Hypothesis Testing II

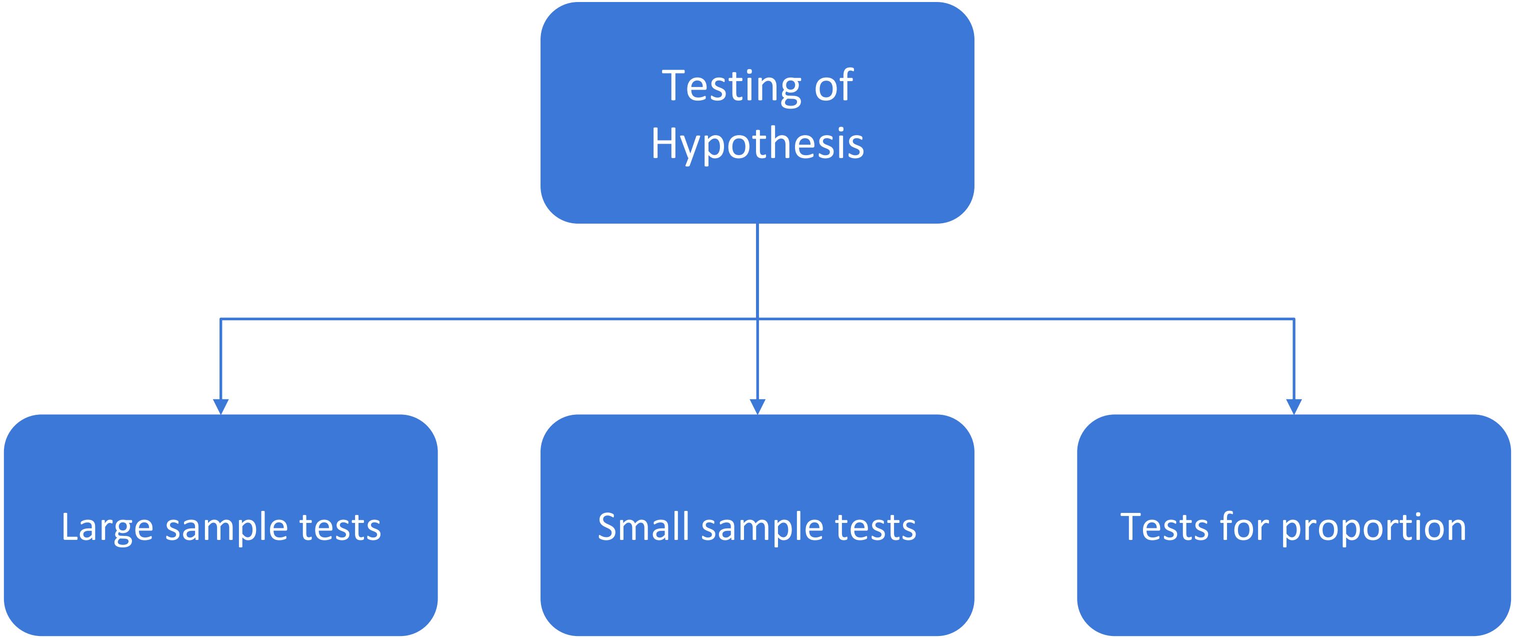
Agenda

- Small Sample Test
  - One Sample
  - Two Sample
- Test for Population Proportion
  - One Sample
  - Two Sample

Last session

- The unknown population parameters are estimated by from sample
- The unknown population parameters are replaced by the sample estimates
- Test the hypothesis for population mean when the population variance is known

In this session…

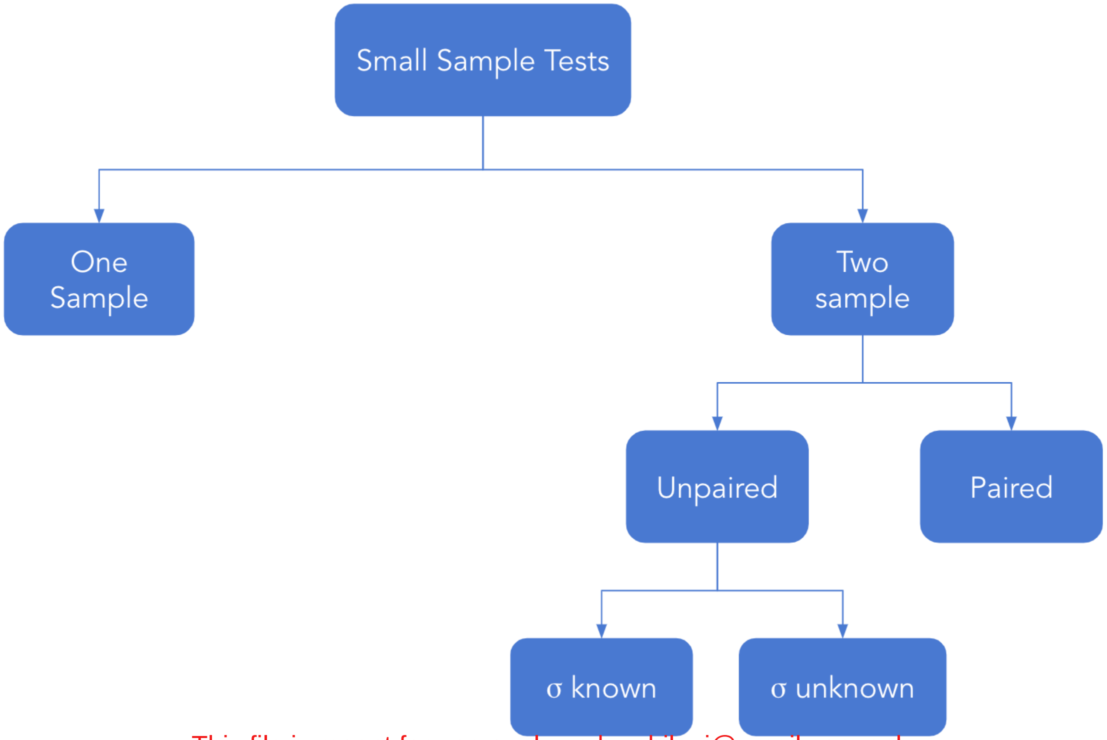


Small Sample Tests

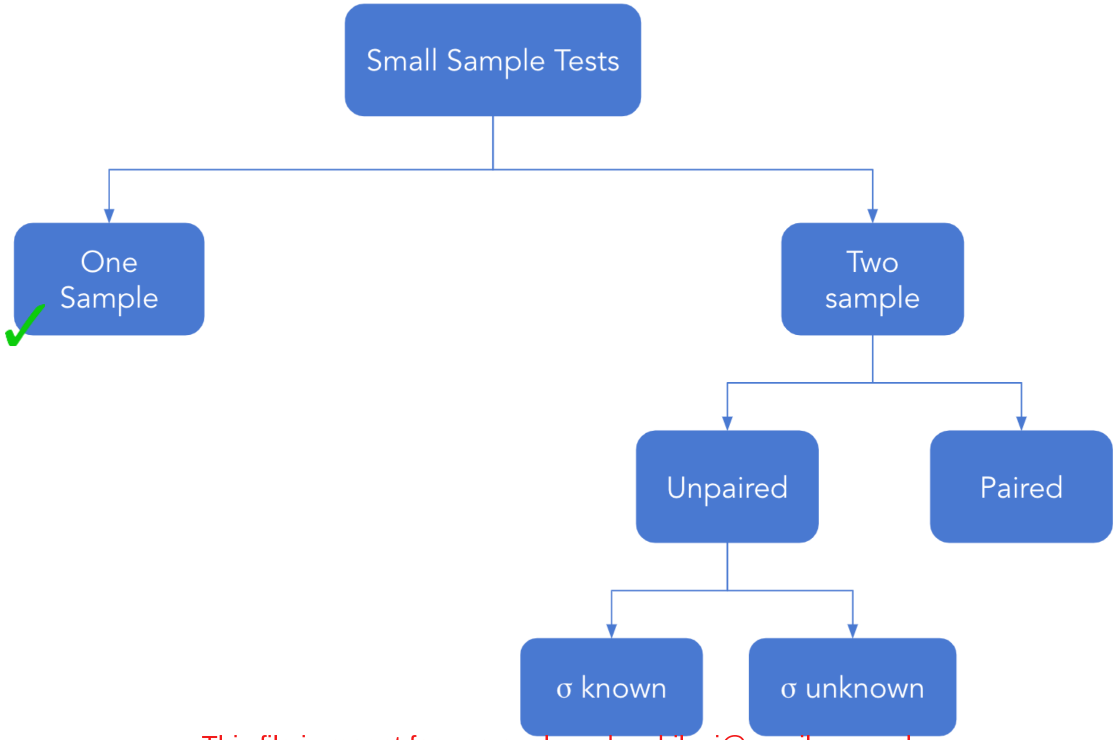
Small sample tests

- For tests $n < 30$ are considered to be small sample tests
- Under the null hypothesis the test statistic follow exact distribution - t distribution, F distribution or χ^2 distribution
- Small sample tests are regarded as exact tests

Small sample tests



Small sample tests

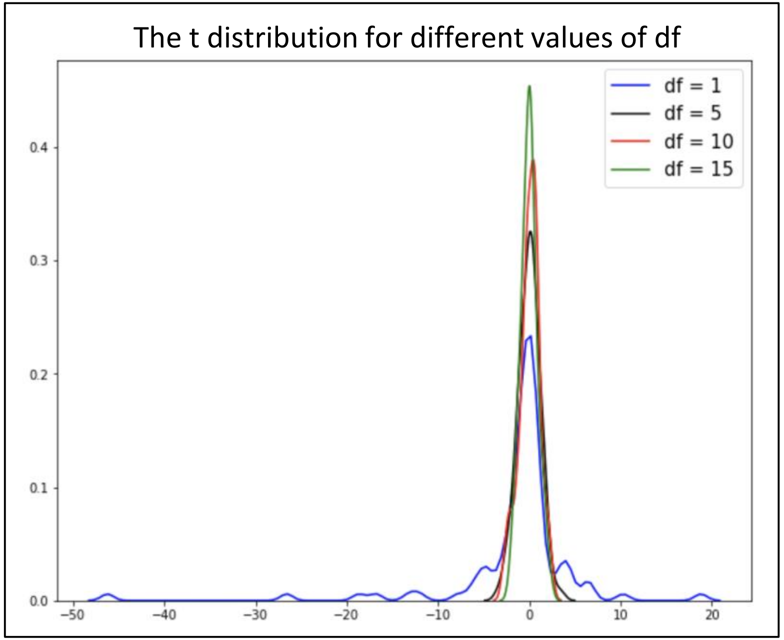


One sample tests

- To test for population mean, if σ^2 is known the Z test is used
- If σ^2 is unknown, use the t-test
- The t-tests are based on the t-distribution
- The unknown σ is replaced by sample standard deviation (s)

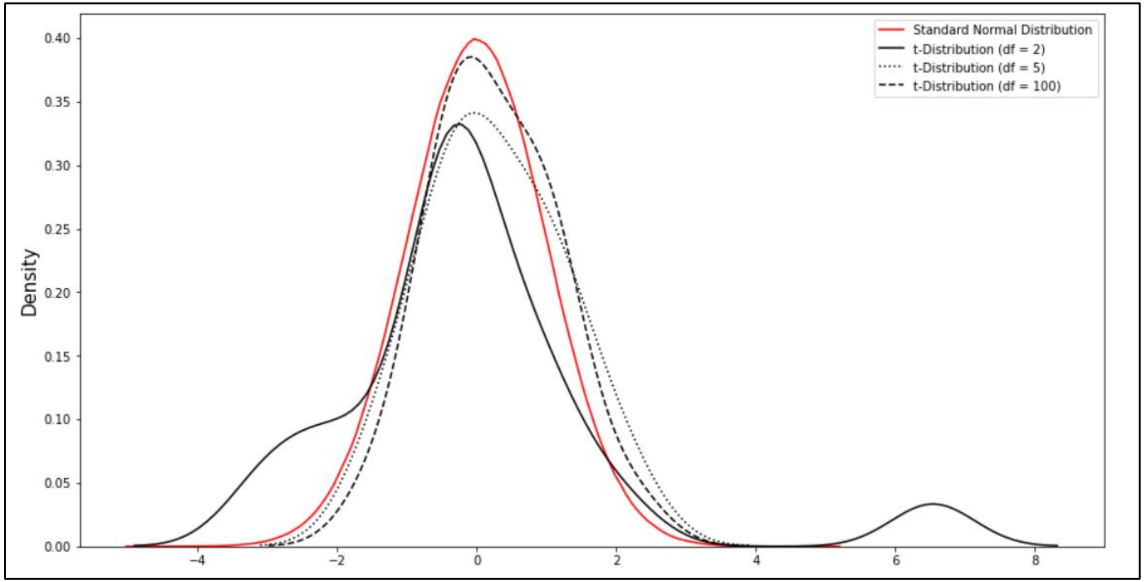
The t-distribution

- It is based on the degrees of freedom
- The t distribution is a symmetric distribution about the mean
- Let *n* denote the degrees of freedom
- Its mean is 0 and variance is $\frac{n}{n-2}$ (for n>2)



The t-distribution

- It is leptokurtic
- For large n it is mesokurtic



Degrees of freedom

Degrees of freedom (d.f.) is the number of independent variables, i.e. they are free to vary.

Example:

Consider a system of equations:

- $Y = 8 + 5Z$
- $Z = X + 10$

Say if the value of Z is known, it is possible to find the value of other variables. Thus the d.f. is 1.

One sample test - hypothesis

- The hypothesis to test whether the population mean is equal to a specified value when σ is unknown

$H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$

- It implies

H_0: The population mean is equal to μ_0 (i.e $\mu = \mu_0$)

against H_1: The population mean is not equal to μ_0 (i.e $\mu \neq \mu_0$)

One sample test - hypothesis

- Like the one sample t- tests, it is possible to test for

$$H_0 : \mu \leq \mu_0 \quad \text{against} \quad H_1 : \mu > \mu_0$$

Or

$$H_0 : \mu \geq \mu_0 \quad \text{against} \quad H_1 : \mu < \mu_0$$

- Failing to reject H_0 implies that the null hypothesis is true

One sample test - test statistic

- The test statistic t given by

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

s^2 is the sample estimate of population variance and is given by

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \quad \text{or} \quad s^2 = \frac{n(\text{sample variance})}{n-1}$$

- Under H_0, the test statistic t follows t distribution with n-1 degrees of freedom

The python code to conduct a t test for population mean is

```
scipy.stats.ttest_1samp(Sample, popmean)
```

Decision rule

	H_1	Based on critical region	Based on p-value	Based on confidence interval
For two tailed test	$\mu \neq \mu_0$	Reject H_0 if $\lvert t \rvert \geq t_{df,\alpha/2}$	Reject H_0 if p-value is less than or equal to the level of significance	Reject H_0 if μ_0 does not lie in the confidence interval
For left tailed test	$\mu < \mu_0$	Reject H_0 if $t \leq -t_{df,\alpha}$		
For right tailed test	$\mu > \mu_0$	Reject H_0 if $t \geq t_{df,\alpha}$		

One sample t-test

Question:

A researcher is studying the growth of bacteria in waters of Lake Beach. The mean bacteria count of 100 per unit volume of water is within the safety level. The researcher collected 10 water samples of unit volume and found the mean bacteria count to be 94.8 with a sample variance of 72.66. Does the data indicate that the bacteria count is within the safety level? Test at the $\alpha = .05$ level. Assume that the measurements constitute a sample from a normal population.

One sample t-test

Solution:

The mean bacteria count of 100 per unit volume of water is within the safety level.
i.e. $\mu = 100$

Let X: bacteria count per unit volume

The researcher collected 10 water samples of unit volume and found the mean bacteria count to be 94.8 with a sample variance of 72.66.
i.e. $n = 10$, $\bar{X} = 94.8$ and $s^2 = 72.66$

One sample t-test

Solution:

To test: The bacteria count is within the safety level

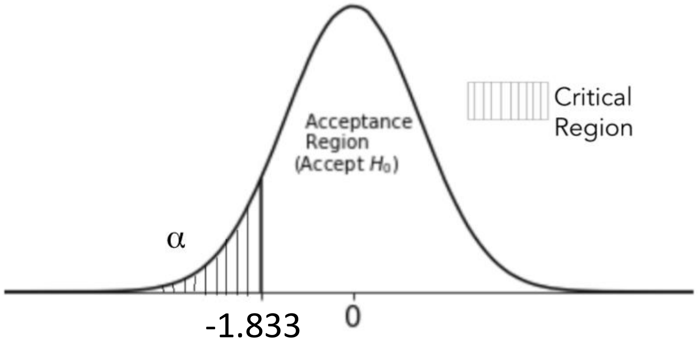
i.e. H_0 : $\mu \geq 100$ against $H_1 : \mu < 100$

The test statistic is

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{94.8 - 100}{\sqrt{72.66/10}} = -1.929$$

Here t follows $t_{9,0.05}$

From the table we have $t_{9,0.05}$ = -1.833



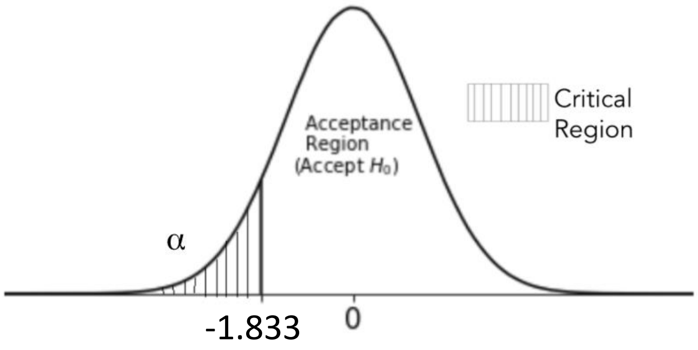
One sample t-test

Solution:

$t_{critical}$ (-1.833) > t_{calc} (-1.929)

Since the critical value is less than the calculated statistic value.

We reject the H_0, and can conclude that the average bacteria per unit volume (true mean) is within the safety levels.



One sample t-test

Python solution: Calculate critical t-value

```
# calculate the t-value for 95% of confidence level
# use 'stats.t.isf()' to find the t-value corresponding to the upper tail probability 'q'
# pass the value of 'alpha' to the parameter 'q', here alpha = 0.05
# pass the degrees of freedom to the parameter, 'df'
# use 'round()' to round-off the value to 2 digits
t_val = round(stats.t.isf(q = 0.05, df = 9), 2)

print('Critical value for one-tailed t-test:', t_val)
```

```
Critical value for one-tailed t-test: 1.83
```

As t-distribution is symmetric, for a left-tailed test if test statistic is less than -1.83, then we reject H_0.

One sample t-test

Python solution: Calculate test statistic

```
# define a function to calculate the t-test statistic
# pass the population mean, sample standard deviation, sample size and sample mean as the function input
def t_test(pop_mean, samp_std, n, samp_mean):

    # calculate the test statistic
    t_score = (samp_mean - pop_mean) / (samp_std / np.sqrt(n))

    # return the t-test value
    return t_score

# calculate the test statistic using the function 't_test'
t_score = t_test(pop_mean = 100, samp_std = (72.66)**(0.5), n = 10, samp_mean = 94.8)
print("t-score:", t_score)
```

```
t-score: -1.9291040236750068
```

As test statistic (=-1.929) < critical value (=-1.83), we reject H_0.

One sample t-test

Python solution: Calculate p-value

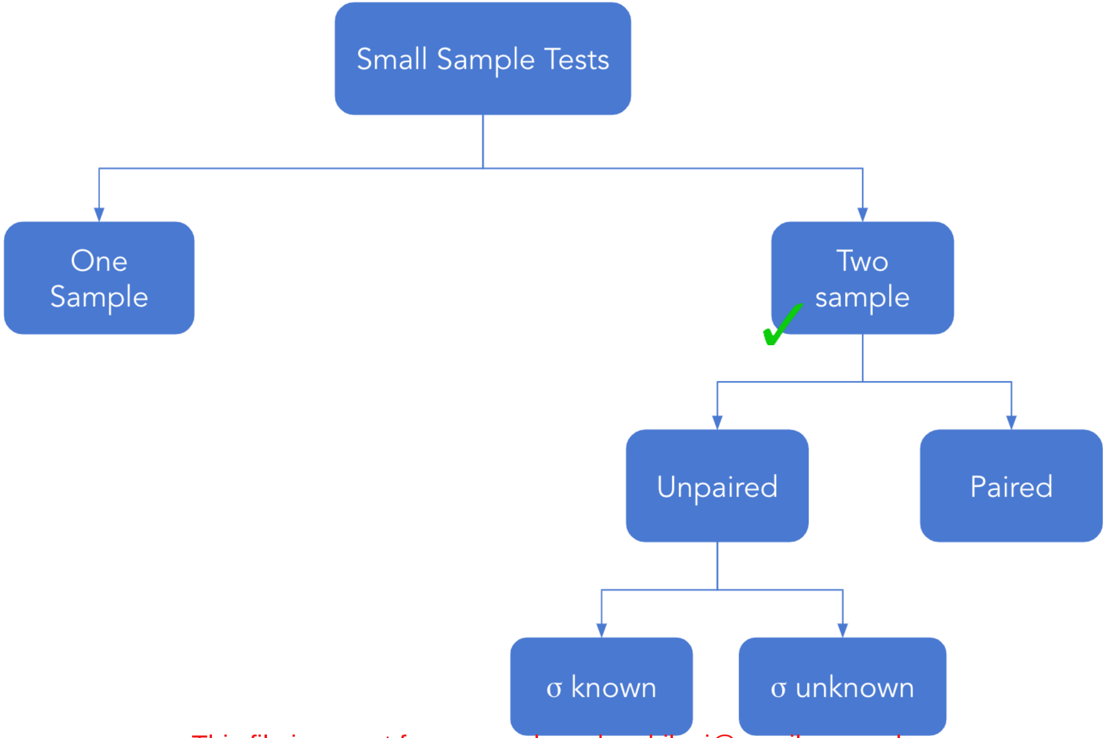
```
# calculate the corresponding p-value for the test statistic
# use 'cdf()' to calculate P(t <= t_score)
# pass the degrees of freedom to the parameter, 'df'
p_value = stats.t.cdf(t_score, df = 9)

print('p-value:', p_value)
```

```
p-value: 0.04289782134327503
```

As p-value < 0.05, we reject H_0.

Small sample tests



Two sample tests

- Like the two sample Z test, the two sample t test are used to compare the equality of means of two populations for unpaired data

- For unpaired data, test relative performance two machineries, investment portfolios

Two sample tests for unpaired data

- To test population means of two populations
- Let there be two different populations such that they follow normal distribution
- Two samples of sizes n_1 and n_2 drawn from normal population $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively
- The two samples are independent of each other

Two sample test - hypothesis

- The hypothesis to test whether the population means are equal

$H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_1$

- It implies

H_0: The two population means are equal (i.e $\mu_1 = \mu_2$)

against H_1: The two population means are not equal μ_0 (i.e $\mu_1 \neq \mu_2$)

- The one sided hypothesis are

$H_0 : \mu_1 \leq \mu_2$ against $H_1 : \mu_1 > \mu_2$ Or $H_0 : \mu_1 \geq \mu_2$ against $H_1 : \mu_1 < \mu_2$

Two sample tests - test statistic

- The test statistic is Z given by

σ_1^2 and σ_2^2 are known

$$Z = \frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Two sample tests - test statistic

- The test statistic is Z given by

σ_1^2 and σ_2^2 are known

$$Z = \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

σ_1^2 and σ_2^2 are unknown

$$t = \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

Where s^2 is the pooled sample variance given by

$$s^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$$

- If σ_1^2 and σ_2^2 are not known then they are replaced the sample estimates s_1^2 and s_2^2 respectively

Two sample tests - test statistic

- If σ_1^2 and σ_2^2 are not known then they are replaced the sample estimates s_1^2 and s_2^2 respectively

- The pooled sample variance (s^2) is used

- Under H_0, the test statistic follows t distribution with $n_1 + n_2 - 2$ df

- Failing to reject H_0 implies that the two population means are equal (i.e $\mu_1 = \mu_2$)

The python code to conduct a t test for two population means which are not paired is

```
scipy.stats.ttest_ind(Sample_1, Sample_2)
```

Two sample test for unpaired data

Question:

An experiment was conducted to compare the pain relieving hours of two new medicines. Two groups of 14 and 15 patients were selected and were given comparable doses. Group 1 was given medicine 1 and group 2 was given medicine 2. Following data is obtained from the two samples. Test whether the two populations give the same mean hours of relief. Assume the data comes from normal distribution has equal variance. [Use $\alpha = 0.01$]

	Medicine 1	Medicine 2
Mean of hours of relief	6.4	7.3
S.D of hours of relief	1.4	1.5

Two sample test for unpaired data

Solution:

Let X: patients receiving medicine 1

Y: patients receiving medicine 2

To test $H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$

The pooled variance is

$$s^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2} = \frac{(13)1.4^2+(14)1.5^2}{14+15-2} = 2.11$$

Two sample test for unpaired data

Solution:

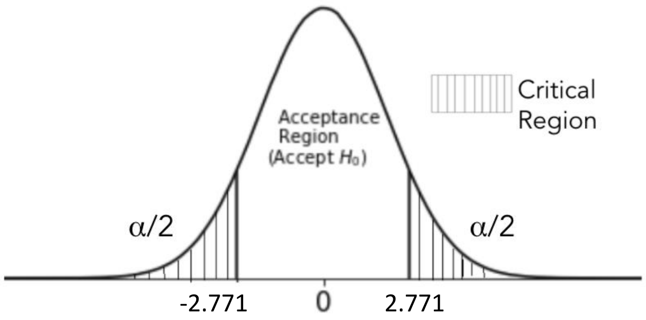
The test statistic is

$$t = \frac{(\bar{X} - \bar{Y})}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(6.4 - 7.3)}{1.4525\sqrt{\frac{1}{14} + \frac{1}{15}}} = -1.667$$

Decision rule: Reject H_0 if $|t_{cal}| \geq t_{n1+n2-2,\, \alpha/2}$

$t_{n1+n2-2,\, \alpha/2} = t_{27,0.005} = 2.771$

Since $2.771 > |-1.667|$, we fail to reject H_0.



The two medicines have the same mean hours of sleep.

Two sample test for unpaired data

Python solution: Calculate critical t-value

```
# calculate the t-value for 99% of confidence level
# use 'stats.t.isf()' to find the t-value corresponding to the upper tail probability 'q'
# pass the value of 'alpha/2' to the parameter 'q', here alpha = 0.01
# pass the degrees of freedom to the parameter, 'df'
# use 'round()' to round-off the value to 2 digits
t_val = np.abs(round(stats.t.isf(q = 0.01/2, df = 27), 2))

print('Critical value for two-tailed t-test:', t_val)
```

```
Critical value for two-tailed t-test: 2.77
```

If test statistic is less than -2.77 or greater than 2.77, then we reject H_0.

Two sample test for unpaired data

Python solution: Calculate test statistic

```python
# size of first sample
n_1 = 14

# mean hours for medicine 1
samp_avg_1 = 6.4

# standard deviation of hours for medicine 1
samp_std_1 = 1.4

# size of second sample
n_2 = 15

# mean hours for medicine 2
samp_avg_2 = 7.3

# standard deviation of hours for medicine 2
samp_std_2 = 1.5

# calculate pooled standard deviation
s = np.sqrt((((n_1-1)*samp_std_1**2) + ((n_2-1)*samp_std_2**2)) / (n_1 + n_2 - 2))

# calculate the test statistic
t_stat = (samp_avg_1 - samp_avg_2) / (s * np.sqrt(1/n_1 + 1/n_2))

# print the test statistic value
print('Test Statistic:', t_stat)
```

```
Test Statistic: -1.667146701465023
```

As test statistic (=-1.667) > critical value (=-2.77), we fail to reject H_0.

Two sample test for unpaired data

Python solution: Calculate p-value

```
# calculate the corresponding p-value for the test statistic
# use 'cdf()' to calculate P(t <= t_stat)
# pass the degrees of freedom to the parameter, 'df'
p_value = stats.t.cdf(t_stat, df = n_1 + n_2 - 2)

# for a two-tailed test multiply the p-value by 2
req_p = p_value*2
print('p-value:', req_p)
```

```
p-value: 0.10704510910964088
```

As p-value > 0.01, we fail to reject H_0.

- The test is carried out under the assumption that the samples are drawn from independent normal population

- If the populations from which the samples are drawn do not have equal variance then the t-test can not be use. In this case the Welch test is used

Two sample tests for paired data

- For paired data effectiveness of some training/treatment is measured

- The observations are recorded on the same individual/item twice resulting in pairs of observations

Example:

A energy drink manufacturing company wants to test if sales increase after they advertise the drink with a life sized picture of a well-known athlete.

Two sample tests for paired data

- Let $\{(X_i, Y_i), i = 1, 2, 3, \ldots, n\}$ where X and Y are paired data
- Let μ_X, μ_Y be the mean of data from X and Y respectively
- Define $d_i = y_i - x_i$
- Let $\mu_d = \mu_y - \mu_x$
- In paired t-test, we test for

$H_0 : \mu_d = \mu_0$ against $H_1 : \mu_d \neq \mu_0$

$H_0 : \mu_d \leq \mu_0$ against $H_1 : \mu_d > \mu_0$

$H_0 : \mu_d \geq \mu_0$ against $H_1 : \mu_d < \mu_0$

Two sample tests for paired data

- Suppose mean of d_i = $\bar{d} = \frac{\sum d_i}{n}$ and variance of d_i = $s^2 = \frac{\sum (d_i - \bar{d})^2}{n-1}$

- The test statistic is

$$t = \frac{\bar{d} - \mu_d}{s/\sqrt{n}}$$

- Under H_0, the test statistic follows t-distribution with n-1 degrees of freedom

- Failing to reject H_0, implies that the null hypothesis is true

The python code to conduct a t test for two population means which are paired is

```
scipy.stats.ttest_rel(Sample_1, Sample_2)
```

Two sample t-test for paired data

Question:

An energy drink distributor claims that a new advertisement poster, featuring a life-size picture of a well-known athlete. For a random sample of 10 outlets, the following data was collected. Test that the null hypothesis that there the advertisement was effective in increasing the sales

Before	33	32	38	45	37	47	48	41	45
After	42	35	31	41	37	36	49	49	48

Test the hypothesis using critical region technique. [Use α = 0.05].

Two sample t-test for paired data

Solution:

To test,

H_0: The advertisement was not effective against H_1: The advertisement was effective

$(\mu_d \leq 0)$

$(\mu_d > 0)$

Compute d

Before	33	32	38	45	37	47	48	41	45
After	42	35	31	41	37	36	49	49	48
$d_i = y_i - x_i$	9	3	-7	-4	0	-11	1	8	3

Two sample t-test for paired data

Solution:

To test,

H_0: The advertisement was not effective against H_1: The advertisement was effective

($\mu_d \leq 0$)

($\mu_d > 0$)

We have mean $\bar{d} = \frac{\sum d_i}{n} = 0.22$ and its variance $s^2 = \frac{\sum (d_i - \bar{d})^2}{n-1} = 43.6944$

The test statistic is

$$t = \frac{\bar{d} - \mu_d}{s/\sqrt{n}} = \frac{0.22 - 0}{6.6101/\sqrt{9}} = 0.0998$$

Two sample t-test for paired data

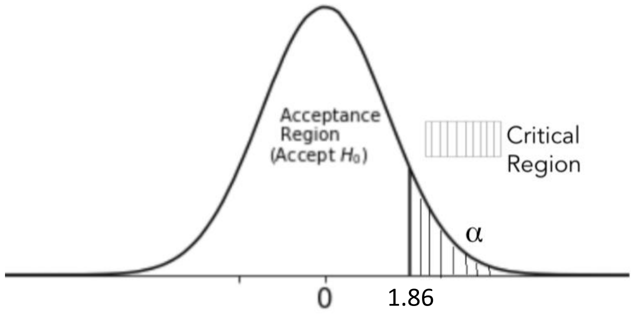
Solution:

Decision Rule: Reject H_0, if $t_{calc} \geq t_{df,\alpha}$

Here $t_{df,\alpha} = t_{n-1,\alpha} = t_{8,0.05} = 1.86$

Since 1.86 > 0.0998, fail to reject H_0

Thus, there is no effect of the advertisement.



Two sample t-test for paired data

Python solution: Calculate critical t-value

```
# calculate the t-value for 95% of confidence level
# use 'stats.t.isf()' to find the t-value corresponding to the upper tail probability 'q'
# pass the value of 'alpha' to the parameter 'q', here alpha = 0.05
# pass the degrees of freedom to the parameter, 'df'
# use 'round()' to round-off the value to 2 digits
t_val = round(stats.t.isf(q = 0.05, df = 8), 2)

print('Critical Value for one-tailed t-test:', t_val)
```

```
Critical Value for one-tailed t-test: 1.86
```

If test statistic is greater than 1.86, then we reject H_0.

Two sample t-test for paired data

Python solution: Calculate test statistic and p-value

```python
# use 'ttest_rel()' to calculate the t-statistic and corresponding p-value for paired samples
# pass the after and before sales to the function
t_stat, p_val = stats.ttest_rel(sales_after, sales_before)

# print the t-test statistic and corresponding p-value
print("Test Statistic:", t_stat)
print("p-value:", p_val)
```

```
Test Statistic: 0.10085458113185983
p-value: 0.9221477146925299
```

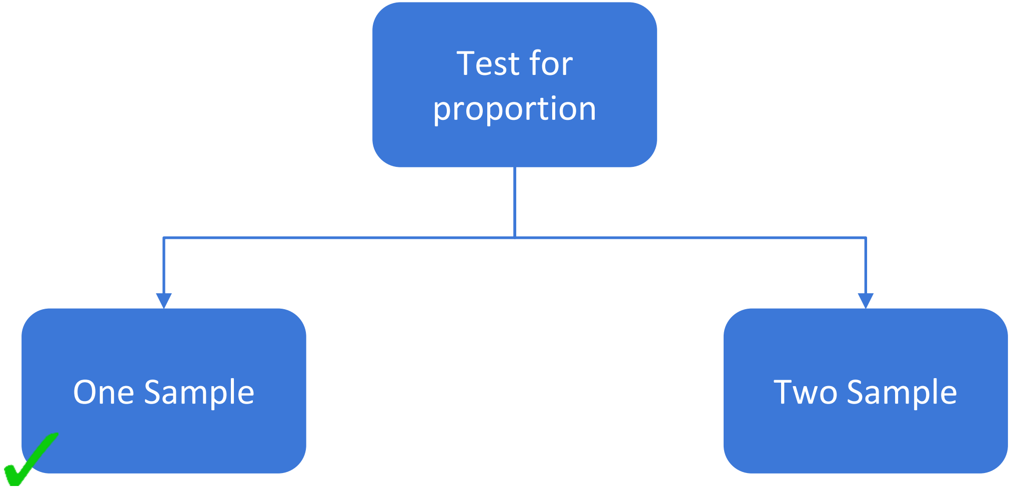
The test statistic (=0.1) < critical value (=1.86), also the p-value > 0.05, thus we fail to reject H_0.

Test for Population Proportion

Test for proportion

- For qualitative data the proportion of a desired characteristic is obtained
- Test for proportion:
    - One sample: Testing population proportion (P) is equal to a specified value (P_0)
    - Two sample: Testing equality of Two population proportions ($P_1 = P_2$)
- Similar to the tests of population mean

Test for proportion



One sample test - hypothesis

- The hypothesis to test the population proportion is equal to a specified value

$H_0 : P = P_0$ against $H_1 : P \neq P_0$

- It implies

H_0: The population proportion is equal to P_0

against H_1: The population proportion is not equal to P_0

- Failing to reject H_0 implies that the population proportion is equal to P_0

Test for proportions

- The test statistic is given by

Sample proportion

Specified proportion

$$Z = \frac{p - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}}$$

Sample size

- Under H_0, the test statistic follows standard normal distribution

One sample test for proportion - decision rule

<table>
<tr><th></th><th>H_1</th><th>Based on critical region</th><th>Based on p-value</th><th>Based on confidence interval</th></tr>
<tr><td>For two tailed test</td><td>$P \neq P_0$</td><td>Reject H_0 if $|Z| \geq Z_{\alpha/2}$</td><td rowspan="3">Reject H_0 if p-value is less than or equal to the level of significance</td><td rowspan="3">Reject H_0 if P_0 does not lie in the confidence interval</td></tr>
<tr><td>For left tailed test</td><td>$P < P_0$</td><td>Reject H_0 if $Z \leq -Z_{\alpha}$</td></tr>
<tr><td>For right tailed test</td><td>$P > P_0$</td><td>Reject H_0 if $Z \geq Z_{\alpha}$</td></tr>
</table>

One sample test for proportion

Question:

From a sample 361 business owners had gone into bankruptcy due to recession. On taking a survey, it was found that 105 of them had not consulted any professional for managing their finance before opening the business. Test the null hypothesis that at most 25% of all businesses had not consulted before opening the business.

Test the claim using p-value technique. [Use $\alpha = 0.05$].

One sample test for proportion

Solution:

From a sample 361 business owners had gone into bankruptcy due to recession.
i.e. $n = 361$

On taking a survey, it was found that 105 of them had not consulted any professional for managing their finance before opening the business.

Let X: business which did not consult before
$x = 105$

The sample proportion $(p) = x/n = 105/361 = 0.2909$

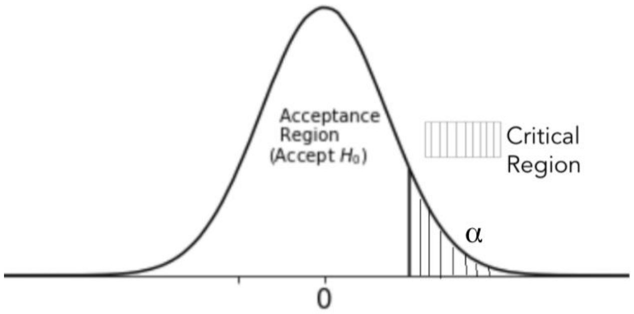
One sample test for proportion

Solution:

To test: The null hypothesis that at most 25% of all businesses had not consulted before opening the business

Here $P_0 = 0.25$

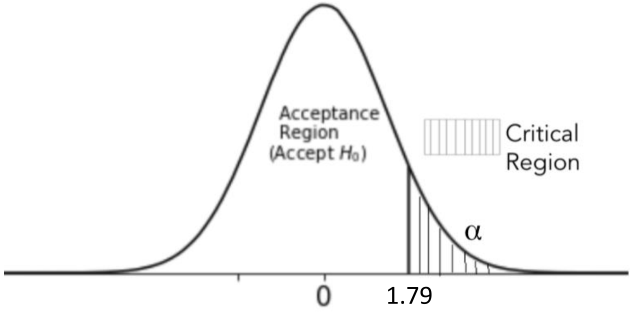
To test, H_0: $P \leq 0.25$ against H_1: $P > 0.25$



One sample test for proportion

Solution:

The test statistic $$Z = \frac{p - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.2909 - 0.25}{\sqrt{\frac{0.25(0.75)}{316}}} = 1.79$$



The p-value = $P(Z > 1.79) = 0.0367$

As p-value < 0.05, reject H_0.

We may conclude that at least 25% of all businesses had not consulted before starting the business.

One sample test for proportion

Python solution: Calculate test statistic

```
# sample size
n = 361

# number of business owners that did not consult before
x = 105

# sample proportion
p_samp = x / n

# hypothesized proportion
hypo_p = 0.25

# calculate test statistic value for 1 sample proportion test
z_prop = (p_samp - hypo_p) / np.sqrt((hypo_p * (1 - hypo_p)) / n)

print('Test statistic:', z_prop)
```

```
Test statistic: 1.7928245201151534
```

One sample test for proportion

Python solution: Calculate p-value

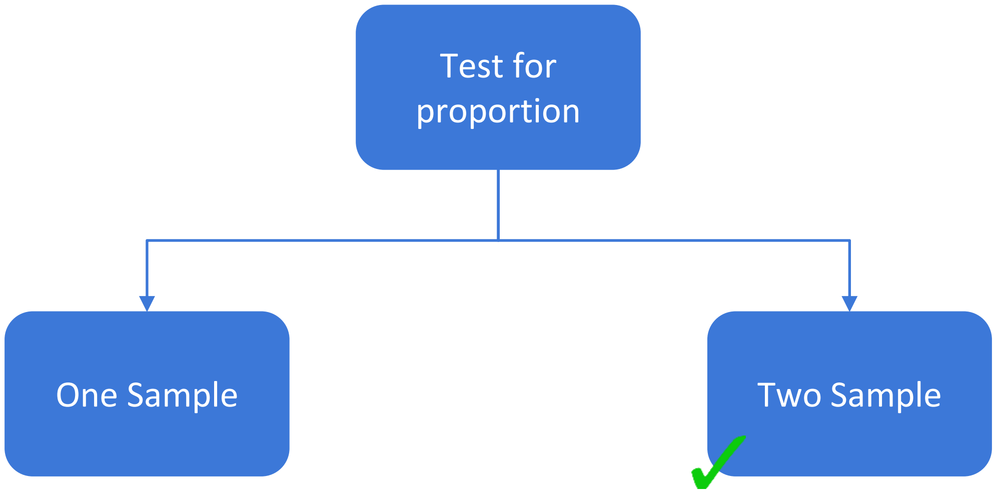
```
# calculate the corresponding p-value for the test statistic
# use 'sf()' to calculate P(Z > z_prop)
p_value = stats.norm.sf(z_prop)

print('p-value:', p_value)
```

```
p-value: 0.03650049373124949
```

As the p-value < 0.05, we reject H_0.

Test for proportion



Two sample tests for population proportion

- Let there be two samples sizes n_1 and n_2 from different populations of such that x_1 and x_2 are the number of specific items in each of them respectively

- Suppose these samples have proportions of specific items p_1 and p_2 respectively

- To test the equality of population proportion from which these samples are chosen

Two sample test - hypothesis

- The hypothesis to test the population proportion

$$H_0 : P_1 = P_2 \quad \text{against} \quad H_1 : P_1 \neq P_2$$

- It implies

H_0: The two population proportions are equal ($P_1 = P_2$)

against H_1: The two population proportions are not equal ($P_1 \neq P_2$)

- Failing to reject H_0 implies that the two population proportions are equal

Test for proportions

- The test statistic is given by

$$Z = \frac{p_1 - p_2}{\sqrt{\bar{P}(1-\bar{P})(\frac{1}{n_1}+\frac{1}{n_2})}}$$

where $\bar{P}$ is the proportion of pooled sample such that

$$\bar{P} = \frac{x_1+x_2}{n_1+n_2} = \frac{n_1p_1+n_2p_2}{n_1+n_2}$$

- Under H_0, the test statistic follows standard normal distribution

The python code to conduct a Z test for two population proportions is

statsmodels.api.stats.proportions_ztest(Sample_1, Sample_2)

Two sample test for proportion - decision rule

	H_1	Based on critical region	Based on p-value	Based on confidence interval
For two tailed test	$P_1 \neq P_2$	Reject H_0 if $\lvert Z \rvert \geq Z_{\alpha/2}$	Reject H_0 if p-value is less than or equal to the level of significance	Reject H_0 if $P_1 - P_2$ does not lie in the confidence interval
For left tailed test	$P_1 < P_2$	Reject H_0 if $Z \leq -Z_\alpha$		
For right tailed test	$P_1 > P_2$	Reject H_0 if $Z \geq Z_\alpha$		

Two sample test for proportion

Question:

Steve owns a kiosk where sells two magazines - A and B in a month. He buys 100 copies of magazine A out of which 78 were sold and 70 copies of magazine B out of which 65 were sold. Is there enough evidence to say that magazine is B is more popular?

Test the claim using p-value technique. [Use $\alpha = 0.05$].

Two sample test for proportion

Solution:

Steve owns a kiosk where sells two magazines - A and B in a month.

Let X: the number of magazines sold

Out of 100 copies of magazine A 78 are sold
Here, x_1 = 78 and n_1 = 100
Let p_1 be the proportion of sell of magazine A
$p_1 = x_1/n_1 = 78/100 = 0.78$

Out of 70 copies of magazine B 65 are sold
Here, x_2 = 65 and n_1 = 70
Let p_2 be the proportion of sell of magazine B
$p_2 = x_2/n_2 = 65/70 = 0.928$

Two sample test for proportion

Solution:

To test, whether magazine B is more popular, i.e

$H_0: P_1 \geq P_2$ against $H_1: P_1 < P_2$

Where P_1: denotes population proportion of magazine A sold

P_2: denotes population proportion of magazine B sold

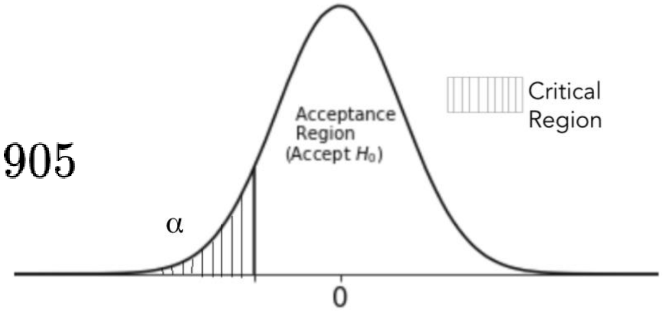
Two sample test for proportion

Solution:

The pooled proportion is $\bar{P} = \frac{x_1+x_2}{n_1+n_2} = \frac{78+65}{100+70} = 0.84$

The test statistic is

$$Z = \frac{p_1 - p_2}{\sqrt{\bar{P}(1-\bar{P})(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{0.78-0.928}{\sqrt{0.84(1-0.84)(\frac{1}{100}+\frac{1}{70})}} = -2.5905$$

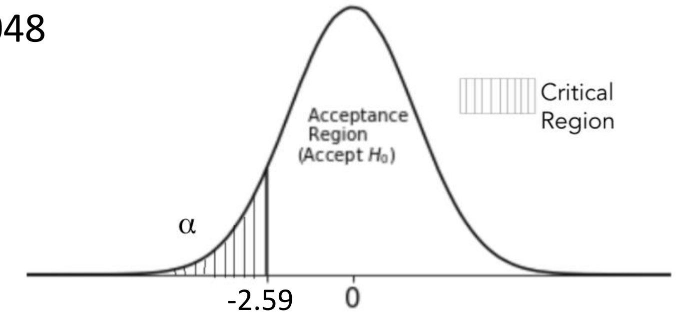


Two sample test for proportion

Solution:

The test statistic Z = -2.5905

The p-value = $P(Z < Z_{calc}$, under $H_0) = P(Z < -2.5905, \mu = 13) = 0.0048$



Since p-value < 0.05, we reject H_0.

Thus there is enough evidence to conclude that magazine is B is more popular.

Two sample test for proportion

Python solution: Calculate test statistic and p-value

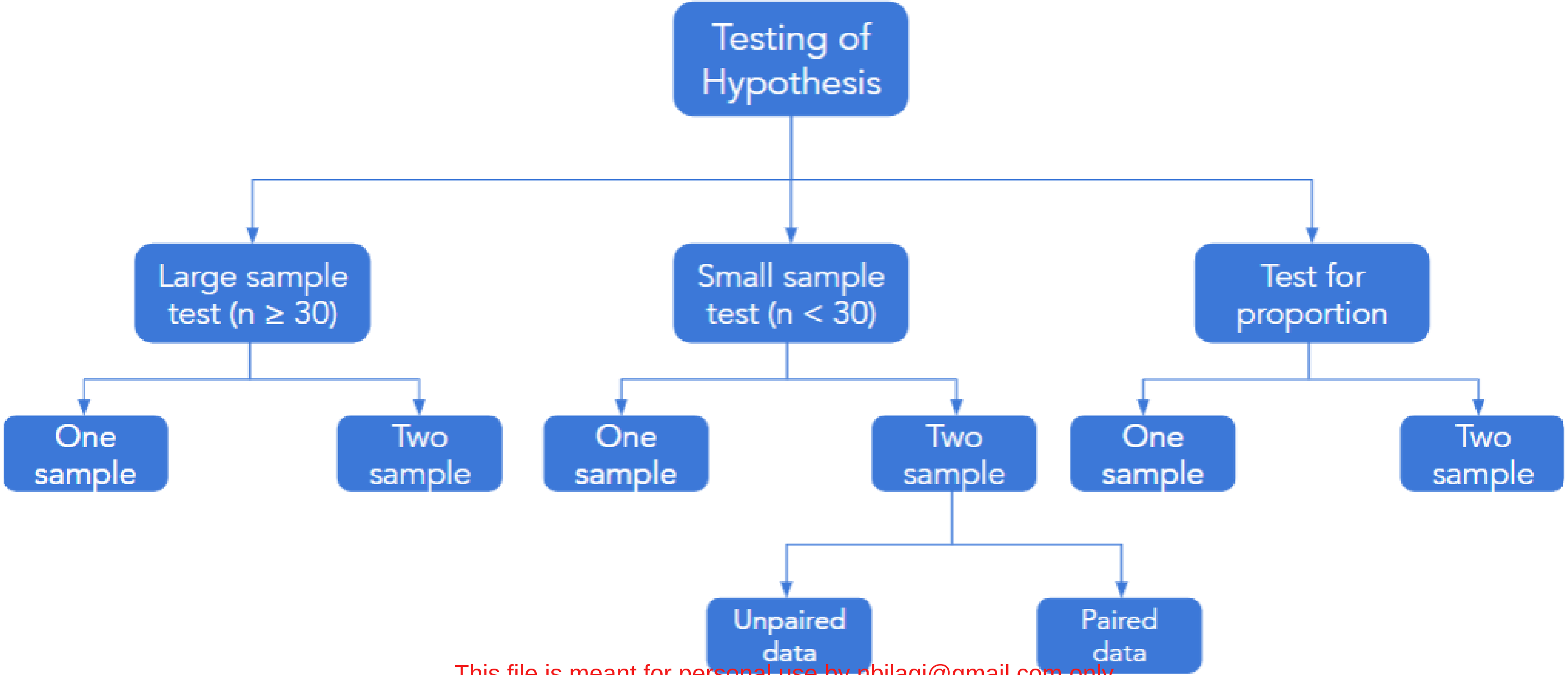
```
# calculate test statistic value for two sample proportion test
# pass the copies sold for both the magazines to the parameter, 'count'
# pass the size of both the samples to the parameter, 'nobs'
# pass the one-tailed condition to the parameter, 'alternative'
z_prop, p_val = sm.stats.proportions_ztest(count = np.array([78, 65]),
                                            nobs = np.array([100, 70]),
                                            alternative = 'smaller')

# print the value of test statistic and the corresponding p-value
print('Test statistic:', z_prop)
print('p-value:', p_val)
```

```
Test statistic: -2.60830803458311
p-value: 0.004549551600547303
```

As the p-value < 0.05, we reject H_0.

Summary


Testing of Hypothesis
Large sample test (n ≥ 30)
Small sample test (n < 30)
Test for proportion
One sample
Two sample
One sample
Two sample
One sample
Two sample
Unpaired data
Paired data

Parametric tests

- The tests considered so far have two features:
    - The probability distribution of the samples was assumed to be known
    - The hypothesis test was about the parameter of the probability distribution
- These tests are known as the parametric tests
- The times when these assumptions are not satisfied, use the non-parametric tests

Thank You