Tutorial 4 (master's theorem)

① $T(n) = 3T(n/2) + n^2$

$a=3, b=2, f(n) = n^2$ ∵ $a \geq 1, b > 1$.

$C = \log_b a = \log_2 3 \approx 1.58$

$n^c = n^{1.58}$,

$f(n) > n^c$

$n^2 > n^{1.58}$ ∴ $T(n) = \Theta(f(n)) = \Theta(n^2)$

② $T(n) = 4T(n/2) + n^2$

$a=4, b=2, f(n) = n^2$

$C = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$f(n) = n^c \Rightarrow n^2 = n^2$

∴ $\Theta(n^c \log n) = \Theta(n^2 \log n)$

③ $T(n) = T(n/2) + 2^n$

$a=1, b=2, f(n) = 2^n$ $[a \geq 1, b = 1]$

$C = \log_2 1 = 0$

$n^0 \not< 2^n$

∴ $\Theta(1)$

④ $T(n) = 2^n T(n/2) + n^n$ (?)

$a = 2^n, b=2, f(n) = n^n$

$c = \log_2 2^n = n \Rightarrow n^c = n^n$

$f(n) = n^c$

∴ $\Theta(n^n \log n)$

⑤ $T(n) = 16T(n/4) + n$

$a=16, b=2, f(n) = n$

$C = \log_4 4^2 = 2$

$n^c = n^2$

$n^c > f(n)$

$n^2 > n$ ∴ $\Theta(n^2)$

⑥ $T(n) = 2T(n/2) + n \log n$ (?)

$a=2,\ b=2,\ f(n) = n \log n$

~~a = log (not possible)~~, $b > 1$

$c = \log_2 2 = 1 \Rightarrow n^c = n$

$\therefore\ n^c < f(n)$

$n < n \log n$

$\therefore\ \Theta(n \log n)$

⑦ $T(n) = 2T(n/2) + n/\log n$

$a = 2;\ b = 2,\ f(n) = \dfrac{n}{\log n}$

if $n=1 \Rightarrow \frac{1}{0} = \infty$

$c = \log_2 2 = 1 \Rightarrow n^c = n$

$n^c > f(n)$

$n > \dfrac{n}{\log n} \Rightarrow \Theta(n)$

⑧ $T(n) = 2T(n/4) + n^{0.51}$

$a=2,\ b=4,\ f(n) = n^{0.51}$

$c = \log_4 2 = 0.5 \Rightarrow n^c = n^{0.5}$

$f(n) > n^c$

$n^{0.51} > n^{0.50}$

$\therefore\ \Theta(f(n)) = \Theta(n^{0.51})$

⑨ $T(n) = 0.5T(n/2) + 1/n$

Master theorem not apply.

⑩ $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$c = \log_4 16 = 2 \Rightarrow n^c = n^2$

$f(n) > n^c$

$\therefore\ \Theta(n!)$

if $n=4$

$n^2 = 16$

$n! = 4 \times 3 \times 2 = 24$

(11) $T(n) = 4T(n/2) + \log n$

$a=4,\ b=2,\ f(n) = \log n$

$c = \log_2 4 = 2 \Rightarrow n^c = n^2$

$\therefore n^c > f(n)$

$n^2 > \log 2 \quad \therefore \Theta(n^2)$

(12) $T(n) = \sqrt{n}\, T(n/2) + \log n$

Master theorem not applied

(13) $T(n) = 3T(n/2) + n$

$a=3,\ b=2,\ f(n)=n$

$c = \log_2 3 = 1.58 \Rightarrow n^{1.58} = n^c$

$\therefore n^c > f(n)$

$n^{1.58} > n \quad \therefore \Theta(n^{1.58})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$a=3,\ b=3,\ f(n) = \sqrt{n}$

$c = \log_3 3 = 1 \Rightarrow n^c = n$

$\therefore n^c > f(n)$

$n > \sqrt{n} \Rightarrow \Theta(n)$

(15) $T(n) = 4T(n/2) + cn$

$a=4,\ b=2,\ f(n) = cn = 2n$

$c = \log_2 2^2 = 2 \Rightarrow n^c = n^2$

$f(n) < n^c$

$2n < n^2$

$\therefore \Theta(n^2)$

) $T(n) = 3T(n/4) + n\log n$

$a=3,\ b=4,\ f(n) = n\log n$

$c = \log_4 3 = 0.79,\ n^c = n^{0.79}$

(17) $T(n) = 3T(n/3) + n/2$

$a=3,\ b=3,\ f(n) = n/2$

$c = \log_3 3 = 1,\quad n^c = n$

$n^c > f(n)$

$\therefore\ \theta(n)$

(18) $T(n) = 6T(n/3) + n^2 \log n$

$a = 6,\ b = 3,\ f(n) = n^2 \log n$

$c = \log_3 6 = 1.63,\quad n^c = n^{1.63}$

$f(n) > n^c$

$\therefore\ \theta(n^2 \log n)$

$4 \times 1 = 4$

$2^{1.63} = 4$

(19) $T(n) = 4T(n/2) + n/\log n$

$a = 4,\ b = 2,\ f(n) = n/\log n$

$c = \log_2 2^2 = 2 \Rightarrow n^c = n^2$

$\therefore\ n^c > f(n)$

$\therefore\ \theta(n^2)$

(20) $T(n) = 64T(n/8) + (-n^2 \log n)$

$a = 64,\ b = 8,\ f(n) = -n^2 \log n$

$c = \log_8 8^2 = 2,\quad n^c = n^2$

$\therefore\ n^c > f(n)$

$\therefore\ \theta(n^2)$

(21) $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$c = \log_3 7 = 1.77 \Rightarrow n^c = n^{1.7}$

$\therefore\ f(n) > n^c$

$\therefore\ \theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1,\ b = 2,\ f(n) = n(2 - \cos n)$

$c = \log_2 1 = 0 \Rightarrow n^0 = 1$

$\therefore\ f(n) > n^c$

$\therefore\ \theta(n(2 - \cos n))$