

DTSC - 5502 Assignment

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③ showing hand calculations that a defined matrix A and its inverse A^{-1} gives back the Identity matrix

given matrix

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A^{-1} = \frac{1}{2(4) - (3)(-1)} \begin{bmatrix} 4 & -3 \\ -(-1) & 2 \end{bmatrix}$$

$$= \frac{1}{8 - (-3)} \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$= \frac{1}{8+3} \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 4/11 & -3/11 \\ 1/11 & 2/11 \end{bmatrix}$$

calculating $A \cdot A^{-1}$.

$$A \cdot A^{-1} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} \frac{4}{11} & -\frac{3}{11} \\ \frac{1}{11} & \frac{2}{11} \end{bmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ then

$$A \cdot A^{-1} = \begin{bmatrix} (a \times p) + (b \times r) & (a \times q) + (b \times s) \\ (c \times p) + (d \times r) & (c \times q) + (d \times s) \end{bmatrix}$$

Similarly for given A and calculated A^{-1}

$$A \cdot A^{-1} = \begin{bmatrix} (2 \times \frac{4}{11}) + (3 \times \frac{1}{11}) & (2 \times -\frac{3}{11}) + (3 \times \frac{2}{11}) \\ (-1 \times \frac{4}{11}) + (4 \times \frac{1}{11}) & (-1 \times -\frac{3}{11}) + (4 \times \frac{2}{11}) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{11} + \frac{3}{11} & -\frac{6}{11} + \frac{6}{11} \\ -\frac{4}{11} + \frac{4}{11} & \frac{3}{11} + \frac{8}{11} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{11} & 0 \\ 0 & \frac{11}{11} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which is identity matrix I

Hence proved that $A \cdot A^{-1} = I$