# Continuous Survey Sample Optimization Using Ad Platform APIs

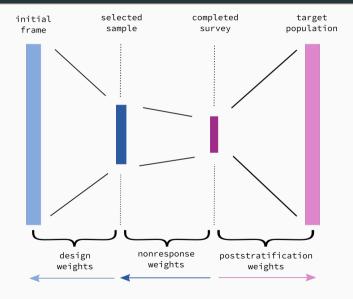
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### Introduction

#### Motivation - weighting



Poststratification weighting can be done in many ways, but we will consider the simplest case.

We want to estimate e a population parameter Y via sampling and measurement. We will assume that the researcher wishes to use the stratified mean for a set of strata  $h \in H$  (mutually exclusive cells) with an assigned weight for each stratum  $W_h$ , which we will denote  $\bar{y}$ :

$$\hat{Y} := \sum_h W_h \bar{y}_h$$

- 1. Commit to poststratification weighting.
- 2. Measure response rate dynamically during the surveying process
- 3. Adjust the selected sample dynamically during the surveying process (dynamic over/under-sampling).
- 4. Make the adjustment to minimize variance subject to budget constraints.

## Optimization

We assume that we are able to measure a set of additional survey responses which we will consider covariates and denote  $x_i \in X$ . We assume the existence of a mapping  $X \to H$  such that the measured covariates are sufficient to assign each individual to one and only one stratum. In addition, we assume that  $x_i$  is measured during recruitment.

The variance of our sample estimate is thus given by:

$$\mathbb{V}[\hat{Y}] = \sum_{h} W_h^2 \frac{s_h^2}{n_h}$$

where  $s_h^2$  denotes the variance of the population parameter of interest Y within stratum h. If the outcome was measured during recruitment, we can estimate this stratum-specific variance.

We simplify the problem by assuming that the variance of the outcome in each stratum is equal (i.e.  $s_h^2 = s^2$ ). With that assumption, we have the following variance of our estimate:

$$\mathbb{V}[\hat{Y}] = s^2 \sum_h \frac{W_h^2}{n_h}$$

Note that, given a fixed n and the assumption of equal variance across strata, this quantity is minimized when  $\frac{n_h}{n} = W_h$ , known as the Neyman allocation.

But we don't have infinite moneys...

Denote the cost to recruit an individual from stratum h as  $P_h$ .

Denote total budget B, such that  $B_h \leq P_h n_h$ .

Denote desired maximum sample size  $N_d$ .

We can then frame the optimization problem of finding the best allocation of budget to minimize the variance of the final estimate as:

$$\underset{n_{i},...,n_{h}}{\operatorname{argmin}} \sum_{h} \frac{W_{h}^{2}}{n_{h}}$$

$$s.t. \sum_{h} P_{h} n_{h} \leq B$$

$$\sum_{h} n_{h} \leq N_{d}$$

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But we don't know the price per respondent...

#### How to measure cost?

We can model the inverse cost  $(\frac{1}{P_h})$ , the number of respondents recruited  $n_{ht}$  given budget spend  $B_h$ , as a Poisson random variable:

$$n_{ht}|B_{ht} \sim Poisson(\lambda)$$
  $\lambda \sim Gamma(\kappa, eta)$ 

We can use closed-form Bayesian updating to obtain a MAP estimator of  $\lambda$  and the implied mean of the predictive distribution  $(1/\lambda)$ .

#### Algorithm

How can we run this optimization problem?

We need an interface for recruitment that targets stratum h and allocates budget  $B_{ht}$  over a specific period of time t. We denote this interface Recruit $(B_t)$  which accepts a budget allocation  $B_t := \{B_{1t}, ..., B_{Ht}\}$ .

Additionally, we require an interface GetResults(t) to collect information on the results of recruitment at time t given budget  $B_t$ . Results should be considered as the number of respondents recruited for each stratum h at time t and will be denoted  $n_{ht}$ .

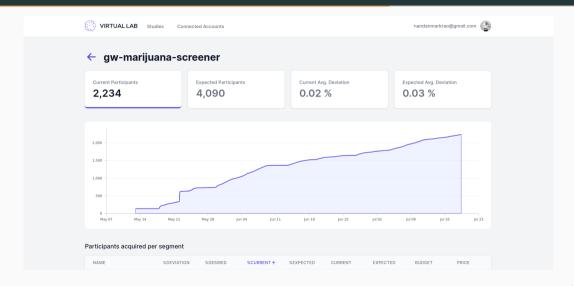
#### **Algorithm**

#### Algorithm 1 Optimizing Stratified Recruitment with Unknown Costs

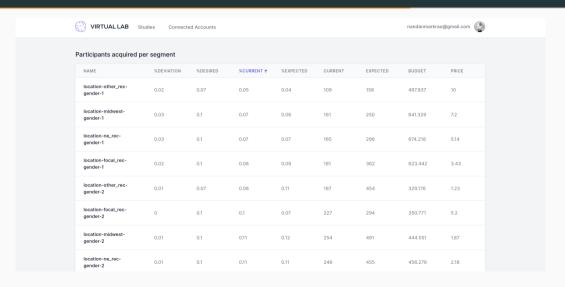
```
procedure ADOPTIMIZATION(W, B, N_d, \kappa, \theta)
   B_0 := [1, ..., 1]
                                                                                       n := [0, ..., 0]
                                                                                        ▷ Results indexed by H strata
   for t \in T do
       p_t := []
                                                                               ▷ Price estimates indexed by H strata
       for h \in H do
           n_{ht} := GetResults(h, t)
           n_b := n_b + n_{bt}
           p_{ht} := \text{EstimatePrice}(\kappa, \theta, B_{ht}, n_{ht})
       end for
       B_{t+1} := \text{Optimize}(W, B, N_d, n, p_t)
       Recruit(B_{t+1})
   end for
end procedure
```

#### **Software**

#### Dashboard



#### Dashboard



#### **Software**

```
Software is fully open source :)
But requires a server cluster to run :(
But is easily installable on kubernetes with helm :)
github.com/vlab-research
```

#### **Governance Structure**

How to make a SaaS sustainable for research purposes?

	Country	Strata	Max CTR	Reach	Respondents	Average Cost
0	India	24	5.09%	873653	9130	\$0.24
1	Libya	176	27.1%	1000294	8338	\$0.3
2	Labanon	48	15.03%	1370234	17399	\$0.57
3	Jordan	192	10.05%	2223793	17223	\$0.6
4	Iraq	144	8.58%	4280255	16015	\$0.61
5	Serbia	48	13.58%	985109	13995	\$0.67
6	Nigeria	23	6.42%	145437	2393	\$0.75
7	US	10	13.61%	16849	1118	\$0.85
8	US	4	6.9%	10616	316	\$0.87
9	Haiti	16	10.0%	1218842	10491	\$0.94
10	Honduras	144	7.4%	909597	4922	\$0.95

	Country	Strata	Max CTR	Reach	Respondents	Average Cost
11	Lebanon	48	13.82%	1650052	14354	\$1.03
12	Papa NG	64	8.71%	118980	1825	\$1.46
13	Iraq	80	8.27%	5616663	6520	\$1.55
14	US	14	6.14%	120647	2553	\$1.66
15	Ukraine	64	5.17%	648512	2394	\$1.69
16	Kyrgyzstan	24	9.63%	489054	3004	\$1.76
17	Djibouti	16	10.84%	313493	2252	\$2.19
18	Kosovo	56	19.93%	663059	6084	\$2.46
19	Chad	32	7.59%	327048	2305	\$2.64
20	Jamaica	16	15.03%	482097	4105	\$2.72
21	Belize	24	9.1%	43684	264	\$2.89

	Country	Strata	Max CTR	Reach	Respondents	Average Cost
22	Serbia	1	6.75%	342403	1737	\$2.99
23	Macedonia	32	24.58%	384956	3156	\$3.19
24	Romania	200	9.31%	785811	1863	\$3.23
25	Macedonia	32	25.17%	538295	4565	\$3.26
26	Jordan	96	7.14%	1304979	2146	\$3.72
27	US	16	3.93%	241134	1429	\$4.55
28	Nigeria	120	7.75%	563939	177	\$5.55
29	Bulgaria	8	3.91%	170205	170	\$6.16
30	Cameroon	80	11.85%	1427793	1712	\$6.72
31	India	160	5.77%	3526970	1639	\$7.85
32	Gambia	2	5.86%	279008	697	\$11.07

#### The end

### Thank you!

https://vlab.digital