Top 4 Problems on PERT | Network Analysis | Networking

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List of top four problems on PERT.

Example 1:

A small project consisting of eight activities has the following characteristics:

Time – Estimates (in weeks)

Activity	Preceding activity	Most optimistic time (a)	Most likely time (m)	Most Pessimestic time (b)
A	None	2	4	12
B	None	10	12	26
C	A	8	9	10
D	A	10	15	20
E	A	7	7.5	11
F	B,C	9	9	9
G	D	3	3.5	7
H	E,F,G	5	5	5

(i) Draw the PERT network for the project.

(ii) Prepare the activity schedule for the project.

(iii) Determine the critical path.

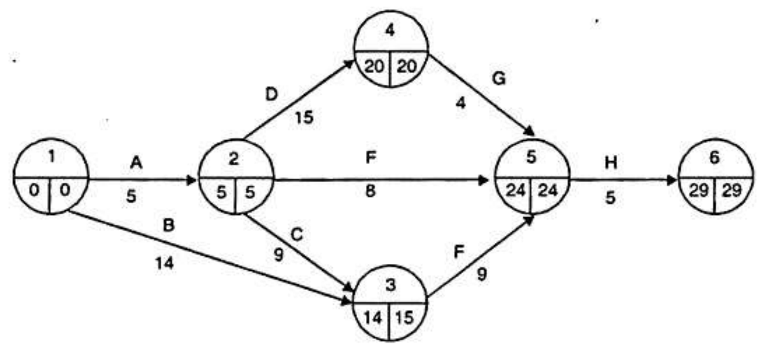
(iv) If a 30- week deadline is imposed, what is the probability that the project will be finished within the time limit?

If the project manager wants to 99% sure that the project is completed on the schedule date, how many weeks before that date should he start the project work?

Solution:

The network diagram for the given data is shown in fig. below. The earliest time and variance of each activity is computed by using the formula.

$$t_e = \frac{a+4m+b}{6} \quad \text{and} \quad \sigma^2_t = \left(\frac{b-a}{6}\right)$$



PERT Network diagram

(ii) Calculation activity duration and scheduling times.

Activity	Time estimates			t_e	(σ_t^2)	Earliest time		latest time	
	a	m	b			Start	finish	Start	Finish
A	2	4	12	5	25/9	0	5	0	5
B	10	12	26	14	64/9	0	14	1	15
C	8	9	10	9	1/9	5	14	6	15
D	10	15	20	15	25/9	5	20	5	20
E	7	7.5	11	8	4/9	5	13	16	24
F	9	9	9	9	0	14	23	15	24
G	3	3.5	7	4	4/9	20	24	20	24
H	5	5	5	5	0	24	29	24	29

(iii) The critical path of the project is 1-2-4-5 -6, critical activities being A, D, G and H.

The expected project length is the sum of duration of each critical activity.
Expected project length = 5 + 15 + 4 + 5

Variance project length is obtained by summing variance of each critical activity.

$$\text{Variance of project} = \frac{25}{9}+\frac{25}{9}+\frac{4}{9}+0=6$$

(Iv) The required probability can be determined by finding the area under the normal curve to the left of X = 30

Now, the probability of completing the project within the 30 week deadline is

$$\begin{aligned} P(X \leq 30) &= 0.5 + P(\mu < x < 30) \\ &= 0.5 + P(0 \leq Z \leq 0.41) \\ &= 0.5 + 0.1591 \\ &= 0.6591 \end{aligned}$$

Where $$Z = \frac{\text{Due date} - \text{Expected date}}{\sigma t}$$

$$Z = \frac{30-29}{\sqrt{6}} = 0.41$$

(v) If the project start T weeks before the due date, the X will represent the ordinate under normal curve to the left of which 99% of area lies.

The area between n and X- being 99-50 or 49% and Z – value corresponding to this is 2033 (From table)

$$\therefore \quad 2.33 = \frac{T-29}{\sqrt{6}}$$

$$T = 29 + 2.33\sqrt{6}$$

$$= 34.7 \text{ weeks}$$

Example 2:

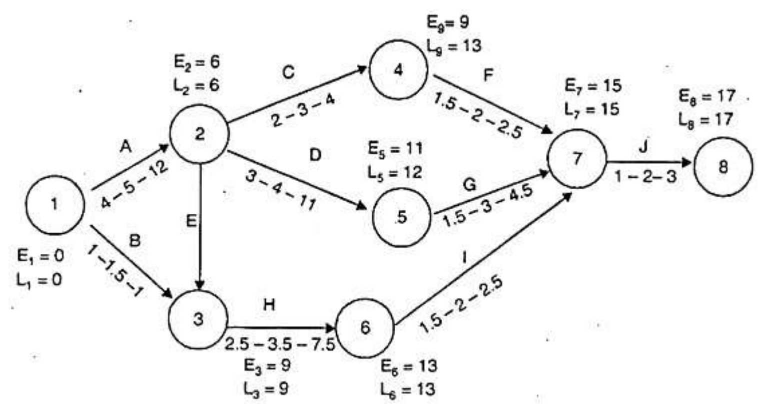
A small project consisting of ten activities has the following characteristics:

Activity	Preceding Activity	Time Estimate weeks		
		Optimistic	Most likely	Pessimestic
A	–	4	5	12
B	–	1	1.5	5
C	A	2	3	4
D	A	3	4	11
E	A	2	3	4
F	C	1.5	2	2.5
G	D	1.5	3	4.5
H	B,E	2.5	3.5	7.5
I	H	1.5	2	2.5
J	F, G, I	1	2	3

Determine the critical path

Solution:

Network for the given project is drawn below:



Value of expected time for each

Activity	time estimate (Weeks)			
	Optimistic t_o	Most likely t_m	Pessimistic t_p	Expected $t_e = \frac{t_0 + 4t_m + t_p}{6}$
A (1–2)	4	5	12	6
B (1–3)	1	1.5	5	2
C (2–4)	2	3	4	3
D (2–5)	3	4	11	5
E (2–3)	2	3	4	3
F (4–7)	1.5	2	2.5	2
G (5–7)	1.5	3	4.5	3
H (3–6)	2.5	3.5	7.5	4
I (6–7)	1.5	2	2.5	2
J (7–8)	1	2	3	2

Time [Earliest & latest] are calculated as follows:

Forward Pass Method	*Backward Pass Method*
$E_1 = 0$	$L_8 = E_8 = 17$
$E_2 = E_1 + t_{1,2} = 0 + 6 = 6$	$L_7 = E_8 - t_{7,8} = 17 - 2 = 15$
$E_3 = \max [E_1 + t_{1-3} ; E_2 + t_{2-3}]$	$L_6 = L_7 - t_{6-7} = 15 - 2 = 13$
$= \max [0+2 ; 6+3] = 9$	$L_5 = L_7 - t_{4-7} = 15 - 3 = 12$
	$L_4 = L_7 - t_{4-7} = 15 - 2 = 13$
$E_4 = E_2 + t_{2-4} = 6+3 = 9$	$L_3 = L_6 - t_{3-6} = 13 - 4 = 9$
$E_5 = E_2 + t_{2-5} = 6 + 5 = 11$	
$E_6 = E_3 + t_{3-6} = 9 + 4 = 13$	$L_2 = \text{Min} [L_3 - t_{2-3}, L_4 - t_{2-4}\ L_5 - t_{2,5}]$
$E_7 = \text{Max} [E_4 + t_{4-7} ; E_5 + t_{5-7} ; E_6 + t_{6-7}]$	$\text{Min} [9-3; 13-3 ; 12-5] = 6$
$= \max [9+2 ; 11+3 ; 13+2] = 15$	$L_1 = \text{Min} [L_2 - t_{1-2} ; L_3 - t_{1-3}]$
	$= \min [6-6 ; 9-2] = 0$
$E_8 = E_7 + t_{4-8} = 15 + 2 = 17$	

As we can see there are two critical paths along which E-values and L-values are similar, but the longest network of critical activities is known as critical path.

Critical path is 1-2-3-6-7-8

Expected length of critical path is = 6 + 3 + 4 + 2 + 2 = 17 weeks

Example 3:

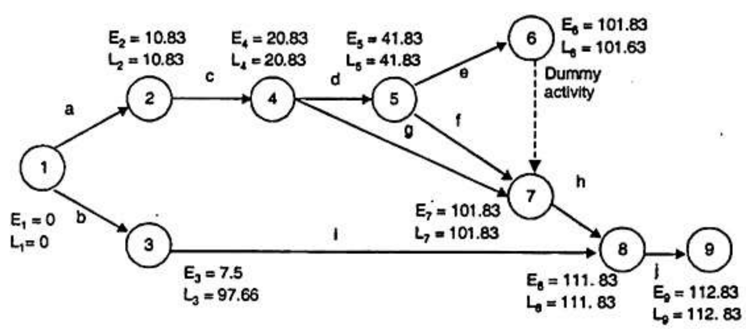
Product manager has planned a list of activities culminating in the inaugurate launch of the new products.

Activity	pert's time estimates days			Immediate Predecessor (s)
	P	M	O	
a	20	10	5	–
b	12	7	5	–
c	12	10	8	a
d	40	20	6	c
e	90	60	30	d
f	14	10	7	d
g	50	30	20	c
h	12	10	8	e, f, g
i	6	4	3	b
j	1	1	1	h,i

What is the probability that product manager will be able to complete the language launch within 80 days-time?

Solution:

Network diagram for given problem is shown in following fig:



Expected time value for each activity of given network is listed in table below along with three variance.

Activity	Time estimates				Variance
	Pessimistic	Most likely	Optimistic	Expected	
	t_p	t_m	t_o	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$
a	20	10	5	10.83	6.25
b	12	7	5	7.5	1.36
c	12	10	8	10	0.44
d	40	20	6	21	32.11
e	90	60	30	60	100
f	14	10	7	10.17	11.36
g	50	30	20	31.67	25
h	12	10	8	10	0.44
i	6	4	3	4.17	0.25
j	1	1	1	1	0

Value of earliest & latest time is calculated on the basis of expected time t_e as follows:

Forward pass method	*Backward pass method*
$E_1 = 0$	$L_9 = E_9 = 112.83$
$E_2 = E_1 + t_{1-2} = 0 + 10.83 = 10.83$	$L_8 = L_9 - t_{8-9} = 112.83 - 1 = 111.83$
$E_3 = E_1 + t_{1-3} = 0 + 7.5 = 7.5$	$L_7 = L_8 - t_{7-8} = 111.83 - 0 = 101.83$
$E_4 = E_2 + t_{2-9} = 10.83 + 10 = 20.83$	$L_6 = L_7 - t_{6-7} = 101.83 - 0 = 101.83$
$E_5 = E_4 + t_{4-5} = 20.83 + 21 = 41.83$	$L_5 = \min [L_6 - t_{5-6}; t_7 - t_{5-7}]$
$E_6 = E_5 + t_{5-6} = 41.83 + 60 = 101.83$	$= \min [101.83 - 60 ; 10.83 - 10.17] = 41.83$
$E_7 = \max [E_4 + t_{4-7}, E_5 + t_{5-7}, F_6 + t_{6-7} =]$	$L_4 = \min [L_5 - t_{4-5}, L_7 - t_{4-7}]$
$= \max [20.83 + 31.67, 41.83 + 10.17$	$\min [41.83 - 21 ; 101.83 - 31.67]$
$+ 101.83 + 0] = 101.83$	$= 20.83$
$E_8 = \max [E_3 + t_{3-8}, E_7 + t_{7-8}]$	$L_3 = L_7 - t_{3,7} = 101.83 - 4.17 = 97.66$
$= \max [7.5 + 4.17; 101.83 + 10] = 111.83$	$L_2 = L_4 - t_{2-4} = 20.83 - 10 = 10.83$
$E_9 = E_8 + t_{8-9}$	$L_1 = \min [L_2 - t_{1-2} ; L_{3} - t_{1-3}]$
$= 111.83 + 1$	$= \min (10.83 - 10.83 ; 97.66 - 7.5)$
$= 112.83$	$= 0$

Value of earliest & latest time is

time t_e as follows:

Hence critical path along with E-value and L- value are same i.e., 1- 2-4-5-6-7-8-9 Expected project duration is 172.83 days

Variance of project length = Sum of variance of each critical activity = 6.25 + 0.44 + 32.11 +100+1.36+.44+0= 140.6

Standard deviation is $\sigma = \sqrt{\text{Variance}}$

$$= \sqrt{140.6}$$

$$= 11.86$$

thus, $Z = \frac{t_s - t_e}{\sigma} = \frac{80 - 112.83}{11.86} = -2.77$

For Z = -2.77 Probability of completing the project with 80 days-time i.e., 0.3%.

Example 4:

A Project is composed of seven activities whose time estimates are listed in the following table. Activities are simplified

Activity		Estimated duration in weeks		
i	j	Optimistic	Most likely	Pessimestic
1	2	1	1	7
1	3	1	4	7
1	4	2	2	8
2	5	1	1	1
3	5	2	5	14
4	6	2	5	8
5	6	3	6	15

Calculate expected project length.

Solution:

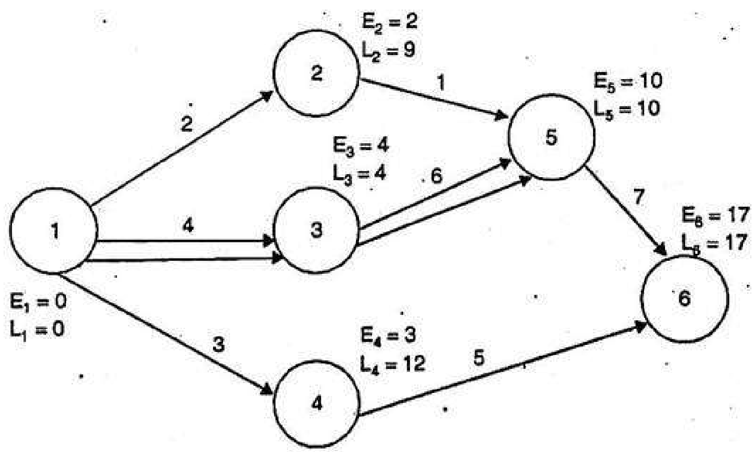
Calculation of expected time for each activity is shown in following table:

Activity	Time estimates (Weaks)			
	Optimistic t_0	Most-likely t_m	Pessimistic t_p	Expected time $t_e = \frac{t_0 + 4t_m + t_p}{6}$
1–2	1	1	7	2
1–3	1	9	7	4
1–4	2	2	8	3
2–5	1	1	1	1
3–5	2	5	14	6
4–6	2	5	8	5
5–6	3	6	15	7

E- Values and L- values are calculated on the basis of expected time are as follows:

Forward pass method	*Backward pass method*
$E_1 = 0$	$L_6 = E_6 = 0$
$E_2 = E_1 + t_{1-2} = 0 + 2 = 2.$	$L_5 = L_6 - t_{5-6} = 17 - 7 = 10$
$E_3 = E_1 + t_{1-3} = 0 + 4 = 4$	$L_4 = L_6 - t_{4-6} = 17 - 5 = 12$
$E_4 = E_1 + t_{1-4} = 0 + 3 = 3$	$L_3 = L_5 - t_{3-5} = 10 - 6 = 4$
$E_5 = \max[E_2 + t_{2-5}; E_3 + t_{3-5}]$	$L_2 = L_5 - t_{2-5} - 10 - 1 = 9$
$= \max[2+1; 4+6] = 10$	$L_1 = \min[L_2 - t_{1-2}; L_3 - t_{1-3}; L_4 - t_{1-4}]$
$E_6 = \max[E_5 + t_{5-6}, E_4 + t_{4-6}]$	$= \min[9-2; 4-4; 12-3] = 0$
$= \max(10+7; 3+5) = 17$	

Network diagram for given project along with E-values and L-values is shown by following Fig:



Critical path for the above network 1-3-5-6 shown by double lines; along with E-values and L-values are same.

Expect project length will be = 4 + 6+ 7

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