This example solves linear regression with Huber modeled as QP.

Try to predict the median housing price using the other features. Use a variety of values of gamma input on the command line.

Boston Housing Data Sources: (a) Origin: This dataset was taken from the StatLib library which is maintained at Carnegie Mellon University. (b) Creator: Harrison, D. and Rubinfeld, D.L. 'Hedonic prices and the demand for clean air', J. Environ. Economics & Management, vol.5, 81-102, 1978. (c) Date: July 7, 1993

In [11]:
```
import gamspy as gp
import gamspy.math as gpm
from gamspy import Sum, Card

import sys
import numpy as np
import pandas as pd
```

In [12]:
```
m = gp.Container(load_from="boston.gdx")

# Sets
i,headr = m.getSymbols(["i", "headr"])
display(headr.records)

# Parameters
[data] = m.getSymbols(["Data"])
display(data.pivot())
```

	uni	element_text
0	CRIM	
1	ZN	
2	INDUS	
3	NOX	
4	RM	
5	AGE	
6	DIS	
7	RAD	
8	TAX	
9	PTRATIO	
10	B	
11	LSTAT	
12	MEDV	
13	CHAS	

	CRIM	ZN	INDUS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	
1	0.00632	18.0	2.31	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396
2	0.02731	0.0	7.07	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396
3	0.02729	0.0	7.07	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392
4	0.03237	0.0	2.18	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394
5	0.06905	0.0	2.18	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396
...	...	...	...	...	...	...	...	...	...	...	
502	0.06263	0.0	11.93	0.573	6.593	69.1	2.4786	1.0	273.0	21.0	391
503	0.04527	0.0	11.93	0.573	6.120	76.7	2.2875	1.0	273.0	21.0	396
504	0.06076	0.0	11.93	0.573	6.976	91.0	2.1675	1.0	273.0	21.0	396
505	0.10959	0.0	11.93	0.573	6.794	89.3	2.3889	1.0	273.0	21.0	393
506	0.04741	0.0	11.93	0.573	6.030	80.8	2.5050	1.0	273.0	21.0	396

506 rows × 14 columns

In [13]:

```
# Standardize data and set up train, test and j in this cell

means = m.addParameter('means',domain=headr)
stddev = m.addParameter('stddev',domain=headr)
means[headr] = Sum(i, data[i,headr])/Card(i)
stddev[headr] = gpm.sqrt(Sum(i, gpm.sqr(data[i,headr]-means[headr]))/Card(i)
```

```
sdat = m.addParameter('sdat',domain=[i,headr])
sdat[i,headr] = (data[i,headr] - means[headr])/stddev[headr]

train = m.addSet('train',description='observation number',domain = i)
train[i] = gp.Number(1).where[i.val <= 450]
# , records=[x for x in range(1, 451)]
test = m.addSet('test',description='observation number', domain = i)
test[i] = ~train[i]
# , records=[x for x in range(451, 507)]
n = m.addSet('n',domain=headr,description='index of independent variables',r
```

In [14]:

```
# Set up huber model and solve in this cell
r = m.addVariable('r','free',domain=i,description='residual')
x = m.addVariable('x','free',domain=n,description='estimates')

b = m.addVariable('b','free')

sigma = m.addParameter('sigma',description="cutoff for quadratic penalty",
    records=1)

t = m.addVariable('t',domain=i,description='residual l_1 error')

upp = m.addEquation('upp',domain=i)
upp[train] = sdat[train,"MEDV"] - Sum(n, sdat[train,n]*x[n]) - r[train] - b

low = m.addEquation('low',domain=i)
low[train] = -t[train] <= sdat[train,"MEDV"] - Sum(n, sdat[train,n]*x[n]) -

hubmod = m.addModel('hubmod',
    equations=[low,upp],
    problem=gp.Problem.QCP,
    sense=gp.Sense.MIN,
    objective=1/2*Sum(train, gpm.sqr(r[train])) + sigma*Sum(train, t[train])
)

x.l[n] = 1

hubmod.solve(options=gp.Options(qcp="conopt"),output=None)
```

Out[14]:

	Solver Status	Model Status	Objective	Num of Equations	Num of Variables	Model Type	Solve
0	Normal	OptimalLocal	54.7179099527434	901	915	QCP	CONOF

In [15]:

```
#LEAST SQUARES
ls = m.addVariable('ls', 'free', domain=i)
beta = m.addVariable('betals', 'free', domain=n)
beta0 = m.addVariable('beta0ls', 'free')

ls_const = m.addEquation('lseq', domain=i)
ls_const[train] = (sdat[train,"MEDV"] - Sum(n, sdat[train, n]*beta[n]) - ls[

least_s = m.addModel('least_s',
    equations=[ls_const],
    problem=gp.Problem.QCP,
```

```python
    sense=gp.Sense.MIN,
    objective=1/2*Sum(train, gpm.sqr(ls[train]))
)

least_s.solve(options=gp.Options(qcp="conopt"), output=None)

huber = 0
least_squares = 0

sigma_range = np.linspace(1, 3, 200)

for sig_val in sigma_range:
    sigma = m.addParameter('sigma', records=sig_val)
    hubmod.solve(options=gp.Options(qcp="conopt"), output=None)
    if hubmod.status in [
        gp.ModelStatus.OptimalGlobal,
        gp.ModelStatus.OptimalLocal
        ]:
        huber = hubmod.objective_value

        if abs(huber - least_s.objective_value) < 1e-4:
            lssigma = sig_val
            print(f"\nMatching Sigma value is: {sig_val}")
            break
```

```
Matching Sigma value is: 2.7688442211055277
```

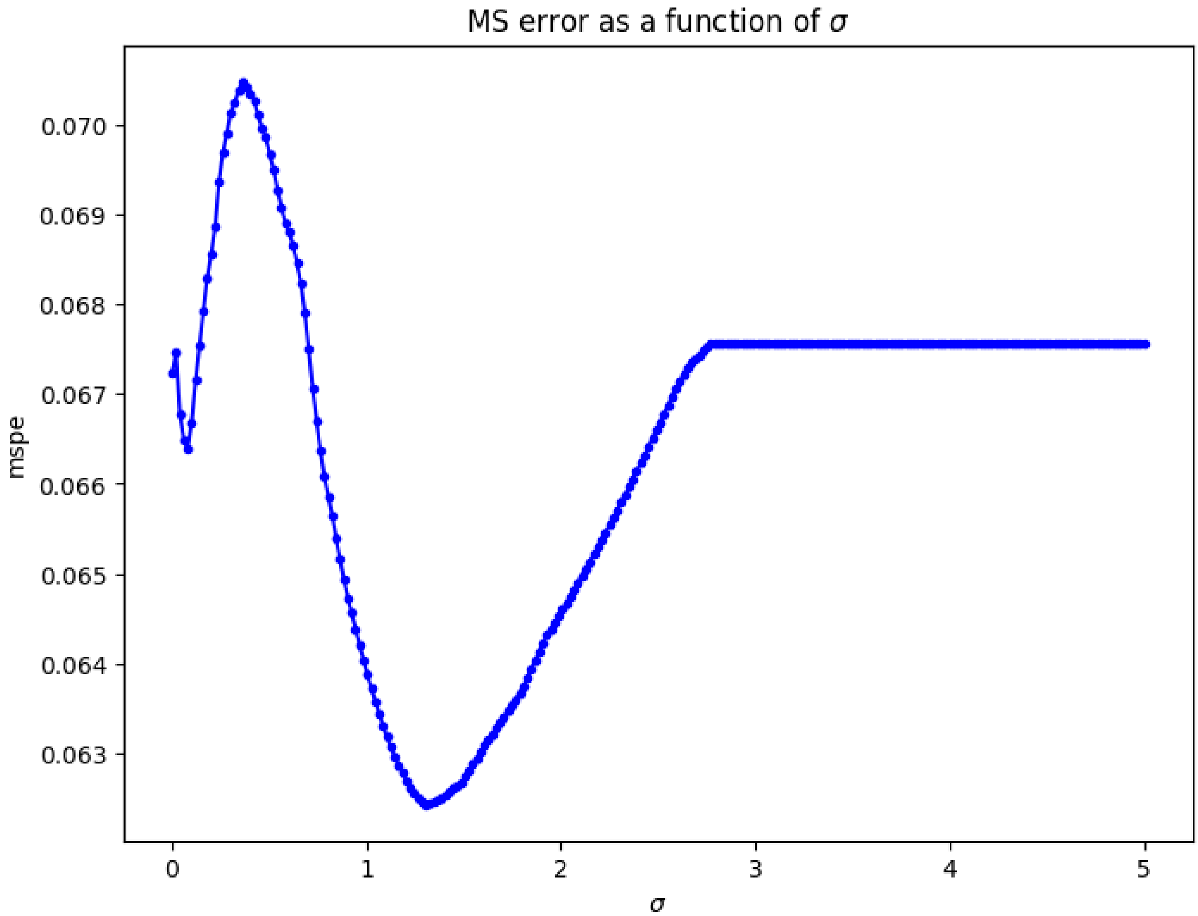
In [16]:

```python
preds = gp.Parameter(m,'pred',domain=i,description='prediction')
mspe = gp.Parameter(m,'mspe',description='mean sqr prediction error')

solsig = []
solmspe = []

ranges = np.linspace(-1, 5, 300)

for sigma2 in ranges:
    sigma[:] = sigma2
    hubmod.solve(options=gp.Options(qcp="conopt"),output=None)
    if hubmod.status in [
        gp.ModelStatus.OptimalGlobal,
        gp.ModelStatus.OptimalLocal,
    ]:
        preds[test] = Sum(n, x.l[n] * sdat[test, n]) + b.l
        mspe[:] = (1/2 * Sum(test, gpm.sqr(preds[test] - sdat[test, "MEDV"]))
        solsig.append(sigma2)
        solmspe.append(mspe.toValue())

# generate values for comparison of sigma and mspe
```

```
[MODEL - WARNING] The solve was interrupted! Solve status: EvaluationInterru
pt. For further information, see https://gamspy.readthedocs.io/en/latest/ref
erence/gamspy._model.html#gamspy.SolveStatus.
[MODEL - WARNING] The solve was interrupted! Solve status: EvaluationInterru
pt. For further information, see https://gamspy.readthedocs.io/en/latest/ref
erence/gamspy._model.html#gamspy.SolveStatus.
```

In [17]:

```
%matplotlib inline
import matplotlib.pyplot as plt

fig, ax = plt.subplots(figsize=(8,6))
ax.set_title("MS error as a function of $\\sigma$")
ax.set_xlabel("$\\sigma$")
ax.set_ylabel("mspe")
ax.plot(solsig,solmspe,"b.-");
```



In [19]:

```
# display lssigma and mspe sigma
min_mspe = np.argmin(solmspe)
min_sigma = solsig[min_mspe]

print(f"lssigma = {lssigma}, This sigma value makes the huber model the same

print(f"\nminsigma = {min_sigma}, This sigma value that makes MSPE as small
```

```
lssigma = 2.7688442211055277, This sigma value makes the huber model the sam
e as the least squares model

minsigma = 1.307692307692308, This sigma value that makes MSPE as small as p
ossible on the test set
```