```
In [37]: import sys
          import numpy as np
          from gamspy import (
              Container, Set, Alias, Parameter, Variable, Equation, Model, Problem, Sense, Opti
              Domain, Number, Sum, Product, Smax, Smin, Ord, Card, Special Values
         options = Options(variable listing limit=0)
         m = Container(options=options)
In [38]: locations = Set(m, "locations", records = ["Burrow", "LeakyCauldron", "Godri
          "LittleWhinging", "Cokeworth", "Egypt"])
          i = m.addAlias('i', locations)
          j = m.addAlias('j', locations)
          k = m.addAlias('k', locations)
          arcs = Set(m, 'arcs', domain=[i, j], records=[
              ("Burrow", "LeakyCauldron"), ("LeakyCauldron", "Burrow"), ("Burrow", "Gd
              ("LeakyCauldron", "GodricHollow"), ("GodricHollow", "LeakyCauldron"), ("
              ("GodricHollow", "LittleHangleton"), ("LittleHangleton", "GodricHollow")
              ("LittleHangleton", "Cokeworth"), ("Cokeworth", "LittleHangleton"), ("Li
              ("Cokeworth", "Egypt"), ("Egypt", "Cokeworth")
          ])
          distances = Parameter(m, 'distances', domain=[i,j], records=[["Burrow", "Lea
                       ["Burrow", "GodricHollow", 60],
                      ["Burrow", "LittleHangleton", 50],
                       ["LeakyCauldron", "GodricHollow", 10],
                      ["LeakyCauldron", "LittleWhinging", 70], ["GodricHollow", "LittleHangleton", 20],
                      ["GodricHollow", "LittleWhinging", 55],
                      ["GodricHollow", "Cokeworth", 40],
                      ["LittleHangleton", "Cokeworth", 50],
                      ["LittleWhinging", "Cokeworth", 10],
["LittleWhinging", "Egypt", 60],
                      ["Cokeworth", "Egypt", 80]
         ])
         distances[i, j].where[distances[j, i] > 0] = distances[j, i]
         x = Variable(m, "x", domain=[i,j], type="positive")
          supply = Parameter(m, "supply", domain=[i], records=[("Burrow", 1), ("Egypt"
          balance = Equation(m, "balance", domain=i)
          balance[i].where[~i.last] = Sum(j.where[arcs[i, j]], x[i, j]) - Sum(j.where[
          floo = Model(m, 'short', equations=m.getEquations(), problem="MIP",
          sense=Sense.MIN,
```

```
objective = Sum(arcs, distances[arcs] * x[arcs])
In [391:
         floo.solve()
             Solver
                                                 Num of
                                                            Num of
Out[39]:
                           Model
                                                                     Model
                                                                                     Solve
                                   Objective
                                                                             Solver
             Status
                           Status
                                              Equations Variables
                                                                      Type
                                                                                      Tim
          0 Normal OptimalGlobal
                                         160
                                                       7
                                                                 25
                                                                        MIP
                                                                             CPLEX
                                                                                      0.00
In [40]: pi = Variable(m, 'pi', type='positive', domain=i)
          dualcons = Equation(m, 'dcons', domain=[i,j])
          dualcons[i, j].where[arcs[i,j]] = pi[i] - pi[j] <= distances[i, j]</pre>
         d = Model(m, name="d", equations=[dualcons], problem="LP", sense=Sense.MAX,
         d.solve(solver='cplex',solver options={'lpmethod': 3, 'netfind': 2, 'preind'
Out[40]:
             Solver
                           Model
                                                 Num of
                                                            Num of Model
                                                                                    Solve
                                   Objective
                                                                             Solver
             Status
                           Status
                                              Equations Variables
                                                                      Type
                                                                                      Tim
          0 Normal OptimalGlobal
                                         160
                                                      25
                                                                  8
                                                                         LP
                                                                             CPLEX
                                                                                      0.00
In [43]:
         # Marginal value from the primal is the shortest distance from egypt
          display(balance.records)
          display(pi.records)
          print("As you can see the marginal values from the primal solution are equal
                        i level marginal lower upper scale
        0
                  Burrow
                            1.0
                                    160.0
                                              1.0
                                                            1.0
                                                      1.0
            LeakyCauldron
                            0.0
                                    120.0
                                              0.0
                                                      0.0
                                                            1.0
        2
             GodricHollow
                            0.0
                                    110.0
                                              0.0
                                                      0.0
                                                            1.0
        3 LittleHangleton
                            0.0
                                    110.0
                                              0.0
                                                      0.0
                                                            1.0
        4
            LittleWhinging
                            0.0
                                      60.0
                                              0.0
                                                      0.0
                                                            1.0
        5
               Cokeworth
                            0.0
                                      70.0
                                              0.0
                                                      0.0
                                                            1.0
                        i level marginal lower upper scale
        0
                  Burrow 160.0
                                       0.0
                                              0.0
                                                      inf
                                                             1.0
            LeakyCauldron 120.0
                                       0.0
                                              0.0
                                                      inf
                                                             1.0
        2
             GodricHollow 110.0
                                       0.0
                                              0.0
                                                      inf
                                                             1.0
        3 LittleHangleton 110.0
                                       0.0
                                              0.0
                                                             1.0
                                                      inf
            LittleWhinging
                           60.0
                                       0.0
                                              0.0
                                                             1.0
        4
                                                      inf
        5
               Cokeworth
                           70.0
                                       0.0
                                              0.0
                                                             1.0
                                                      inf
        6
                    Egypt
                            0.0
                                      -0.0
                                              0.0
                                                      inf
                                                             1.0
```

As you can see the marginal values from the primal solution are equal to the variable values from the dual solution!

```
In [44]: DistEgypt = Parameter(m,'DistEgypt')
DistEgypt[:] = balance.records.loc[balance.records['i'] == 'LeakyCauldron',
print(f"The distance (length) from the Leaky Cauldron to Egypt is {DistEgypt
```

The distance (length) from the Leaky Cauldron to Egypt is 120.0 miles