# Quiz 1: Probability and Statistics

## 5 marks

1. The CDF of a random variable $X$ is defined as $F_X(x) = \mathbb{P}(\omega \in \Omega : X(w) \leq x) = \sum_{x \leq x_1} p_X(x)$ where $p_X$ is the PMF. Prove that

$$\mathbb{P}(a < X \leq b) = F_X(b) - F_X(a)$$

2. A geometric random variable $X$ with parameter $p$ has PMF given by $p_X(k) = (1-p)^{k-1}p$. Derive the expression for its mean and variance.

3. Two cards are chosen from a standard deck of 52 cards. Suppose that you win 2 Rs for each heart selected, and lose 1 Rs for each spade selected. Other suits (clubs or diamonds) bring neither win nor loss. Let $X$ denote your winnings. Determine the probability mass function $p_X(x)$.

4. For a random variable $X$ with mean $\mu$, its variance $Var(X)$ is defined as $E[(X-\mu)^2]$. Prove that $Var(aX+b) = a^2Var(X)$ for arbitrary constants $a$ and $b$.

## 10 marks

1. Let random variable $X$ denote the outcome of a dice. Plot the Cumulative Distribution function (CDF) of $X$. Also find the mean and variance of $X$. Additionally prove that (prove! do not numerically verify. Start with either RHS or LHS and prove the other side.)

$$\sum_{x \in \{1,2,\ldots 6\}} x p_X(x) = 1 + \sum_{x \in \{1,2,\ldots 6\}} (1 - F_X(x))$$

(Hint: Write the CDF on the rhs in terms of PMF) The RHS is an alternative formula to get the expectation of non-negative random variables using the CDF.