Sem2 Stats2

Number of Questions : 12

Section Marks : 50

Question Number : 247 Question Type : MCQ

Correct Marks : 0

Question Label : Multiple Choice Question

THIS IS QUESTION PAPER FOR THE SUBJECT "SEMESTER 2: STATISTICS FOR DATA SCIENCE 2"

ARE YOU SURE YOU HAVE TO WRITE EXAM FOR THIS SUBJECT?
CROSS CHECK YOUR HALL TICKET TO CONFIRM THE SUBJECTS TO BE WRITTEN.

(IF IT IS NOT THE CORRECT SUBJECT, PLS CHECK THE SECTION AT THE TOP FOR THE SUBJECTS REGISTERED BY YOU)

Options :

A. ✔ Yes

B. ✖ No

Question Number : 248 Question Type : MCQ

Correct Marks : 0

Question Label : Multiple Choice Question

Use the following values of F_Z if needed:

$F_Z(0.13) = 0.55, F_Z(1.88) = 0.96995, F_Z(-1.88) = 0.03005, F_Z(0.4) = 0.65542, F_Z(1.64) = 0.95,$
$F_Z(-1.64) = 0.05, F_Z(1.9) = 0.97128, F_Z(1.33) = 0.90824, F_Z(-1.33) = 0.09176$

List of options:

1. $P(Z > a) = 1 - F_Z(a)$
2. $E[aX + bY] = aE[X] + bE[Y]$
3. $p^* = \arg\max_p \prod_{i=1}^{10} f_X(x_i; p)$
4. $P(|\overline{X} - \mu| > \delta) \leq \dfrac{\sigma^2}{n\delta^2}$
5. $\int_a^b x^n\, dx = \dfrac{x^n}{n+1}\bigg|_a^b$
6. Central Limit Theorem
7. If X and Y are independent, then $\mathrm{Var}[aX + Y] = a^2\mathrm{Var}[X] + \mathrm{Var}[Y]$.
8. $X \sim \mathrm{Geometric}(p), f_X(k) = (1-p)^{k-1}p$.
9. Sum of n i.i.d. Bernoulli(p) is Binomial(n, p)
10. $P(E) = \iint_{Supp(E)} f_{XY}(x, y)\, dx\, dy$
11. $P(a < X < b) = F_X(b) - F_X(a)$
12. $L(x_1, \ldots, x_{10}) = \prod_{i=1}^{10} f_X(x_i; p)$
13. $\iint_{Supp} f_{XY}(x, y)\, dx\, dy = 1$
14. $F_Z(-a) = 1 - F_Z(a), Z \sim \mathrm{Normal}(0, 1)$
15. $X \sim \mathrm{Poisson}(\lambda), f_X(k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$.
16. If X and Y are independent, then $E[XY] = E[X]E[Y]$.
17. For an event $E, P(E^c) = 1 - P(E)$.

Discrete random variables:

Distribution	PMF ($f_X(k)$)	CDF ($F_X(x)$)	$E[X]$	$\mathrm{Var}(X)$
Uniform(A) $A = \{a, a+1, \ldots, b\}$	$\frac{1}{n},\ x = k$ $n = b - a + 1$ $k = a, a+1, \ldots, b$	$\begin{cases} 0 & x < 0 \\ \frac{k-a+1}{n} & k \leq x < k+1 \\ & k = a, a+1, \ldots, b-1, b \\ 1 & x \geq n \end{cases}$	$\frac{a+b}{2}$	$\frac{n^2-1}{12}$
Bernoulli(p)	$\begin{cases} p & x = 1 \\ 1-p & x = 0 \end{cases}$	$\begin{cases} 0 & x < 0 \\ 1-p & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$	p	$p(1-p)$
Binomial(n, p)	${}^nC_k p^k (1-p)^{n-k}$, $k = 0, 1, \ldots, n$	$\begin{cases} 0 & x < 0 \\ \sum_{i=0}^{k} {}^nC_i p^i (1-p)^{n-i} & k \leq x < k+1 \\ & k = 0, 1, \ldots, n \\ 1 & x \geq n \end{cases}$	np	$np(1-p)$
Geometric(p)	$(1-p)^{k-1} p$, $k = 1, \ldots, \infty$	$\begin{cases} 0 & x < 0 \\ 1-(1-p)^k & k \leq x < k+1 \\ & k = 1, \ldots, \infty \end{cases}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson(λ)	$\frac{e^{-\lambda}\lambda^k}{k!}$, $k = 0, 1, \ldots, \infty$	$\begin{cases} 0 & x < 0 \\ e^{-\lambda} \sum_{i=0}^{k} \frac{\lambda^i}{i!} & k \leq x < k+1 \\ & k = 0, 1, \ldots, \infty \end{cases}$	λ	λ

Continuous random variables:

Distribution	PDF ($f_X(k)$)	CDF ($F_X(x)$)	$E[X]$	Var(X)
Uniform$[a,b]$	$\frac{1}{b-a}$, $a \leq x \leq b$	$\begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x \geq b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exp(λ)	$\lambda e^{-\lambda x}$, $x > 0$	$\begin{cases} 0 & x \leq 0 \\ 1-e^{-\lambda x} & x > 0 \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal(μ, σ^2)	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$, $-\infty < x < \infty$	No closed form	μ	σ^2
Gamma(α, β)	$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$, $x > 0$		$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Beta(α, β)	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ $0 < x < 1$		$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

1. **Markov's inequality:** Let X be a discrete random variable taking non-negative values with a finite mean μ. Then,
$$P(X \geq c) \leq \frac{\mu}{c}$$

2. **Chebyshev's inequality:** Let X be a discrete random variable with a finite mean μ and a finite variance σ^2. Then,
$$P(\mid X - \mu \mid \geq k\sigma) \leq \frac{1}{k^2}$$

3. **Weak Law of Large numbers:** Let $X_1, X_2, \ldots, X_n \sim$ iid X with $E[X] = \mu$, Var$(X) = \sigma^2$.
Define sample mean $\overline{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}$. Then,
$$P(|\overline{X} - \mu| > \delta) \leq \frac{\sigma^2}{n\delta^2}$$

4. **Using CLT to approximate probability:** Let $X_1, X_2, \ldots, X_n \sim$ iid X with $E[X] = \mu$, Var$(X) = \sigma^2$.
Define $Y = X_1 + X_2 + \ldots + X_n$. Then,
$$\frac{Y - n\mu}{\sqrt{n}\sigma} \approx \text{Normal}(0, 1).$$

5. **Bias of an estimator:** Bias$(\hat{\theta}, \theta) = E[\hat{\theta}] - \theta$.

6. **Method of moments:** Sample moments, $M_k(X_1, X_2, \ldots, X_n) = \frac{1}{n}\sum_{i=1}^{n} X_i^k$

7. **Likelihood of i.i.d. samples:** Likelihood of a sampling $x_1, x_2, \ldots, x_n$, denoted
$$L(x_1, \ldots, x_n) = \prod_{i=1}^{n} f_X(x_i; \theta_1, \theta_2, \ldots)$$

8. **Maximum likelihood (ML) estimation:**
$$\theta_1^*, \theta_2^*, \ldots = \arg\max_{\theta_1^*, \theta_2^*, \ldots} \prod_{i=1}^{n} f_X(x_i; \theta_1, \theta_2, \ldots)$$

Options :

A. ✔ Useful Data has been mentioned above.

B. ✖ This data attachment is just for a reference & not for an evaluation.

Question Number : 249 Question Type : MCQ

Correct Marks : 3

Question Label : Multiple Choice Question

Let X and Y be independent Poisson random variables with mean 1. Then

$P(\min(X, Y) = 0)$ equals

Options :

A. ✖ e^{-2}

B. ✖ $1 + e^{-2} - 2e^{-1}$

C. ✖ $2e^{-1}$

D. ✔ $2e^{-1} - e^{-2}$

Question Number : 250 Question Type : MSQ

Correct Marks : 4

Question Label : Multiple Select Question

Suppose the moment generating function of a random variable X is given by $M_X(\lambda) = e^{8\lambda^2}$ and $X_1, X_2, \ldots, X_9$ are 9 i.i.d. samples with distribution X. Let Y be a random variable defined as $Y = X_1 + X_2 + \cdots + X_9$. Which of the following options are true?

Options :

A. ✖ $X \sim \text{Normal}(0, 8)$.

B. ✔ $X \sim \text{Normal}(0, 16)$.

C. ✔ The moment generating function of Y is given by $M_Y(\lambda) = e^{72\lambda^2}$.

D. ✖ The moment generating function of Y is given by $M_Y(\lambda) = 9e^{8\lambda^2}$.

Question Number : 251 Question Type : MSQ

Correct Marks : 5

Question Label : Multiple Select Question

Suppose $X_1, X_2, \ldots, X_9 \sim$ i.i.d. Poisson(λ). Let $\hat{\lambda}_1 = \frac{1}{8}\sum_{i=1}^{8}\frac{X_i + X_{i+1}}{2}$ and

$\hat{\lambda}_2 = \frac{1}{9}\sum_{i=1}^{8}\frac{X_i + X_{i+1}}{2}$ be two estimators of λ. Which of the following statements are correct?

Options :

A. ✖ Both $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are biased estimators of λ.

B. ✔ $\hat{\lambda}_1$ is an unbiased estimator of λ.

C. ✖ $\hat{\lambda}_2$ is an unbiased estimator of λ.

D. ✔ The variance of $\hat{\lambda}_1$ is $\frac{15\lambda}{128}$.

E. ✔ The bias of $\hat{\lambda}_2$ is $\frac{-\lambda}{9}$.

Question Number : 252 Question Type : SA

Correct Marks : 2

Question Label : Short Answer Question

A random sample of size 64 is collected from a normal population with mean μ and variance σ^2. Suppose the expected value and the variance of the sample mean is 50 and 0.25, respectively. Find the value of $\mu - 11\sigma$.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

6

Question Type : COMPREHENSION

Question Numbers : (253 to 254)

Question Label : Comprehension

Answer the given subquestions.

Sub questions

Question Number : 253 Question Type : SA

Correct Marks : 5

Question Label : Short Answer Question

An urn contains a certain number of balls and one of them is numbered 1. Let the probability of drawing a 1 be p. Independent draws are made with replacement until ball 1 is drawn. The experiment is conducted 10 times and number of draws made are as follows: 4, 3, 1, 5, 7, 5, 4, 2, 6, 8. Find the maximum likelihood estimate of p. Enter your answer correct to two decimal places.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

0.20 to 0.24

Question Number : 254 Question Type : SA

Correct Marks : 0

Question Label : Short Answer Question

Select three facts/formulae/steps from the list of options that you will use for solving the previous question.

Note: "This question is optional. We will check your answer to this question if you make a mistake in the previous one. Please enter your answers in comma-separated format. E.g. 1, 2, 3". Check the list of options given in the data attachment.

Response Type : Alphanumeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Set

Answers Case Sensitive : No

Text Areas : PlainText

Possible Answers :

3, 8, 12

Question Type : COMPREHENSION

Question Numbers : (255 to 257)

Question Label : Comprehension

At a particular petrol pump, petrol is stocked in a bulk tank each week. Let random variable X denote the proportion of the tank's capacity that is filled for a given week, and let Y denote the proportion of the tank's capacity that is sold in the same week. The petrol pump cannot sell more than what was stocked in a given week. Assume the joint density function of X and Y is given by

$$f_{XY}(x,y) = \begin{cases} c & \text{if } 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Based on the above data, answer the given subquestions.

Sub questions

Question Number : 255 Question Type : SA

Correct Marks : 2

Question Label : Short Answer Question

Find c so that f_{XY} is a valid PDF.

NOTE: Enter your answer to the nearest integer.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

2

Question Number : 256 Question Type : SA

Correct Marks : 2

Question Label : Short Answer Question

Find the probability that in a given week the amount of petrol sold is less than half the amount that is stocked. Enter the answer correct to one decimal place.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

0.5

Question Number : 257 Question Type : SA

Correct Marks : 0

Question Label : Short Answer Question

Select three facts/formulae/steps from the list of options that you will use for solving question number 255 & 256.

Note: "This question is optional. We will check your answer to this question if you make a mistake in the previous one. Please enter your answers in comma-separated format. E.g. 1, 2, 3". Check the list of options given in the data attachment.

Response Type : Alphanumeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Set

Answers Case Sensitive : No

Text Areas : PlainText

Possible Answers :

5, 10, 13, 17

Question Type : COMPREHENSION

Question Numbers : (258 to 259)

Question Label : Comprehension

Answer the given subquestions

Sub questions

Question Number : 258 Question Type : SA

Correct Marks : 4

Question Label : Short Answer Question

A truck can hold 50 containers (identical in shape and size) and can safely carry 2700 kg. The average weight of the containers is 50 kg with standard deviation of 15 kg. What is the approximate probability that 50 containers will overload the truck? Write your answer correct to 3 decimal places.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

0.029 to 0.031

Question Number : 259 Question Type : SA

Correct Marks : 0

Question Label : Short Answer Question

Select three facts/formulae/steps from the list of options that you will use for solving the previous question.

Note: "This question is optional. We will check your answer to this question if you make a mistake in the previous one. Please enter your answers in comma-separated format. E.g. 1, 2, 3". Check the list of options given in the data attachment.

Response Type : Alphanumeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Set

Answers Case Sensitive : No

Text Areas : PlainText

Possible Answers :

1, 2, 6, 7, 14

Question Type : COMPREHENSION

Question Numbers : (260 to 262)

Question Label : Comprehension

Let the joint PDF of two random variables X and Y be given by

$$f_{XY}(x,y) = \begin{cases} kxy & \text{if } 0 < x < 1, 0 < y < 1. \\ 0 & \text{otherwise} \end{cases}$$

Based on the above data, answer the given subquestions.

Sub questions

Question Number : 260 Question Type : SA

Correct Marks : 2

Question Label : Short Answer Question

Find the value of k so that f_{XY} is a valid PDF.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

4

Question Number : 261 Question Type : SA

Correct Marks : 2

Question Label : Short Answer Question

Find $P\left(Y \leq \frac{X}{2}\right)$. Enter the answer correct to three decimal places.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

0.125

Question Number : 262 Question Type : SA

Correct Marks : 3

Question Label : Short Answer Question

Find $P\left(Y \leq \frac{X}{4} | Y \leq \frac{X}{2}\right)$. Enter the answer correct to two decimal places.

Hint:

Use $\int\limits_{x=a}^{b} x^n \, dx = \frac{1}{n+1}(b^{n+1} - a^{n+1})$

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

0.25

Question Type : COMPREHENSION

Question Numbers : (263 to 265)

Question Label : Comprehension

A fair die is rolled *n* times. Let *S* denote the total number of times six is obtained.

Based on the above data, answer the given subquestions.

Sub questions

Question Number : 263 Question Type : SA

Correct Marks : 3

Question Label : Short Answer Question

Use Weak Law of Large Numbers to find the minimum value of *n* such that

$$P\left(\left|\frac{S}{n}-\frac{1}{6}\right|>0.1\right)<0.1$$

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

139

Question Number : 264 Question Type : SA

Correct Marks : 4

Question Label : Short Answer Question

Use CLT to find the value of *n* such that

$$P\left(\left|\frac{S}{n}-\frac{1}{6}\right|>0.1\right)<0.1$$

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

38

Question Number : 265 Question Type : SA

Correct Marks : 0

Question Label : Short Answer Question

Select four facts/formulae/steps from the list of options that you will use for solving question number 263 & 264.

Note: "This question is optional. We will check your answer to this question if you make a mistake in the previous one. Please enter your answers in comma-separated format. E.g. 1, 2, 3". Check the list of options given in the data attachment.

Response Type : Alphanumeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Set

Answers Case Sensitive : No

Text Areas : PlainText

Possible Answers :

1, 2, 4, 6, 7, 9, 11, 14

Question Type : COMPREHENSION

Question Numbers : (266 to 268)

Question Label : Comprehension

Let X be a discrete random variable with the following probability mass function:

X	0	1	2	3
$f_X(x)$	$\theta/2$	$(1-\theta)/2$	$\theta/2$	$(1-\theta)/2$

where $0 < \theta < 1$. Suppose we want to estimate the parameter θ from i.i.d. samples of X.

Based on the above data, answer the given subquestions.

Sub questions

Question Number : 266 Question Type : MCQ

Correct Marks : 2

Question Label : Multiple Choice Question

Calculate the first moment of *X*.

Options :

A. ✖ $\theta + 2$

B. ✔ $2 - \theta$

C. ✖ $1 + \frac{\theta}{2}$

D. ✖ $\theta - 2$

Question Number : 267 Question Type : SA

Correct Marks : 3

Question Label : Short Answer Question

Find the estimate of θ for the samples 0, 1, 2, 3, 3, 2, 0, 0, 2, 1 using method of moments. Write your answer correct to one decimal place.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

0.6

Question Number : 268 Question Type : SA

Correct Marks : 4

Question Label : Short Answer Question

Find the maximum likelihood estimate of θ for the samples 1, 2, 3, 2, 1. Write your answer correct to one decimal place.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

0.4