# MA2.101: Linear Algebra (Spring 2022)

Mid-semester Exam

Tuesday, 17 May 2022

## Attention

1. Mid-sem exam is 15% of total marks for this course (50% of the total marks is allocated to the first part). 2 marks out of 15 (assuming total of 100 marks for full course) allocated for Mid-sem exam is from the in-house Mid-sem exam that asked you to submit a question with solution by 8 PM, 14 May 2022.

2. Total points for in-class Midsem exam questions is 65: $Q1 = 23, Q2 = 11, Q3 = 11, Q4 = 20$. 65 points $\equiv$ 13 marks allocated for in-class Mid-sem exam.

3. You may opt to submit solution of either Question 1 or Question 4 by Tuesday midnight (11:59:59 PM, 17 May 2022) via moodle. However, total points allocated for given question will be reduced to half of the original.

## Question 1

Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation whose action is given by

$$T(x, y, z) = (x + 2y + z, z, -x + y - 5z, 3x + 2z)$$

- What are the standard ordered bases for $\mathbb{R}^3$ and $\mathbb{R}^4$. (2 points).

- Write the matrix representation of $T$ relative to the standard ordered bases of $\mathbb{R}^3$ and $\mathbb{R}^4$. (12 points).

- If the standard ordered basis for $\mathbb{R}^4$ is $\{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4\}$ (in the given sequence). Then find the matrix representation of $T$ relative to the standard ordered basis for $\mathbb{R}^3$ and the ordered basis $\{\epsilon_2, \epsilon_1, \epsilon_3, \epsilon_4\}$ (in the given sequence) for $\mathbb{R}^4$. (7 points)

- Is T invertible? Why or why not? (2 point)

→ every element in V maps to only one unique element in W
isomorphic
the inverse statement map from
suppose $T(\alpha) =$

## Question 2

If a linear transformation $T : V \to W$ is an isomorphism, then the inverse map $T^{-1} : W \to V$ is also an isomorphism. Give a formal proof of this statement. (11 points)

## Question 3

Let $A$ be an $m \times n$ matrix representing a linear transformation with respect to a pair of ordered bases. Suppose that the nullspace of $A$ is a plane in $\mathbb{R}^3$ and the range is spanned by a nonzero vector $\vec{v} \in \mathbb{R}^5$.

?? → 8 or 3 ??  ~~Rank = dim~~  $T(\alpha)$

- Determine $m$ and $n$. (2 points)  $R(A) + N(A) = \dim(V)$  $A[\alpha]_B$
- Find the rank and nullity of $A$. (6 points)
- Write the expression for rank-nullity theorem for given $A$. (3 points)

idek anymore

## Question 4

Consider a linear transformation $T : \mathbb{R}^3 \to P_2$ from $\mathbb{R}^3$ to the vector space $P_2$ of polynomials in $x$ with real coefficients, where the degree of polynomial is at most 2:

$$T(a, b, c) = (a - b)x^2 + cx + (a + b + c). \tag{1}$$

Find the matrix representation of $T$ with respect to the pair of ordered bases $\{(1, 0, 0), (1, 1, 0), (0, -1, 1)\}$ and $\{x + 1, x^2 - x, x^2 + x + 1\}$ for $\mathbb{R}^3$ and $P_2$, respectively. (20 points)