Quiz: Probability and Statistics (50 Marks)

Instruction:

- Please state reasons wherever applicable.
- Those who want to replace their Quiz marks will answer Section 1.
- Those who want to replace their midsem marks will answer both sections.

SECTION 1: 6 marks each

1. Find the stationary distribution π for Markov Chain with the following transition probability matrix.(4 marks) State if π is unique. (1 mark) Is the chain irreducible? Give reasons (1mark)

$$P = \begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.9 & 0.1 \\ 0.1 & 0.9 & 0 \end{bmatrix}$$

$\pi = \pi P$

$(\pi_1, \pi_2, \pi_3) = (\pi_1, \pi_2, \pi_3)$

2. If A and B are exponential random variables with parameters a and b respectively, then prove that $P(A < B) = E[e^{-bA}]$. Further show that this is equal to $\frac{a}{a+b}$.

3. Derive the expression for the Moment generating function of the following random variables
   - Standard Normal with mean 0 and variance 1 (3 marks)
   - Poisson random variable with parameter λ. (3 marks)

4. Let X be an exponential random variable with parameter 1. Find

   really??

   - Conditional pdf and cdf of X given $X > 1$. (3 marks)
   - $E[X|X > 1]$ (3 marks)

5. Let X and Y be independent and identically distributed discrete random variables taking values on positive integers. Their pmf is $p(x) = C2^{-x}$ for $x \geq 1$ ~~$x = 1$~~. Find

   $x \geq 1$

   - The value of C that makes it a valid pmf. (2 marks)
   - $P(min\{X, Y\} \leq x)$ (2 marks)
   - $P($ X divides Y $)$ (2 marks)

$C/2$, $C/4$, $C/8$

$\frac{C}{2} + \frac{C}{4} + \frac{C}{8} \dots$

$\frac{e^{-\lambda_1}\lambda_1^k}{k!} + \frac{e^{-\lambda_2}\lambda_2^k}{k!}$

$\frac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^k}{k!}$

SECTION 2: 5 marks each

1. Let X and Y be independent Poisson random variables with parameters λ_1 and λ_2. Let $Z = X + Y$. Then find the pmf of Z.

2. let $X_1, X_2, \ldots,$ be a sequence of i.i.d uniform $U[0, 1]$ random variables. Let $Y_n = min(X_1, X_2, \ldots, X_n)$. Prove the following independently.
   - $Y_n \to 0$ in distribution (2.5 marks)
   - $Y_n \to 0$ in probability (2.5 marks)

P_X converges

$P(|X_n - X| \geq \epsilon) = 0$

use all samples to generate sample of binomial!

3. let $u_1, u_2, \ldots, u_n$ denote n samples drawn from a $U[0, 1]$ random variable. Describe a procedure to use them to obtain one sample from a $Binomial(n, p)$ random variable.

4. Let X have a Poisson distribution with parameter Λ, where Λ itself is an exponential random variable with parameter μ. Show that X has a geometric distribution.

$-\mu e^{-\mu x}$

$\frac{e \quad (\mu e)}{k!}$