1.1 Activity, $N = \{A, B, C, D, E, F, G, H, I, J, K,\}$

Precedence : $P = \{ (A,B), (A,C), (B,D), (B,G), (C,D), (C,F), (D,E)$
$(D,I), (E,H), (F,J), (G,H), (H,I), (I,J), (J,K) \}$

a_i: Durations of activity $i \in N$

DV: x_i : Time activity starts

z = makespan = max $\{x_1 + a_1, x_2 + a_2 .. x_{|N|} + a_{|N|}\}$

Objective minimize z

Constraints

① $z \geq x_i + a_i \quad \forall i \in N$

② $x_j \geq x_i + a_i \quad \forall (i,j) \in P$

③ $x_i \geq 0 \quad \forall i \in N$

1.2 Optimal Objective value is 29 hours, which is the minimum amount of time required to defeat the final boss.

Activity ->	A	B	C	D	E	F	G	H	I	J	K
Start Time ->	0	2	2	5	7	5	3	11	14	18	24

1.3 y_i : amount of time to reduce duration of activity $i \in N$

c_i = spell cost for activity $i \in N$

Objective $\min \sum_{i \in N} c_i y_i$ [minimize the spell cost]

Constraints

① Force

$z \leq 20$

② Total duration of campaign

$z \geq x_i + a_i - y_i \quad \forall i \in N$

③ Precedence

$x_j \geq x_i + a_i - y_i \quad \forall (i,j) \in P$

④ can be reduced a max of their total duration or 2 hours

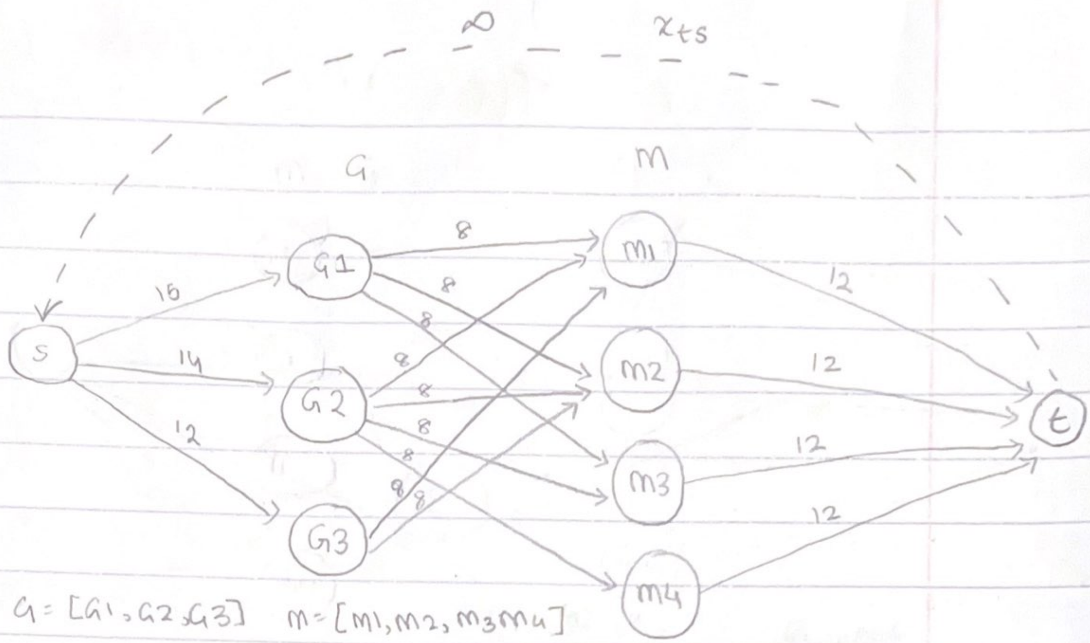
$y_i \leq \min(a_i, 2) \quad \forall i \in N$

⑤ $y_i \geq 0 \quad \forall i \in N$

1-4 Optimal objective value is \$33 with a total of 20 hours of game play

Amt of time to reduce for A: 0.0 hours
B: 0.0 hours
C: 2.0 hours
D: 0.0 hours
E: 2.0 hours
F: 0.0 hours
G: 0.0 hours
H: 2.0 hours
I: 2.0 hours
J: 1.0 hours
K: 0.0 hours

2-1



$G = [G1, G2, G3]$ $M = [M1, M2, M3 M4]$

N, Nodes: $\{s, G_1, G_2, G_3, m_1, m_2, m_3, m_4, t\}$

A, Arcs : $\{(s, G1), (s, G2), (s, G3)$

$(G_1, m_1), (G_1, m_2), (G_1, m_3)$

$(G_2, m_1), (G_2, m_2), (G_2, m_3), (G_2, m_4), (G_3, m_1), (G_3, m_2)$

$(m_1, t), (m_2, t), (m_3, t), (m_4, t)$

$(t, s)\}$

d_i, demand hours needed per group = [15, 14, 12] $i \in$

m_j, monthly hour capacity = 12 $j \in m$

H_{ij}, max hours spent on each campaign in each month = 8 $\forall (i, j) \in A$

Decision Variable

x_{ij}: Flow on arc (i, j), $\forall (i, j) \in A$

Objective: Max x_{ts}

Constraints:

⋆ Capacity.

$x_{s, G1} \leq 15$

$x_{s, G2} \leq 14$

$x_{s, G3} \leq 12$

$$G1 \begin{cases} 0 \le x_{G1,m1} \le 8 \\ 0 \le x_{G1,m2} \le 8 \\ 0 \le x_{G1,m3} \le 8 \end{cases}$$

$$G2 \begin{cases} 0 \le x_{G2,m1} \le 8 \\ 0 \le x_{G2,m2} \le 8 \\ 0 \le x_{G2,m3} \le 8 \\ 0 \le x_{G2,m4} \le 8 \end{cases}$$

$$G3 \begin{cases} 0 \le x_{G3,m1} \le 8 \\ 0 \le x_{G3,m2} \le 8 \end{cases}$$

$x_{m1,t} \le 12$

$x_{m2,t} \le 12$

$x_{m3,t} \le 12$

$x_{m4,t} \le 12$

* Non Negativity : $x_{ij} \ge 0 \quad (i,j) \in A$
* Flow balance

$$\sum_{j \in N:(i,j) \in A} x_{ij} = \sum_{j \in N:(i,j) \in A} x_{ji} \qquad \forall i \in N$$

Node s : $x_{s,G1} + x_{s,G2} + x_{s,G3} = x_{ts}$

Node G1 : $x_{G1,m1} + x_{G1,m2} + x_{G1,m3} = x_{s,G1}$

Node G2 : $x_{G2,m1} + x_{G2,m2} + x_{G2,m3} + x_{G2,m4} = x_{s,G2}$

Node G3 : $x_{G3,m1} + x_{G3,m2} = x_{s,G3}$

Node m1 : $x_{m1,t} = x_{G1,m1} + x_{G2,m1} + x_{G3,m1}$

Node m2 : $x_{m2,t} = x_{G1,m2} + x_{G2,m2} + x_{G3,m2}$

Node m3 : $x_{m3,t} = x_{G1,m3} + x_{G2,m3}$

Node m4 : $x_{m4,t} = x_{G2,m4}$

Node t : $x_{ts} = x_{m1,t} + x_{m2,t} + x_{m3,t} + x_{m4,t}$

let $= \{s, G_1, G_2, G_3, m_1, m_2, m_3, m_4, t\} = 1:9$

2-2 Max Flow Value = 41

Flow on arc (1,2) = 15

(1,3) = 14

(1,4) = 12

(2,5) : 3

(2,6) = 4

(2,7) : 8

(3,5) : 5

(3,6) : 0

(3,7) = 1

(3,8) = 8

(4,5) = 4

(4,6) = 8

(5,9) = 12

(6,9) = 12

(7,9) = 9

(8,9) = 8

(9,1) = 41.0

Since we have 4 months, which is 4x12 = 48 hours in total, it is possible to complete the three campaigns by the end of month 4, since we only need 41 hours.

2-3 ∴ The cut is $\{(5,9), (6,9), (7,9), (8,9)\}$

$12 + 12 + 9 + 8 = 41$

3) 3-1 Arcs → (1,2), (2,4), (4,5), (4,6), (6,7), (7,3), (7,8)

Total cost → $2 + 5 + 3 + 4 + 16 + 1 + 5$

= \$26

3 Minimum Spanning Tree

3-1 ***Problem*** Use the algorithm from class to find the minimum weight spanning tree in the network below. (Start from node 1 when running the algorithm.) Write down the selected arcs *in the order in which they are selected in the algorithm.* Also report the total cost of the minimum weight spanning tree you found with the algorithm.