

1

$$\begin{aligned}
Y_t &= e^{at} X_t \\
\frac{d}{dt} Y_t &= ae^{at} X_t + e^{at} \frac{dX_t}{dt} + \frac{1}{2} * 0 \\
dY_t &= ae^{at} X_t dt + e^{at} dX_t \\
dY_t &= ae^{at} X_t dt + e^{at} (a(m - X_t) dt + \sigma dB_t) \\
dY_t &= ae^{at} X_t dt + ame^{at} dt - ae^{at} X_t dt + e^{at} \sigma dB_t \\
dY_t &= ame^{at} dt + e^{at} \sigma dB_t \\
\int_0^t dY_s ds &= \int_0^t ame^{as} ds + \int_0^t e^{as} \sigma dB_s \\
Y_t - Y_0 &= \frac{1}{a} ame^{at} - \frac{1}{a} ame^{a*0} + \int_0^t e^{as} \sigma dB_s \\
e^{at} X_t - X_0 &= me^{at} - me^0 + \int_0^t e^{as} \sigma dB_s \\
X_t &= X_0 e^{-at} + me^{at} e^{-at} - me^{-at} + e^{-at} \sigma \int_0^t e^{as} dB_s \\
X_t &= m - (X_0 - m) e^{-at} + e^{-at} \sigma \int_0^t e^{as} dB_s
\end{aligned}$$

2

$$\begin{aligned}
\mathbb{E}[X_t] &= \mathbb{E} \left[m - (X_0 - m) e^{-at} + e^{-at} \sigma \int_0^t e^{as} dB_s \right] \\
\mathbb{E}[X_t] &= m - (X_0 - m) e^{-at} + e^{-at} \sigma \int_0^t \mathbb{E} \left[e^{as} dB_s \right]
\end{aligned}$$