Chp-3

* Replacement of items that Deteriorate with time when many value is not counted & counted.

1)

Year	1	2	3	4	5
Resale value	84,000	60,000	40,800	28,000	19,300
maintainance cost	8,000	8,540	9,760	11,400	13,600
cost of labour spares	28,000	32,000	36,000	42,000	50,000

A m/c cost is RS 1,20,000 ?

Sol

Replacement of end of year (n)	maintainance cost (m)	cost of labour spares (l)	Running cost = (m+l)	Cummlative Running cost (C.R.C)	Capital cost (c)	Resale value (or) scrap value (s)	Depreciation cost = C−S	Total cost T.C = CRC+D.C	Avg annual cost T.C÷n
1	8000	28,000	36,000	36,000	1,20,000	84,000	36000	72000	72000
2	8540	32000	40,540	76,540	1,20,000	60,000	60,000	136540	68270
3	9760	36,000	45760	122300	1,20,000	40,800	79200	201500	67166.6
4	11400	42,000	53400	175700	1,20,000	28,000	92000	267700	66925
5	13600	50,000	63600	239300	1,20,000	19,300	100700	340000	68000

At the end of 4th year the machine was replaced.

\#

2) A taxi owner estimates from his past records that the cost per year for opperating a taxi whose purchase price is 60,000 is are given below:

Age in years	1	2	3	4	5
operating cost	10,000	12,000	15,000	18,000	20,000

After 5 years the operating cost 6000 k where k = 6, 7, 8, 9, 10 that is age in years. If the resale value decreases by 10% of the purchase price each year what is the optimum replacement policy when the cost of money is zero.

Sol operating cost = 6000 k

k = 6, 7, 8, 9, 10 years

operating cost 6th year = 6000 × 6 = 36,000

" " 7th " = 6000 × 7 = 42,000

" " 8th " = 6000 × 8 = 48,000

" " 9th " = 6000 × 9 = 54,000

" " 10th " = 6000 × 10 = 60,000

Replacement of the end of year (n)	Running cost (R.C)	Cummulative Running cost (C.R.C)	Capital cost (C)	Resalve value (RV) Scrap value (S)	Depreciation cost (D.C) = C − S	Total cost (T.C) = CRC + DC	Avg annual cost (T.C ÷ n)
1	10,000	10,000	60000	54,000	6000	16,000	16000
2	12,000	22,000	60000	48,000	12,000	34,000	17000
3	15,000	37,000	60000	42,000	18,000	55,000	18333.33
4	18,000	55,000	60000	36,000	24,000	79,000	19750
5	20,000	75,000	60000	30,000	30,000	105000	21000
6	36,000	111000	60000	24,000	36,000	147,000	245000
7	42,000	153000	60000	18,000	42000	1,95,000	27857.1
8	48,000	201000	60000	12,000	48000	249000	31125
9	54,000	255000	60000	6,000	54000	309000	34333.3
10	60,000	315000	60000	0	60000	375000	37500

At the end of the first year the m/c was replaced.

3) a) m/c A cost Rs 9000 Annual operating cost are Rs 200 for the first year & then increase by Rs 2000 every year. Determine the best age at which we replace the machine.

If the optimum replacement policy followed what will be the avg yearly cost of owning & operating the m/c.

b) m/c B cost Rs, 10000 Annual operating cost are Rs 400 for the first year & then increase by Rs 800 every year.

You know have a m/c of type A which is one year old. Should you replaced with B. If so when?

Sol a) For machine A :-

$C = 9000$ Rs : Rc_1 = 1st year = 200

Every year increase by 2000 Rs

$Rc_2 = 200 + 2000 = 2200 \rightarrow$ 2nd year.

$Rc_3 = 2200 + 2000 = 4200 \rightarrow$ 3rd year

$Rc_4 = 4200 + 2000 = 6200 \rightarrow$ 4th year

$Rc_5 = 6200 + 2000 = 8200 \rightarrow$ 5th year

We take only 5 years.

Replacement of End of year	Running cost	Comm Running cost	capital cost	Dec cost = C - S = C	Total cost = CRC + DC	Avg. Annual cost
1	200	200	9000	9000	9200	9200
2	2200	2400	9000	9000	11400	5700
3	4200	6600	9000	9000	15600	5200
4	6200	12800	9000	9000	21800	5450
5	8200	21,000	9000	9000	30,000	6000

Lowest m/c A for Avg. annual cost = 5200 Rs

b) For m/c B :-

$C = 10,000$ Rs ; $Rc_1 \rightarrow 1^{st}$ year = 400

Every year increased by 800 Rs

$Rc_2 \rightarrow 2^{nd}$ year = 1200

$Rc_3 \rightarrow 3^{rd}$ year = 2000

$Rc_4 \rightarrow 4^{th}$ year = 2800

$Rc_5 \rightarrow 5^{th}$ year = 3600

$Rc_6 \rightarrow 6^{th}$ year = 4400

we take 6 years.

n	R.C	C.R.C	C	D.C = C−S	T.C = CRC+ D.C	Avg An cost
1	400	400	10,000	10,000	10,400	10,4000
2	1200	1600	10,000	10,000	11,600	5,800
3	2000	3600	10,000	10,000	13,600	4533
4	2800	6400	10,000	10,000	16,400	4100
5	3600	10,000	10,000	10,000	20000	4000
6	4400	14,400	10,000	10 000	24,400	4066.7

Lowes for m/c B Avg. Annual cost = 4000 Rs.

when we can replace m/c 'A' by m/c 'B'

Lowest Avg. annual cost for A = 5200

Lowest Avg. annual cost for B = 4000

Lowest Avg. annual cost for m/c A > Lowest Avg. annual cost for m/c B

Note :- only this type of problems we satisfy this equation.

Replacement at which year :-

→ Compare the differential total cost value for m/c A with lowest Avg Annual cost for m/c B.

→ If differential total cost exceeds lowest Avg annual cost replace at ~~the~~ that end of the previous year.

machine - A

S.NO	Total cost for current year (a)	Total cost for previous year (b)	Differential T-C = a-b
1	9200	-	4000
2	11400	9200	2200 < 4000
3	15600	11400	4200 > 4000
4	21800	15600	6200 > 4000
5	30,000	21,800	8200 > 4000

Differential total cost values for m/c A exceeds lowest Avg. Annual cost for m/c B from 3rd year onwards. so replace the m/c A by m/c B at the end of 2nd year itself.

* **Present worth factor** :

1) The initial cost of an item is 15,000 Rs & maintainance or running cost for d/f years are given below.

year	1	2	3	4	5	6	7
running	2500	3000	4000	5000	6500	8000	10,000

what is the replacement policy to be adopted if the capital cost is work 10% & their is no scrap value (salvage & Resale value).

So/

year (n)	Running cost	Present worth factor	Present value of Running cost	comm P.V.R.C	Depression cost = C−S	Total cost = D.C + CPVRC	Comm present worth factor	Avg. Annual cost
1	2500	1	2500	2500	15000	17500	1	17500
2	3000	0.909	2727	5227	15000	20227	1.909	10595
3	4000	0.826	3305	8532	15000	23532	2.735	8603
4	5000	0.7513	3755	12286	15000	27287	3.486	7826
5	6500	0.6830	4439	16725	15000	31727	4.169	7609
6	8000	0.620	4960	21685	15000	36687	4.789	7660
7	10,000	0.564	5640	27325	15000	42327	5.353	7906

at the end of 5th year the m/c was replaced.

2) The cost of a new m/c is 5000 the running cost of the n^{th} year is given by $R_n = 500(n-1)$ where $n = 1, 2, 3, ----$ suppose money is worth 5% per year. After How many years will it be economical to replace the m/c with a new one.

Sol

At the end of 5th year the m/c w[illegible] replaced.

year (n)	Running cost	Present worth factor	Present value of Running cost	Cummlative value of Running cost	Depression cost $= C - S$	Total cost = CPRC+DC	Cummulative present worth factor	Weighted Avg annual cost
1	0	1	0	0	5000	5000	1	5000
2	500	0.952	475	475	5000	5475	1.952	2804.8
3	1000	0.906	902	1377	5000	6377	2.854	2234.4
4	1500	0.863	1285.5	2662.5	5000	7662.5	3.71	2064.8
5	2000	0.822	1629	4291.5	5000	9291.5	4.525	2053.1
6	2500	0.783	1932.5	6224	5000	11224	5.298	2118.5
7	3000	0.745	–	–	5000	–	–	–
8	3500	0.710	–	–	5000	–	–	–

$$P.W.F = \frac{1}{1+r} = \frac{1}{1+\frac{5}{100}} = \frac{1}{\frac{100+5}{100}} = \frac{100}{105} = 0.952$$

3) Let the value of money assume to the 10% per year & suppose m/c A is replaced after every 3 years where as m/c B is replaced every 6 years the yearly cost of both m/c's is as given below.

year	1	2	3	4	5	6
m/c A	1000	200	400	1000	200	400
m/c B	1700	100	200	300	400	500

Determine which m/c should be purchased.

Sol: Given data:-

interest rate = 10%

present worth factor, $= \frac{1}{1+\cancel{r}} = \frac{1}{1+10\%}$

$= \frac{1}{1+\frac{10}{100}} \Rightarrow \frac{1}{\frac{100+10}{100}} = \frac{100}{110} = 0.909$

for m/c A :-

Total cost after discount $= (1000\times1) + \left(200\times\frac{10}{11}\right) + \left[400\times\left(\frac{10}{11}\right)^2\right] + \left(1000\times\left\{\frac{10}{11}\right\}^3\right) + \left[200\times\left(\left(\frac{10}{11}\right)^4\right)\right] + \left[400\times\left(\frac{10}{11}\right)^5\right] = 2648.1$ RS

for m/c B :-

Total cost after discount $= (1700\times1) + \left[(100\times\left(\frac{10}{11}\right)\right]$ $\left[200\times\left(\frac{10}{11}\right)^2\right] + \left[300\times\left(\frac{10}{4}\right)^3\right] + \left[400\times\left(\frac{10}{11}\right)^4\right] + \left[500\times\left(\frac{10}{11}\right)^5\right] = 2764.7$ Rs.

∴ we take the m/c A.

* Replacement of items which falls suddenly & complete break down :-

a) Individual replacement b) Group replacement :-

1) The following failure rate have been observed for a certain type of light bulbs.

week	1	2	3	4	5
% failed by the end of week	10	25	50	80	100

They are 1000 bulbs is used & its cost RS 2 to replace an individual bulb which has burnt out. If all bulbs replaced simulataneously it would cost 50 ps/bulb. It is proposed to replace all the bulbs at fixed interval of time wheather they have burnt out or not. And to continue replacing burnt out bulbs as and when the fail. At what interval should all the bulbs be replaced. At what ~~interval should~~ ~~all the~~ group replacement price per bulb would a policy of strictly individual replacement become preparable to be adopted policy.

Sol Individual replacement :-

Let P_i be the probability of failure in the i^{th} week.

P_1 = Probability of failure in first week.

$$P_1 = \frac{10}{100} = 0.1$$

P_2 = probability of failure in second week

$$P_2 = \frac{25-10}{100} = \frac{15}{100} = 0.15.$$

P_3 = probability of failure in the third week

$$P_3 = \frac{50-25}{100} = 0.25$$

P_4 = probability of failure in the fourth week.

$$P_4 = \frac{80-50}{100} = 0.30$$

P_5 = probability of failure in the fifth week.

$$P_5 = \frac{100-80}{100} = 0.20$$

$$\sum_{i=1}^{5} P_i = P_1 + P_2 + P_3 + P_4 + P_5$$

$$P_1 = 0.1 + 0.15 + 0.25 + 0.30 + 0.20 = 1.$$

$$\sum_{i=1}^{5} P_i = 1$$

expected life of bulbs $= \sum_{i=1}^{5} i\, P_i$

$= 1 \times P_1 + 2 \times P_2 + 3 \times P_3 + 4 \times P_4 + 5 \times P_5$

$= (1 \times 0.1) + (2 \times 0.15) + (3 \times 0.25) + (4 \times 0.30) + (5 \times 0.20)$

$= 3.35$ weeks.

Avg. no. of bulbs per week $= \frac{\text{Total no. of bulbs}}{\text{Expected life bulbs}}$

$= \frac{1000}{3.35} = 298.6 \simeq 299$ bulbs.

Total cost in individual replacement

$= 299 \times 2 = 598$ Rs.

<u>Group replacement</u> :-

let N_i be the no. of replacement in i^{th} week.

N_0 be the total no. of bulbs at the begining $= 1000$

N_1 = No. of replacement at the end of 1^{st} week

$= N_0 P_1 = 1000 \times 0.1 = 100$ bulbs.

2)

N_2 = No. of replacements at the end of 2^{nd} week.

$= N_0 P_2 + N_1 P_1 = (1000 \times 0.15) + (100 \times 0.1) = 160$ bulbs

N_3 = No. of replacements at the end of 3^{rd} week

$= N_0 P_3 + N_1 P_2 + N_2 P_1$

$= (1000 \times 0.25) + (100 \times 0.15) + (160 \times 0.1) = 281$ bulbs

N_4 = No. of replacements at the end of 4^{th} week

$= N_0 P_4 + N_1 P_3 + N_2 P_2 + N_3 P_1$

$= (1000 \times 0.3) + (100 \times 0.25) + (160 \times 0.15) + (281 \times 0.1)$

$= 377.1$ bulbs.

Sol

N_5 = No. of replacements at the end of 5^{th} week

$= N_0 P_5 + N_1 P_4 + N_2 P_3 + N_3 P_2 + N_4 P_1$

$= (1000 \times 0.20) + (100 \times 0.3) + (160 \times 0.25) + (281 \times 0.15)$

$+ (377.1 \times 0.1) = 350$ bulbs.

End of the week	Total cost of group replacement	Avg. cost per week
1	$(100 \times 2) + (1000 \times 0.5) = 700$	$700 \div 1 = 700$
2	$(100 \times 2) + (160 \times 2) + (1000 \times 0.5) = 1020$	$1020 \div 2 = 510$
3	$(100 \times 2) + (160 \times 2) + (281 \times 2) + (1000 \times 0.5) = 1582$	$1582 \div 3 = 527.3$
4	$(100 \times 2) + (160 \times 2) + (281 \times 2) + (377.1 \times 2) + (1000 \times 0.5) = 2338$	$2338 \div 4 = 584.3$
5	$(100 \times 2) + (160 \times 2) + (281 \times 2) + (377.1 \times 2) + (350 \times 2) + (1000 \times 0.5) = 3038$	$3038 \div 5 = 607.5$

At the end of 2^{nd} week we need to go for group replacement ##

UNIT - III
REPLACEMENT.

Introduction:-

The study of replacement is concerned with situations that arise when some items such as machines, men, electric-light bulbs, etc., need replacement due to their deteriorating efficiency, failure or breakdown. The deteriorating efficiency or complete breakdown may be either gradual or all of a sudden. For example, an electric light bulb fails all of a sudden, pipeline get blocked, parts of machines become faulty. These are some situations that need most economic replacement policy for replacing faulty units or to take some remedial to restore efficiency.

Suppose an item goes on performing and with decreasing efficiency, then it requires more money to be spent in order to increase the operating cost, repair cost and so on. In such a situation the replacement of an old item with new one is the only alternative. Thus, the problem of replacement is to decide the best policy to determine an age at which the replacement is most economical instead of continuous increase in cost.

The need for replacement arises in:

(i) We may decide either to wait for complete failure of the item or to replace earlier due to high expense.

(ii) The expensive items may be considered individually to decide whether we should replace now or when it should be next replaced.

(iii) Whether the replaced item is of same type or of different type of the item.

The main objective of replacement is to direct the organisation for maximising its profit.

Replacement problems can be classified as:

(i) When the equipment deteriorates with time due to constant use and needs increased operating and maintenance cost.

(ii) When equipments such as light bulbs, tubes, radio and television parts, and so on, do not give any indication of deterioration with time, but fail completely all of a sudden.

(iii) Existing working staff in an organisation reduces gradually due to retirement, death and so on.

Failure Mechanism of Items:-

(i) Gradual Failure:- It is progressive in nature, that is, the lifetime increases, but its efficiency deteriorates causing

(a) increased maintenance & operating costs.

(b) decreased productivity

(c) decrease in the value of the equipment, that is, resale or salvage value.

(ii) Sudden Failure:- This type of failure occurs after some period of service rather than deterioration distribution which may be progressive, retrogressive or random in nature.

(a) Progressive Failure:- If the probability of failure increases with the increase in its life, then the failure is said to be progressive.

Eg:- Electric light bulbs, automobile tubes.

(b) Retrogressive failure:- If the probability of failure in the beginning of the life of an item is more and due to change of time, the chances of failure decreases, then the failure is said to be retrogressive.

(c) Random Failure:- The constant probability of failure is associated with items that fail from random causes like physical shocks not related to age. In such a case, virtually all items fail before ageing has any effect.

Replacement of Items that deteriorate with time

Generally, the cost of maintenance and repair of certain items increases with time. When years go by, these costs become so high that it is more economical to replace the item by a new one.

Value of Money Does not change with time:-

The cost of maintenance of a machine is given as a function increasing with time and its scrap value is constant.

(a) If a time is continuous variable, then the average annual cost will be minimised by replacing the machine when the average cost to date becomes equal to the current maintenance cost.

(b) If time is a discrete variable, then the average annual cost will be minimised by replacing the machine when the next period's maintenance cost becomes greater than the current average cost.

Proof:- Let

C - Capital cost of the item

S - scrap value of the item.

n - number of years that the equipment would be in use.

f(t) - Maintenance cost function

A(n) - Average total annual cost.

(a) When t is a continuous variable: If the item is used for n years, then the total cost used during this period is:

Total cost = Capital cost − Scrap value + Maintenance cost

$$= C - S + \int_0^n f(t)\,dt.$$

Average annual total cost is

$$A(n) = \frac{\text{Total cost}}{n} = \frac{C-S}{n} + \frac{1}{n}\int_0^n f(t)\,dt.$$

Now, we find such time n for which A(n) is minimum. therefore, differentiating A(n) w.r.t 'n'

$$\frac{dA(n)}{dn} = \frac{1}{n} f(n) - \frac{1}{n^2}\int_0^n f(t)\,dt - \frac{C-S}{n^2}.$$

$= 0$ for minimum of $A(n)$.

$$\Rightarrow f(n) = \frac{C-S}{n} + \frac{1}{n}\int_0^n f(t)\,dt.$$

$$= A(n)$$

$$\therefore \frac{d^2}{dn^2}\left[A(n)\right] \geq 0 \text{ at } f(n) = A(n).$$

(b) When time t is a discrete variable: Since the ~~item~~ time t is taken as discrete, it can take the values 1, 2, 3, ...

Then,
$$A(n) = \frac{C-S}{n} + \frac{1}{n}\sum_{t=1}^{n} f(t)$$

By using finite differences $A(n)$ will be minimum if the relationship is satisfied:

$$A(n+1) - A(n) \geq 0 \text{ and } A(n) - A(n-1) \leq 0.$$

$$A(n+1) - A(n) = \left[\frac{C-S}{n+1} + \frac{1}{n+1}\sum_{t=1}^{n+1} f(t)\right] - \left[\frac{C-S}{n} + \frac{1}{n}\sum_{t=1}^{n} f(t)\right].$$

$$= \frac{1}{n+1}\left[C-S+\sum_{t=1}^{n} f(t) + f(n+1)\right] - \frac{1}{n}\left[C-S+\sum_{t=1}^{n} f(t)\right]$$

$$= \frac{f(n+1)}{n+1} + \sum_{t=1}^{n} f(t)\left[\frac{1}{n+1} - \frac{1}{n}\right] + (C-S)\left[\frac{1}{n+1} - \frac{1}{n}\right]$$

$$= \frac{f(n+1)}{n+1} - \frac{1}{n(n+1)}\left[\sum_{t=1}^{n} f(t) + (C-S)\right].$$

Since $A(n+1) - A(n) \geq 0$,

$$\frac{f(n+1)}{n+1} \geq \frac{1}{n(n+1)}\left[\sum_{t=1}^{n} f(t) + (C-S)\right]$$

$$\Rightarrow f(n+1) \geq A(n).$$

Similarly, $A(n) - A(n-1) \leq 0 \Rightarrow f(n) \leq A(n-1)$.

$\therefore$ Replace the machine at the end of n years when the maintenance cost in the $(n+1)^{th}$ year is more than the average total cost in the n^{th} year & the n^{th} year maintenance cost is less than the previous year's average total cost.

Problem:- The cost of a machine is 6,100/- and its scrap value is only 100/-. The maintenance costs are found from experience to be:

Year	1	2	3	4	5	6	7	8
Maintenance cost in Rs.	100	250	400	600	900	1250	1600	2000

When shoud machine be replaced?

Sol:-

Total cost in a year = Capital cost − scrap value + Maintenance cost.

Replacement at the end of year (n).	Maintenance cost $f(n)$.	Total Maintenance cost $\Sigma f(n)$.	Difference b/w price value & scrap value $C-S$.	Total cost $\Sigma f(n) + C-S$	Average cost $\frac{\Sigma f(n)}{n} + \frac{C-S}{n}$
1	100	100	6000	6100	6100
2	250	350	6000	6350	3175
3	400	750	6000	6750	2250
4	600	1350	6000	7350	1837
5	900	2250	6000	8250	1650
6	1250	3500	6000	9500	1583
7	1600	5100	6000	11,100	1586
8	2000	7100	6000	13,100	1638

Here, it may be observed that the average cost per year is minimum in the 6th year and the maintenance cost in the 7th year becomes greater than average cost for six years. So, machine should be replaced at the end of the 6th year.

Value of Money changes with constant rate during the period:- As money value changes with time, calculate the present value or present worth of the money to be spent in a few years. One rupee a year from ~~no~~ now is equivalent to $(1.1)^{-1}$ rupee at the interest rate of 10% per year. One rupee spent two years from now is equivalent to $(1.1)^{-2}$ today. Hence one rupee spent n years from now is equivalent to $(1.1)^{-n}$ today. The quantity $(1.1)^{-n}$ is called the present worth factor of one rupee spent in n years from now.

Generally, if r is the rate of interest per year, then $(1+r)^{-n}$ is called the present worth factor of one rupee spent in n years time from now. The expression $(1+r)^{n}$ is known as the payment compound amount factor of one rupee spent in n years time.

Discount rate or Depreciation value: The present worth factor of unit amount to be spent after one year is given by $v = (1+r)^{-1}$, where r is the interest rate. Then, v is called discount rate or depreciation value.

Problem:- Let the value of money be assumed to be 10% per year and suppose that machine A is replaced after every 3 years whereas machine B is replaced after every 6 years. The yearly costs of both the machines are given below:

Year	1	2	3	4	5	6
Machine A	1000	200	400	1000	200	400
Machine B	1700	100	200	300	400	500.

Determine which machine should be purchased.

Sol:- Since money carries the rate of interest, the present worth factor is:

$$v = (1+r)^{-1} = \left(1 + \frac{10}{100}\right)^{-1} = \left(\frac{11}{10}\right)^{-1} = \frac{10}{11}.$$

Total discount cost of A for 3 years is:

$$= Rs\left[1000 + 200 \times \frac{10}{11} + 400 \times \left(\frac{10}{11}\right)^2\right]$$

$= Rs.\ 1,512.$

The total discount cost of B for 6 years is:

$$= Rs\left[1700 + 100 \times \left(\frac{10}{11}\right) + 200 \times \left(\frac{10}{11}\right)^2 + 300\left(\frac{10}{11}\right)^3 + 400 \times \left(\frac{10}{11}\right)^4 + 500 \times \left(\frac{10}{11}\right)^5\right].$$

$= Rs.\ 2,765.$

Average yearly cost of A $= \frac{1513}{3} = Rs.\ 504$

Average yearly cost of B $= \frac{2765}{6} = Rs.\ 461.$

Average yearly

This shows that apparent advantage with B, the comparison is unfair because the periods of consideration are different. So, if we consider 6-year period for machine A, then the total discounted cost of A is

$$= 1000 + 200 \times \left(\frac{10}{11}\right) + 400 \times \left(\frac{10}{11}\right)^2 + 1000 \times \left(\frac{10}{11}\right)^3 + 200 \times \left(\frac{10}{11}\right)^4 + 400 \times \left(\frac{10}{11}\right)^5$$

$$= \text{Rs. } 2,647.$$

Hence, the average yearly cost of A is $\frac{2647}{6} = 441$ which is lesser than the average yearly cost of B. Hence, machine A should be purchased.

Replacement of Items that fail Completely and Suddenly :- It is not easy to predict when a particular equipment will fail and its time of failure. This difficulty can be overcome by determining the probability distribution of failures. Assume that the failures occur only at the end of the period, t. The objective is to find the value of t for which the total cost after replacement of an equipment is minimum. Replacement policies are:

1) Individual Replacement policy

2) Group Replacement Policy.

Individual Replacement Policy:- Under this policy, an item is replaced immediately on its failure.

Mortality Theorem:- A large population is subject to a given mortality law for a very long period of time. All deaths are immediately replaced by births and there are no other entries or exits. Then, the age distribution ultimately becomes stable and that the number of deaths per unit time becomes constant.

Proof:- Assume that death occurs just before the age of $(k+1)$ years, where k is an integer. That is, the lifespan of any item is between $t=0$ to $t=k$. Define,

$f(t)$ = number of births at time t.

$p(x)$ = probability of death just before the age $x+1$, that is, failure at the age x.

$$\Rightarrow \sum_{x=0}^{k} p(x) = 1.$$

Group Replacement Policy:- It is concerned with those items that either work or fail completely. It happens that a system contains a large number of items that are increasingly liable to failure with age. In this case it is advisable to replace all items irrespective of the fact that the items have failed or not failed, with a provision that if any item fails before optimal time, it may be individually replaced. Such a policy is called group replacement policy and is best when the value of any individual item is so small that the cost of keeping records of individual ages cannot be justied.

(a) Group replacement should be made at the end of the period if the cost of individual replacements for the period t is greater than the average cost per period through the end of period, t.

(b) Group replacement is not advisable at the end of period t if the cost of individual replacements at the end of period t-1 is less than the average cost per period through the end of ith period.

Problem:- Let p(t) be the probability that a machine in a group of 30 machines would breakdown in period t. The cost of repairing a breakdown machine is Rs. 200. Preventive maintenance is performed by the servicing team on all the 30 machines at the end T units of time. Preventive maintenance cost is Rs. 15 per machine. Find optimal T which will minimise the expected total cost per period of serviing, given

$$p(t) = \begin{cases} 0.03 & \text{for } t=1 \\ p(t-1)+0.01 & \text{for } t=2,3,\dots,10 \\ 0.13 & \text{for } t=11,12,\dots \end{cases}$$

<u>Sol:-</u>

t	1	2	3	4	5	6	7	8	9	10	11	12
p(t)	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0

as $p(1)+p(2)+\dots p(11) = 0.88 < 1$.

if we add $p(12) = 0.13$,

then $p(1)+\dots+p(12) = 1.01 > 1$, where the sum of all probabilities can never be greater than 1, so consider $p_{12}=0$, $p_{13}=0$, and so on.

This means that a machine which has already lasted up to the 11^{th} period is sure to fail in the 12^{th} period. Let N_i be the number of machines at the end of i^{th} period. Then,

$$N_0 = 30$$

$$N_1 = N_0 p_1 = 30 \times 0.03 = 0.9 \approx 1$$

$$N_2 = N_0 p_2 + N_1 p_1 = (30 \times 0.04) + (1 \times 0.03) \approx 1.23 \approx 1$$

$$N_3 = N_0 p_3 + N_1 p_2 + N_2 p_1 = 30 \times 0.05 + 1 \times 0.04 + 1 \times 0.03 \approx 2$$

similarly, $N_4 = 2$, $N_5 = 2$, $N_6 = 3$, $N_7 = 3$, $N_8 = 4$, $N_9 = 4$, $N_{10} = 5$, $N_{11} = 6$.

As the expected life of each machine;
$\sum_{i=1}^{11} i p_i = 6.41$ time units, the average number of machines failed per period is $\frac{30}{6.41} \simeq 5$.
Hence, cost of individual replacement =
Rs. 5×200 = Rs. 1,000.

Group maintenance cost is:

End of period	Cost of group maintenance (in Rs)	Average cost of group maintenance per period (in Rs)
1	$(30 \times 15) + (1 \times 200) = 650$	650
2	$(30 \times 15) + (2 \times 200) = 850$	425
3	$(30 \times 15) + (4 \times 200) = 1250$	417
4	$(30 \times 15) + (6 \times 200) = 1650$	412
5	$(30 \times 15) + (8 \times 200) = 2050$	$\boxed{410}$
6	$(30 \times 15) + (11 \times 200) = 2650$	442

Since the minimum cost occurs in the 5th period it is optimal to maintain all the machines upto the 5th period.