1 Experiment 1

1.1 Experimental Design

This consists of playing one round of either 3 or 15 boxes. Users win 0 the first round. Then are asked the probability of winning at least one the next round. Then they are asked if they would like to play another round, where they will keep their winnings, but they will forfeit a 1USD consolation prize they could have now.

1.2 Proof of Optimality

Let's first consider the case where one makes 3 attempts and gets a payoff of 0 each time. What can we say about the outcome of our next 3 attempts? And in the case where one makes 15 attempts and gets a payoff of 0 each time, what can we then say about the outcome of our next 15 attempts? Our estimated expected payoff is 0 in both cases, but in which case are we most confident about that expectation?

The problem is clearly a discrete 0,1 problem: either you are rejected or you are accepted. The problem of learning then becomes one of estimating the free (p) parameter of a binomial distribution, given you have sent n applications. Given your estimate, $\hat{p}$, the probability of winning at least one of the next n attempts is given by:

$$P(win) = 1 - (1 - \hat{p})^n \tag{1}$$

We will formulate the process of estimating $\hat{p}$ as one of Bayesian learning, with a Beta prior, paramaterized by α and β, on the value of p. Given n failures, the corresponding posterior Beta distribution is paramaterized by:

$$\begin{aligned} \alpha'_n &= \alpha \\ \beta'_n &= \beta + n \end{aligned}$$

The PDF of the posterior estimate of p after n failed attempts is therefore given by:

$$P(x|n) = \frac{x^{\alpha-1}(1-x)^{\beta-1+n}\Gamma(\alpha+\beta+n)}{\Gamma(\alpha)\Gamma(\beta+n)}$$

Which can be plugged back into 1 and take expectations to give us the expected payoff:

$$\mathbb{E}[win] = \int_0^1 1 - (1-x)^n \, P(x|n)dx$$

Which is increasing in n for any valid values of α and β. Any reasonable estimate of p compatible with correct Bayesian updating will, therefore, suggest that players estimate a greater probability of winning given another n chances, after n failures, as n increases.

2 Experiment 3

2.1 Experimental Design

2.2 Proof of Optimality

In this game, players two unknown variables to estimate, both of which effect the probability of winning:

1. The "group" the player belongs to ($a_i, i \in \{L, H\}$)
2. The (unkown) probability of payout, t, given that they are in the group a_H.

Rolls are modelled as Bernouli random variables:

$$k_{n+1} \sim Bernouli(p)$$
$$p := P(a_H|n)P(t|n)$$

Clearly, the two variables are not uniquely identifiable without further assumptions. In general, the difference between updating one's beliefs about one's own characteristics versus updating one's beliefs about the world is extremely important in the general question of how people search for jobs. For our current purposes, however, we treat them as one and the same: participants are given no information as to the underlying probabilities of each roll, and as such must estimate everything at once.

We can formulate the Bayesian posteriors, after n failures, as follows (assuming independence of a_i and t):

$$P(a_i|n,t) = \frac{P(n|a_i,t)P(a_i)}{\sum_i P(n|a_i,t)P(a_i)}$$
$$P(t|n,a_i) = \frac{P(n|a_i,t)P(t)}{\int_{t=0}^{1} P(n|a_i,t)P(t)dt}$$

We can formulate the game formally in the form of a recursive dynamic programming

$$V(\hat{P}_{H,n}, k_n) = \begin{cases} W, \text{for } k_n = 1 \\ \max \left\{ \mathbb{E}[V(\hat{P}_{H,n+1}, k_{n+1})] \ , \ \mathbb{E}[V_q] \right\}, \text{for } k_n = 0 \end{cases}$$

Where W donates the winnings from the game (\$5), and V_q the value of quitting and pursuing ones outsides option (presumably working on other MTurk tasks). Clearly, this is an optimal stopping time problem, with the conditions for stopping given by:

$$\mathbb{E}[V(\hat{P}_{H,n+1}, k_{n+1})] \leq \mathbb{E}[V_q]$$

Which happens when the opportunity cost of playing "one more roll" exceeds the expected payoff, given the current estimation of p. As such, we can directly examine the marginal case:

$$\hat{P}_{H,n}W \leq rc$$

Where t is the time (seconds) it takes to play one more roll, and c is the opportunity cost (dollars / second) of the outside option's wage.