Math 2550 - Midterm Test 3
L Section - Spring 2021

Carefully explain your answers.
Calculators not allowed.
No notes of any kind - closed book.
Four questions, 10 points each. 100%=40.

Make sure to match pages to questions when uploading to Gradescope.

Question 1

Find the maximum of

$$f(x, y, z) = e^{xyz}$$

subject to the constraint

$$\left(\frac{x}{3}\right)^4 + \left(\frac{y}{2}\right)^4 + \left(\frac{z}{2}\right)^4 = 1.$$

Also determine x, y, z all positive giving the maximum.

(10 points)

Question 2

Consider

$$\int_0^6 \int_{(x/6)^{1/5}}^1 f(x,y)\ dy\ dx$$

where

$$f(x,y) = \sin\left(y^6\right).$$

(a) Sketch the region of integration.

(2 points)

(b) Interchange the order of integration and hence evaluate the integral.

(5 points)

(c) Find the average of $f(x,y)$ over the given region of integration.

(3 points)

Question 3

Let D be the solid defined as follows: It is bounded above by the surface $z = 3 + e^{x^2+y^2}$ and below by the surface $z = e^{x^2+y^2} - 7$. Its base R in the $xy-$plane is bounded by the curves $y = 9 - x^2$ and $y = 3x^2 - 7$.

(a) Sketch R.

(3 points)

(b) Calculate the volume of D.

(7 points)

Question 4

Let R be the region $\{(x, y) : x, y \leq 0 \text{ and } 4 \leq x^2 + y^2 \leq 25\}$.

(a) Sketch R.

(2 points)

(b) Describe R in polar coordinates.

(2 points)

(c) Let

$$f(x, y) = \frac{\cos\left(\sqrt{x^2 + y^2}\right)}{\sqrt{x^2 + y^2}}.$$

Use polar coordinates to evaluate

$$\int\int_R f(x, y)\, dx\, dy. \qquad \text{(6 points)}$$