UNIT- III

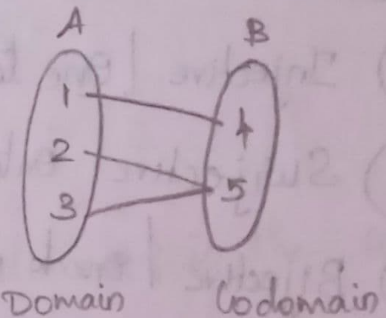
FUNCTIONS AND RELATIONS:-

FUNCTIONS:-

If A & B are any 2 sets, then a function from A & B is a rule that assigns to each element of A exactly one element of B.

Eg:

1) $A = \{1, 2, 3\}$ $B = \{4, 5\}$.

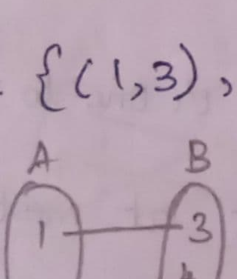


$R = \{(1,4), (2,5), (3,5)\}$.

It is a function.

2) $A = \{1, 2\}$; $B = \{3, 4, 5\}$

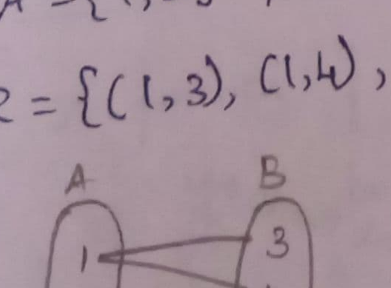
$R = \{(1,3), (2,5)\}$.



It is a function.

3) $A = \{1, 2\}$; $B = \{3, 4, 5\}$.

$R = \{(1,3), (1,4), (2,5)\}$.



It is not function because, the domain(1) has two codomain.(3 & 4)

If A & B are two sets, then a function from A to B is a set function of order pairs in $A \times B$ with the property that for each $a \in A$ there exist a unique $b \in B$. with $(a, b) \in F$.

∴ denoted $f : \underset{\text{Domain}}{A} \rightarrow \underset{\text{Codomain}}{B}$.

TYPES OF FUNCTION:-

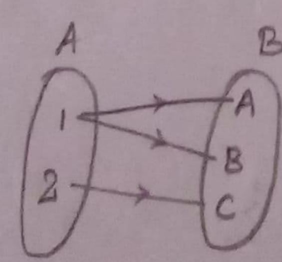
1) Injective / One to one function.
2) Surjective / Onto function.
3) Bijective / One to one correspondence.
4) Many to one.
5) Identity function.
6) Constant function.

1) INJECTIVE / ONE TO ONE FUNCTION:-

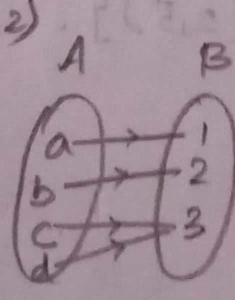
A function $f : A \rightarrow B$ is injective or one to one function if for every $b \in B$, there exist at most one $a \in A$.

Eg 1) $A = \{1, 2\}$, $B = \{A, B, C\}$.

$R = \{(1, A), (1, B), (2, C)\}$.



Eg 2)

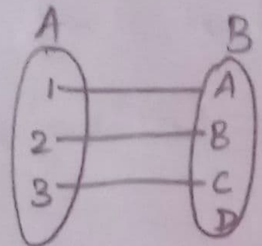


Not one to one

*) It is not function and it is not one to one function.

Eg 3). $A = \{1, 2, 3\}$, $B = \{A, B, C, D\}$

$R = \{(1, A), (2, B), (3, C)\}$

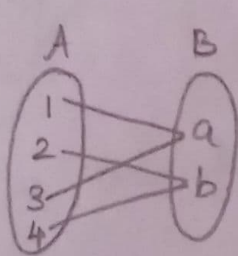


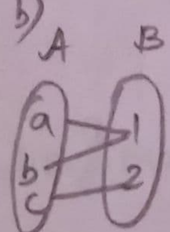
It is one - one but not onto function.

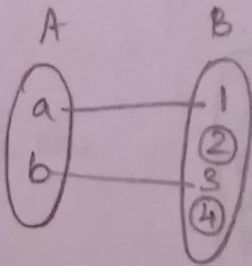
It is one to one function. If codomain has an empty element is also fine and A elements should have different image in B.

2) SURJECTIVE | ONTO FUNCTION:-

A function is onto if each element in the codomain is an image of some pre image

a) 

b) 

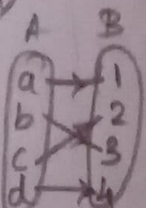


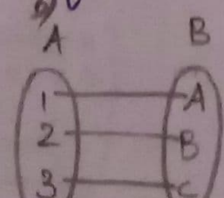
It is onto both a&b.

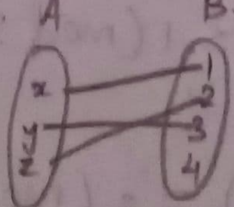
It is not onto function.

3) BIJECTIVE FUNCTION:-

A function $f: A \to B$ is bijective or one - to one correspondent iff f is both injective & surjective.

a) 

b) 



It is bijective function(a,b). Not Bijective.

Q) Let N be the set of natural numbers including zero. Determine which of the following function are one to one, onto, and which are one to one onto.

a) $f: N \to N$ $f(j) = j^2 + 2$.

Sol Let $j = \{1, 2, 3\}$.

$f(j) = j^2 + 2$

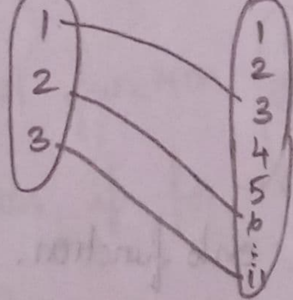
$= 1^2 + 2 = 1 + 2 = 3$

$f(j) = 2^2 + 2$

$= 4 + 2 = 6$.

$f(j) = 3^2 + 2$

$= 9 + 2 = 11$



It is one to one function.

b) $f: N \to N$ $f(j) = j \pmod 3$.

$j = 1$:

$j \pmod 3 = 1 \pmod 3 = 1$

$j = 2$

$j \pmod 3 = 2 \pmod 3 = 2$

$j = 3$

$(3 \pmod 3) = 0.$

$j = 4.$

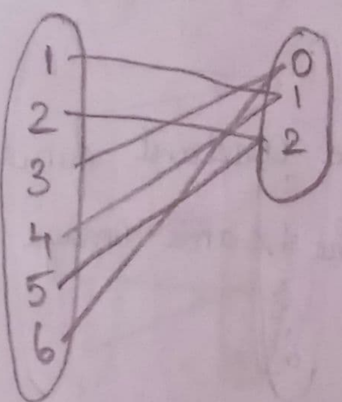
$4 \pmod 3 = 1$

$j = 5$

$5 \pmod 3 = 2$

$J = 6$

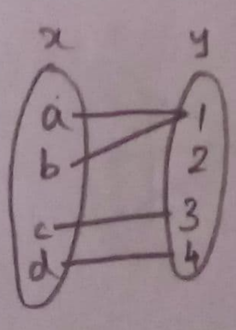
$6 \pmod 3 = 0.$

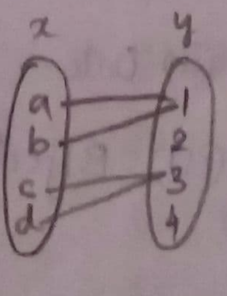


It is onto function & many to one

4) <u>Many to one function:-</u>

A function $f: x \to y$ is said to be many-to-one function if two or more elements of x are associated to the same element of set Y.

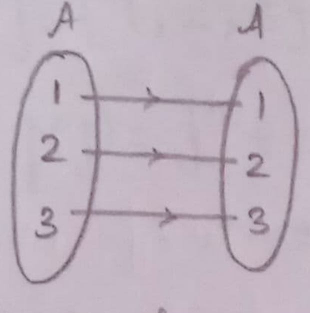




5) Identity function:

A function $f: A \to A$ is said to be identity function, if the image of every of A (under f) is itself i.e $f(A) = A$.

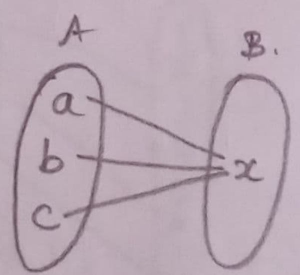
The identity function on set A is denoted by I_A.



Identity function

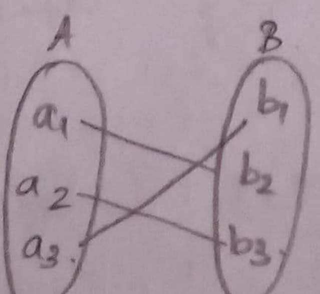
6) Constant function:-

A function $f: A \to B$ is said to be constant function, if all the elements of set A have the same image in set B.



Q) Consider the following figure. Identify which are one-to one, onto, bijective, many to one, constant and identity functions.

(i)

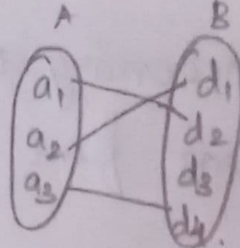


Ans.

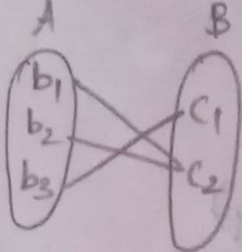
(i) It is one to one.

(ii) Onto

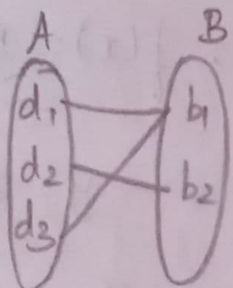
(iii) Bijective.

(ii) 

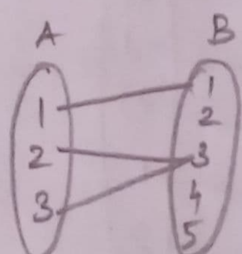
Ans:- i) It is one to one
ii) Not onto
iii) Not bijective.

(iii) 

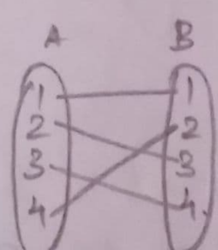
Ans (i) Not one to one
(ii) Onto
(iii) Not bijective.
(iv) Many to one.

(iv) 

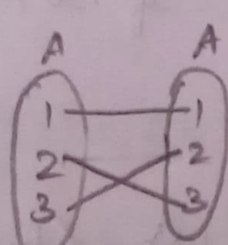
Ans (i) Not one to one
(ii) Onto
(iii) Not bijective
(iv) Many to one.

(v) 

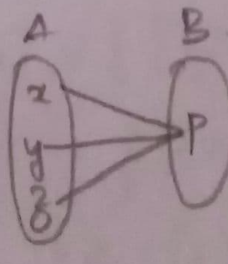
Ans (i) Not one to one.
(ii) Not onto.
(iii) Not bijective.

(vi) 

Ans (i) One to one
(ii) Onto
(iii) Bijective.

(vii) 

Ans (i) Identity function.
(ii) One to one.
(iii) Onto
(iv) Bijective.

(viii) 

Ans) (i) Constant.
(ii) Not one - to one.
(iii) Onto
(iv) Not bijective. (v) Many to one.

Q) If X and Y are finite sets, find a necessary condition for the existence of one to one mapping from X to Y.

Ans Consider a function $f(x) = x - 3$.

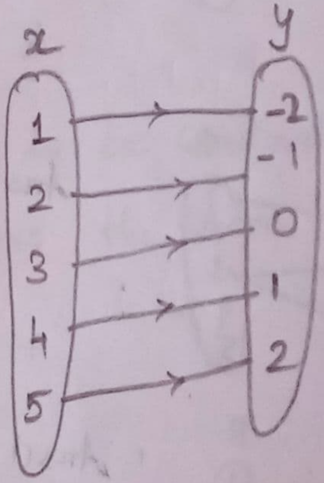
$x = 1$, $f(x) = x - 3$.
$= 1 - 3 = -2 \Rightarrow f(y)$.
$y = -2$.

$x = 2$, $f(x) = 2 - 3$
$= -1$.
$y = -1$.

$x = 3$; $f(x) = 3 - 3$
$= 0$.
$y = 0$.

$x = 4$; $f(x) = 4 - 3$
$= 1$.
$y = 1$.

$x = 5$; $f(x) = 5 - 3$
$= 2$.
$y = 2$.

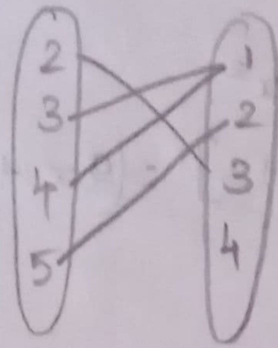


Q) Do the following sets define functions? If so, give their domain and range in each case.

a) $\{\langle 1, \langle 2,3\rangle\rangle, \langle 2, \langle 3,4\rangle\rangle, \langle 3, \langle 1,4\rangle\rangle, \langle 4, \langle 1,4\rangle\rangle\}$

Domain $= \{1, 2, 3, 4\}$.
Codomain $= \{1, 2, 3, 4\}$
Range $= \{1, 2, 3, 4\}$.

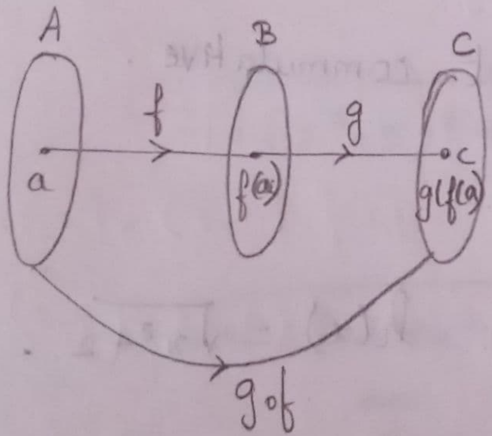
b) $\{(2,3), (4,1), (3,1), (5,2)\}$



Domain = $\{2,3,4,5\}$. Range = $\{1,2,3\}$.

COMPOSITION OF FUNCTIONS :-

Let $f: A \to B$ & $g: B \to C$. The composition of f and g, denoted by $g \circ f$ (g of f) [composite] is a new function from A to C and is given by $\boxed{g \circ f(x) = g[f(x)]}$ $\forall x \in A$.



'A' nu parayana oru set nu. 'a' element eduthal athunte image nu parayunnathu f(a). athu 'B' set aanu verunnathu. So, B lu ulla elements ok f(a), f(b) angana aayirikum. Apo B to C oru function define cheyyumbo, g(f(a)) na eduka. Apo A to C ne nammal g composite f (g of)

So, $g \circ f = g \circ f[x] = g[f(x)]$

Q1) Eg: $f(x) = 2x+3$. $g(x) = x^2$.

Find $f \circ g$ & $g \circ f$.

Sol $f \circ g[x] = f[g(x)]$

$= f[x^2] = \underline{2x^2+3}$. $\left[\because f(x) = 2x+3. \quad f(x^2) = 2x^2+3\right]$

$gof[x] = g[f(x)]$

$= g\overbrace{[2x+3]}^{g\,(x)}$

$= (2x+3)^2$ $[\because g(x) = x^2.$
$g[2x+3] = (2x+3)^2]$

$(a+b)^2 = a^2+b^2+2ab$

$= (2x)^2 + 3^2 + 2\times 2x \times 3.$

$= 4x^2 + 9 + 12x.$

$= 4x^2 + 12x + 9.$

So, $fog \neq gof$. f composite of g is not equal to g composite of g

*) Suppose, if the question arises like, ~~can~~ check whether composition of 'function is commutative?

commutative means $fog = gof$.

*) But in this eg; it is not commutative.

Q2) $f, g, h : R \to R$.
$f(x) = x^2$, $g(x) = x+5$. $h(x) = \sqrt{x^2+2}$.. Is,
$(hog)of = ho(gof)$?

sol $(hog)of$.

$(hog)[f(x)] = hog[x^2]$

$= h[g(x)^2]$ $\cancel{\because g(x)}$

$= h\overbrace{[x^2+5]}^{x}$ $\because g(x) = x+5$.
$g(x^2) = x^2+5$.

$= \sqrt{(x^2+5)^2 + 2}$ $\because \left[h(x) = \sqrt{x^2+2}\right]$

$\because h(x^2+5)$

$\because \sqrt{(x^2+5)^2+2}$..

$= \sqrt{x^4+10x^2+25+2}$

$= \sqrt{x^4+10x^2+27}$. $\longrightarrow$ (1)

$ho(gof)$.

$ho[gof(x)] = ho[g(f(x))]$

$= ho[g(x^2)]$ $\because f(x) = x^2$.

$= ho[x^2+5]$ $\because g(x) = x+5$.

$g(x^2) = x^2+5$.

$= \sqrt{(x^2+5)^2+2}$ $\because h(x) = \sqrt{x^2+2}$

$= \sqrt{(x^2+5)^2+2}$

$= \sqrt{x^4+10x^2+27}$ $\longrightarrow$ (2)

From (1) = (2).

$(hog)of = ho(gof)$.

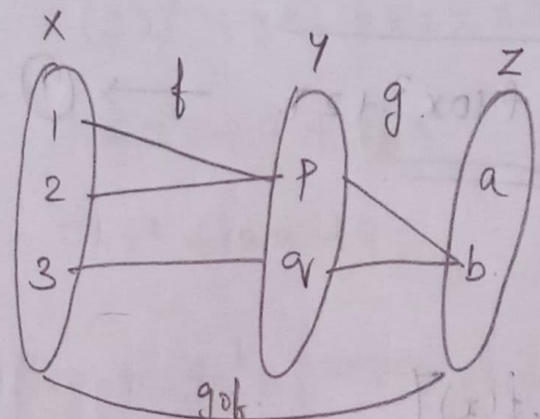
*) You may get the question like f, h, g, are associative.

So, while like $(hog)of = ho(gof)$. and prove it.

Q3) Let $X = \{1,2,3\}$, $Y = \{p, q\}$, and $Z = \{a, b\}$. Also let $f : X \to Y$ be $f = \{(1,p), (2,p), (3,q)\}$

$g : Y \to Z$ be given by $g = \{(p,b), (q,b)\}$. Find $g \circ f$

Sol



$$g \circ f = \{(1,b), (2,b), (3,b)\}.$$

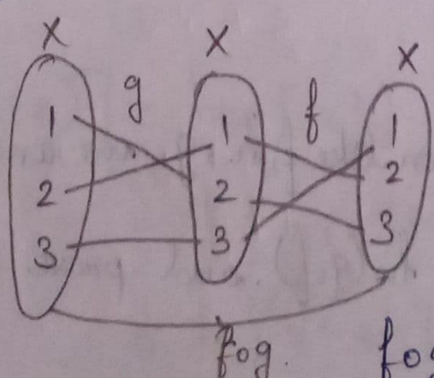
Q4) Let $X = \{1,2,3\}$ and f, g, h and s be functions from X to X given by.

$f = \{(1,2), (2,3), (3,1)\}$ $g = \{(1,2), (2,1), (3,3)\}$

$h = \{(1,1), (2,2), (3,1)\}$ $s = \{(1,1), (2,2), (3,3)\}$.

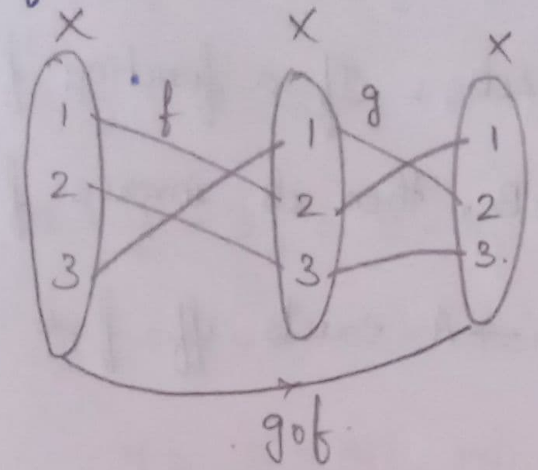
Find $f \circ g$; $g \circ f$; $f \circ h \circ g$; $s \circ g$; $g \circ s$; $s \circ s$; and $f \circ s$.

Sol. fog.



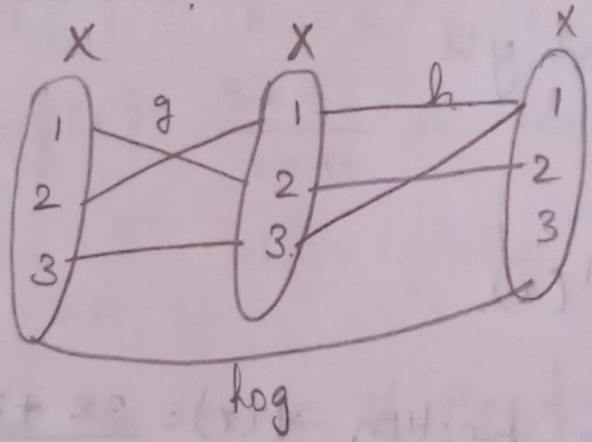
$f \circ g = \{(1,3), (2,2), (3,1)\}$.

gof.

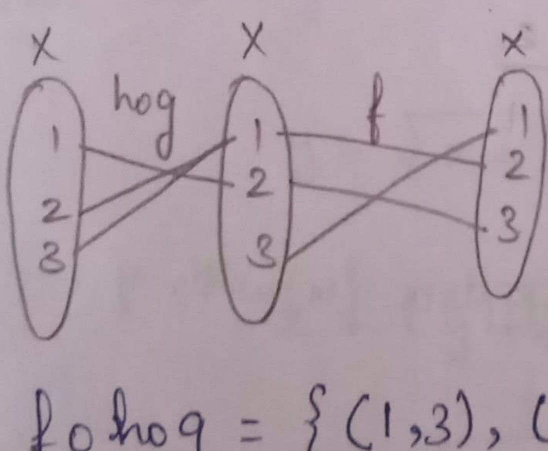


$gof = \{(1,1), (2,3), (3,2)\}$.

fohog.



$hog = \{(1,2), (2,1), (3,1)\}$.



$fohog = \{(1,3), (2,2), (3,2)\}$.

INVERSE FUNCTION :-

Let A and B are two sets. If a function 'f' mapping from A to B i.e, $f : A \rightarrow B$, then its inverse f^{-1} mapping from $B \rightarrow A$ i.e, $f^{-1} = B \rightarrow A$ exists iff f is one-to-one and onto.

Steps to follow for finding the inverse of a function :-

(i) Replace $f(x)$ with y.

(ii) Interchange x's and y's.

(iii) Solve for y.

(iv) Replace y with $f^{-1}(x)$.

Q1) Find the inverse of the function $f(x) = \dfrac{3x+2}{2x+1}$.

Sol Given that $f(x) = \dfrac{3x+2}{2x+1}$.

$y = \dfrac{3x+2}{2x+2}$ => Replace $f(x)$ with y

$x = \dfrac{3y+2}{2y+1}$. => Interchang x's with y, & y with x

$x(2y+1) = 3y+2$

$2xy + x = 3y+2$

solve for 'y'

$2xy - 3y = 2 - x.$

$y(2x-3) = 2-x$

$y = \dfrac{2-x}{2x-3}.$

$f^{-1}(x) = \dfrac{2-x}{2x-3}.$ → Replace y with $f^{-1}(x)$

$\therefore f(x) = \dfrac{3x+2}{2x+1}$ its inverse is $f^{-1}(x) = \dfrac{2-x}{2x-3}.$

Q2). Find the inverse of the function $f(x) = \sqrt{x+4} - 3.$

Sol. Given that $f(x) = \sqrt{x+4} - 3.$

$y = \sqrt{x+4} - 3.$

$x = \sqrt{y+4} - 3.$

$x+3 = \sqrt{y+4}.$

squaring on both sides.

$(x+3)^2 = (\sqrt{y+4})^2$

$$x^2 + 2(x)(3) = 3^2 = y+4$$

$$x^2 + 6x + 9 = y+4$$

$$x^2 + 6x + 9 - 4 = y.$$

$$x^2 + 6x + 5 = y.$$

$$\therefore y = x^2 + 6x + 5.$$

$$\therefore f^{-1}(x) = x^2 + 6x + 5.$$

$\therefore f(x)$ inverse is $f^{-1}(x) = \underline{x^2 + 6x + 5}$

Q3) Find the inverse of the function $f(x) = \dfrac{5x-3}{2x+1}$.

Sol. Given that $f(x) = \dfrac{5x-3}{2x+1}$.

$$y = \frac{5x-3}{2x+1}$$

$$x = \frac{5y-3}{2y+1}.$$

$$x(2y+1) = 5y-3$$

$$2xy + x = 5y-3$$

$2xy + x - 5y = -3.$

$2xy - 5y = -3 - x.$

$y(2x-5) = -3-x.$

$y = \frac{-3-x}{2x-5} = \frac{-(x+3)}{2x-5}.$

$\therefore f^{-1}(x) = \frac{-(x+3)}{2x-5}.$

Inverse of $f(x)$ is $f^{-1}(x) = \frac{-(x+3)}{2x-5}$.

Q4) Let $X = \{1, 2, 3\}$, $Y = \{p, q, r\}$, and $f : X \to Y$ be given by $f = \{(1, p), (2, q), (3, q)\}$. Find f^{-1}.

Sol $f^{-1} = \{(p, 1), (q, 2), (q, 3)\}$.

f^{-1} is not a function.

Q5) Let R be the set of real numbers and let $f : R \to R$ be given by $f = \{(x, x^2) \mid x \in R\}$. Find f^{-1}.

Sol $f^{-1} = \{(x^2, x) \mid x \in R\}$ is not a function.

Q6) Let R be the set of real numbers and let $f : R \to R$ be given by $f = \{(x, x+2) \mid x \in R\}$. Find f^{-1}

Sol. $f^{-1} = \{(x+2, x) \mid x \in R\}$ is a function from R to R.

CLOSURE OF RELATIONS:-

*) Sometimes a given relation R defined in set A may not be reflexive, symmetric or transitive.

*) By adding more ordered pairs to R we can make it reflexive, symmetric or transitive.

*) Closure of relation is obtained by minimum number of ordered pairs to make the relation reflexive, symmetric, or transitive.

Definition:-

The closure of a relation R with respect to property P is the relation obtained by adding the minimum number of ordered pairs to R to obtain property P.

Types of closures:-

1) **Reflexive Closure:-**

The reflexive closure of a relation R on A is obtained by adding (a, a) to R for each $a \in A$.

2) **Symmetric Closure:-**

The symmetric closure of R is obtained by adding (b, a) to R for each $(a, b) \in R$.

3) <u>Transitive closure</u>:-

The transitive closure of R is obtained by repeatedly adding (a,c) to R for each $(a,b) \in R$ and $(b,c) \in R$.

Eg;

$A = \{1, 2, 3, 4\}$.

$R = \{(1,1), (2,3), (1,4), (2,2), (3,1), (4,2)\}$.

Find reflexive, symmetric and transitive closure?

<u>sol</u>. $R = \{(1,1), (2,3), (1,4), (2,2), (3,1), (4,2)\}$.

Reflexive closure <u>r(R)</u>:- $A = \{1, 2, 3, 4\}$. adding reflexive for each element of 'A' for 1, 2, 3, & 4. So (11), (22) (33) (44) are added for relation.

$r(R) = \{(1,1), (2,2), (3,3), (4,4), (2,3), (3,1), (4,2)\}$.

Symmetric closure s(R):- check for symmetric i.e, if (2,3) is there add (3,2) ....

$s(R) = \{\overbrace{(1,1), (2,3), (1,4), (2,2), (3,1), (4,2)}^{\text{added elements in 'R'}}, (3,2), (4,1), 1,3), (2,4)\}$.

Transitive closure R^* :-

$R^* = \{\overbrace{(1,1), (2,3), (1,4), (2,2), (3,1), (4,2)}^{\text{added elements in 'R'}}, (2,1), (1,2), (3,4), (3,2), (4,3), (4,1)\}$.

(2,3), (3,1) | (1,4) (4,2) | (3,1), (1,4) | (3,1), (1,2).

(4,2) (2,3) | (4,2), (2,1)

PARTIAL ORERED SET (POSET):-

PARTIAL ORDERING RELATION:-

A relation 'R' on a set 'P' is said to be partial ordering (or) partial ordering relation iff R is reflexive, anti-symmetric and transitive.

Partial ordering is denoted by the symbol '$\leq$'.

Partial ordered set (or) POSET:-

If '$\leq$' is a partial ordering on a set 'P', then the order pair $(P, \leq)$ is called a "partial ordered set" or "POSET".

Totally ordered relation:-

Let $(P, \leq)$ is a partially ordered set. If for every two elements $a, b \in P$, we have either $a \leq b$ or $b \leq a$ (comparable), then '$\leq$' is called a simple ordering (or) linear ordering on 'P' and $(P, \leq)$ is called a totally ordered set (or) simply ordered set (or) a chain.

Note:- $(P, \leq)$ is a dual of $(P, \geq)$

$(P, \geq)$ is a dual of $(P, \leq)$.

$(P, \leq)$ is a partially ordered set we can take any 2 elements in set P (i.e) a & b, $a, b \in P$ then these two elements all comparable either $a \leq b$ or $b \leq a$

This relation is called simple ordering or linear ordering. then the set is called a totally ordered set.

Eg: Show that the relation "greater than or equal to" is a partially ordering relation on a set of integers.

sol Set of integer $Z = \{ \cdots -3, -2, -1, 0, 1, 2, 3 \cdots \}$.

A relation $\geq$ is a partially ordering relation if it satisfies 3 properties.

1) '$\geq$' is Reflexive

2) '$\geq$' is anti symmetric.

3) '$\geq$' is transitive.

1) Reflexive :-

For every integer 'a', $a \in Z$.

aRa (or) $(a,a) \in R$.

$a \geq a$. (or) $(a,a) \in R$.

$\therefore$ '$\geq$' satisfies reflexive property.

2) Antisymmetric property :-

For any two ~~be~~ integers, a, b $a, b \in Z$.

If aRb, bRa, Then $a=b$.

If $a \geq b$, $b \geq a$, Then $a = b$.

$\therefore$ "$\geq$" satisfies anti-symmetric property.

3. Transitive property:-

For any three integers a, b and c

$a, b, c \in Z$.

If aRb, bRc then aRc.

If $a \geq b$, $b \geq c$, then $a \geq c$.

$\therefore$ '$\geq$' satisfies transitive property.

Now the relation "$\geq$" satisfies the above 3 properties.

Hence '$\geq$' is a POSET (or) partially ordered set.

HASSE DIAGRAM:-

*) A partial ordering ($\leq$) on a set 'P' can be represented by means of a diagram known as Hasse diagram of $(P, \leq)$. diagramatic representation of partial order.

*) In Hasse diagram, each element is represented by a small circle or a dot. • or o

*) In Hasse diagram, we represent the vertices by dots (•) (o) or small circles, we does not put arrows on edges (—) and we does not draw self-loops at vertices.

*) In digraph of partial order, there is an edge from vertex 'A' to vertex 'B' and there is an edge from vertex 'B' ~~and~~ to vertex 'C', then there should be an

edge from vertex 'A' to vertex 'C'. As such, we need not exhibit an edge from 'A' to 'C' explicitly, It will automatically understood.

suppose, edge from A — B & B C, A — B — C

no need draw from A to C. it is automatically understood that it is connected with one another

Q1) Eg: Let $X = \{2, 3, 6, 12, 24, 36\}$ and the relation '$\leq$' be such that $x \leq y$ if x divides y. Draw the Hasse diagram.

Sol: $X = \{2, 3, 6, 12, 24, 36\}$.

$2 \to 6$, take the 1st element 2 and compare with 3. Goto next element 2 is < 3 but 2 cannot divide 3. Goto next element i.e, 6, so, it is ≮ 2 & it is divisible by 2. compare with each element in the set and write it.

*) $2 \to 6, 12, 24, 36$.

The ordered pairs are $(2,6), (2,12), (2,24), (2,36)$.

$(2,2)$ → not taken because self loop is not drawn in hesse diagram

Take next element 3 and compare with each elements of sets and list below.

*) $3 \to 6, 12, 24, 36$.

The orderd pairs are $(3,6), (3,12), (3,24), (3,36)$.

$(3,3)$

*) $6 \to 12, 24, 36$.

The ordered pairs are $(6,12), (6,24), (6,36), (6,6)$

*) $12 \to 24, 36$.

The ordered pairs are $(12,24), (12,36), (12,12)$

*) 24 → (24, 24)

There is no ordered pairs because, 24 is not divisible by 36. It satisfy ~~24 ≥ 24~~ $24 \le 24$, but it is not divisible.

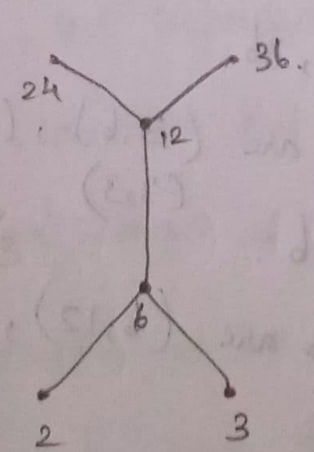
*) 36 → (36, 36).

R = {(2,2), (2,6), (2,12), (2,24), (2,36), (3,6), (3,12), (3,24), (3,36), (3,3), (6,6), (6,12), (6,24), (6,36), (12,24), (12,36), (12,12), (24,24), (36,36)}

*) This relation 'R' is called as partially ordered relation.
(2,2), (3,3), (6,6), ... (36,36) are present
So, this relation 'R' satisfies, reflexive property, anti-symmetric property and transitive property.
(2,6) is there but no (6,2) like that.
(2,12) & (12,24) is there so, (2,24) is present so it is partially ordered relation.

*) Whenever the relation satisfies partially ordered relation then it is said as partially ordered set or POSET.

*) Hasse diagram:



No need to draw edges from 2 to 12 or (2,24), (2,36) it is connected with 6 & 12 so, draw dia like.

(2, 3, 6, 12, 24, 36)

$2 \le 6$ & divides 6 so draw line then check with 6 to 12, 12 to 24, 12 to 36

*) This the Hasse diagram of Relation 'R'.

The above relation is represented in hasse diagram.

Q2) Draw the Hasse diagram for Relation $[\{1,2,3,4,6,9\}, /]$

Divisibility

sol $A = \{1,2,3,4,6,9\}$.

R is divisibility relation.

$[A, R]$ or $[A, /]$

A is partially ordered set, and 'R' Relation is a divisibility relation.

~~Hasse diagram:~~

*) $1 \rightarrow 1, 2, 3, 4, 6, 9.$ $\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{6}{1}, \frac{9}{1}$ so, 1, 2, 3, 4, 6, 9.

the ordered pairs are (1,1), (1,2), (1,3), (1,4), (1,6), (1,9).

*) $2 \rightarrow 2, 4, 6,$ $\frac{2}{2}, \frac{4}{2}, \frac{6}{2}$

the ordered pairs are (2,2), (2,4), (2,6)

*) $3 \rightarrow 3, 6, 9.$ $\frac{3}{3}, \frac{6}{3}, \frac{9}{3}.$

the ordered pairs are (3,3), (3,6), (3,9).

*) $4 \rightarrow 4.$ $\frac{4}{4}.$

the ordered pairs are (4,4)

*) $6 \rightarrow 6.$

the ordered pair is (6,6).

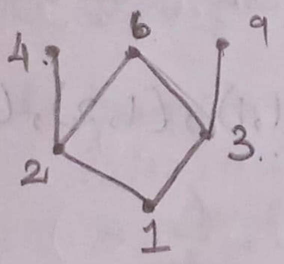
*) $9 \rightarrow 9.$

the ordered pair is (9,9).

$R = \{ (1,1), (1,2), (1,3), (1,4), (1,6), (1,9), (2,2),$
$(2,4), (2,6), (3,3), (3,6), (3,9), (4,4), (6,6),$
$(9,9) \}.$

The relation 'R' satisfies reflexive, anti symmetric and transitive. So, the relation 'R' is partially ordering relation on a set 'A'. So, it is partially ordered set.

Hasse diagram:



Q3) Draw the Hasse diagram for $[\{1,2,3,6,9,18\} |]$

sol. $A = \{1, 2, 3, 6, 9, 18\}.$

R = R is divisibility relation.

*) $1 \rightarrow 1, 2, 3, 6, 9, 18.$
Ordered pair:- $(1,1), (1,2), (1,3), (1,6), (1,9),$
$(1,18).$

*) $2 \rightarrow 2, 6, 18.$
ordered pairs: $(2,2), (2,6), (2,18).$

*) $3 \rightarrow 3, 6, 9, 18$.

Ordered pairs :- $(3,3), (3,6), (3,9), (3,18)$.

*) $6 \rightarrow 6, 18$.

Ordered pairs :- $(6,6), (6,18)$

*) $9 \rightarrow 9, 18$.

Ordered pairs :- $(9,9), (9,18)$.

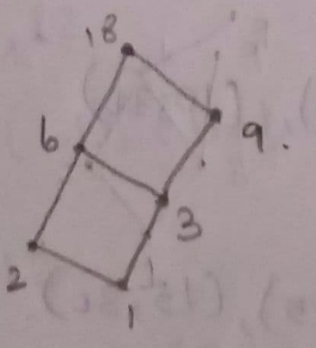
*) $18 \rightarrow 18$.

Ordered pair = $(18,18)$.

*) $R = \{(1,1), (1,2), (1,3), (1,6), (1,9), (1,18),$
$(2,2), (2,6), (2,18), (3,3), (3,6), (3,9),$
$(3,18), (6,6), (6,18), (9,9), (9,18), (18,18)\}$.

*) The relation R is called as partially ordered relation.
So, this relation 'R' satisfies, reflexive property, antisymmetric and transitive property. It is also said as partially ordered set or POSET.

*) Hasse diagram.



Q4) Draw the Hasse diagram for $[\{1,2,3,5,6,10,15,30\}, |]$

sol. Let $A = \{1,2,3,5,6,10,15,30\}$.

Here relation R is divisibility relation.

*) $1 \rightarrow 1, 2, 3, 5, 6, 10, 15, 30$.

Ordered pairs $= (1,1), (1,2), (1,3), (1,5), (1,6), (1,10),$
$(1,15), (1,30)$.

*) $2 \rightarrow 2, 6, 10, 30$.

Ordered pairs $= (2,2), (2,6), (2,10), (2,30)$.

*) $3 \rightarrow 3, 6, 15, 30$.

Ordered pairs $= (3,3), (3,6), (3,15), (3,30)$.

*) $5 \rightarrow 5, 10, 15, 30$.

Ordered pairs $= (5,5), (5,10), (5,15), (5,30)$.

*) $6 \rightarrow 30, 6$

ordered pairs $= (6,6), (6,30)$.

*) ~~E~~ $10 \rightarrow 10, 30$.

ordered pairs $= (10,10), (10,30)$.

*) $15 \rightarrow 15, 30$.

Ordered pairs $= (15,15), (15,30)$.

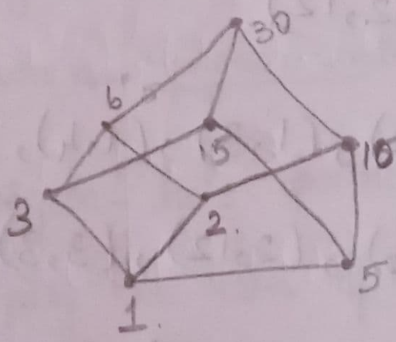
*) 30 → 30.

Ordered pairs = (30,30)

R = {(1,1), (1,2), (1,3), (1,5), (1,6), (1,10), (1,15), (1,30), (2,2), (2,6), (2,10), (2,30), (3,3), (3,6), (3,15), (3,30), (5,5), (5,10), (5,15), (5,30), (6,6), (6,30), (10,10), (10,30), (15,15), (15,30), (30,30)}

Write the stmt which is written in prior question.

Hasse diagram.:



Q5) Draw the Hass diagram for [{1,2,3,4,6,12}, |]

sol A = {1, 2, 3, 4, 6, 12}

Relation R is divisibility on set A.

*) 1 → 1, 2, 3, 4, 6, 12.

Ordered pairs = (1,1), (1,2), (1,3), (1,4), (1,6), (1,12)

*) 2 → 2, 4, 6, 12.

Ordered pairs = (2,2), (2,4), (2,6), (2,12).

*) $3 \rightarrow 3, 6, 12$.

Ordered pairs = $(3,3), (3,6), (3,12)$.

*) $4 \rightarrow 4, 12$.

Ordered pairs = $(4,4), (4,12)$.

*) $6 \rightarrow 6, 12$.

Ordered pairs = $(6,6), (6,12)$.

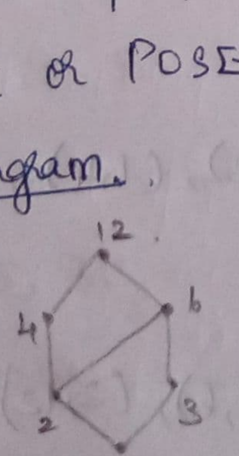
*) $12 \rightarrow 12$

Ordered pairs = $(12,12)$.

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,12), (2,2),$
$(2,4), (2,6), (2,12), (3,3), (3,6), (3,12), (4,4),$
$(4,12), (6,6), (6,12), (12,12)\}$.

*) the relation 'R' is called a partially ordered relation. So, this relation 'R' satisfies, reflexive, anti-symmetric and transitive property. It is also said as partially ordered set or POSET.

*) Hass diagram.



THE PIGEON HOLE PRINCIPLE:-

*) If 'm' pigeons occupy 'n' pigeon-holes, where $m > n$ then atleast one pigeon-hole must contain two or more pigeon.

Suppose, m - pigeons

n - pigeon holes.

m should be greater than 'n' so, one hole contains 2 or more pigeons.

*) If m pigeons occupy 'n' pigeon holes and if $m > n$, then two or more pigeons occupy the same pigeon-hole. This statement is known as "pigeon-hole principle".

*) To apply the pigeon-hole principle, we must decide which objects will play the roles of pigeons and which objects will play the role of pigeon holes.

Applications:-

1) If 6 pigeons occupy 4 pigeon-holes, then atleast one pigeon-hole contain 2 or more pigeons in it.

2) If 8 children are born on the same week, then two or more children are born on the same day of the week.

Generalized pigeon-hole principle :-

If k-pigeons are assigned to n-pigeon holes, then one of the pigeon-hole must contain atleast

$\boxed{\left\lceil \frac{k-1}{n} \right\rceil + 1}$ pigeons.

k -> pigeon
n - p-holes

Q1) Suppose there are 26 students and 5 cars to transport them. show that atleast one car must have 4 or more passengers.

Ans.

$k = 26$ (pigeons) (students)

$n = 5$ (pigeon holes) cars.

Apply generalized pigeon hole principle.

$$\left\lceil \frac{k-1}{n} \right\rceil + 1 = \left\lceil \frac{26-1}{5} \right\rceil + 1 = \frac{25}{5} = 5+1 = \underline{\underline{6}}.$$

One car must have 4 or more passegers is correct. We got 5 passegers.

Q2) Prove that if any 30 peoples are selected, then we may choose a subset of 5 so that all 5 were born on the same day of the week.

sol. $k = 30$

$n = 7$ (days of the week)

Apply generalized pigeon-hole principle.

$$\cancel{k \geq N} \quad \left\lceil \frac{k-1}{n} \right\rceil + 1 = \left\lceil \frac{30-1}{7} \right\rceil + 1.$$

$$= 4 + 1 = \underline{\underline{5}}.$$

5 people were born on the same day of the week.

Q3) Suppose a department contain 13 professors. Then two of the professors were born in the same month.

sol We have 13 professor & 12 months.

$k = 13$ and $n = 12$,

$k > n$

$13 > 12$.

Apply generalized pigeon-hole principle

$$\left\lceil \frac{k-1}{n} \right\rceil + 1 = \left\lceil \frac{13-1}{12} \right\rceil + 1$$

$$= \frac{12}{12} + 1 = 1 + 1 = \underline{\underline{2}}$$

∴ Atleast one month is with two professors born on same month.

Q4) Suppose a laundry bag contains many red, white and blue stocks of socks. Then, one need only to grab 4 socks to be sure of getting a pair with same colour.

Sol. $k = 4$ (Socks to be grabbed).

$n = 3$ different colours.

$k > n$.

$4 > 3$.

The generalized pigeon hole

$$\left\lceil \frac{k-1}{n} \right\rceil + 1 = \frac{4-1}{3} + 1 = \frac{3}{3} + 1 = \underline{\underline{2}}.$$

which means that there will be a pair of socks with same colour.

Q5 Among any group of 367 people, there must be atleast two with same birthday, because there are atleast only 366 possible birthdays.

Sol. $k = 367$ (people)

$n = 366$ (possible b'days).

Apply generalized pigeon hole

$$\left\lceil \frac{k-1}{n} \right\rceil + 1 = \frac{367-1}{366} + 1 = 1+1 = \underline{2} \text{ person.}$$