CNOT Gate 作業

先完成 (A), (B), (C), (D)：(還沒學過矩陣乘法的同學可以參考連結)

如果 $q_0 = |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ 而 $q_1 = |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$,

(A) $q_1 q_0 = |10\rangle = |?\rangle \otimes |?\rangle = \begin{pmatrix} ? \times \begin{pmatrix}?\\?\end{pmatrix} \\ ? \times \begin{pmatrix}?\\?\end{pmatrix}\end{pmatrix} = \begin{pmatrix}?\\?\\?\\?\end{pmatrix}$, 參考連結

也就是 $a_{00} = ?, a_{01} = ?, a_{10} = ?, a_{11} = ?$. 若讓 q_1 以 CNOT 控制 q_0,

$$CNOT|10\rangle = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}a_{00}\\a_{01}\\a_{10}\\a_{11}\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}?\\?\\?\\?\end{pmatrix} = \begin{pmatrix}?\\?\\?\\?\end{pmatrix}$$ ，也就是 $|??\rangle$ (如下)

同樣地，

(B) $|00\rangle = |?\rangle \otimes |?\rangle = \begin{pmatrix} ? \times \begin{pmatrix}?\\?\end{pmatrix} \\ ? \times \begin{pmatrix}?\\?\end{pmatrix}\end{pmatrix} = \begin{pmatrix}?\\?\\?\\?\end{pmatrix}$

$$CNOT|00\rangle = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}?\\?\\?\\?\end{pmatrix} = \begin{pmatrix}?\\?\\?\\?\end{pmatrix} = |??\rangle$$

(C) $|01\rangle = |?\rangle \otimes |?\rangle = \begin{pmatrix} ? \times \begin{pmatrix}?\\?\end{pmatrix} \\ ? \times \begin{pmatrix}?\\?\end{pmatrix}\end{pmatrix} = \begin{pmatrix}?\\?\\?\\?\end{pmatrix}$

$$CNOT|01\rangle = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}?\\?\\?\\?\end{pmatrix} = \begin{pmatrix}?\\?\\?\\?\end{pmatrix} = |??\rangle$$

(D) $|11\rangle = |?\rangle \otimes |?\rangle = \begin{pmatrix} ? \times \begin{pmatrix}?\\?\end{pmatrix} \\ ? \times \begin{pmatrix}?\\?\end{pmatrix}\end{pmatrix} = \begin{pmatrix}?\\?\\?\\?\end{pmatrix}$

$$CNOT|11\rangle = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}?\\?\\?\\?\end{pmatrix} = \begin{pmatrix}?\\?\\?\\?\end{pmatrix} = |??\rangle$$

但是請注意：$a_{00}, a_{01}, a_{10}, a_{11}$ 可能是滿足 $a_{00}^2 + a_{01}^2 + a_{10}^2 + a_{11}^2 = 1$ 的任何實數組合 (各 qubit 可能處於不同的疊加態。例如，若

$$q_0 = |\alpha\rangle = \frac{\sqrt{3}}{2}|0\rangle + ?\,|1\rangle = \frac{\sqrt{3}}{2}\begin{pmatrix}1\\0\end{pmatrix} + ?\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}\frac{\sqrt{3}}{2}\\?\end{pmatrix},$$

$$q_1 = |\beta\rangle = \frac{3}{5}|0\rangle + ?\,|1\rangle = \frac{3}{5}\begin{pmatrix}1\\0\end{pmatrix} + ?\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}?\\?\end{pmatrix},$$

$$q_1 q_0 = |\beta\alpha\rangle = |\beta\rangle \otimes |\alpha\rangle = \begin{pmatrix} ? \times \begin{pmatrix}?\\?\end{pmatrix} \\ ? \times \begin{pmatrix}?\\?\end{pmatrix}\end{pmatrix} = \begin{pmatrix}?\\?\\?\\?\end{pmatrix}$$, (注意這四個數的平方和為 1)

$$CNOT\,|\beta\alpha\rangle = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}?\\?\\?\\?\end{pmatrix} = \begin{pmatrix}?\\?\\?\\?\end{pmatrix} = ?\,|00\rangle + ?\,|01\rangle + ?\,|10\rangle + ?\,|11\rangle$$