Competitive Search Equilibrium

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In this paper, I construct an equilibrium for markets with frictions, which is competitive in the sense that all agents are price takers and maximize utility subject to a set of market parameters. I show that the equilibrium allocation is socially optimal. I also show how the competitive search equilibrium can be achieved if employers with vacancies can advertise publicly the wages they pay.

I. Introduction

For decades, economists have debated whether the unemployment rate generated by the market is socially efficient. Since Friedman (1968) and Phelps (1971) introduced the notion of the natural rate of unemployment, a large body of research has developed on this issue. A commonly held view is that the market does not generate a socially efficient unemployment rate since search externalities are not reflected in the wage rate (Mortensen 1982; Hosios 1990; Pissarides 1990). In the present paper I challenge this view. I introduce what I call a competitive search equilibrium and find that the associated equilibrium allocation is socially efficient. I also show how phenomena frequently observed in the labor market may lead to the existence of the competitive search equilibrium.

The starting point for my analysis is a model with two-sided search developed by Diamond (1982), Mortensen (1982), Pissarides

(1984), and others that provides a natural framework for analyzing the issue of the welfare properties of the natural rate of unemployment. The main innovation in those papers is that market frictions are modeled by an exogenously given matching function that relates the number of matches per unit of time to the stock of workers and firms engaged in searching. The matching function thus captures the technology that brings agents together in the market. Wages are set by decentralized bargaining between the worker and the firm after they are matched. Since finding a new trading partner is a costly and time-consuming process for both workers and firms, there is a surplus associated with the match, and this surplus is split according to the (asymmetric) Nash sharing rule.

A necessary condition for an efficient allocation of resources is that the private and social return to search coincide. This is typically not the case when wages are determined by ex post bargaining. Although there exists a sharing rule that leads to efficiency, there are no "forces" or mechanisms that equate the actual and the optimal allocation. One agent's search behavior affects searching agents of the opposite type positively and of the same type negatively, and the wage that prevails from bargaining does not fully reflect these externalities. Greenwald and Stiglitz (1988) show that the inefficiencies can be interpreted as a consequence of missing markets.

In the present paper, I first construct an equilibrium for markets with frictions that captures features normally associated with competitive equilibria, in that all the agents exhibit price-taking behavior and maximize income subject to a given set of market parameters. I assume that a market maker can separate the market into submarkets. Each submarket consists of a subset of the unemployed workers and the firms with vacancies. The firms in a given submarket search for workers in the same submarket, and vice versa for the workers. The matching technology is the same in all submarkets, and the arrival rate of job offers to workers (applicants for vacant jobs) thus depends positively (negatively) on the ratio of searching workers to vacancies, or the labor market tightness, in the submarket in question. The market maker also determines the wages. In each submarket, all jobs pay the same wage, but the wages differ across submarkets.

Both unemployed workers and employers with vacancies are free to choose which submarket to enter. As I assume identical workers, it follows that all submarkets must give the unemployed workers the same expected utility. The ratio of searching workers to vacancies, the "labor market tightness," is therefore high in markets with low wages and vice versa. When determining which submarket to enter, firms thus face a trade-off between wage costs and search costs. I

assume that firms are heterogeneous and find that high-productivity firms generally enter submarkets with higher wages than low-productivity firms do.

The market maker sets wages so that it is impossible to create a new submarket, which would attract workers and in which vacant jobs yield strictly higher expected income. This resembles the no-surplus conditions in Makowski (1980) and Ostroy (1980) and implies that wages are set such that the values of the vacancies are maximized subject to the expected income received by the searching workers. Using the same argument as in the standard theory for competitive markets, I find that the marginal rate of substitution between labor market tightness and wages is the same for workers and firms and implicitly determines a market price of search time. The agents in the economy act as though they maximize income given this price, and as a result the social and the private return from entry coincide. The equilibrium allocation of resources is therefore socially efficient, with respect to both the distribution of searching agents on submarkets and the number of agents entering the market. If the model has more than one equilibrium, they are all equivalent from a welfare point of view.

The competitive search equilibrium is a theoretical construct. In the second part of the paper I show that it can be implemented under a reasonable set of assumptions. The important issue is that workers have information about wages prior to their search, or at least at an early stage in the search process. I think that this is common, at least in parts of the labor market, because of reputation effects or because firms, when advertising a vacancy, announce the wage they intend to pay.

I assume that all firms with vacancies publicly (and credibly) announce their wage offers. I also assume that the market is so large that the workers never have the capacity to apply for more than a fraction of the vacant jobs and therefore select a subset of them to apply for. In equilibrium, all workers must be indifferent between which subset to choose. I find that firms that announce a certain wage and the workers that apply for jobs with this wage in effect form a submarket. Firms choose a wage that maximizes expected profit subject to their perceived relationship between the announced wage and the arrival rate of workers. With reasonable assumptions on the firms' out-of-equilibrium beliefs, the resulting equilibrium is identical to the competitive search equilibrium described above.

In the present paper I provide a micro foundation of one mechanism (out of several) that is regarded as a rationale for efficiency wages, namely that firms may offer a high wage in order to attract

workers. In macroeconomics, the existence of efficiency wages is often used as an explanation for nonclearing markets and an inefficient allocation of resources. In contrast, I find that the unemployment generated is an optimal response to the frictions in the market.

The paper is organized as follows: After the matching framework is laid down in Section II, the competitive search equilibrium is presented in Section III and wage announcements in Section IV. In Section V, I study the relationship between productivity and wages and show how to extend the model to allow for heterogeneous workers. In Section VI, I relate my work to the existing literature and discuss some features of the model more broadly, in particular my assumptions about the matching technology and the information structure in the economy. Section VII concludes the paper.

II.   The Matching Framework

The model is set in continuous time. The labor market consists of a continuum of workers and firms. The measure of workers is normalized to one. The measure of jobs is endogenously determined through entry, and there is a sunk cost $k \geq 0$ associated with the opening of a vacancy. When the cost is incurred, the productivity of the vacancy is drawn from a discrete probability distribution F with mass points at $y_1, \ldots, y_n$. Both workers and firms are risk neutral.[1]

Let $x(u, v)$ denote the flow of new worker-firm matches, where u is the measure of unemployed workers searching for a measure v of vacancies. The matching function $x(u, v)$ captures the frictions in the market. As discussed later on, the sources of the frictions are costs and time delays such as those due to the completion and processing of applications, unmodeled heterogeneities, and information imperfections (though not concerning wages).

Following standard assumptions, let x be concave and homogeneous of degree one in (u, v) with continuous derivatives. Let $p = x(u, v)/u = x(1, \theta) = p(\theta)$ denote the transition rate from unemployment to employment for an unemployed worker, and $q = x(u, v)/v = q(\theta)$ the arrival rate of workers for a vacancy, where θ is the labor market tightness v/u. Let

$$\lim_{\theta \to 0} p(\theta) = \lim_{\theta \to \infty} q(\theta) = 0$$

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[1] It is convenient to think of a firm as consisting of one job only, so that a firm either has one employee and is producing or has one vacancy and is searching for a worker. However, as shown in Pissarides (1990), a firm can also be thought of as consisting of many jobs and having constant returns to scale both in production and in searching.

and

$$\lim_{\theta \to \infty} p(\theta) = \lim_{\theta \to 0} q(\theta) = \infty.$$

When the labor market tightness goes to zero, the arrival rates of trading partners for firms and workers go to infinity and zero, respectively. When θ goes to infinity, the opposite holds.

When matched, the worker-firm pair starts production immediately and continues production until the job is destroyed. This happens at a constant and exogenous probability rate s. After separation, the worker joins the unemployment pool; the remaining vacancy is worthless and is therefore destroyed.

A submarket consists of a subset of the unemployed workers and the firms with vacant jobs that are searching for each other. The number of matches in submarket i is $x(u_i, v_i)$, where u_i is the measure of workers and v_i the measure of vacant jobs. The arrival rates of trading partners for workers and vacancies in this market are thus $p(\theta_i)$ and $q(\theta_i)$, respectively, where $\theta_i = v_i/u_i$. Both workers and firms are free to move between submarkets. We shall assume throughout that all submarkets contain a continuum of searching agents.

III. Competitive Search Equilibrium

In this section, we shall assume that there exists a market maker who determines the number of submarkets. He also determines the wage in each submarket, so that all vacancies in the same market offer the same wage, whereas the wage offers differ across submarkets. All agents have full information about the market maker's actions. We shall first study the behavior of workers and firms separately and then analyze the properties of the competitive search equilibrium.

A.   Workers

Let $m \geq 1$ denote the number of submarkets and $(w_1, \ldots, w_m)$ the corresponding set of wages, where w_i is the wage in the ith submarket and $w_i \geq w_j$ for $i > j$.

Let U_i denote the expected discounted income (or asset value) for an unemployed worker in submarket i. Then the asset value (or Bellman) equation for U is

$$rU_i = z + p(\theta_i)(E_i - U_i). \tag{1}$$

Here z denotes the unemployment income, θ_i the labor market tightness in submarket i, r the discount factor, and E_i the expected income when the worker is employed at wage w_i. The expected income flow when unemployed is equal to the current income z plus the expected capital gain from job search, $p(E - U)$. Similarly, E_i can be written as

$$rE_i = w_i - s(E_i - U_i), \tag{2}$$

where s is the separation rate. Substituting out E gives

$$rU_i = \frac{(r + s)z + w_i p(\theta_i)}{r + s + p(\theta_i)} \tag{3}$$

for $w \geq z$. If $w < z$, the workers do not search, and their expected income is z/r.

The workers enter the submarkets that yield the highest expected income. Since workers are identical, this implies that all submarkets that attract workers yield the same expected income. Denote this income by U. Substituting in for U in (3) and rearranging yields

$$p(\theta_i) = \frac{rU - z}{w_i - rU}(r + s). \tag{4}$$

For a given U, this equation defines a unique relationship between the wage and the labor market tightness in each submarket. Denote this relationship by $\theta(w; U)$. From equation (4) it follows that $\theta(w; U)$ is continuous, strictly decreasing in w, and strictly increasing in U on $(rU, \infty) \times (z, \infty)$. In a submarket with low wages, the gain from finding a job is low, and the workers are compensated by a high arrival rate of job offers, that is, a high labor market tightness θ. As the wage approaches rU, the gain from finding a job approaches zero, and the labor market tightness goes to infinity. Submarkets with wages less than or equal to rU do not attract any workers and are therefore shut down. On the other hand, as the wage goes to infinity, the gain from finding a job goes to infinity, and the labor market tightness goes to zero.

B. Firms

As mentioned earlier, the productivity of a vacancy is determined after the fixed cost is incurred. If the productivity is too low, the vacancy is destroyed immediately and without costs. Otherwise, the firm joins one of the existing submarkets and starts searching for a worker.

Denote by $V(y_i, w, \theta)$ the expected discounted value, or asset

value, of a vacancy with productivity y_i in a submarket in which the wage is w and the labor market tightness is θ. Similarly, let $J(y_i, w)$ denote the expected discounted income when the job is filled. The asset value equation that determines V is given by

$$rV(y_i, w, \theta) = -c + q(\theta)[J(y_i, w) - V(y_i, w, \theta)].$$

The expected income flow associated with a vacancy is thus equal to the current income flow, $-c$ (the search cost), plus the expected gain from search, $q(\theta)(J - V)$. Similarly, J is given by

$$rJ(y_i, w) = y_i - w - sJ(y_i, w). \tag{5}$$

If we substitute the expression for J into the asset value equation for V, we get

$$(r + q)V(y_i, w, \theta) = q(\theta)\frac{y_i - w}{r + s} - c. \tag{6}$$

Each vacancy enters a submarket that maximizes its asset value V.

C. Equilibrium

Now we want to determine the set of wages that exists in equilibrium. In the spirit of Makowski's (1980) and Ostroy's (1980) definitions of competitive equilibrium, the equilibrium allocation is required to be a no-surplus allocation. More specifically, suppose that W^* is the equilibrium set of wages. Then there does not exist a wage w' such that, for some i, $V(y_i, w', \theta(w')) > V(y_i, w^*, \theta(w^*))$ for all $w^* \in W$. If such a wage w' did exist, a submarket with this wage would make workers as well off as and some firms strictly better off than in equilibrium.[2] For technical reasons we shall also assume that the market maker shuts down submarkets in which trade does not occur. The wage w in each submarket thus solves the problem

$$\max_{w} V(y_i, w, \theta(w; U)) \tag{7}$$

for some i. Lemma 1 states that the maximization problem is well defined.

[2] Instead of relying on a market maker, we can obtain the same results using club theory. Regard the submarkets as profit-maximizing, price-taking clubs, which potentially can earn profit by charging entry fees from workers or vacancies or both. Free entry of clubs implies that the entry fees are bid down to zero. The resulting allocation must be a no-surplus allocation. If not, there exists a wage w' such that a club with wages w' could charge strictly positive entry fees and still attract members, thus making a strictly positive profit. See Sandler and Tschirhart (1980) and Scotchmer and Wooders (1987) for a discussion of the economics of clubs.
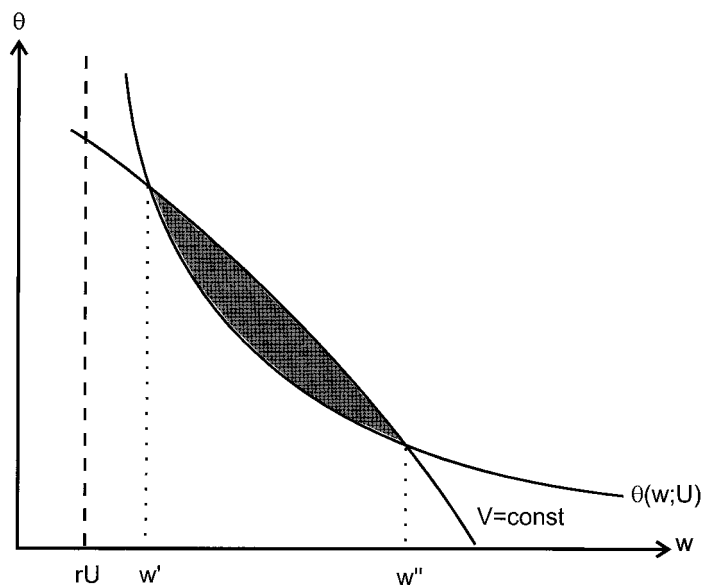
FIG. 1.—Indifference curves for the workers $\theta(w; U)$ and the iso-profit curves for the firms (V = const.) are drawn as convex and concave curves, respectively, in the w-θ space.

LEMMA 1. Let $V_i^s \equiv \sup_w V(y_i, w, \theta(w; U))$, and assume that $V_i^s \geq 0$. Then the maximization problem (7) for y_i is well defined, and V_i obtains its maximum on the interior of $[rU, y_i]$.

The situation is depicted in figures 1 and 2. The workers' indifference curve and the iso-profit curve for a given type of vacancies are drawn as convex and concave curves, respectively, in the w-θ space.[3] The workers' indifference curve is downward sloping since workers prefer both high wages and high labor market tightness. The firms' iso-profit curve is also downward sloping since firms prefer both low wages and low labor market tightness.

Equation (7) implies that any equilibrium wage must be located at a tangency point between the two curves. Figure 1 shows a situation in which this is not the case: w' can obviously not give rise to a no-surplus equilibrium since all combinations of θ and w in the shaded area will make both firms and workers better off. In the club

[3] The restrictions on the matching technology are not sufficient to ensure that $\theta(w)$ is convex and iso-profit curves are concave. However, this is not important for the analysis since we know from lemma 1 that the equilibrium is always at a tangency point between the two curves.
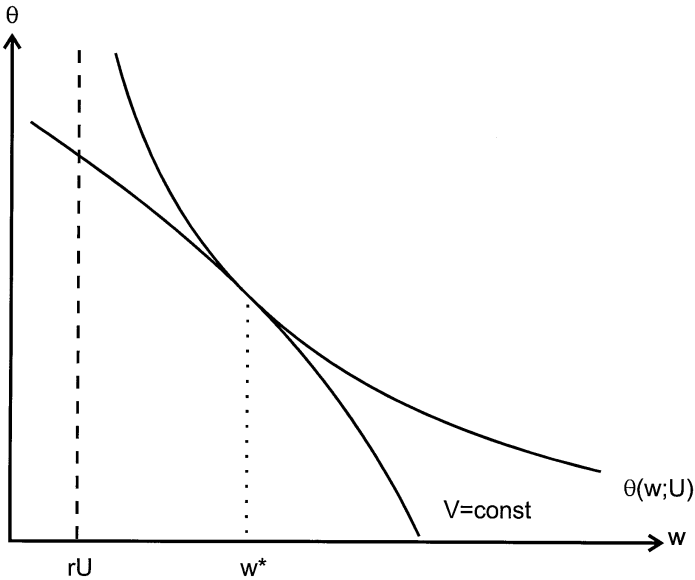
Fig. 2.—Equilibrium

analogy, if w' were the going wage, a club could create profit by charging a wage in the interval (w', w'').

This contrasts with w^* in figure 2. At this point, the marginal rates of substitution between θ and w are equal for firms and workers. The inverse slope of the tangency line can be thought of as a price of labor market tightness in terms of wages; if the tangent is regarded as the choice set for both the workers and the firms, they will choose (w^*, θ^*).

A vacancy is maintained if and only if its asset value is positive. Let $\hat{\imath}$ denote the lowest vacancy type that is maintained. The expected income of opening a vacancy is then

$$\bar{V}(U) = \sum_{i=\hat{\imath}}^{n} f_i \max_{w} V(y_i, w, \theta(w; U)), \qquad (8)$$

where, as before, $f_i = \Pr(y = y_i)$. The following lemma then holds.

LEMMA 2. $\bar{V}(U)$ is continuous and strictly decreasing in U.

The expected income when unemployed, U, plays an important role in the analysis. If U increases, $\theta(w; U)$ strictly increases for all $w > rU$. As a result, the value of all types of vacancies falls, and thus also the expected value of opening a vacancy.

Since there is free entry in the model, vacant jobs are created until

the expected income of a vacancy is equal to the creation cost k. In equilibrium, therefore,

$$\bar{V}(U) = k. \tag{9}$$

To close the model, let us also include a steady-state relationship between the stock of unemployed and the labor flows into and out of the various submarkets. This gives a multidimensional version of the Beveridge curve, showing the relationship between the number of searching workers and the number of searching firms prevailing in equilibrium. Note that in the steady state, the flow of workers entering the unemployment pool is given by $(1 - u)s$, where u is the aggregate unemployment rate and is equal to the flow of vacancies entering the market. Furthermore, the inflow of vacancies must be equal to the outflow in each submarket; the latter is given by $u_i p(\theta_i)$, where u_i denotes the measure of unemployed workers in the submarket in question. Therefore, $u_i p(\theta_i) = (1 - u)s\tilde{f}_i$, where $\tilde{f}_i = f_i/(1 - F_{i-1})$, with $F_i = \Pr[y \le y_i]$. Together with the fact that $\sum_i u_i = u$, this equation determines $u_i, \ldots, u_n$ and u given $\theta_i, \ldots, \theta_n$.

In summary, an equilibrium E^* of the model is given by the following equations:[4]

$$\bar{V}(U) = k, \tag{10}$$

$$w_i = \operatorname*{argmax}_{w} V(y_i, w, \theta(w; U)), \quad i \ge \hat{\imath}, \tag{11}$$

$$rU = \frac{(r + s)z + p(\theta_i)w_i}{r + s + p(\theta_i)}, \quad i \ge \hat{\imath}, \tag{12}$$

$$u_i p(\theta_i) = \tilde{f}_i(1 - u)s, \quad i \ge \hat{\imath}, \tag{13}$$

$$\sum_{i=\hat{\imath}}^{n} u_i = u. \tag{14}$$

The next proposition shows that an equilibrium exists if the economy is productive enough.

PROPOSITION 1. If $\sum_{i=1}^{n} f_i \max[y_i - z, 0]/(r + s) > k$, the equilibrium defined above exists.

The structure of the equilibrium is almost recursive. The key variable U is determined in the first equation by the entry condition. Given U, the second set of equations determines the wages in each submarket, and the third set of equations the corresponding values

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[4] To simplify the exposition we shall assume that, in the case in which (7) has more than one solution, the market maker opens a submarket for only one of the optimal wages. This ensures that there are exactly as many submarkets as there are maintained vacancies.

of θ. The last equations determine the unemployment rate and the distribution of unemployed workers over submarkets. Note also that although the equilibrium is not necessarily unique (since [11] can have more than one solution for each i), the value of U is; that is, in all equilibria the unemployed workers get the same expected income.

D. Optimality

Now the time has come to look at the welfare properties of the model. The optimality criterion is as in Pissarides (1990), the discounted aggregate production net of search costs and creation costs of vacancies. Let a denote the flow of new vacancies created and N_i the measure of workers working in firms with productivity y_i. The social optimum then maximizes

$$W = \int_0^\infty e^{-rt} \sum_{i=\hat{i}}^n (N_i y_i + z u_i - c v_i - a k) \, dt \qquad (15)$$

with respect to a, $\hat{i}$, and $u_{\hat{i}}, \ldots, u_n$, given that the paths of the state variables $N_{\hat{i}}, \ldots, N_n, v_{\hat{i}}, \ldots, v_n$ are governed by the differential equations

$$\dot{N}_i = v_i q\left(\frac{v_i}{u_i}\right) - s(1 - N_i), \quad i \geq \hat{i}, \qquad (16)$$

and

$$\dot{v}_i = a f_i - v_i q\left(\frac{v_i}{u_i}\right), \quad i \geq \hat{i}, \qquad (17)$$

and given the constraint

$$\sum_{i=\hat{i}}^n (u_i + N_i) = 1. \qquad (18)$$

PROPOSITION 2. All equilibria satisfying (10)–(14) are optimal.

The result can be separated into two parts. First, the equilibrium allocation gives an efficient allocation of searching workers and firms on submarkets. Second, an efficient number of vacancies are opened.[5] Note also that if there is more than one equilibrium solu-

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[5] From the Hamiltonian defined in the Appendix, it follows that the asset value of a vacancy coincides with its social value in all submarkets. From this it follows directly that the optimality result still holds if we endogenize search intensities. See Moen (1995a, chap. 7) for details.

tion, the proposition implies that they are all equivalent from a welfare point of view.

In the competitive search equilibrium, vacancies maximize expected profit subject to the constraint that they provide the searching workers an expected income U. Furthermore, a vacancy affects other vacancies only through the market parameter U. Hence all externalities are internalized, and efficiency is obtained.

In the Appendix I show how the core equilibrium equations can be rewritten to a more familiar form. Define the match surplus S_i as $S_i \equiv E_i + J_i - U - V_i$, and let $\eta = \theta q'(\theta)/q$ (i.e., the elasticity of q with respect to θ). Then (11) and (13) can be written as

$$rV_i = -c + q(\theta_i)(1 - \eta_i)S_i, \quad i \geq \hat{\imath}, \tag{19}$$

and

$$S_i = \frac{y_i + sU}{r + s} - U - V, \quad i \geq \hat{\imath}. \tag{20}$$

From Hosios (1990) and Pissarides (1990), the equations can be recognized as the equilibrium equations with ex post bargaining, when the workers' share of the surplus (or bargaining power) β in submarket i is equal to η_i. Furthermore, both Hosios and Pissarides show that the model with ex post bargaining, and with homogeneous firms, gives rise to an optimal allocation of resources exactly when $\eta = \beta$. With heterogeneous agents, the bargaining model is never efficient since optimality requires that vacancies of different types be separated into submarkets (with different θ).

IV. Wage Announcements

In this section, I demonstrate how the competitive search equilibrium can be realized if one assumes that firms, when advertising a vacancy, also announce the wage they are going to pay.[6]

Suppose that a set $W^a = w_1, \ldots, w_m$ of wages is announced in equilibrium by a measure $v_1, \ldots, v_m$ of vacancies. Again assume that the unemployed workers have the capacity to search for only a fraction of the jobs and thus choose a subset of jobs to apply for. The set of firms announcing a certain wage and the set of workers

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[6] As mentioned in the Introduction, the important issue is that workers know the wages at an early stage in the search process. This may be due to reputation effects.

applying for jobs paying this wage thus form a submarket.[7] If u_j is the measure of workers in submarket j, the number of matches in this market is $x(u_j, v_j)$.

In equilibrium, workers are indifferent about which subset of firms to apply for. Hence all submarkets must give the same expected income for unemployed workers. By arguing in the same way as in the last section, we therefore find that the labor market tightness in submarket i is given by $\theta(w_i; U)$.

Firms announce the wages that maximize the expected value of their vacancy, given their beliefs about the relationship between the wage they announce and the arrival rate of workers. Denote these beliefs by $q^e(w)$. The firms thus choose w so as to maximize the expected income V given by (6), with $q^e(w)$ substituted in for q. In any rational expectations equilibrium, $q^e(w) = q(\theta(w))$ for all wages w actually announced in equilibrium. However, for the model to have predictive power, we must also specify the firms' out-of-equilibrium beliefs, that is, the beliefs about the arrival rates of workers for wage announcements that are not made in equilibrium.

A trembling-hand type of argument indicates that it may be reasonable to require that $q^e(w) = q(\theta(w; U))$ for all w, not just for wages announced in equilibrium. To see this, let w' be any wage not announced in equilibrium. Then assume that a set of firms deviates and announces w'. Let the measure of the deviating firms be so small that the effects on U can be ignored. Then the arrival rate of workers facing the deviating firms is exactly $q(\theta(w; U))$.

In Moen (1995b), using a refinement of the equilibrium concept along the lines of Gale's (1994) notion of a stable equilibrium, I show that all equilibrium wage offers must maximize the expected income of the vacancy subject to the condition that $\theta = \theta(w; U)$. Here I give only a heuristic argument, based on figure 3. The figure shows a rational expectations equilibrium in which the expectations differ from $q(\theta(w))$. For simplicity set $n = 1$. The situation with heterogeneous firms can be analyzed in a similar way.

For expositional clarity, assume that the vacancy in question has expectations over θ rather than q. Since $q(\theta)$ is monotone in θ, this assumption is innocuous. The curve given by $\theta^e(w)$ shows an arbitrary set of expectations. Given these expectations, all firms announce w' in equilibrium. Since $\theta^e(w') = \theta(w')$, this is a rational

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[7] If workers search for jobs with different wages, they will belong to more than one submarket, and their contribution to u will be adjusted relative to their search effort in each market. To simplify the exposition, we shall assume that all workers join one submarket only. This is not necessary for the results, though. Note that since firms cannot announce more than one wage at any one time, a vacant job can belong to only one submarket by definition.
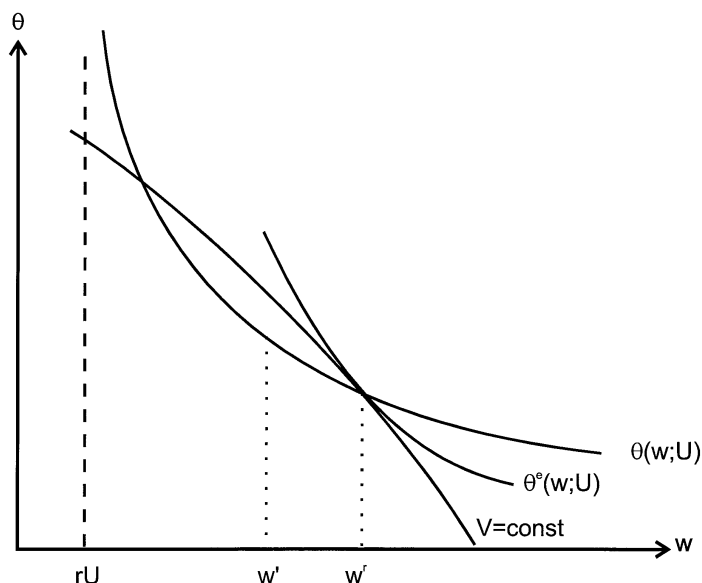
Fɪɢ. 3.—Rational expectations equilibrium with beliefs given by $\theta(w; U)$

expectations equilibrium. Assume that a small subset of the firms deviates and instead announces a wage w^*.[8]

The deviating firms then face a labor market tightness $\theta(w^*; U)$, and from the figure we can see that they obtain a strictly higher expected income than the nondeviating firms. Furthermore, when this is observed by the other firms, they update their expectations and change the announced wage. Hence, a deviation by an arbitrarily small subset of vacancies changes the equilibrium substantially; we say that the equilibrium with $w = w'$ is unstable. A similar argument can be used for all sets of beliefs that lead to an announced wage different from the one that maximizes $V(w, y, \theta(y; U))$. All announced wages thus satisfy the equilibrium condition (11). Since the remaining parts of the two models are identical, we have shown the following proposition.

Pʀᴏᴘᴏsɪᴛɪᴏɴ 3. Suppose that the firms' beliefs are as described above. Then the set of announced wage equilibria and the set of competitive equilibria are equivalent.

From proposition 2 it thus follows that all the announced wage equilibria are efficient.

The result hinges on the assumption that wages are not renegoti-
ated ex post, that is, after the match has occurred. The firms have
announced their wage offers publicly, and it is easy to imagine that
this is binding because of reputation effects, trade unions, and so
forth. However, this does not apply to workers. It follows from equa-
tions (19) and (20) that the worker can improve his wage through
conventional Nash bargaining if and only if his share of the surplus
(or bargaining power) exceeds η, the elasticity of q with respect to
θ. This suggests that a wage announcement is more likely to occur
in labor markets in which the workers' bargaining power is relatively
small.

V.   Examples and Extensions

In this section, I first study the relationship between productivity and
wages (insider-outsider effects). Then I extend the model to allow
for heterogeneous workers. The results are stated as properties of
the competitive equilibrium solution, but they also hold for the
model with wage announcements.

A.   Wage Distributions

The wage given by (11) can be expressed as $w = \delta(y; U)$. The func-
tion[9] δ relates the exogenous distributions of productivities over
firms and the distribution of wages, and it can be shown that δ has
the following properties.
    PROPOSITION 4. $\delta(y)$ is strictly increasing in y, $\lim_{y \to \infty} \delta(y; U) =$
∞; and when $c = 0$, $\lim_{y \to rU} \delta(y; U) = rU$.
    The intuition behind the first part of the proposition is illustrated
in figure 4, where the iso-profit curves for two vacant jobs with differ-
ent productivities are drawn. The search costs in terms of forgone
production increase with productivity, so the rate of substitution be-
tween wages and labor market tightness is lower (the iso-profit curve
is flatter) for high- than for low-productivity firms. High-productivity
firms are therefore more willing than low-productivity firms to trade
off low wages for low labor market tightness and thus join submar-
kets with higher wages.
    For a general functional form of the matching function, I cannot
say much specific about the properties of δ. It might even be discon-
tinuous in y, since (11) can have more than one solution. However,

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[9] Since the firm's maximization problem may have more than one solution, $\delta(y)$
is actually a correspondence. To simplify the exposition, let $\delta(y)$ denote the smallest
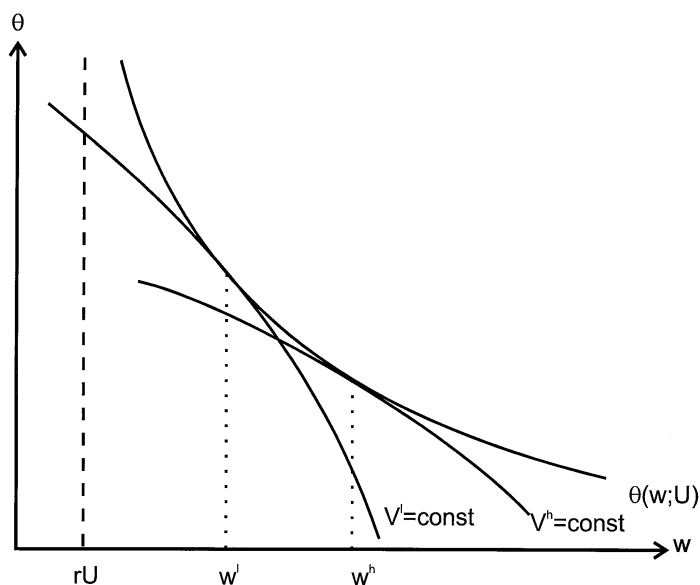value of w when the maximization problem has more than one solution.

F<small>IG</small>. 4.—Equilibrium with heterogeneous firms and homogeneous workers. The figure shows the indifference curves for high- (h) and low- (l) productivity firms and the wages they are offering in equilibrium.

if we assume that the matching function is of the Cobb-Douglas type (which is often assumed in the literature and fits data reasonably well), it can be shown that $\delta'(y)$ is decreasing in y (see Moen 1995b). Hence the influence of a firm's productivity on the wages it pays is highest at low productivity levels.

B. Extensions

In this subsection, I indicate how the model can be extended to allow for heterogeneous workers. More specifically, assume that the workers differ in z, their income (or utility) when unemployed. Also assume that η is nondecreasing, which implies that the equilibrium is unique in the previous model with homogeneous workers.

Denote each worker's unemployment income by z_i, $i = 1, \ldots, k$, with z_i increasing in the index i, and let U_i denote the expected discounted income for a worker of type i. Then U_i is increasing with i. Furthermore, for each worker i, define $\theta_i(w; U_i)$ from the indifference requirement (4) for each i; that is, define a separate relationship between wages and labor market tightness for each type of workers. Finally, define
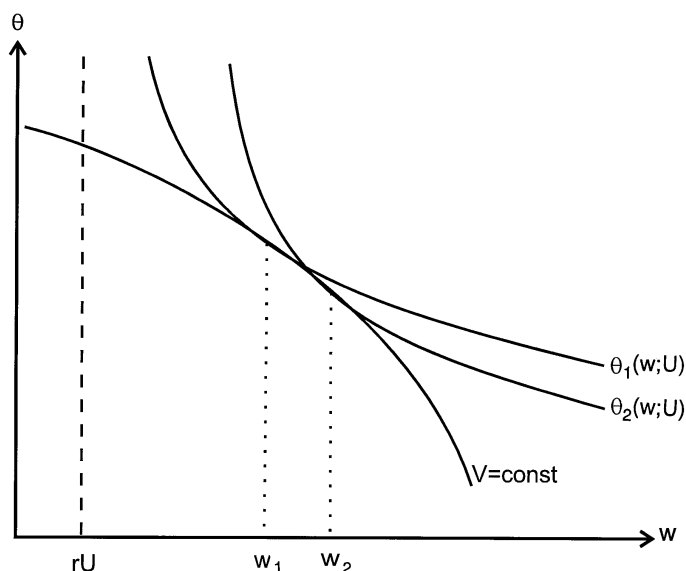
FIG. 5.—Equilibrium with heterogeneous workers and homogeneous firms. $\theta_1(w;$ $U)$ and $\theta_2(w; U)$ denote the indifference curves to workers with unemployment incomes z_1 and z_2, respectively, where $z_1 < z_2$; w_1 and w_2 are equilibrium wages.

$$\tilde{\theta}(w; U_1, \ldots, U_k) = \min_{i \in k} \theta_i(w; U_i).$$

The no-surplus condition implies that all firms with vacancies maximize profit subject to $\tilde{\theta}(w; U_1, \ldots, U_k)$. Entry of firms implies that $V(U_1, \ldots, U_k) = k$. Workers of all types join the submarkets that give them the highest expected income.

The definition of the competitive equilibrium is considerably more complicated than with homogeneous workers. The entry condition gives only one restriction on $U_1, \ldots, U_k$. The model is thus not recursive, and one has to keep track of inflows and outflows of both vacancies and workers of different types to be able to solve for $U_1, \ldots, U_k, \theta_1, \ldots, \theta_k$, and the set of wages. Instead of going through a general analysis, I give some examples and show some results in a less rigorous setting.

Figure 5 shows the equilibrium in the case in which z can take two values z_1 and z_2, whereas firms are homogeneous. The curves $\theta_1(w;$ $U)$ and $\theta_2(w; U)$ represent the indifference requirements for workers of types 1 and 2, respectively, and $\tilde{\theta}(w)$ consists of $\theta_1(w; U_1)$ to the left and $\theta_2(w; U_2)$ to the right of the intersection between the two lines. Note that θ_1 is flatter than θ_2, reflecting that the gain from getting a job more quickly is higher the lower the unemployment
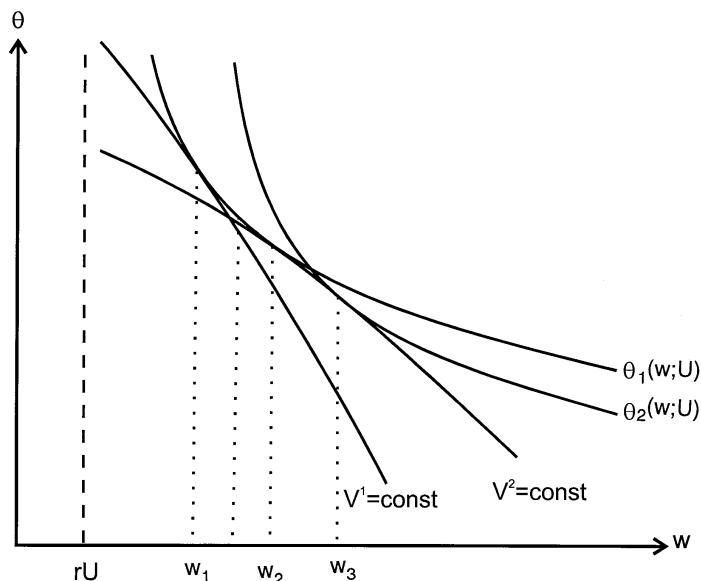
Fig. 6.—Equilibrium with heterogeneous workers and homogeneous firms. In the submarket with wage w_1, workers are of type 1 (low unemployment income) and firms are of type 1 (low productivity). In the submarkets with wages w_2 and w_3, workers are of type 2 (high unemployment income) and firms are of types 1 and 2 (low and high productivity, respectively).

benefit is. In equilibrium there are two active submarkets with wages w_1 and w_2, respectively, $w_1 < w_2$. The wage and the productivity in each of the submarkets are the same as though only one of the types was present. Hence there are no "spillovers" between the types.

Figure 6 shows an equilibrium of the model with two types of both firms and workers. The equilibrium consists of three active submarkets. In the submarket with the highest wage (lowest labor market tightness), workers with high unemployment income (and low waiting costs) search for high-productivity jobs (high waiting costs). The opposite holds for the submarket with the lowest wage, and the third submarket attracts high-productivity firms and workers with low income when unemployed.

In general, a given type of firm can search for more than one type of worker. However, the flavor of the results above carries over to the general case with n types of firms and k types of workers. The results are summarized in proposition 5.[10]

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[10] A less straightforward extension is to allow for differences in the workers' productivities. An efficient allocation of resources then requires that workers with different productivities be separated into different submarkets. This may occur if firms, when announcing wages, also announce skill requirements (university degrees

PROPOSITION 5. (1) With homogeneous firms, the equilibrium described above is unique. The equilibrium vectors $(w_1^*, \ldots, w_k^*)$ and $(U_1^*, \ldots, U_k^*)$ are such that, for all i, (w_i^*, U_i^*) corresponds to the (unique) equilibrium values in the original model with workers of type i only. (2) In any equilibrium with heterogeneous firms, the wage in all submarkets joined by firms with productivity y_i is strictly greater than the wage in submarkets joined by firms with productivity $y_j < y_i$. (3) Workers with unemployment income i join submarkets with a strictly higher wage than workers with unemployment income $z_j < z_i$.

VI. Discussion

In this section I first relate my work to the existing literature. Then I discuss more broadly my assumptions about the matching technology and the information structure in the economy. I also discuss whether my main findings have any empirical support.

There does not exist anything similar to the competitive search equilibrium in the literature. However, price advertisements occur in some search models of the retail market (Butters 1977; Robert and Stahl 1993). Most of them differ from mine in that they have congestion effects on only one side of the market, so that customers always visit the supplier with the lowest price. An exception is the model by Peters (1991). He studies a nonstationary market in which the agents exit the market when matched. He focuses on the construction of a matching technology in which the agents are matched randomly and the match probability for a seller is influenced by the price that he advertises. Peters shows how the price announcements can be embodied into the matching technology. Wage announcements in models of the labor market are not common. An exception can be found in Montgomery (1991), which uses wage announcement to explain interindustry differences in wages. Note also that the analysis is similar to the one in Rosen (1986), describing a competitive, frictionless market with equalizing wage differences caused by differences in nonpecuniary characteristics between jobs.

In comparison with the early search literature, the assumption that wage offers are available prior to the search process may seem strange. In Mortensen (1971) and Phelps (1971), the main reason for search activity is to collect information about wage offers by dif-

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etc.). Separation may also occur if the matching technology is such that workers with high productivity have higher job-finding rates than workers with low productivity when operating in the same market. This may deter low-productivity workers from entering the market for high-productivity workers, where the competition from the latter is too stiff. The issues are on my agenda for future work.

ferent firms. But with the introduction of the matching function, this element of the search process is downplayed. Diamond (1971) and Pissarides (1985, 1987), among others, obtain a unique equilibrium wage known to all agents in the economy, thereby eliminating completely the incomplete wage information as a reason for search.

The frictions in the models of Pissarides and others can be associated with incomplete information about the location of jobs. This is not ruled out in the competitive equilibrium either. If wages are known through reputation effects, a worker may search down firms sequentially in order to find a job, knowing through rumors and so forth the wages paid in each firm. However, if workers learn about wages through job advertisements, they obviously have already localized a job slot when they know the wage offer, so in this case imperfect information about the localization of jobs cannot be a reason for search.

Layard, Nickell, and Jackman (1991) argue that a worker's job search can be divided into two parts: First he collects information about vacancies, which come with different preassigned wages and conditions. Then he applies for some of the vacancies he has heard of and accepts the first offer he gets.[11] I focus on the second part of the process as the main contributor to costs and time delays and assume that the frictions underlying the matching function are due to the completion and processing of applications, interviews, time lags due to selection by the firm, and so forth. They may also be due to imperfect information about job characteristics other than wages and to other unmodeled heterogeneities.[12] Similar assumptions can be found in search models with wage announcement of the retail market (see Butters 1977; Robert and Stahl 1993).

The matching technology is regarded as exogenous in the model. An extension of the model to allow for endogenous search intensities is straightforward and would not change the equilibrium of the model in any fundamental way.[13] Furthermore, the modeling of the matching technology is general and includes as special cases the

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[11] Akerlof, Rose, and Yellen (1988) find that only 8 percent of job seekers have rejected a previous job offer.

[12] An interesting generalization of the model would be to allow for match-specific differences in productivities. I conjecture that this would not change my equilibrium in any fundamental way since firms would face the same type of relationship between wages and labor market tightness when deciding on wages.

[13] If we follow Pissarides (1990) and define the arrival rate of, say, job offers to workers as $eq(\theta)$, where e is the search intensity, the worker chooses e to maximize $rU = z + eq(E - U) - a(e)$, where $a(e)$ denotes the convex search costs. Since $q(E - U)$ is the same in all submarkets, all workers choose the same search intensity. The search intensity of vacant jobs can be endogenized in a similar way, resulting in higher search intensity for high- than for low-productivity firms. From the comment in n. 5, we know that this will not influence the optimality result either.

matching functions that result from specific matching processes such as the urn-ball process. Still, the frictions per se are regarded as exogenous. Since the model highlights the importance of the agents' search technology, it would be attractive to have as primitives, say, a technology for the production and processing of applications. This is on the agenda for future work.

The major empirical implication of the model is that the firms that offer high wages attract more applicants and speed up the search process compared to firms that offer low wages. There are some difficulties in testing this hypothesis though, since firms offering high-wage jobs tend to search for workers with specific skills who are difficult to find. Still, empirical findings support our hypothesis. Holzer, Katz, and Krueger (1988) find that higher wages lead to more applicants per vacancy (although the effect is weak). Larsen and Devold (1995) find that higher wages significantly increase the number of applicants (5 percent significance level). Kaufman (1984) finds that employers certainly believe that there is a relationship between the wage and the arrival rate of workers.

VII. Conclusion

In this paper, I have introduced a new equilibrium concept for search models, the competitive search equilibrium, which generates a socially efficient allocation of resources. I have demonstrated how the equilibrium can be realized if firms can publicly announce wages for their vacant jobs. Finally, I have analyzed the resulting wage distribution and have suggested how the model can be extended in various directions.

I have employed a stripped-down matching framework, and in future work I would like to extend the model to allow for things such as match-specific productivities on the job search and heterogeneities in worker productivity. However, I still think that my analysis illustrates that market frictions do not necessarily lead to an inefficient allocation of resources.

Appendix

Proof of Lemma 1 and Lemma 2

Define $\hat{V}(w, y_i, U) \equiv V(w, y_i, U, \theta(w; U))$ for $w > rU$ and $\hat{V}(rU, y_i, U) = -c/r$. From (6) it follows that $\hat{V}$ is continuous in w at $w = rU$. Hence $\hat{V}$ is continuous in all its arguments at the entire domain.

The solution to the maximization problem (7) must be on the interval $[rU, y_i]$. The maximization problem (7) can thus be written as

$$\max_{w \in [rU, y]} \hat{V}(w, y_i, U). \tag{A1}$$

Since $\hat{V}$ is continuous, the problem is well defined. Furthermore, since $\hat{V}(rU, y, rU)$ and $\hat{V}(y, y, rU)$ are both less than zero, the solution must be on the interior of $[rU, y]$ since by assumption the supremum V^s is greater than zero. Since $\hat{V}$ is continuous in all its arguments, the maximum defined by (A1) and thus $\overline{V}$ defined by (8) are continuous in U. Finally, since $\theta(w; U)$ is strictly decreasing in U for all $w > rU$, it follows from the envelope theorem that $\overline{V}(U)$ is strictly decreasing in U as well (for $\overline{V} \geq 0$).

Proof of Proposition 1

It is sufficient to show that there exists a U such that $\overline{V}(U) = k$. The existence of a solution to (11) then follows from lemma 1, and the rest of the equations are well defined by definition.

We know that $\overline{V}(U)$ is continuous and strictly increasing in U. Furthermore, $\overline{V}(U) = 0$ if $rU > y_n$ (the productivity in the most productive firm). It is therefore sufficient to show that there exists a U such that $V(U) > k$.

It is sufficient to show that $\lim_{U \to z} \overline{V}(U) > k$. From (4) it follows that $\lim_{rU \to z} p(\theta) = 0$ for all $w > rU$ and hence that $\theta \to 0$ as well. The value of a vacancy with productivity $y > z$ thus converges to $(y - z)/(r + s)$. From (8) it follows that

$$\lim_{U \to z} \overline{V}(U) = \sum_{i=1}^{n} f_i \frac{\max[y_i - z, 0]}{r + s}.$$

Hence the proposition follows.

Proof of Proposition 2

I first show that the market solution satisfies the necessary conditions for optimality in the special case in which the elasticity of η is nondecreasing. Then I show necessary conditions in the general case. Finally I give sufficient conditions.

Let $\tilde{\imath}$ denote a given cutoff level, so that a vacancy is announced iff $i \geq \tilde{\imath}$. The current-value Hamiltonian associated with the maximization of (15) subject to (16)–(17) is then given by

$$\begin{aligned}
H = &\sum_{i=\tilde{\imath}}^{i} N_i y_i + z \sum_{i=\tilde{\imath}}^{n} u_i - c \sum_{i=\tilde{\imath}}^{n} v_i - ak \\
&+ \sum_{i=\tilde{\imath}}^{n} \lambda_i \left[v_i q\left(\frac{v_i}{u_i}\right) - s N_i \right] \\
&+ \sum_{i=\tilde{\imath}}^{n} \gamma_i \left[aF_i - v_i q\left(\frac{v_i}{u_i}\right) \right]
\end{aligned} \tag{A2}$$

$$+ \alpha \left[1 - \sum_{i=\tilde{\imath}}^{n} (u_i + N_i) \right],$$

where λ_i and γ_i are the adjoint functions corresponding to N_i and v_i, respectively, and α denotes the multiplier for the constraint (18). Necessary conditions for the steady-state optimal solution are given by

$$u_i = \operatorname*{argmax}_{u_i} H(u, a, v, N) \quad \forall \; i, \tag{A3}$$

$$a = \operatorname*{argmax}_{a} H(u, a, v, N), \tag{A4}$$

$$\frac{\partial H}{\partial N_i} = r\lambda_i \Rightarrow \frac{y - \alpha}{r + s} \quad \forall \; i, \tag{A5}$$

$$\frac{\partial H}{\partial v_i} = r\gamma_i \Rightarrow r\gamma_i = -c + q(1 - \eta)(\lambda_i - \gamma_i). \tag{A6}$$

When η is nondecreasing in θ, H is concave in u, and (A2) is determined by the unique set of first-order conditions. Since H is linear in a, we can thus write (remember that $\eta = -q'(\theta)\theta/q$, and thus $\partial vq(v/u)/\partial u = \theta q(\theta)\eta$, etc.)

$$\frac{\partial H}{\partial a} = 0 \Rightarrow k = \sum_{i=\tilde{\imath}}^{n} \gamma_i F_i, \tag{A7}$$

$$\frac{\partial H}{\partial u_i} = 0 \Rightarrow z + \eta_i \theta_i q(\theta_i)(\lambda_i - \gamma_i) = \alpha \quad \forall \; i. \tag{A8}$$

To determine $\tilde{\imath}$, note that the derivative of W with respect to v_i is given by γ_i; therefore, a vacancy must be announced if and only if

$$\gamma_i \geq 0 \Rightarrow -c + q_i(1 - \eta_i)(\lambda_i - \gamma_i) \geq 0. \tag{A9}$$

The set of first-order conditions is thus equivalent to the equilibrium conditions (19) and (20), with α, γ_i, and λ_i substituted in for rU, V_i, and $S_i + \gamma_i$. Since we know that the equilibrium is unique when η is nondecreasing, this means that the market solution satisfies the first-order conditions.

The proof in the general case follows the same lines as in the case in which η is nondecreasing. However, a solution to (A7) is not necessarily solving (A2), and we therefore have to work with (A7) directly. Thus

$$u_i = \operatorname*{argmax}_{u_i} H \Rightarrow u_i = \operatorname*{argmax}_{u_i} \left[zu_i + (\lambda_i - \gamma_i) v_i q\left(\frac{v_i}{u_i}\right) - \alpha u_i \right]. \tag{A10}$$

We want to show that this problem is equivalent to the problem of maximizing V_i given $\theta(w; U)$. It turns out to be convenient to rewrite the maximization problem to the equivalent problem of maximizing V with respect to the *share b* of the surplus that is allocated to the worker, that is (with subscripts suppressed),

$$\max_{b} \; -c + q(\theta)(1 - b) S_i$$

subject to

$$rU = z + \theta q(\theta) bS,$$

where the last equation is the workers' indifference constraint. Since there is a one-to-one relationship between b and θ, we can substitute out b from the maximand and maximize with respect to θ instead. The maximization problem can thus be written as

$$\max_{\theta} \left[-c + q(\theta) S - \frac{rU - z}{\theta} \right], \tag{A11}$$

which can be rewritten as

$$-cv_i + v_i[\max_{u_i}(zu_i + v_i qS - u_i rU)].$$

Note that the problem has the same form as (A9). The set of first-order conditions is therefore again equivalent to the market equilibrium conditions. Therefore, all market equilibria satisfy the necessary conditions for optimality.

To show sufficiency, we use Arrow's sufficiency theorem (see Seierstad and Sydsæter [1987, p. 289, theorem 6] for details). Write $\hat{H}(N, v) = H(N, v, \lambda^*, \gamma^*)$, where the asterisk indicates that we are using the values derived by the necessary conditions. It is then sufficient to show that $\hat{H}$ is concave in N, v. First note that $\hat{H}$ is linear in N and that $\hat{H}_{Nv} = 0$. Further we know from (A5) that $\hat{H}_v$ is positive and (since $vq(\theta) = x(u, v)$) $\hat{H}_v = -c + x_v(u, v)(\lambda^* - \gamma^*)$. Since x is concave in v, this gives sufficiency.

Derivation of Equations (19) and (20)

If we substitute $p(\theta) = \theta q(\theta)$ into (4), we get (here we suppress U in the θ-function and subscript i)

$$\theta(w) q(\theta(w)) = \frac{rU - z}{w_i - rU} (r + s). \tag{A12}$$

Taking derivatives with respect to w in (6), setting the total derivative of V equal to zero, gives

$$q'(\theta_i) \frac{d\theta}{dw} \left(\frac{y - w}{r + s} - V_i \right) = \frac{q(\theta)}{r + s}. \tag{A13}$$

Taking the derivative of (A12) with respect to w and setting $V'(w) = 0$ gives

$$\frac{d\theta}{dw} q(1 - \eta) = -\frac{rU - z}{(w - rU)^2}(r + s),$$

where $\eta = \eta(\theta_i) = -\theta q'(\theta)/q$ and so $(d/d\theta)\,\theta q(\theta) = q(1 - \eta)$. Substituting out $(rU - z)/(w - rU)$ by virtue of (A12) gives

$$(1 - \eta)\frac{d\theta}{dw} = -\frac{\theta}{w - rU}.$$

Inserting this into (A13) yields

$$\frac{\eta}{1 - \eta} = \frac{w - rU}{y - w - (r + s)V(y)} \tag{A14}$$

$$= \frac{E_i - U}{J - V}, \tag{A15}$$

where, as before, J and E denote the asset values of an occupied job and an employed worker, given by equations (2) and (5), respectively. Defining the match surplus S as $S = J - V + E - U$, we find that $J - V = (1 - \eta)S$ and $E - U = \eta S$, which inserted into (A13) and (A14) gives (19) and (20).

Proof of Lemma 4

Let $y_1 > y_2$ and $w_1 > w_2$, and define $\Delta_i = V(w_1, y_i) - V(w_2, y_i)$, $i = 1, 2$. Then we have

$$\Delta_1 - \Delta_2 = \frac{q_1}{r + q_1}\left(\frac{y_1 - y_2}{r + s}\right) - \frac{q_2}{r + q_2}\left(\frac{y_1 - y_2}{r + s}\right) > 0,$$

where $q_i = q(\theta(w_i))$ and thus $q_1 > q_2$. This means that a firm with high productivity always gains strictly more than a firm with lower productivity when increasing the wage; thus $\delta(y)$ is nondecreasing in y. Further, since the partial derivative of V with respect to w is continuous, the optimal wages cannot be equal. It follows that $\delta(y)$ is strictly increasing in y.

Now we want to show that

$$\lim_{y \to \infty} \delta(y) = \infty.$$

We know that w solves $\max_w V(y, w, \theta(w; U))$. The first-order condition for the maximum is $V_w = -V_\theta\theta'(w)$. Taking derivatives (6) thus gives

$$\frac{q}{(r + q)(r + s)} = \frac{y_i - w}{r + s} q(\theta)|\theta'(w; U)|. \tag{A16}$$

The left-hand side is smaller than $1/(r + s)$ for all y. The right-hand side goes to infinity if w does not, and the result thus follows.

When $c = 0$, the value of a vacancy is strictly positive if and only if $y > rU$, and then $y > \delta(y) > rU$. Hence $\lim_{y \to rU} \delta(y) = rU$.

Proof of Proposition 5

Parts 2 and 3 follow directly from a revealed preference argument analogous to the argument in the proof of lemma 4. We therefore concentrate on the proof of the first part.

In any equilibrium with homogeneous firms, we know that workers of different types join submarkets with different wages. If workers of type k

join submarket i, we know that $w_i \in \text{argmax}_w V(w, \theta_i(w; U_i))$ and that $V(w_k, y_i) = k$ in all submarkets. But then the result follows.

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