Assignment 2: SPA 611

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1 Mean Power vs. Voltage Variance

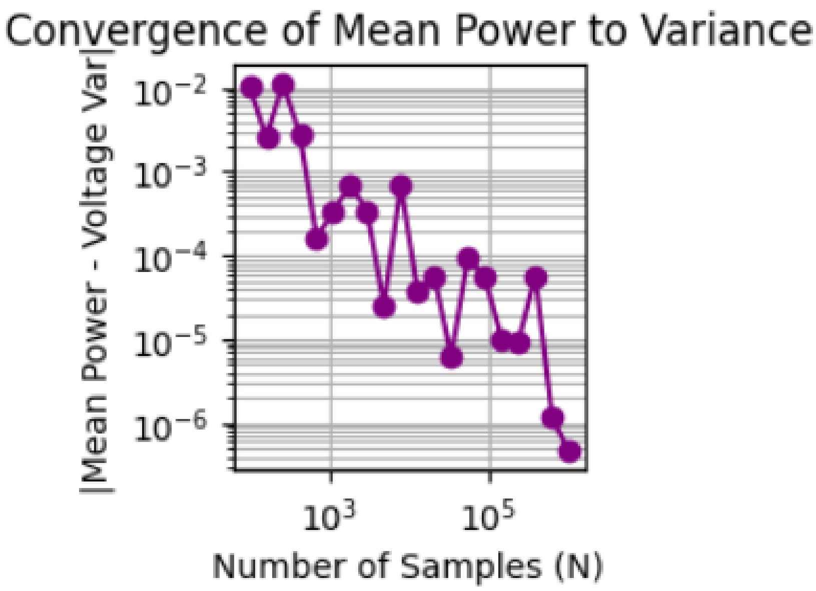

Figure 1: .

Gaussian voltage:

$$v \sim \mathcal{N}(0, \sigma^2) \tag{1}$$

Power (for $R = 1\,\Omega$):

$$p = v^2 \tag{2}$$

$$\langle p \rangle = E[v^2] = \mathrm{Var}(v) + (E[v])^2 \tag{3}$$

Since $E[v] = 0$:

$$\boxed{\langle p \rangle = \sigma^2} \tag{4}$$

Inference: Mean detected power equals the signal variance — foundation of total power radiometry.

Code Explanation

Gaussian samples were generated numerically:

```
v = np.random.normal(0, sigma, N)
```

Instantaneous power was computed using the square-law detector model:

```
p = v**2
```

The equality between measured power and variance was verified using:

```
np.mean(p)
np.var(v)
```

To confirm statistical convergence, the experiment was repeated for increasing sample sizes:

```
sample_sizes = np.logspace(2, 6, 20, dtype=int)
for N in sample_sizes:
    v = np.random.normal(0, 2, N)
    v_var = np.var(v)
    p_mean = np.mean(v**2)
```

Observations

- For small N, measured power fluctuates due to finite sampling.
- As N increases, $\langle p \rangle$ converges to σ^2.
- The convergence follows the Law of Large Numbers.
- This validates that radiometers estimate signal variance through time averaging.

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2 Effect of RMS Voltage

Tested $\sigma = \{0.5, 1.0, 2.0\}$.

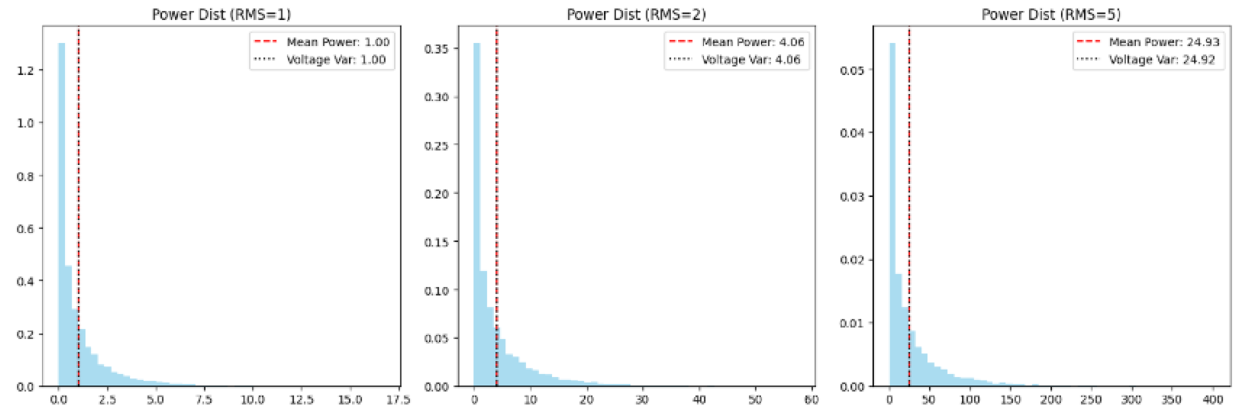

Figure 2: Power histograms for different RMS voltages.

Observations:

- $p = v^2$ follows χ_1^2 (highly right-skewed).
- Increasing σ stretches the distribution.
- Mean shifts to σ^2; shape unchanged.

Code Explanation

Independent Gaussian signals were generated for each RMS level:

```
for sigma in [0.5, 1.0, 2.0]:
    v = np.random.normal(0, sigma, N)
    p = v**2
    plt.hist(p, bins=..., density=True)
```

Histograms allow visualization of how signal strength affects the detected power distribution.

Observations

- RMS controls only the *scale* of the distribution, not its shape.
- The detector responds linearly to power ($\propto \sigma^2$).
- Stronger signals simply expand the measurable dynamic range.

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3 Addition of a Slowly Varying Sine (RFI)

$$v(t) = n(t) + A\sin(2\pi f t) \tag{5}$$

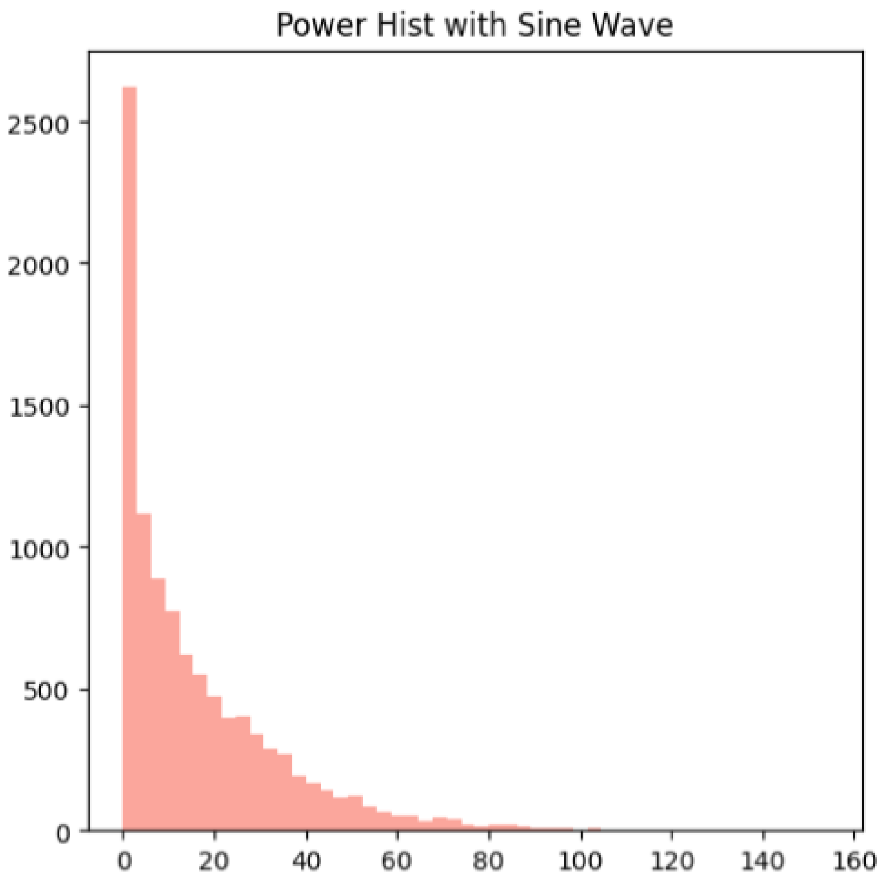

Figure 3: Power histogram after adding sinusoidal interference.

Mean power:

$$\boxed{\langle p \rangle = \sigma_{\text{noise}}^2 + \frac{A^2}{2}} \tag{6}$$

Effect:

- Histogram shifts right (extra deterministic power).
- Reduced probability near zero.
- Distribution broadens.

Code Explanation

A deterministic sinusoidal interference term was added:

```
t = np.arange(N)
noise = np.random.normal(0, sigma, N)
v = noise + A*np.sin(2*np.pi*f*t)
p = v**2
```

The histogram of p was compared to the pure-noise case.

Observations

- The sine wave injects constant additional energy.
- Even weak interference biases radiometric measurements.
- This models real Radio Frequency Interference (RFI) contamination.

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4 Distribution of Estimated Variance

For N samples:

$$\hat{\sigma}^2 \sim \frac{\sigma^2}{N-1}\chi^2_{N-1} \tag{7}$$

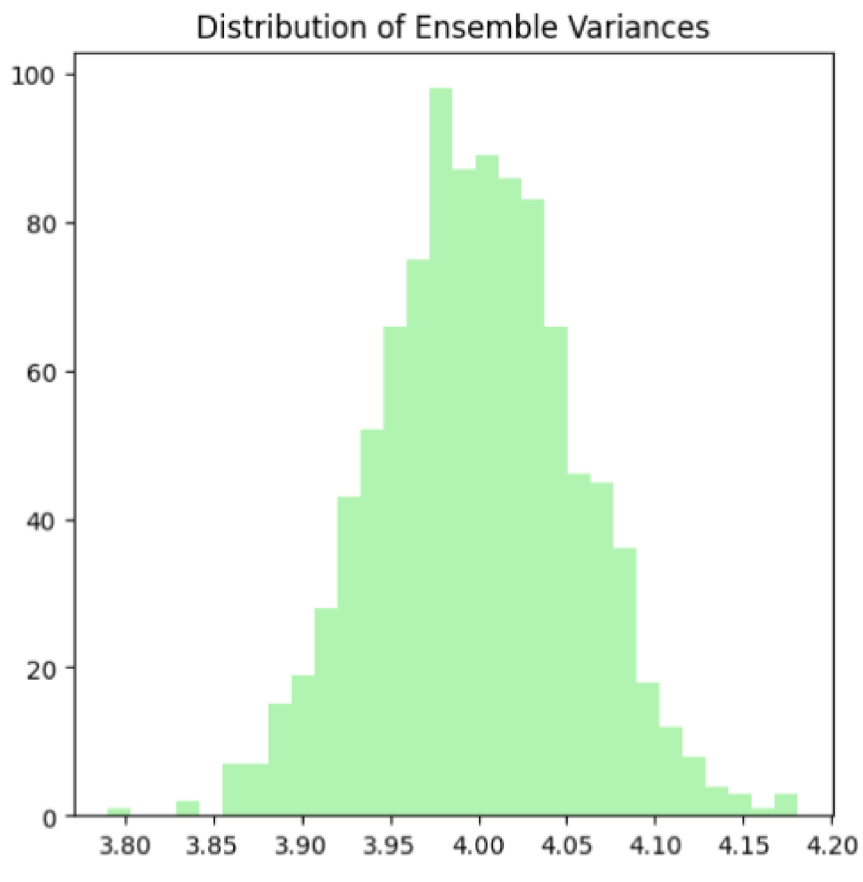

Figure 4: Distribution of variance estimates across ensembles.

Key Points:

- $\hat{\sigma}^2$ fluctuates around true σ^2.
- Larger $N \Rightarrow$ narrower distribution.
- Measurement uncertainty:

$$\Delta\sigma^2 \propto \frac{1}{\sqrt{N}} \tag{8}$$

Code Explanation

Multiple independent ensembles were simulated:

```
for m in range(M):
    v = np.random.normal(0, sigma, N)
    est_var.append(np.var(v, ddof=1))
```

Each ensemble produces one variance estimate; their spread forms the histogram.

Observations

- Variance itself is a random variable for finite observations.
- Increasing integration time reduces estimator noise.
- This statistical fluctuation defines radiometer sensitivity.

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Conclusion:

Square-law detection converts stochastic voltage into measurable power whose mean equals the signal variance. Finite-time measurements introduce statistical uncertainty that decreases as $1/\sqrt{N}$, explaining why longer integration improves precision in practical radiometric systems.