~~Assignment~~ 2-1 Id No :- 11686577

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1. Is the following function differentiable at the given point? why?

$f(x) = (x-5)^{2/3}$ at $x=5$.

Sol:-

To determine $f(x) = (x-5)^{2/3}$ is differentiable at $x=5$

$f'(x)$ existence & continuity at the point should be find.

Derivative of $f(x)$ using power rule:-

$$f'(x) = \frac{d}{dx}\left((x-5)^{2/3}\right) = \frac{2}{3}(x-5)^{-1/3} \times 1$$

at $x=5$

$$f'(5) = \frac{2}{3}(5-5)^{-1/3}$$

$$= \frac{2}{3}(0^{-1/3})$$

$\therefore$ $f'(5)$ is undefined because of the power $-\frac{1}{3}$.

$f'(x)$ is undefined at $x=5$ so, $f(x) = (x-5)^{2/3}$ is not differentiable at $x=5$.

Finally, the derivative of the function is not continuous and thus lacks a clearly defined tangent line at that location.

2. Find the critical points of the following function :-

$$f(x) = (2x-1)\,x^{2/3}$$

Sol :-

To find the critical points of the function $f(x) = (2x-1)\,x^{2/3}$, we'll first find the derivative $f'(x)$ and then solve for x when $f'(x) = 0$

i. $f'(x) = ?$

using product rule, $f'(x)$ of given function is :

$$f'(x) = (2x-1)\cdot \frac{2}{3}x^{-1/3} + x^{2/3}\cdot 2$$

$$f'(x) = \frac{4}{3}x^{2/3}(2x-1) + 2x^{2/3}$$

ii. Set $f'(x)$ to zero and solve for x :-

$$0 = \frac{4}{3}x^{2/3}(2x-1) + 2x^{2/3}$$

$$0 = x^{2/3}\left(\frac{4}{3}(2x-1) + 2\right)$$

solve for x :

$$\frac{4}{3}(2x-1) + 2 = 0$$

$$4(2x-1) + 6 = 0$$

$$8x - 4 + 6 = 0$$

$$8x + 2 = 0$$

$$4\,\cancel{8x} = -\cancel{2}$$

$$\boxed{x = -\frac{1}{4}}$$

$\therefore$ The critical point of the function $f(x) = (2x-1)\,x^{2/3}$ is $x = -\frac{1}{4}$.

3. Find the local extrema for the following function by the first derivative test.

$$f(x) = x^3/(x+1)$$

Sol:- Given, $f(x) = \dfrac{x^3}{x+1}$

To locate the function's local extrema, the first derivative test, take the following actions.

i. Find the derivative $f'(x)$ of the function $f(x)$

ii. Set $f'(x)=0$ and Solve x to find critical points

iii. Analyze the intervals and determine whether there are any local extrema by doing a first derivative test on the intervals surrounding the crucial points.

Let's go over each step in turn:

Step 1:- Find $f'(x)$

By quotient rule, differentiate $f'(x)$:-

$$f(x) = \frac{x^3}{x+1}$$

$$f'(x) = \frac{(x+1)(3x^2) - (x^3)(1)}{(x+1)^2}$$

Simplify $f'(x)$:

$$f'(x) = \frac{3x^2 + 3x^2 - x^3}{(x+1)^2}$$

$$f'(x) = \frac{6x^2 - x^3}{(x+1)^2}$$

Step 2:- Set $f'(x)=0$ & find critical points

$$6x^2 - x^3 = 0$$

$$x^2(6-x) = 0$$

set all factors equal to zero & Solve x

i. $x^2 = 0$: $x = 0$

ii. $6 - x = 0$: $x = 6$.

Now, we have two critical points $x=0$ & $x=6$.

Step 3:- Apply the first derivative test to the interval analysis:

We'll make a sign table and look at the inclination of $f'(x)$ at regular intervals around the crucial spots.

For $x < -1$:

Choose $x = -2$ (value less than -1)

$$f'(-2) = \frac{6(-2)^2 - (-2)^3}{(-2+1)^2}$$

$$= \frac{24+8}{1} = 32 > 0$$

For $-1 < x < 0$:

$x = -0.5$ (value between -1 & 0)

$$f'(-0.5) = \frac{6(-0.5)^2 - (-0.5)^3}{(-0.5+1)^2} = \frac{1.5 - 0.125}{0.25}$$

$$= \frac{1.375}{0.25} = 5.5 > 0$$

For $0 < x < 6$:

$x = 1$ (value between 0 & 6):

$$f'(1) = \frac{6(1)^2 - (1)^3}{(1+1)^2} = \frac{6-1}{4} = \frac{5}{4} > 0$$

For $x > 6$:

$x = 7$ (value > 6).

$$f'(7) = \frac{6(7)^2 - (7)^3}{(7+1)^2} = \frac{294 - 343}{64} = \frac{-49}{64} < 0$$

In light of the first derivative test, now:

→ At $x=0$, $f'(x)$ changes from positive to negative, indicating a local maximum.

→ At $x=6$, $f'(x)$ changes from positive to negative, indicating a local maximum.

Finally, the function $f(x)=\frac{x^3}{x+1}$ has local maxima at $x=0$ and $x=6$.

4. Find the local extrema for the following function by the second derivative test:

$$f(x)=x^4-2x^2+3$$

Sol:- To locate the function's local extrema follow these

$$f(x)=x^4-2x^2+3$$

Steps to use the second derivate test.

i. Calculate the first derivate of $f(x)$ in relation to x:

$$f'(x)=4x^3-4x.$$

ii. Calculate the second derivative of $f(x)$ by calculating the derivative of $f'(x)$:

$$f''(x)=12x^2-4.$$

iii. Set $f'(x)=0$ to identify the crucial points

$$4x^3-4x=0$$

iv. 4 is a multiplier from the equation, $4x$

$$4x(x^2-1)=0$$

v. Solve for x:

$$4x=0 \quad \text{or} \quad x^2-1=0$$

$4x = 0$ gives $x = 0$

for 2nd equation, $x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x = 1$ (or) $x = -1$

∴ There are 3 critical points

$x = 0$, $x = 1$ and $x = -1$.

i. Now, assess the second derivative to ascertain the nature of the extrema at these crucial points:

a) for $x = 0$:

$f''(0) = 12(0)^2 - 4 = -4$

$f''(0)$ is negative, this emphazises a local maximum at $x = 0$.

b) for $x = 1$:

$f''(1) = 12(1)^2 - 4 = 12 - 4 = 8$.

$f''(1)$ is positive, this indicates a local minimum at $x = 1$.

c) for $x = -1$:

$f''(-1) = 12(-1)^2 - 4 = 12 - 4 = 8$

∴ $f''(-1)$ is also positive, this represents a local minimum at $x = -1$.

Finally, the function $f(x) = x^4 - 2x^2 + 3$ has a local maximum at $x = 0$ and local minima at $x = 1$ and $x = -1$.

5. Determine the global minimum and maximum of the given function.

$$f(x) = x^2 (10-x)^{2/3} \text{ on } [2, 10.5]$$

Sol:- To determine the global minimum and maximum of the function $f(x) = x^2(10-x)^{2/3}$ on the break interval $[2, 10.5]$. We should first examine the end points and the crucial points inside the interval $[2, 10.5]$.

i. calculate the critical points:

To determine the critical points, we should calucculate the derivative of $f(x)$ and set it equal to $= 0$.

$$f(x) = x^2(10-x)^{2/3}$$

$$f'(x) = 2x(10-x)^{2/3} + \frac{2}{3}x^2(10-x)^{-1/3}(-1).$$

$$f'(x) = 2x(10-x)^{2/3} - \frac{2}{3}x^2(10-x)^{-1/3}$$

Set $f'(x) = 0$ for finding critical points

$$2x(10-x)^{2/3} - \frac{2}{3}x^2(10-x)^{-1/3} = 0$$

multiply both sides by 3

$$3\left[2x(10-x)^{2/3} - \frac{2}{3}x^2(10-x)^{-1/3}\right] = 0$$

$$6x(10-x)^{2/3} - 2x^2(10-x)^{-1/3} = 0$$

factor outing the common terms

$$2x(10-x)^{-1/3}[3(10-x) - x] = 0$$

$$2x(10-x)^{-1/3}(30 - 3x - x) = 0$$

we have 2 factors

$2x = 0$

$x = 0$

$(10-x)^{-1/3} \; (30-4x) = 0$

$(10-x)^{-1/3} = 0$ has no real solutions

$30 - 4x = 0$

$4x = 30$

$x = \frac{30}{4} = 7.5$

$\boxed{x = 7.5}$

$\therefore$ we have 2 critical points

$x = 0$ and $x = 7.5$

ii. Evaluate $f(x)$ at the critical points & end points

a) $f(2)$:

$f(2) = 2^2 (10-2)^{2/3} = 4(8)^{2/3} = 4(4) = 16$

b) $f(7.5)$:

$f(7.5) = (7.5)^2 (10-7.5)^{2/3}$

$= (7.5)^2 (2.5)^{2/3}$

$= (56.25)(2.924)$

$= 164.97$

c) $f(10.5) = (10.5)^2 (10-10.5)^{2/3}$

$= (10.5)^2 (-0.5)^{2/3}$

$= (110.25)(0.7937)$

$= 87.70$

$\therefore$ we are having

$f(2) = 16$

$f(7.5) \approx 164.97$

$\rightarrow f(10.5) \approx 87.70$

iii. To determine the global minimum and maximum

compare the data.

$\rightarrow$ minimum value of $f(x)$ in the interval $[2, 10.5]$

is approximately 16. at $x=2$.

$\rightarrow$ maximum value of $f(x)$ within $[2, 10.5]$ interval

is 164.97 approximately at $x = 7.5$ //

$\therefore$ Finally, consequently the global minimum

of $f(x)$ in $[2, 10.5]$ interval represents

roughly 16 at $x=2$ and approximately is the

global maximum 164.97 at $x = 7.5$. //