1 Pricing and Hedging Asian Options

1.1

1.1.1

1.1.2 Boundary Conditions

We begin by adding a parameter to our Y function to make it a bit more expressive:

$$Y_T = \int_0^T S_u du$$
$$Y_{0,T} = \int_0^T S_u du$$

We will re-write our payoff as a function of two variables by utilizing the fact that Y is an integral, and as such can be decomposed into the sum of two parts:

$$H(Y_T) = \left(\frac{1}{T}Y_T - K\right)^+$$
$$H(Y_{0,t}, Y_{t,T}) = \left(\frac{1}{T}Y_{0,t} + \frac{1}{T}Y_{t,T} - K\right)^+$$

This allows us to use the fact that $Y_{0,t}$ is $\mathcal{F}_t$ measurable and $Y_{t,T}$ is independent of $\mathcal{F}_t$ given S_t:

$$\tilde{V}_t = e^{-r(T-t)}\ \mathbb{E}\big[H(y, Y_{t,T})|_{S_t, y=Y_{0,t}}\big]$$

And we solve with $y = Y_t$ for our two boundary conditions:

$$v(t,0,y) = e^{-r(T-t)}\ \mathbb{E}\big[\left(\frac{y}{T} + \frac{Y_{t,T}}{T} - K\right)^+|_{S_t, y=Y_t}\big]$$

$$v(T,x,y) = e^{-r(T-t)}\ \mathbb{E}\big[\left(\frac{y}{T} + \frac{Y_{T,T}}{T} - K\right)^{+} |_{S_t, y=Y_t}\big]$$
$$v(T,x,y) = e^{-r(T-t)}\ \mathbb{E}\big[\left(\frac{y}{T} + 0 - K\right)^{+} |_{S_t, y=Y_t}\big]$$
$$v(T,x,y) = e^{-r(T-t)}\ \mathbb{E}\big[\left(\frac{y}{T} - K\right)^{+} |_{S_t, y=Y_t}\big]$$