# ASSIGNMENT 2-1

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① Is the following function differentiable at the given point? why?

$$f(x) = (x-5)^{2/3} \quad \text{at } x=5$$

Sol: the function $f(x)$ possesses differentiability at the point $x=5$, we need compute its derivative and subsequently verify the existence of the derivation at point

$$f(x) = (x-5)^{2/3}$$

Applying the differentiation

$$f'(x) = \frac{d}{dx}\left((x-5)^{2/3}\right)$$

$$f'(x) = \frac{2}{3}(x-5)^{-1/3}$$

at point $x=5$

$$f'(5) = \frac{2}{3}(5-5)^{-1/3}$$

$$f'(5) = \frac{2}{3}(0)^{-1/3}$$

$\therefore f'(5)$ is undefined

denoted as $f'(x)$ does not have a defined value when $x$ is set to 5

$f(x) = (x-5)^{2/3}$ does not possess the property of differentiablity at the point $x=5$

② find the critical points of the following function

$$f(x) = (2x-1)\,x^{2/3}$$

Sol:

we use product rule

$$f(x) = u(x) \cdot v(x)$$

$$u(x) = 2x-1 \qquad v(x) = x^{2/3}$$

$$f'(x) = u(x)\,v'(x) + v(x)\,u'(x)$$

$$u'(x) = 2$$

$$v'(x) = \frac{2}{3}x^{-1/3}$$

$$f'(x) = (2x-1)\,\frac{2}{3}x^{-1/3} + 2x^{2/3}$$

$$= (2x-1)\,\frac{2}{3}x^{-1/3} + 2x^{2/3}$$

$$= \frac{(2x-1)*2}{3x^{1/3}} + 2x^{2/3} = 0$$

$$= \frac{2(2x-1) + 6x^{2/3}\,x^{1/3}}{3x^{1/3}} = 0$$

$$= \frac{4x-2+6x^{1}}{3x^{1/3}} = 0$$

$$4x-2+6x = 0$$

$$10x - 2 = 0$$

$$x = \frac{\cancel{2}\;1}{\cancel{10}\;5}$$

$$\boxed{x = \frac{1}{5}}$$

$\therefore$ $x = \frac{1}{5}$ is a critical point

③ find the local extrema for the following function by the first derivative test:

$$f(x) = \frac{x^3}{(x+1)}$$

sol:

Applying the diffentation

$$f'(x) = \frac{(x+1)\frac{d}{dx}(x^3) - x^3\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)(3x^2) - x^3(1)}{(x+1)^2}$$

$$= \frac{3x^3 + 3x^2 - x^3}{(x+1)^2}$$

$$= \frac{2x^3 + 3x^2}{(x+1)^2}$$

for critical numbers

$$f'(x) = 0$$

$$\frac{2x^3 + 3x^2}{(x+1)^2} = 0$$

$$x + 1 \neq 0$$

$$\Rightarrow x \neq -1$$

$$x^2(2x+3) = 0$$

$$\Rightarrow x = 0, -3/2$$

sign of $f'(x)$ :-

$f'(x)$ is $-ve$ $\quad$ $f'(x) = +ve$ $\quad$ $f'(x) = +ve$

$x = -3/2$ $\quad$ $x = 0$

for $x < -3/2 \Rightarrow f(x)$ is decreasing

& for $x > -3/2 \Rightarrow f(x)$ is increasing

$\therefore$ local minima at $x = -3/2$

local minima $= \left(-3/2, 27/4\right)$

(4) find the local extrema for the following function by the second derivative test.

$$f(x) = x^4 - 2x^2 + 3$$

sol:

$$f(x) = x^4 - 2x^2 + 3$$

$$f'(x) = \frac{d}{dx}(x^4 - 2x^2 + 3)$$

$$= 4x^3 - 4x + 0$$

$$f'(x) = 4x(x^2 - 1)$$

$$f''(x) = \frac{d}{dx}(f'(x))$$

$$f''(x) = \frac{d}{dx}(4x^3 - 4x)$$

$$f''(x) = 12x^2 - 4$$

for critical point $f'(x) = 0$

$$4x(x^2 - 1) = 0$$

$x = 0, -1$ and $1$

$$f''(0) = 12(0)^2 - 4 = -4$$

$$f''(-1) = 12(-1)^2 - 4 = 8$$

$$f''(1) = 12(1)^2 - 4 = 8$$

At $x = 0$, the second derivative is negative $(-4)$

This tell you that $f$ is concave down where $x$ equals to $0$ and therefore that theri's a local max at 0, The second derivative is positive (8) where $x=-1$ and $x=1$

$f(-1)$ and $f(1)$ has local minima at $(-1,2)$ and $(0,2)$

$$f(-1) = (-1)^4 - 2\times(-1)^2 + 3 = 2$$

$$f(1) = (1)^4 - 2\times(1)^2 + 3 = 2$$

$f(0)$ has local maxima at $(0,3)$

$$f(0) = 0^4 - 2\times 0^2 + 3 = 3$$

⑤ Determine the global minimum and maximum of the given function
$f(x) = x^2 (10-x)^{2/3}$ on $[2, 10.5]$

Sol:

Apply the differention

$$f'(x) = \frac{d}{dx}\left(x^2 (10-x)^{2/3}\right)$$

$$= \frac{d}{dx}(x^2)\,(10-x)^{2/3} + \frac{d}{dx}(10-x)^{2/3}\,x^2$$

$$= 2x(10-x)^{2/3} - \frac{2}{3\sqrt[3]{10-x}}\,x^2$$

$$= \frac{-8x^2+60x}{3\sqrt[3]{10-x}}$$

for critial point $f'(x)=0$

$$-8x^2+60x=0$$

$$-x(8x-60)=0$$

$x=0$ $\qquad$ $8x=60$

$$x = \frac{60}{8} = \frac{30}{4} = \frac{15}{2}$$

$$x = \frac{15}{2}$$

The local extremum points belongs to the intenal $[2, 10.5]$

$$f(0) = 0^2 (10-0)^{2/3} = 0$$

$$f\left(\frac{15}{2}\right) = \left(\frac{15}{2}\right)^2 \left(10-\frac{15}{2}\right)^{2/3} = 103.61339$$

$$f(2) = 2^2 (10-2)^{2/3} = 16$$

$$f(10.5) = (10.5)^2 (10-2)^{2/3} = 69,45315$$

Maximum value of the function $f\left(\frac{15}{2}\right)$
$= 103.61339$

minimum value of the function $f(0)$
$= 0$