f, her market wage rate, unless F is at the margin of entering the labor force,[56] and l_{pm} and l_{pf} are the time allocated to portfolio management by M and F, respectively. If the production function for Z were homogeneous of the first degree in time and goods, $Z = S/C(p, w_m, \hat{w}_f, A_f, A_m)$.

Then,

$$\frac{\partial Z}{\partial K_i} = C^{-1} \left[r + K \left(\frac{\partial r}{\partial l_{pm}} \frac{\partial l_{pm}}{\partial K_i} + \frac{\partial r}{\partial l_{pf}} \frac{\partial l_{pf}}{\partial K_i} \right) - \frac{\partial l_{pm}}{\partial K_i} w_m - \frac{\partial l_{pf}}{\partial K_i} \hat{w}_f \right]$$

$$+ TC^{-1} \frac{d\hat{w}_f}{dK_i} - SC^{-2}C^f \frac{d\hat{w}_f}{dK_i} - C^{-1}l_{pf} \frac{\partial \hat{w}_f}{\partial K_i} \tag{A8}$$

$$= rC^{-1} > 0, \quad i = m \text{ or } f \tag{A9}$$

[56] An earlier draft of this section developed the analysis using the shadow price of F, but contained some errors. I owe the present formulation to H. Gregg Lewis.

since $C^f = t_f Z^{-1} = (T - l_{pf})Z^{-1}$, $K_m + K_f = K$, and $\hat{w}_f = (\partial r/\partial l_{pf})K$ and $w_m = (\partial r/\partial l_{pm})K$ with an optimal allocation of time. Similarly,

$$\frac{\partial Z}{\partial w_m} = TC^{-1} + \frac{TC^{-1}d\hat{w}_f}{dw_m}$$

$$+ C^{-1}\left(\frac{\partial r}{\partial l_{pm}}\frac{\partial l_{pm}}{\partial w_m}K + \frac{\partial r}{\partial l_{pf}}\frac{\partial l_{pf}}{\partial w_m}K - l_{pm} - \frac{\partial l_{pm}w_m}{\partial w_m} - l_{pf}\frac{d\hat{w}_f}{dw_m} - \frac{\partial l_{pf}}{\partial w_m}\hat{w}_f\right)$$

$$- SC^{-2}C^m - SC^{-2}C^f\frac{\partial \hat{w}_f}{\partial w_m} = l_m C^{-1} > 0, \tag{A10}$$

and

$$\frac{\partial Z}{\partial A_i} = -SC^{-2}C_{a_i} + TC^{-1}\frac{\partial \hat{w}_f}{\partial A_i} - SC^{-2}C^f\frac{\partial w_f}{\partial A_i} - l_{pf}\frac{\partial \hat{w}_f}{\partial A_i}$$

$$+ \text{ terms whose sum is zero}$$

$$= -SC^{-2}C_{a_i} > 0 \qquad i = m \quad \text{or} \quad f, \tag{A11}$$

if A_i does not directly affect r. Note that equations (A9)–(A11) are exactly the same as those when F does participate—equations (24), (28), and (31).

Then,

$$\frac{\partial^2 Z}{\partial K_f \, \partial K_m} = C^{-1}\left|\frac{\partial r}{\partial l_{pm}}\frac{\partial l_{pm}}{\partial K_m} + \frac{\partial r}{\partial l_{pf}}\frac{\partial l_{pf}}{\partial K_m}\right| - rC^{-2}C^f\frac{\partial \hat{w}_f}{\partial K_m}. \tag{A12}$$

The first term is positive, but the second one is negative since

$$\frac{\partial w_f}{\partial K_m} > 0, \qquad \frac{\partial \hat{w}_f}{\partial K_f} > 0, \qquad \left(\text{and } \frac{\partial \hat{w}_f}{\partial w_m} > 0\right). \tag{A13}$$

A proof of (A13) follows from the derived demand equation for t_f. Of course,

$$\frac{\partial^2 Z}{\partial w_m \, \partial w_f} = 0. \tag{A14}$$

Moreover,

$$\frac{\partial^2 Z}{\partial K_m \, \partial A_f} = -rC^{-2}C_{a_f} - C^{-2}C^f\frac{\partial \hat{w}_f}{\partial A_f}. \tag{A15}$$

The first term is necessarily positive and the second would be nonnegative if $\partial \hat{w}_f/\partial A_f \leq 0$. It can easily be shown that $\partial \hat{w}_f/\partial A_f = 0$ if A_f has a factor-neutral effect on output and $\partial \hat{w}_f/\partial A_f < 0$ if A_f is own-time augmenting. Consequently, there is some presumption that

$$\frac{\partial^2 Z}{\partial K_m \, \partial A_f} > 0. \tag{A16}$$

The general expression for the cross-derivative of Z with respect to A_m and A_f can be found by differentiating equation (A11). We consider here only the case where the effects are factor-neutral, so that

$$Z = g(A_m, A_f)f(x, t_m, t_f), \tag{A17}$$

or the optimal Z is $Z = gS/[K(p, w_m, \hat{w}_f)]$, with

$$g_i = \frac{\partial g}{\partial A_i} > 0, \qquad \text{and} \qquad g_{mf} = \frac{\partial^2 g}{\partial A_m \, \partial A_f} > 0. \qquad i = m, f. \tag{A18}$$

By substituting into (A11),

$$\frac{\partial Z}{\partial A_i} = Z \frac{g_i}{g} > 0. \qquad (A19)$$

Therefore,

$$\frac{\partial^2 Z}{\partial A_m \, \partial A_f} = \frac{g_m}{g^2} g_f Z + \frac{g_{mf} Z}{g} - \frac{g_m g_f Z}{g^2} = \frac{g_{mf} Z}{g} > 0. \qquad (A20)$$

3. Own-Time-Augmenting Effects

By own-time augmenting is meant that the household production function can be written as $Z = f(x, t'_f, t'_m)$, where $t'_f = g_f(A_f) t_f$, and $t'_m = g_m(A_m) t_m$ are the time inputs of F and M in "efficiency" units, and

$$\frac{dg_f}{dA_f} = g'_f > 0, \qquad \text{and} \qquad \frac{dg_m}{dA_m} = g'_m > 0, \qquad (A21)$$

indicates that an increase in the trait raises the number of efficiency units. The optimal Z can be written as $Z = S/C(p, w'_m, w'_f)$, where $w'_m = w_m/g_m$ and $w'_f = w_f/g_f$ are wage rates in efficiency units. Therefore,

$$\frac{\partial Z}{\partial A_m} = -t'_m C^{-1} \frac{\partial w'_m}{\partial A_m} > 0, \qquad (A22)$$

since $\partial w'_m/\partial A_m < 0$. Hence,

$$\frac{\partial^2 Z}{\partial A_m \, \partial A_f} = - \frac{\partial w'_m}{\partial A_m} C^{-1} \left(\frac{\partial t'_m}{\partial A_f} - \frac{\partial w'_f}{\partial A_f} t'_m t'_f S^{-1} \right). \qquad (A23)$$

The term outside the parenthesis and the second term in it are positive. The first term in the parenthesis might well be negative,[57] but Gregg Lewis has shown in an unpublished memorandum that $\partial^2 Z/\partial A_m \, \partial A_f$ is necessarily positive if the elasticity of substitution between the time of M and F is less than 2.

4. Sorting by Income and Nonmarket Productivity

If M differed only in K_m and F only in A_f, and if all M and F participated in the labor force, $\partial Z/\partial K_m = rC^{-1} > 0$, and

$$\frac{\partial^2 Z}{\partial K_m \, \partial A_f} = -rC^{-2} C_{a_f} > 0 \qquad \text{since } C_{a_f} < 0. \qquad (A24)$$

If M differed only in w_m, $\partial Z/\partial w_m = C^{-1} l_m > 0$, and

$$\frac{\partial^2 Z}{\partial w_m \, \partial A_f} = -C^{-2} C_{a_f} l_m + C^{-1} \frac{\partial l_m}{\partial A_f}. \qquad (A25)$$

The first term on the right is positive, and the second would also be if $\partial l_m/\partial A_f \geq 0$, that is, if an increase in A_f does not reduce the time M spends in the market sector. Even if it does, the cross-derivative is still positive if the first term dominates.

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[57] There is some evidence suggesting, e.g., that men with more educated wives generally work more hours (Benham 1972).

In particular, equation (A25) is necessarily positive if the effect of A_f is independent of the input of goods and time. For, if A_f were independent, $C = b(A_f)K(p, w_m, w_f)$. Since $l_m = (\partial C/\partial w_m) Z = (\partial K/\partial w_m)SK^{-1}$, then,

$$\frac{\partial l_m}{\partial A_f} = 0. \tag{A26}$$

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