TRANSPORTATION PROBLEM

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OBJECTIVES OF TRANSPORTATION PROBLEM

- Transportation problem works in a way of **minimizing** the **cost function.**
- The **cost function** is the amount of money spent to the logistics provider for **transporting** the **commodities** from production or **supplier place to the demand place.**
- It includes the **distance between the two locations**, the path followed, **mode of transport**, the number of **units that are transported**, the **speed of transport**, etc.
- To transport the commodities with **minimum transportation cost** without any compromise in supply and demand.

INTRODUCTION TO TRANSPORTATION PROBLEM

- The transportation problem is a special type of linear programming problem, where the objective is to minimize the cost of distributing a product from a number of sources to a number of destinations.
- Transportation deals with the transportation of a commodity (single product) from '*m*' sources (origin or supply or capacity centres) to '*n*' destinations (sinks or demand or requirement centres).
- It is assumed that, level of supply of each source and the amount of demand at each destination are known . The unit transportation cost of commodity from each source to each destination are known.
- The objective is to determine the amount to be shifted from each source to each destination such that the total transportation cost is minimum.

MATHEMATICAL FORMULATION OF A TRANSPORTATION PROBLEM

Let us assume that there are m sources and n destinations. Let a_i be the supply (capacity) at source i, b_j be the demand at destination j, c_{ij} be the unit transportation cost from source i to destination j and x_{ij} be the number of units shifted from source i to destination j.

Then the transportation problem can be expressed mathematically as,

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$ subject to the constraints,

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, \cdots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \cdots n$$

$$x_{ij} \geq 0, \forall i, j$$

REMARK

- The two sets of constraints will be consistent, if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, (ie.,***Total Supply = Total Demand***) which is the necessary and sufficient for a transportation problem to have a feasible solution.
- Problem satisfying this condition are called as ***balanced transportation problem***. If not, then the transportation problem is called an ***unbalanced*** one.
- If the given transportation problem is an unbalanced problem one has to balance the transportation problem first.
- When the total supply is greater than the total demand, a dummy is destination is included in the matrix with zero cost elements and the excess supply is entered as a constraint for the dummy destination.
- When the total demand is greater than the total supply, a dummy source is included in the matrix with zero cost elements and the excess demand is entered as a constraint for the dummy source.

STANDARD TRANSPORTATION TABLE

Transportation problem is explicitly represented by the following transportation table,

Destination

		D_1	D_2	D_3	$\dots$	D_j	$\dots$	D_n	Supply
Source	S_1	c_{11}	c_{12}	c_{13}	$\cdots$	c_{1j}	$\cdots$	c_{1n}	a_1
	S_2	c_{21}	c_{22}	c_{23}	$\cdots$	c_{2j}	$\cdots$	c_{2n}	a_2
	S_3	c_{31}	c_{32}	c_{33}	$\cdots$	c_{3j}	$\cdots$	c_{3n}	a_3
		$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
	S_i	c_{i1}	c_{i2}	c_{i3}	$\cdots$	c_{ij}	$\cdots$	c_{in}	a_i
		$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
	S_m	c_{m1}	c_{m2}	c_{m3}	$\cdots$	c_{mj}	$\cdots$	c_{mn}	a_m
Demand		b_1	b_2	b_3		b_j		b_n	$\sum a_i = \sum b_j$

The mn squares are called ***cells***. The various a's and b's are called the ***constraints*** (rim conditions)

BASIC DEFINITIONS

- **Feasible Solution:** A set of non-negative values x_{ij}, $i = 1, 2 \ldots m$; $j = 1, 2 \ldots n$ that satisfies the constraints (rim conditions) is called a ***feasible solution***.
- **Basic Feasible Solution:** A feasible solution to a (m x n) transportation problem that contains no more that $m + n - 1$ non-negative independent allocations is called a ***basic feasible solution*(BFS)** to the transportation problem.
- **Non-Degenerate Basic Feasible Solution:** A basic feasible solution to a (m x n) transportation problem is said to be a ***non-degenerate basic feasible solution*** if it contains exactly $m + n - 1$ non-negative allocations in independent positions.
- **Degenerate Basic Feasible Solution:** A basic feasible solution that contains less than $m + n - 1$ non-negative allocations is said to be a ***degenerate basic feasible solution***.
- **Optimal Solution:** A feasible solution (not necessarily basic) is said to be an ***optimal solution*** if it minimises the total transportation cost.
- The number of basic variables in an m x n balanced transportation problem is at the most $m + n - 1$.

TYPES OF TRANSPORTATION PROBLEM

- **Balanced Transportation Problem:** where the total supply equals to the total demand

- **Unbalanced Transportation Problem:** where the total supply is not equal to the total demand

PHASES OF SOLUTION OF TRANSPORTATION PROBLEM

- **Phase I**- obtains the initial basic feasible solution
- **Phase II**-obtains the optimal basic solution

Optimality condition: m+n-1 no of allocation

Where m – number of rows and n –
number of columns

INITIAL BASIC FEASIBLE SOLUTION

The following methods are to find the initial basic feasible solution for the transportation problems.

- North West Corner Rule (NWCR)
- Least Cost Method
- Vogle's Approximation Method (VAM)

METHOD 1: NORTH-WEST CORNER RULE

Step 1: The first assignment is made in the cell occupying the upper left-hand (north-west) corner of the transportation table. The maximum possible amount is allocated there. That is, $x_{11} = \min \{a_1, b_1\}$.

i. If $\min \{a_1, b_1\} = a_1$, then put $x_{11}= a_1$, decrease b_1 by a_1 and move vertically to the 2nd row (to the cell (2, 1)) and cross out the first column.

ii. If $\min \{a_1, b_1\} = b_1$, then put $x_{11}= b_1$, decrease a_1 by b_1 and move horizontally right (to the cell (1, 2)) and cross out the first column.

iii. If $\min \{a_1, b_1\} = a_1 = b_1$, then put $x_{11}= a_1 = b_1$ and move diagonally to the cell (2, 2) cross out the first row and the first column.

Step 2: Repeat the procedure until all the rim requirements are satisfied.

METHOD 2: LEAST COST METHOD (OR) MATRIX MINIMA METHOD (OR) LOWEST COST ENTRY METHOD

Step 1: Identify the cell with smallest cost and allocate $x_{ij} = \min\{a_i, b_j\}$.

i. If $\min\{a_i, b_j\} = a_i$, then put $x_{11} = a_i$, cross out the i^{th} row and decrease b_j by a_i and go to step (2).

ii. If $\min\{a_i, b_j\} = b_j$, then put $x_{11} = b_j$, cross out the j^{th} column and decrease a_i by b_j, and go to step (2).

iii. If $\min\{a_i, b_j\} = a_i = b_j$, then put $x_{11} = a_i = b_j$, cross out either ith row or jth column but not both and go to step (2).

Step 2: Repeat step (1) for the resulting reduced transportation table until all the rim requirements are satisfied.

METHOD 3: VOGEL'S APPROXIMATION METHOD (OR) UNIT COST PENALTY METHOD

Step1: Find the difference (penalty) between the smallest and next smallest costs in each row (column) and write them in brackets against the corresponding row (column).

Step 2: Identify the row (or) column with largest penalty. If a tie occurs break the tie arbitrarily. Choose the cell with smallest cost in that selected row or column and allocate as much as possible to this cell and cross out the satisfied row or column and go to step (3).

Step 3: Again compute the column and row penalties for the reduced transportation table and then go to step (2). Repeat the procedure until all the rim requirements are satisfied.

PROBLEM 1: Obtain initial feasible solution for the following Transportation table using (i) North West Corner rule. (ii) Least Cost Method. (iii) VAM Method.

Source	Destination			Supply
	A	B	C	
1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	

Source	Destination			Supply
	A	B	C	
1	**2**	**7**	**4**	5
2	**3**	**3**	**1**	8
3	**5**	**4**	**7**	7
4	**1**	**6**	**2**	14
Demand	7	9	18	34

The given problem is balanced $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 34$,

ie.,***Total Supply = Total Demand***

To find the IBFS using (i) North West Corner rule:

The number of allocations = 6 = 4+3-1 = m+n-1, is a non-degenerate solution.

The IBFS is = (2*5)+(3*2)+(3*6)+(4*3)+(7*4)+(2*14) = 153.

Source	Destination			Supply
	A	B	C	
1	2 $\boxed{5}$	7	4	~~5~~ 0
2	3 $\boxed{2}$	3 $\boxed{6}$	1	~~8~~ ~~6~~ 0
3	5	4 $\boxed{3}$	7 $\boxed{4}$	~~7~~ ~~4~~ 0
4	1	6	2 $\boxed{14}$	~~14~~ 0
Demand	~~7~~ ~~2~~ 0	~~9~~ ~~3~~ 0	~~18~~ ~~14~~ 0	34

To find the IBFS using (ii) Least Cost Method

The number of allocations = 6 = 4+3-1 = m+n-1, is a non-degenerate solution.

The IBFS is = (7*2)+(4*3)+(1*8)+(4*7)+(1*7)+(2*7) = 83.

Source	Destination			Supply
	A	B	C	
1	2	7 $\boxed{2}$	4 $\boxed{3}$	~~5~~ ~~2~~ 0
2	3	3	1 $\boxed{8}$	~~8~~ 0
3	5	4 $\boxed{7}$	7	~~7~~ 0
4	1 $\boxed{7}$	6	2 $\boxed{7}$	~~14~~ ~~7~~ 0
Demand	~~7~~ 0	~~9~~ ~~2~~ 0	~~18~~ ~~10~~ ~~3~~ 0	34

To find the IBFS using (iii) VAM

The number of allocations = 6 = 4+3-1 = m+n-1, is a non-degenerate solution.

The IBFS is = (2*5)+(3*2)+(1*6)+(4*7)+(1*2)+(2*12) = 76.

Source	Destination A	Destination B	Destination C	Supply	Row Panalty
1	2 (5)	7	4	~~5~~ 0	2
2	3	3 (2)	1 (6)	~~8~~ ~~2~~ 0	2 2 2 2 3 3
3	5	4 (7)	7	~~7~~ 0	1 1 3 3 4
4	1 (2)	6	2 (12)	~~14~~ ~~12~~ 0	1 1 4
Demand	~~7~~ ~~2~~ 0	~~9~~ ~~2~~ 0	~~18~~ ~~6~~ 0	34	

Column Panalty

A	B	C
1	1	1
2	1	1
	1	1
	1	6
	1	
	3	

the difference (penalty) between the smallest and next smallest costs in each row (column)

OPTIMAL BASIC SOLUTION - MODI METHOD - MODIFIED DISTRIBUTION METHOD

Step 1: Find the initial basic feasible solution of the given problem by Northwest Corner Rule (or) Least Cost Method (or) VAM.

Step 2: Check the number of occupied cells. If there are less than $m + n - 1$, there exists degeneracy and we introduce a very small positive assignment of in suitable independent positions, so that the number of occupied cells is exactly equal to $m + n - 1$.

Step 3: Find the set values u_i, v_j ($i = 1, 2, \ldots m$; $j = 1, 2, \ldots n$) from the relation $c_{ij} = u_i + v_j$ for each occupied cell (i, j), by starting initially with $u_i = 0$ or $v_j = 0$ preferably for which the corresponding row or column has maximum number of individual allocations.

Step 4: Find $u_i + v_j$ for each unoccupied cell (i, j) and enter at the upper right corner of the corresponding cell (i, j).

Step 5: Find the cell evaluations $d_{ij} = c_{ij} - (u_i + v_j)$ for each unoccupied cell (i, j) and enter at the upper left corner of the corresponding cell (i, j).

Step 6: Examine the cell evaluations d_{ij} for all unoccupied cells (i, j) and conclude that,

•If all $d_{ij} > 0$, then the solution under the test is optimal and unique.

•If all $d_{ij} > 0$, with at least one $d_{ij} = 0$, then the solution under the test is optimal and an alternative optimal solution exists.

•If at least one $d_{ij} < 0$, then the solution is not optimal. Go to the next step.

Step 7: Form a new BFS by giving maximum allocation to the cell for which d_{ij} is most negative by making an occupied cell empty. For that draw a closed path consisting of horizontal and vertical lines beginning and ending at the cell for which d_{ij} is most negative and having its other corners at some allocated cells. Along this closed loop indicate $+\theta$ and $-\theta$ alternatively at the corners. Choose minimum of the allocations from the cells having $-\theta$. Add this minimum allocation to the cells with $+\theta$ and subtract this minimum allocation from the allocation to the cells with $-\theta$.

Step 8: Repeat steps (2) to (6) to test the optimality of this new basic feasible solution.

Step 9: Continue the above procedure till an optimum solution is attained.

PROBLEM 1: Solve the Transportation table.

Source	Destination			Supply
	A	B	C	
1	2	2	3	10
2	4	1	2	15
3	1	3	1	40
Demand	20	15	30	

Source	Destination			Supply
	A	B	C	
1	2	2	3	10
2	4	1	2	15
3	1	3	1	40
Demand	20	15	30	65

The given problem is balanced $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 65,$

ie.,***Total Supply = Total Demand***

To find the IBFS using VAM

The number of allocations = 5 = 3+3-1 = m+n-1, is a non-degenerate solution.

The IBFS is = (2*10)+(1*5)+(2*10)+(1*20)+(1*20) = 85.

Source	Destination A	Destination B	Destination C	Supply
1	2	2 10	3	~~10~~ 0
2	4	1 5	2 10	~~15~~ ~~5~~ 0
3	1 20	3	1 20	~~40~~ ~~20~~ 0
Demand	~~20~~ 0	~~15~~ ~~5~~ 0	~~30~~ ~~10~~ 0	65

Row Penalty

Row 1	1	1	1		
Row 2	1	1	1	1	1
Row 3	2	2			

Column Penalty

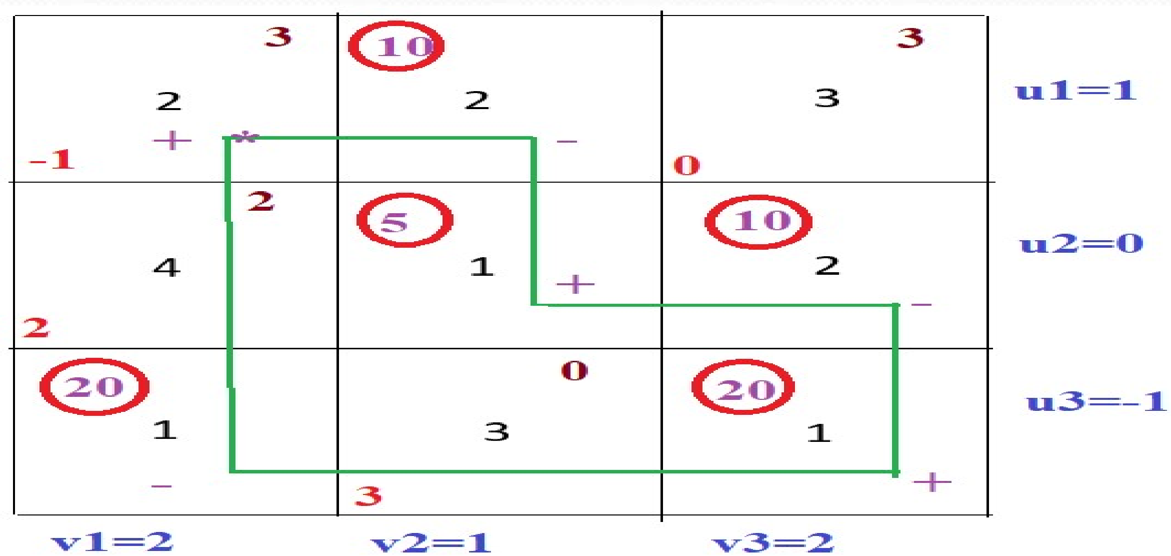
A	B	C
1	1	1
	1	1
	1	1
	1	2
	1	

To find the optimal solution using MODI Method:

- Find the set values u_i, v_j ($i = 1, 2, \dots m$; $j = 1, 2, \dots n$) from the relation $c_{ij} = u_i + v_j$ for each occupied cell (i, j), by starting initially with $u_i = 0$ or $v_j = 0$ preferably for which the corresponding row or column has maximum number of individual allocations.
- Find $u_i + v_j$ for each unoccupied cell (i, j) and enter at the upper right corner of the corresponding cell (i, j).
- Find the cell evaluations $d_{ij} = c_{ij} - (u_i + v_j)$ for each unoccupied cell (i, j) and enter at the upper left corner of the corresponding cell (i, j).

3 2 -1	10 2	3 3 0	u1=1
2 4 2	5 1	10 2	u2=0
20 1	0 3 3	20 1	u3=-1
v1=2	v2=1	v3=2	

Form a new BFS by giving maximum allocation to the cell for which d_{ij} is most negative by making an occupied cell empty. For that draw a closed path consisting of horizontal and vertical lines beginning and ending at the cell for which d_{ij} is most negative and having its other corners at some allocated cells. Along this closed loop indicate $+\theta$ and $-\theta$ alternatively at the corners. Choose minimum of the allocations from the cells having $-\theta$. Add this minimum allocation to the cells with $+\theta$ and subtract this minimum allocation from the allocation to the cells with $-\theta$.



10 2	1 2 1	2 3 1	u1= 0
2 4 2	15 1	∈ 2	u2=0
10 1	2 3 1	30 1	u3=-1
v1=2	v2=1	v3=2	

Since all dij > 0 the above allocation is optimal unique solution.

The Optimal solution = (2*10)+(1*15)+(2*0)+(1*30)+(1*10) = 75.

ASSIGNMENT QUESTIONS

1. Obtain initial feasible solution for the following Transportation table using (i) North West Corner rule. (ii) Least Cost Method. (iii) VAM Method.

Source	Destination			Supply
	A	B	C	
1	7	3	4	2
2	2	1	3	3
3	3	4	6	3
Demand	4	1	3	

2. Solve the Transportation table.

Source	Destination				Supply
	A	B	C	D	
1	5	4	2	6	20
2	8	3	5	7	30
3	5	9	4	6	50
Demand	10	40	20	30	

MCQ QUESTIONS WITH ANSWER

(1) To find initial feasible solution of a transportation problem the method which starts allocation from the lowest cost is called__________method.

- (a) north west corner
- (b) **least cost**
- (c) south east corner
- (d) Vogel's approximation

(2) In a transportation problem, the method of penalties is called ____ method.

- (a) least cost
- (b) south east corner
- (c) **Vogel's approximation**
- (d) north west corner

(3) When the total of allocations of a transportation problem match with supply and demand values, the solution is called____________________solution.

- (a) non-degenerate
- (b) degenerate
- (c) **feasible**
- (d) infeasible

(4) When the allocations of a transportation problem satisfy the rim condition (m + n – 1) the solution is called_________________solution.

- (a) degenerate
- (b) infeasible
- (c) unbounded
- (d) **non-degenerate**

(5) When there is a degeneracy in the transportation problem, we add an imaginary allocation called________________________in the solution.

- (a) dummy
- (b) penalty
- (c) **epsilon**
- (d) regret

(6) If M + N – 1 = Number of allocations in transportation, it means___. (Where 'M' is number of rows and 'N' is number of columns)

(a) **There is no degeneracy**
(b) Problem is unbalanced
(c) Problem is degenerate
(d) Solution is optimal

(7) Which of the following considers difference between two least costs for each row and column while finding initial basic feasible solution in transportation?

(a) North west corner rule
(b) Least cost method
(c) **Vogel's approximation method**
(d) Row minima method

(8) The solution to a transportation problem with m-sources and n-destinations is feasible if the numbers of allocations are ___________.

(a) m+n
(b) mn
(c) m-n
(d) **m+n-1**

(9) When the total demand is equal to supply then the transportation problem is said to be __________

(a) **balanced**
(b) unbalanced
(c) maximization
(d) minimization

(10) When the total demand is not equal to supply then it is said to be ___________.

(a) balanced
(b) **unbalanced**
(c) maximization
(d) minimization

(11) The allocation cells in the transportation table will be called ____________ cell

(a) **occupied**

(b) unoccupied

(c) no

(d) finite

(12) In the transportation table, empty cells will be called ___________.

(a) occupied

(b) **unoccupied**

(c) basic

(d) non-basic

(13) In a transportation table, an ordered set of___________or more cells is said to form a loop

(a) 2

(b) 3

(c) **4**

(d) 5

(14) To resolve degeneracy at the initial solution, a very small quantity is allocated in ____________ cell

(a) occupied

(b) basic

(c) non-basic

(d) **unoccupied**

(15) For finding an optimum solution in transportation problem ____________ method is used.

(a) **Modi**

(b) Hungarian

(c) Graphical

(d) simplex

THANK YOU