Homework - 2

Question 1

The decision of choosing the class such that the risk is minimum is given by:

$$d = \underset{\substack{j=\{1,\dots,c\}\\ i=\{1,\dots,c\}}}{\operatorname{argmin}} \{\lambda_{ij} P(\omega_i | x)\}$$

$$= \underset{\substack{j=\{1,\dots,c\}\\ i=\{1,\dots,c\}}}{\operatorname{argmin}} \{\lambda_{i1} P(\omega_1 | x), \dots, \lambda_{ii} P(\omega_i | x), \dots \lambda_{cc} P(\omega_c | x)\}$$

$$= \underset{\substack{j=\{1,\dots,c\}\\ i=\{1,\dots,c\}}}{\operatorname{argmin}} \{\lambda_{i1} P(\omega_1 | x), \dots, 0, \dots, \lambda_{cc} P(\omega_c | x)\}$$

$= 0$ for $j=i$ which is the minimum value obtained.

But, we will choose $j=i$ only if $P(\omega_i | x)$ is the highest amongst all the probabilities possible. Hence, to choose $j=i$, we need that $\underline{P(\omega_i | x) \geq P(\omega_j | x)}$ for all $j \in \{1,\dots,c\}$

The average risk of choosing class ω_i (for $i \in \{1,\dots,c\}$) is given by:

$$R(D=i | x) = \sum_{j=1}^{c} \lambda_{ij} \frac{P(x | \omega_j) P(\omega_j)}{P(x)}$$

$$= \sum_{j=1}^{c} \lambda_{ij} P(\omega_j | x)$$

$$= \sum_{j=1}^{i-1} \lambda_s P(\omega_j | x) + \lambda_{ii} P(\omega_i | x) + \sum_{j=i+1}^{c} \lambda_s P(\omega_j | x)$$

$$= \sum_{j=1, j\neq i}^{c} \lambda_s P(\omega_j | x) + 0$$

$$= \lambda_s \sum_{j=1, j\neq i}^{c} P(\omega_j | x) = \lambda_s \left(1 - P(\omega_i | x)\right)$$

Here, again we see that as $P(\omega_i|x)$ increases, the ~~risk~~ average risk ~~decr~~ decreases.

Now, for $i = c+1$, the risk is given by:

$$R(D=c+1|x) = \lambda_r$$

Hence, the minimum risk is achieved if we decide ~~that~~ class ω_i if,

$$R(D=i|x) \leq R(D=c+1|x)$$

$$\lambda_s(1 - P(\omega_i|x)) \leq \lambda_r$$

$$1 - P(\omega_i|x) \leq \frac{\lambda_r}{\lambda_s}$$

$$\therefore P(\omega_i|x) \geq 1 - \frac{\lambda_r}{\lambda_s}$$, and reject otherwise.

<u>Case 1</u>: If $\lambda_r = 0$:

If the cost of rejecting is 0 then we will always reject since that will give the least risk.

$$P(\omega_i|x) \geq 1 \Rightarrow P(\omega_i|x) = 0 \quad \text{at } i = c+1$$

<u>Case 2</u>: If $\lambda_r > \lambda_s$:

If the cost of rejecting is greater than the cost of choosing any other value, then we will never reject.

Note: The answers for questions 2 and 3 have can be accessed through GitHub using the following link:

https://github.com/nandayvk/EECE_5644_Intro_to_ML_Coursework/tree/master/HW2/Solutions