Homework 5

1. Plane Assignment

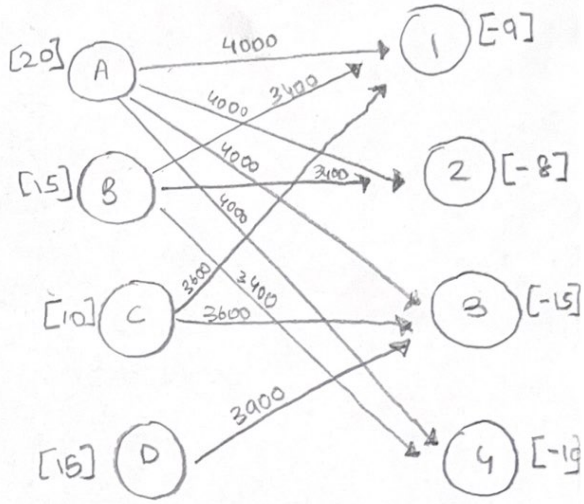
Decision Variables

$x_{ij} \rightarrow$ number of planes i [$I = A, B, C, D$] flying on route j [$J = 1, 2, 3, 4$]

Nodes, N = A, B, C, D , 1, 2, 3, 4   Arcs A = [(A,1), (A,2), (A,3), (A,4), (B,1), (B,2) (B,4), (C,1) (C,3), (D,3)]

Objective

Minimize $\quad 4000 \sum_{j \in J}^{4} x_{Aj} + 3400 \sum_{j \in J} x_{Bj} + 3600 \sum_{j \in J} x_{Cj} + 3900\, x_{D3}$



Constraints

1. Route Requirements

For R1 : $x_{A1} + x_{B1} + x_{C1} \geq 9$

For R2 : $x_{A2} + x_{B2} \geq 8$

For R3 : $x_{A3} + x_{C3} + x_{D3} \geq 15$

For R4 : $x_{A4} + x_{B4} \geq 10$

2. Number of plane's available

For A : $\sum_{j \in J} x_{Aj} \leq 20$

For B : $\sum_{j \in J} x_{Bj} \leq 15$

For C : $\sum_{j \in J} x_{Cj} \leq 10$

For D $\quad x_{D3} \leq 15$

3. Non negativity

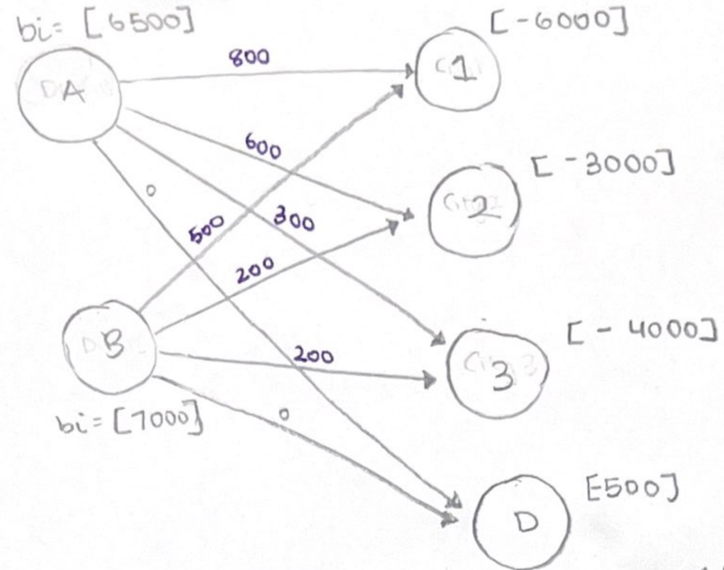
$x_{ij} \geq 0 \quad \forall\, i \in [A, B, C, D]$
$\qquad\qquad j \in [1, 2, 3, 4]$

2. MNCF

b_i = net supply at node i

$\boxed{c_{ij}}$ = arc cost, flow i to j

u_{ij} = arc capacity, from i to j



Nodes, N =

A → Detroit

B → Dallas

1 → City 1

2 → City 2

3 → City 3

D → Dummy Node

$U_{ij} = 3000 \quad \forall (ij) \in A$

Production Cost i = [2000, 1800]

Arcs, A = [(A,1), (A,2), (A,3), (A,D), (B,1), (B,2), (B,3), (B,D)]

c_{ij} = cost of flow per unit on arc $(i,j) \in A$

b_i = net supply on node $i \in N$

$L_{ij} = 0$, $U_{ij} = 3000 \quad \forall (ij) \in A$

Decision Variables

x_{ij} = Flow on arc $(i,j) \in A$

Objective minimize $Z = \sum_{(i,j)\in A} c_{ij} x_{ij} + \sum_{(i,j)\in A} 2000 \cdot x_{Aj} + \sum_{(i,j)\in A} 1800 \cdot x_{Bj}$

Subject to

NA: $x_{A1} + x_{A2} + x_{A3} + x_{AD} \le 6500$

NB: $x_{B1} + x_{B2} + x_{B3} + x_{BD} = 7000$

N1: $x_{A1} + x_{B1} = -6000$

N2: $x_{A2} + x_{B2} = -3000$

N3: $x_{A3} + x_{B3} = -4000$

ND: $x_{AD} + x_{BD} = -500$

$x_{ij} \geq 0 \quad \forall (ij) \in A$

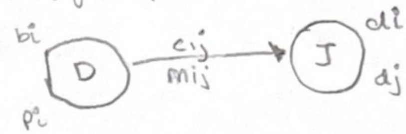
$0 \le x_{ij} \le 3000 \quad \forall (ij) \in A$

∴ optimal Objective = $30200000

3. General MCNF

Sets : Nodes D : Plants, $i \in D$

Arcs : A : (i,j), flow from $i \in D$ to $j \in J$



J : shipped to cities, $j \in J$

Parameters

c_{ij} = cost of flow of cars from plant $i \in D$ to city $j \in J$

m_{ij} = maximum cars that can be sent from plant $i \in D$ to city $j \in J$

b_i = net supply by plant $i \in D$

d_j = demand by city $j \in J$

p_i = cost of producing a car in plant $i \in D$

d_j = number of cars required by city $j \in J$

Decision Variables

x_{ij} : Number of cars shipped from plant $i \in D$ to city $j \in J$

Objective

minimize the total cost

Cost of shipping = $\sum_{(i,j) \in A} c_{ij} \cdot x_{ij}$

Cost of producing = $\sum_{(i,j) \in A} p_i \cdot x_{ij}$

$\therefore$ minimize $\sum_{(i,j) \in A} c_{ij} x_{ij} + \sum_{(i,j)} p_i \cdot x_{ij}$

Constraints

1) Each plant can produce maximum b_i cars

$$\sum_{(i,j) \in A} x_{ij} \leq b_i \quad \forall i \in D$$

2) City $j \in J$ must recieve d_j cars

$$\sum_{(i,j) \in A} x_{ij} = d_j \quad \forall j \in J$$

3) Balanced

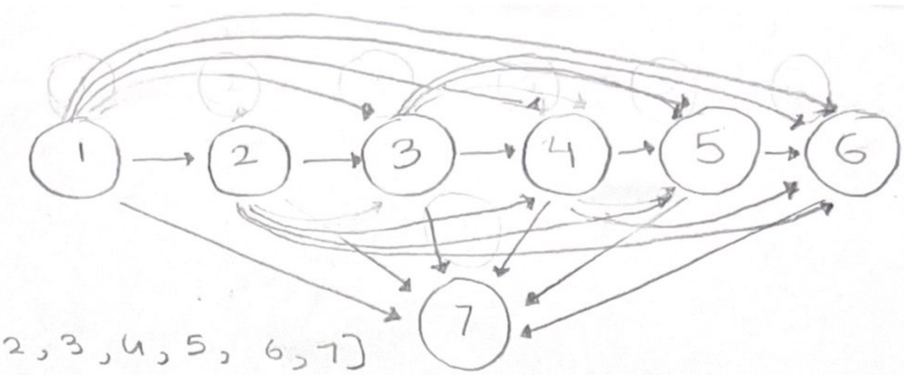
$$\sum_{i \in D} b_i = \sum_{j \in J} d_j$$

4) At most m_j cars must be sent

$$x_{ij} \leq m_{ij} \quad \forall (i,j) \in A$$

5) $x_{ij} \geq 0 \quad \forall (i,j) \in A$

4. Making Boxes



N, Nodes = [1, 2, 3, 4, 5, 6, 7]

where 1, ... 6 are the box's

7 is the destination

A, Arc's = [(1, 2), (1, 3), (1, 4), (1, 5), (1, 6) (1, 7)
(2, 3), (2, 4), (2, 5), (2, 6), (2, 7)
(3, 4), (3, 5), (3, 6), (3, 7)
(4, 5), (4, 6), (4, 7)
(5, 6), (5, 7)
(6, 7)]

From each box, an arc to all boxes - (i, j) with size of $i > j$

DV : $x_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is selected to satisfy demand } (i,j) \in A \text{ and } i > j \\ 0 & \text{otherwise} \end{cases}$

Demand D_i = [40, 30, 50, 70, 20, 40]

C_{ij}, Costs = Fixed cost + Variable cost

$i \downarrow$ \ $j \rightarrow$	1	2	3	4	5	6
1	$10 + \frac{33}{10} \times 40$ = \$142	$10 + \frac{33}{10} \times 70$ = \$241	$10 + \frac{33}{10} \times (120)$ = \$406	$10 + \frac{3.3}{10} \times (190)$ = \$637	$10 + \frac{3.3}{10} \times (210)$ = \$703	$10 + \frac{33}{10} \times (250)$ = \$835
2		$10 + \frac{30}{10} \times 30$ = \$100	$10 + \frac{30}{10} \times (80)$ = \$250	$10 + \frac{30}{10} \times (150)$ = \$460	$10 + \frac{30}{10} \times (170)$ = \$520	$10 + \frac{30}{10} \times (210)$ \$640
3			$10 \times \frac{26}{10} \times 50$ = \$140	$10 + \frac{26}{10} \times (120)$ = \$322	$10 + \frac{26}{10} \times (140)$ = \$374	$10 + \frac{26}{10} \times (180)$ = \$478
4				$10 + \frac{25}{10} \times (70)$ = \$185	$10 + \frac{25}{10} \times (90)$ = \$235	$10 + \frac{25}{10} \times (130)$ = \$335
5					$10 + \frac{20}{10} \times 20$ = \$50	$10 + \frac{20}{10} \times 60$ = \$130
6						$10 + \frac{19}{10} \times 40$ = \$86

obj = Minimize $\sum_{(i,j)\in A} c_{ij}\, x_{ij}$

Constraints :

Capacity : $0 \le x_{ij} \le 1$

Flow Balance

Node 1 : $x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} = 1$

Node 2 : $x_{23} + x_{24} + x_{25} + x_{26} + x_{27} - x_{12} = 0$

Node 3 = $x_{34} + x_{35} + x_{36} + x_{37} - x_{13} - x_{23} = 0$

Node 4 = $x_{45} + x_{46} + x_{47} - x_{14} - x_{24} - x_{34} = 0$

Node 5 = $x_{56} + x_{57} - x_{15} - x_{25} - x_{35} - x_{45} = 0$

Node 6 = $x_{67} - x_{16} - x_{26} - x_{36} - x_{46} - x_{56} = 0$

Node 7 = $-x_{17} - x_{27} - x_{37} - x_{47} - x_{57} - x_{67} = -1$