UNIT IV: NETWORK MANAGEMENT – CRASHING OF A PROJECT

COST CONSIDERATION IN PERT/CPM

PROJECT COST

In order to include the cost factors in project scheduling, we must first define the cost duration relationship for various activities in the project. The total cost of any project comprises of direct and indirect costs.

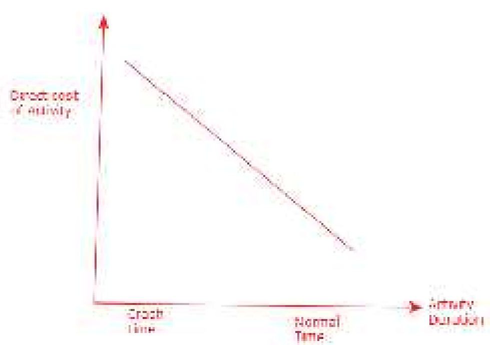
PROJECT COST ANALYSIS

So far we have dealt with how to find project completion time in PERT and CPM networks. In CPM network, when the time required by an activity is deterministic in nature, we may come across a situation that we may have to reduce the activity duration. This is not possible in PERT activity; because activity duration is probabilistic in nature and we have three time estimates. Which time (either to, tm or tp) is to be reduced is a question. Hence activity time crashing is possible in critical path network only.

Before crashing the activity duration, we must understand the costs associated with an activity.

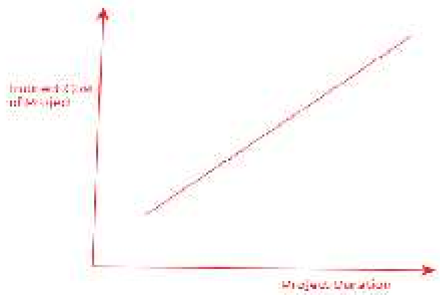
Direct Cost

Direct costs are the costs that can be identified with activity. For example, labour costs, material cost etc. When an activity whose duration is to be reduced (crashed), we have to supply extra resources, specially manpower. Let us say an activity takes 7 days with 2 men. If 4 men works it can be done in 4 days. The cost of 2 workmen increases. As we go on reducing the activity time, cost goes on increasing as shown in figure



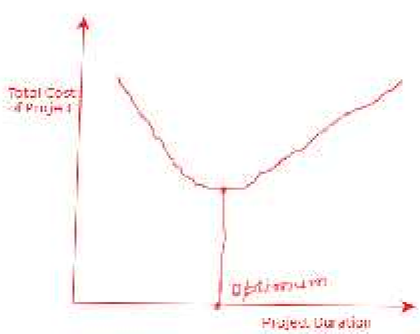
Indirect Cost

These are the costs, which cannot be identified with the activity. Say the salary of a manager, who is in-charge of many projects. Exact amount of his salary that should be charged to a particular project cannot be estimated correctly as it is very difficult to say how much time he has spent on each project. We can express all indirect costs put together in terms of an amount per time period, for example say Rs. 100/- per day, as the indirect costs are expressed as so much of amount per time period, as the duration of project goes on reducing the indirect cost also goes on decreasing as in figure



Total cost

The total cost which is the sum of direct cost and indirect cost is shown in figure. As the project duration goes on reducing the total cost reduces from B to C and if duration is still crashed the total cost increases to A. Hence our problem here is to find out the optimal duration of the project and optimal cost.



Cost Slope

Cost slope is the slope of the direct cost curve, approximated as a straight line. It is given by

Cost slope = (Crash cost – Normal cost)/(Normal time – Crash time) = $(C_c - C_n)/(T_n - T_c)$

i.e., it represents the rate of increase in the cost of performing the activity per unit reduction in time and is called cost/time trade off. It varies from activity to activity. The total cost of project is the sum total of the project's direct and indirect costs.

Time – Cost Optimization Algorithm

Step 1: Find the normal critical path and identify the critical activities.

Step 2: calculate the cost slope for the different activities by using the formula:
Cost slope = (Crash cost – Normal cost)/(Normal time – Crash time) = $(C_c - C_n)/(T_n - T_c)$

Step 3: Rank the activities. The activity whose cost slope is minimum is to be ranked 1, the next minimum as rank 2 and so on, i.e., the ranking takes place in ascending order of cost slope.

Step 4: By crashing the activities on the critical path, other paths also become critical and are called parallel paths.
In such case, project duration can be reduced by crashing activities simultaneously on the parallel critical path.

Step 5: find the total cost of the project at each step.

Step 6: Continue the process until all the critical activities are fully crashed or no further crashing is possible.

In the case of indirect cost, the process of crashing is repeated until the total cost is minimum, beyond which it may increase.

This minimum cost is called the optimum project cost and the corresponding time, the optimum project time.

Solved Example of Crashing of a Project

Q1. The normal and crash times and direct costs for the activities of a project are shown below:

Activities	Time		Cost	
	Normal	Crash	Normal	Crash
1 – 2	8	4	3000	6000
1– 3	5	3	4000	8000
2 – 4	9	6	4000	5500
3 – 5	7	5	2000	3200
2 – 5	5	1	8000	12000
4 – 6	3	2.5	10000	11200
5 – 6	6	2	4000	6800
6 – 7	10	7	6000	8700
5 – 7	9	5	4200	9000

(a) Draw the network diagram

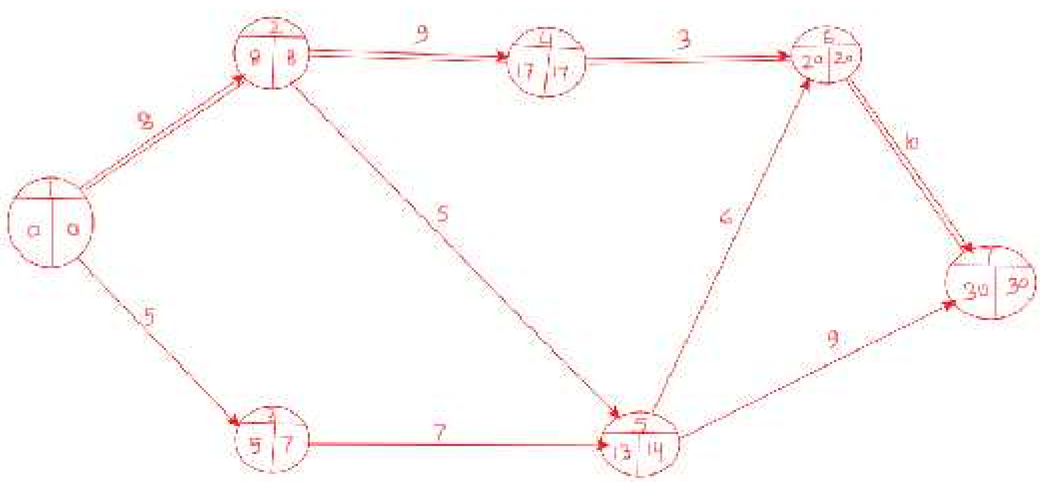
(b) Determine the critical path

(c) Find the minimum cost project schedule if the indirect costs are Rs2000 per week.

Step 1:

Network Diagram & critical Path

Figure – 1



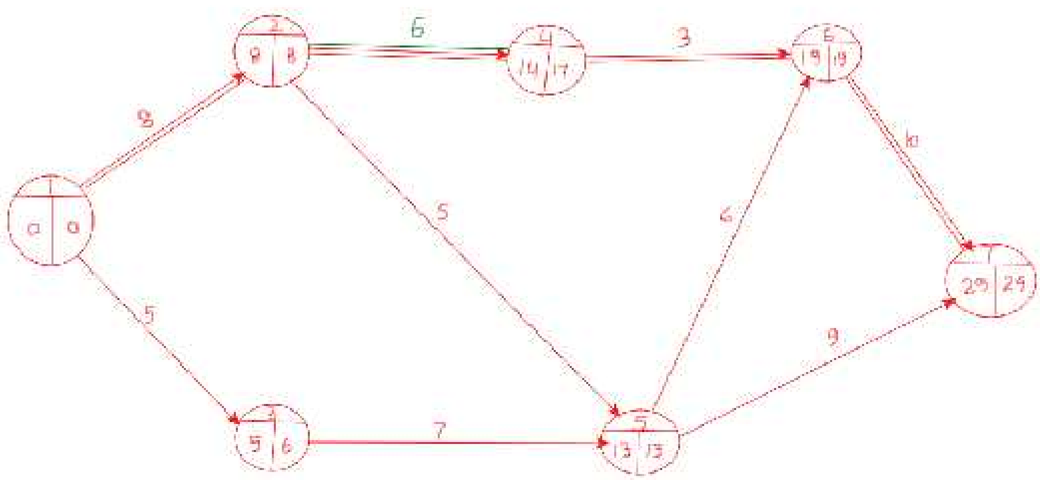
Critical path is (1 -2-4-6-7). The Project duration is 30 weeks and total direct cost is Rs.45200

Step 2 & 3: Cost Slope & Ranking

Activities	Time		Cost		Cost Slope	Rank
	Normal	Crash	Normal	Crash		
1 – 2	8	4	3000	6000	**750**	**IV**
1– 3	5	3	4000	8000	**2000**	**VIII**
2 – 4	9	6	4000	5500	**500**	**I**
3 – 5	7	5	2000	3200	**600**	**II**
2 – 5	5	1	8000	12000	**1000**	**VI**
4 – 6	3	2.5	10000	11200	**2400**	**IX**
5 – 6	6	2	4000	6800	**700**	**III**
6 – 7	10	7	6000	8700	**900**	**V**
5 – 7	9	5	4200	9000	**1200**	**VII**

Examining the time cost slope of activities on the critical path we find that activity (2-4) has the lowest cost slope; in other word cost to expedite per week is the lowest for activity (2-4). Hence, activity (2-4) is crashed. Project network after such crashing is shown below:

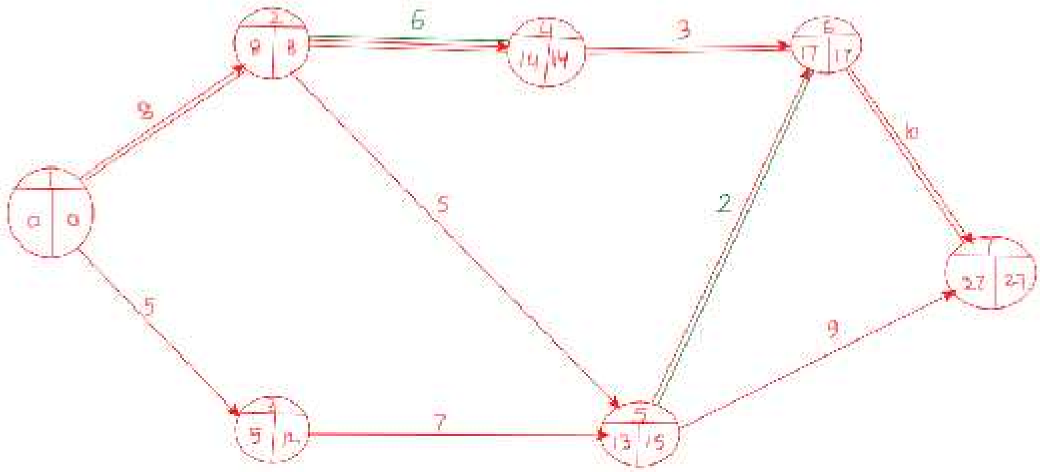
Figure – 2



The new critical path is (1-2-5-6-7), with length of 29 weeks, and the total direct cost is Rs.46700

Examining the time cost slope of activities on the new critical path (1-2-5-6-7) we find that activity (5-6) has the lowest cost slope; in other word cost to expedite per week is the lowest for activity (5-6). Hence, activity (5-6) is crashed. Project network after such crashing is shown below:

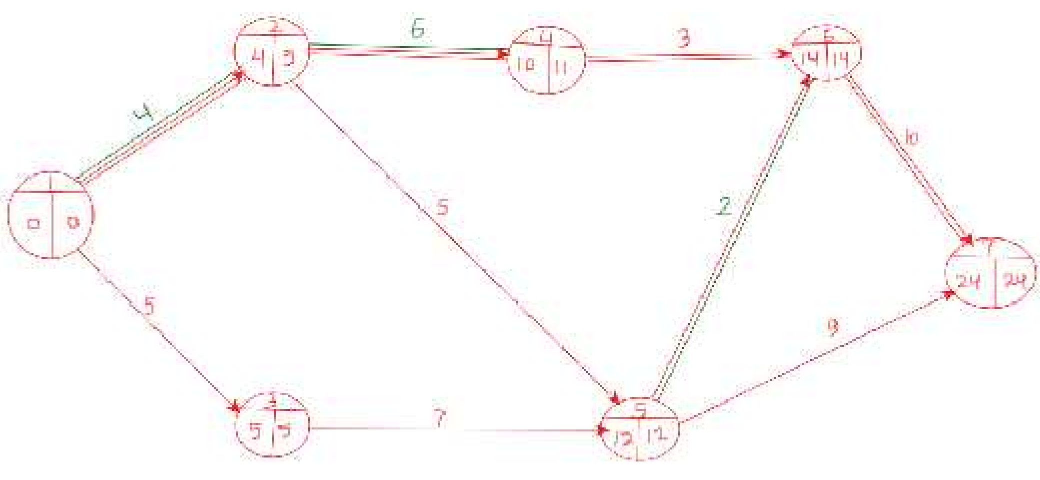
Figure – 3



The new critical path is (1-2-4-6-7), with length of 27 weeks, and the total direct cost is Rs.49500

Examining the time cost slope of activities on the new critical path (1-2-4-6-7) we find that activity (1-2) has the lowest cost slope. Hence, activity (1-2) is crashed. Project network after such crashing is shown below:

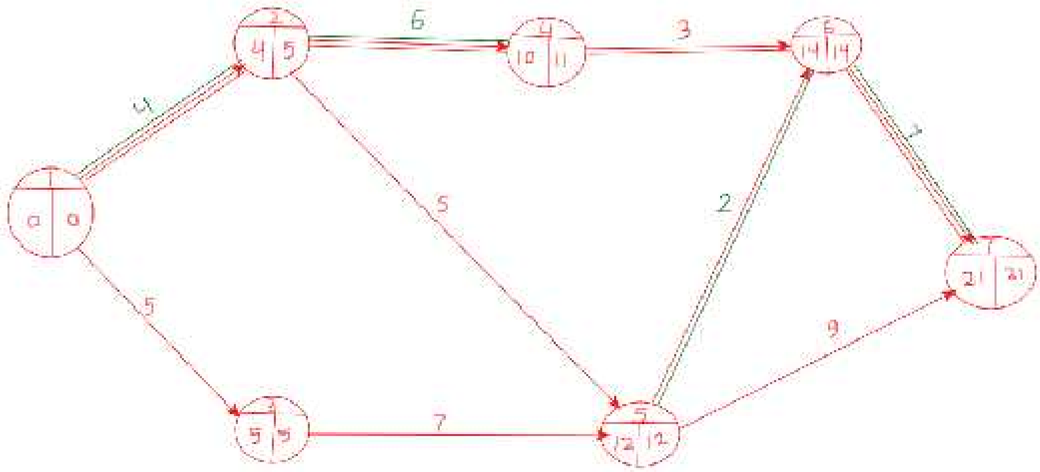
Figure – 4



The new critical path is (1-3-5-6-7), with length of 24 weeks, and the total direct cost is Rs.52500

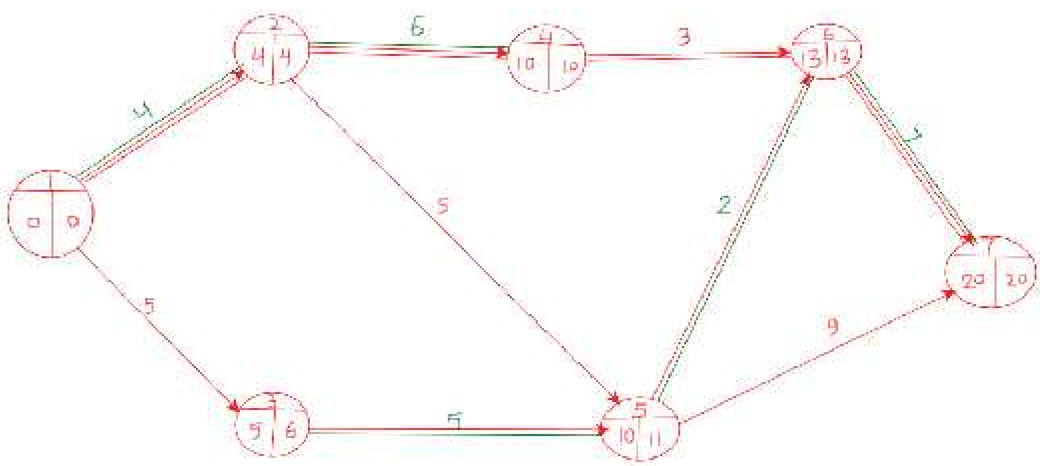
Examining the time cost slope of activities on the new critical path (1-3-5-6-7) we find that activity (6-7) has the lowest cost slope. Hence, activity (6-7) is crashed. Project network after such crashing is shown below:

Figure – 5



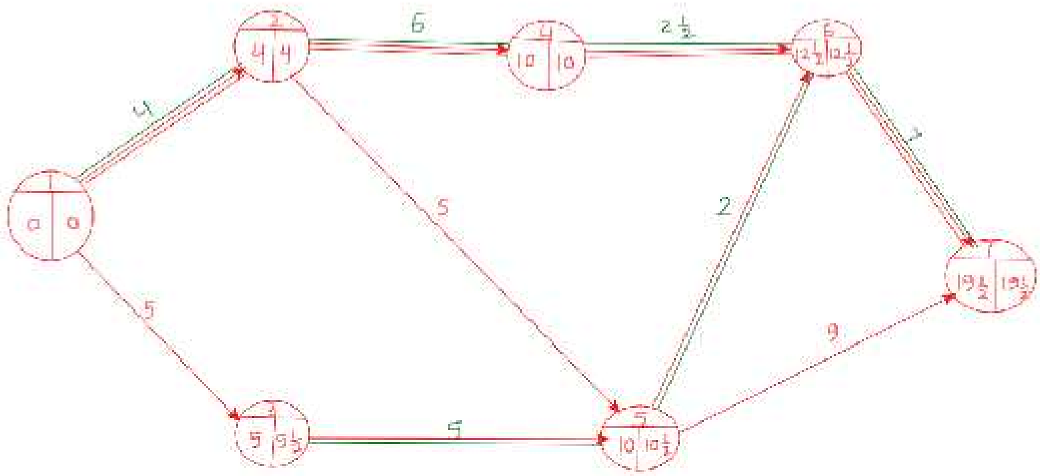
The new critical path is (1-3-5-6-7), and (1-3-5-7), both with length of 21 weeks, and the total direct cost is Rs.55200. Examining the time cost slope of activities on the new critical path (1-3-5-6-7), and (1-3-5-7) we find that activity (3-5) has the lowest cost slope. Hence, activity (3-5) is crashed. Project network after such crashing is shown below:

Figure – 6



The new critical path is (1-2-4-6-7), with length of 20 weeks, and the total direct cost is Rs.56400. Examining the time cost slope of activities on the new critical path (1-2-4-6-7) we find that activity (4-6) has the lowest cost slope. Hence, activity (4-6) is crashed. Project network after such crashing is shown below:

Figure – 7



The new critical path is (1-2-4-6-7), with length of 19 ½ weeks, and the total direct cost is Rs.57600. Since all the activities on the critical path (1-2-4-6-7) are crashed, there is no possibility of further time reduction.

Project duration and Total Cost

Activities crashed	Project duration in weeks	Total direct cost	Total Indirect Cost	Total Cost
None	30	45200	60000	105200
(2-4)	29	46700	58000	104700
(2-4) and (5-6)	27	49500	54000	103500
(1-2), (2-4) and (5-6)	24	52500	48000	100500
(1-2), (2-4), (5-6) and (6-7)	21	55200	42000	97200
(1-2), (2-4), (3-5), (5-6) and (6-7)	20	56400	40000	96400
(1-2), (2-4), (3-5), (5-6), (4-6) and (6-7)	19.5	57600	39000	96600

If the objective is to minimize the total cost of the project, the pattern of crashing suggested by figure – 6 is optimal. If the objective is to minimize the total duration of the project, the pattern of crashing suggested by figure – 7 is optimal. In real life situations, however, both the factors may be important. In addition, factors like strain on resources and degree of manageability are also important. The final decision would involve a careful weighing and balancing of these diverse factors, some quantitative, some qualitative.

Q2.

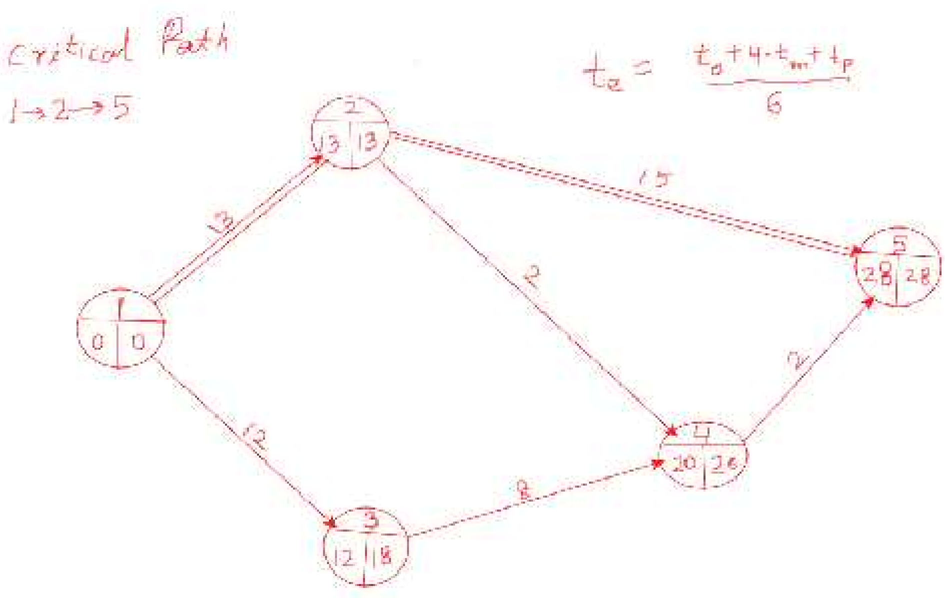
Activities	Optimistic time (in days)	Most likely time (in days)	Pessimistic time (in days)
1 – 2	9	12	21
1 – 3	6	12	18
2 – 4	1	1.5	5
3 – 4	4	8.5	10
2 – 5	10	14	24
4 – 5	1	2	3

a) Draw the network diagram
b) Determine the critical path
c) Calculate event slacks and activity floats

Solution: first of find the estimated time (t_e) by using the formula given below:

$$t_e = \frac{(t_o + 4t_m + t_p)}{6}$$

Activities	Optimistic time	Most likely time	Pessimistic time	Estimated time
1 – 2	9	12	21	13
1 – 3	6	12	18	12
2 – 4	1	1.5	5	2
3 – 4	4	8.5	10	8
2 – 5	10	14	24	15
4 – 5	1	2	3	2



Steps:

(1) Calculate the earliest occurrence time (EOT) for each event.
(2) Calculate the latest occurrence time (LOT) for each event.
(3) Obtain the critical and slack path
(4) Compute the activity floats

Event slack in days

Events	$LOT_i = Min(LOT_j - d_{ij})$	$EOT_i = Max(EOT_j - d_{ij})$	Slack = LOT - EOT
5	28	28	0
4	26	20	6
3	18	12	6
2	13	13	0
1	0	0	0

Activity Float

Activity	Duration	$EST_{ij}=EST_i$	$EFT_{ij}=EFT_j$	$LST_{ij}=LST_i$	$LFT_{ij}=LFT_j$	Total float	Free float	Independent float
1 – 2	13	0	13	0	13	0	0	0
1 – 3	12	0	12	6	18	6	0	0
2 – 4	2	13	15	24	26	11	5	5
3 – 4	8	12	20	18	26	6	0	-6
2 – 5	15	13	28	13	28	0	0	0
4 – 5	2	20	22	26	28	6	6	0

Total float: $TF_{ij} = LOT_j - EOT_i - d_{ij} = LS - ES$ **OR** $LF - EF$
Free float: $FF_{ij} = EOT_j - EOT_i - d_{ij} = TF -$ **(Head event slack)**
Independent float: $IF_{ij} = EOT_j - LOT_i - d_{ij} = FF -$ **Tail event slack**

Q3. A small Project is composed of seven activities, whose time estimates (in weeks) are listed in table as follows:

Activity	T_o	T_m	T_p
1 to 2	**1**	**1**	**7**
1 to 3	**1**	**4**	**7**
2 to 4	**2**	**2**	**8**
2 to 5	**1**	**1**	**1**
3 to 5	**2**	**5**	**14**
4 to 6	**2**	**5**	**8**
5 to 6	**3**	**6**	**15**

you are required to:

1. Draw the project network

2. Find the expected duration and variance of each activity

3. Calculate the earliest and latest occurrence of each event and the expected project length

4. Calculate the variance and standard deviation of project length.

5. what is the probability that the project will be completed,

(a) 4 weeks earlier than expected?

(b) Not more than 4 weeks later than expected?

(c) If the project's due date is 19 weeks, what is the probability of meeting the due date?

Solution:

Activity	to	tm	tp	te	σ^2
1 to 2	1	1	7	2	1
1 to 3	1	4	7	4	1
2 to 4	2	2	8	3	1
2 to 5	1	1	1	1	0
3 to 5	2	5	14	6	4
4 to 6	2	5	8	5	1
5 to 6	3	6	15	7	4

expected duration = 17
critical path = 1 -3 - 5 - 6
project length variance = 9
S.D. = 3

5(a) **the probability that the project will be completed in 13 weeks = P(Z=<13))**
Z = (Ts - Te)/σ = -1.33333
area under the normal curve for the region z=< -1.33 9.18%

5(b) **the probability that the project will be completed in 21 weeks = P(Z=<21))**
Z = (Ts - Te)/σ = 1.333333
area under the normal curve for the region z=< 1.33 90.82%

the probability that the project will be completed in 19 weeks = P(Z=<19))
5(c) **Z = (Ts - Te)/σ = 0.666667**
area under the normal curve for the region z=< 0.67 74.86%

Exercise:

Q1. A building project consists of the following activities:
A = Lay foundation
B = Erect framework
C = Install millwork
D = Install wiring
E = Install plumbing
F = plaster walls
G = install siding
H = Decorate the interior
I = Finish the exterior
The interrelationship among these activities is as follows:

1. A should precede B
2. B should precede C, D, E, F, and G

3. C, D, E, and F should precede H
4. G should precede I

Draw the network diagram for the above activities for a building project

Q2.

Activities	Optimistic time	Most likely time	Pessimistic time
1 – 2	4	6	10
1 – 3	3	7	12
1 – 4	5	6	9
1 – 7	2	4	6
2 – 4	6	10	20
2 – 6	3	4	7
2 -7	5	9	15
3 – 4	3	7	12
4 – 5	2	4	5
5 – 6	1	3	6
3 – 7	2	5	8
6 – 7	1	2	6

(a) Draw the network diagram
(b) Determine the critical path
(c) Calculate event slacks and activity floats
(d) Find the standard deviation of the critical path duration
(e) Compute the probability of completing the project in 30 weeks

Q3. A small Project is composed of seven activities, whose time estimates (in weeks) are listed in table as follows:

job	to	tm	tp
1 to 2	1	7	13
1 to 6	2	5	14
2 to 3	2	14	26
2 to 4	2	5	8
3 to 5	7	10	19
4 to 5	5	5	17
6 to 7	5	8	29
5 to 8	3	3	9
7 to 8	8	17	32

you are required to:

1. Draw the project network
2. Find the expected duration and variance of each activity
3. Calculate the earliest and latest occurrence of each event and the expected project length
4. Calculate the variance and standard deviation of project length.
5. what is the probability that the project will be completed in 40weeks

Q4. The normal and crash times and direct costs for the activities of a project are shown below:

Activities	Time		Cost	
	Normal	Crash	Normal	Crash
1 – 2	5	2	6000	9000
2– 4	6	3	7000	10000
1 – 3	4	2	1000	2000
3 – 4	7	4	4000	8000
4 – 7	9	5	6000	9200
3 – 5	12	3	16000	19600
4 – 6	10	6	15000	18000
6 – 7	7	4	4000	4900
7 – 9	6	4	3000	4200
5 – 9	12	7	4000	8500

(a) Draw the network diagram
(b) Determine the critical path
(c) Find the minimum cost project schedule if the indirect costs are Rs1000 per week.

Q5. The normal and crash times and direct costs for the activities of a project are shown below:

Activities	Time		Cost	
	Normal	Crash	Normal	Crash
1 – 2	8	6	100	200
1 – 3	4	2	150	350
2– 4	2	1	50	90
2 – 5	10	5	100	400
3 – 4	5	1	100	200
4 – 5	1	1	80	100

(a) Draw the network diagram
(b) Determine the critical path
(c) Find the minimum cost project schedule if the indirect costs are Rs70 per day.