CNOT Gate 作業

先完成 (A), (B), (C), (D): (還沒學過矩陣乘法的同學可以參考連結)

如果
$$q_0=|0
angle=egin{pmatrix}1\\0\end{pmatrix}$$
 而 $q_1=|1
angle=egin{pmatrix}0\\1\end{pmatrix}$,

$$\text{(A) } q_1q_0=\ket{10}=\ket{?}\otimes\ket{?}=\begin{pmatrix}?\times\begin{pmatrix}?\\?\\?\\?\times\begin{pmatrix}?\\?\end{pmatrix}\end{pmatrix}=\begin{pmatrix}?\\?\\?\\?\end{pmatrix}, 參考連結$$

也就是 $a_{00}=?,a_{01}=?,a_{10}=?,a_{11}=?$. 若讓 q_1 以 CNOT 控制 q_0 ,

$$CNOT|\mathbf{10}\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix}, 也就是 |??\rangle (如下)$$

同樣地,

$$(B) \left| 00 \right\rangle = \left| ? \right\rangle \otimes \left| ? \right\rangle = \begin{pmatrix} ? \times \begin{pmatrix} ? \\ ? \\ ? \\ ? \times \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix} \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix}$$

$$CNOT|00\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix} = |??\rangle$$

$$\text{(C) } |01\rangle = |?\rangle \otimes |?\rangle = \begin{pmatrix} ? \times \begin{pmatrix} ? \\ ? \\ ? \\ ? \times \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix} \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{pmatrix}$$

$$CNOT|01\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix} = |??\rangle$$

$$\text{(D) } |11\rangle = |?\rangle \otimes |?\rangle = \begin{pmatrix} ? \times \begin{pmatrix} ? \\ ? \\ ? \\ ? \times \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix} \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{pmatrix}$$

$$CNOT | \mathbf{11} \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix} = | ? ? \rangle$$

但是請注意: $a_{00},a_{01},a_{10},a_{11}$ 可能是滿足 $a_{00}^2+a_{01}^2+a_{10}^2+a_{11}^2=1$ 的任何實數組合 (各 qubit 可能處於不同的疊加態。例如,若

$$q_0 = \ket{lpha} = rac{\sqrt{3}}{2}\ket{0} + ?\ket{1} = rac{\sqrt{3}}{2}egin{pmatrix}1\0\end{pmatrix} + ?egin{pmatrix}0\1\end{pmatrix} = egin{pmatrix}rac{\sqrt{3}}{2}\?\end{pmatrix},$$

$$q_1=|eta
angle=rac{3}{5}\left|0
ight
angle+?\left|1
ight
angle=rac{3}{5}\left(egin{matrix}1\0\end{pmatrix}+?\left(egin{matrix}0\1\end{pmatrix}=\left(egin{matrix}?\?\end{pmatrix},$$

$$q_1q_0=|etalpha
angle=|eta
angle\otimes|lpha
angle=egin{pmatrix}?\timesigg(?\?)\?\timesigg(?\?)\end{pmatrix}=igg(?\?)\?\end{pmatrix}$$
 , (注意這四個數的平方和為 1)

$$CNOT\ket{etalpha} = egin{pmatrix} 1 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 \ 0 & 0 & 0 & 1 \ 0 & 0 & 1 & 0 \end{pmatrix} egin{pmatrix} ? \ ? \ ? \ ? \ ? \ \end{cases} = ?\ket{00} + ? + ?\ket{10} + ?\ket{11}$$