1. Garden Diet

1-1 Decision Variables

x_1 = quantity (in units) of plant 1

x_2 = quantity (in units) of plant 2

x_3 = quantity (in units) of plant 3

x_4 = quantity (in units) of plant 4

x_5 = quantity (in units) of plant 5

Objective : Minimize the total cost

Minimize $5.5x_1 + 8.5x_2 + 7.0x_3 + 6.2x_4 + 5.9x_5$

Constraints

1) $x_1 + x_2 + x_3 + x_4 + x_5 = 100$
2) $4x_1 + 3.5x_2 + 6x_3 + 3.0x_4 + 5.0x_3 \leq 550$
3) $10x_1 + 36x_2 + 15x_3 + 18x_4 + 12x_3 \geq 1000$
4) $0x_1 + 6x_2 + 4x_3 + 8x_4 + 6x_5 \geq 500$
5) $x_1, x_2, x_3, x_4, x_5 \geq 0$

$\therefore$ The LP is

$$\boxed{\begin{array}{ll} \text{minimize} & 5.5x_1 + 8.5x_2 + 7.0x_3 + 6.2x_4 + 5.9x_5 \\ \text{subject to} & x_1 + x_2 + x_3 + x_4 + x_5 = 100 \\ & 4x_1 + 3.5x_2 + 6x_3 + 3.0x_4 + 5.0x_3 \leq 550 \\ & 10x_1 + 36x_2 + 15x_3 + 18x_4 + 12x_3 \geq 1000 \\ & 0x_1 + 6x_2 + 4x_3 + 8x_4 + 6x_5 \geq 500 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \\ & x_3 \geq 0 \\ & x_4 \geq 0 \\ & x_5 \geq 0 \end{array}}$$

1-2 Optimal values:-

$x_1 = 16.66$

$x_2 = 0$

$x_3 = 0$

$x_4 = 0$

$x_5 = 83.83$

$\therefore$ minimum cost $= 5.5(16.66) + 5.9(83.33) = \$583.33$

Prof. Smith can plant 16.66 of plant 1 and 83.83 of plant 5 in order to have a minimum total cost of \$583.33

2. General Garden

2-1 Set of plants

$P = \{Plant1, Plant2, Plant3, Plant4, Plant5\}$

Parameters

c_p = dollars per plant in $p \in P$

a_{ip} = properties $i \in I$ obtained by one unit of plant in $p \in P$

b_i = Properties $i \in I$ met or exceeded ($\geq$

m = Total amount of plants to be purchased

u_p : maximum number of unit plants that can be purchased of each type $p \in P$

Decision Variables

x_p : Number of units purchased for plant p, for each $p \in P$

Objective Function $\min \sum_{p \in P}^{n} c_p \cdot x_p$

∴ The LP is

$$\min \sum_{p \in P}^{n} c_p \cdot x_p$$

$$\text{subject to} \quad \sum_{p \in P} a_{ip} \cdot x_p \geq b_i \quad \forall i \in I$$

$$\sum_{p \in P} x_p = m \quad \forall_p \in P$$

$$0 \leq x_p \leq u_p \quad \forall p \in P$$

2-2 Optimal value :-

$P_1 = 100$

$P_2 = 0$

$P_3 = 13.25$

$P_4 = 13.19$

$P_5 : 0$

$P_6 = 0$

Optimal objective value = \$736.505

By planting 100 of P_1, 13.25 of P_2 and 13.19 of P_3, our minimum cost of planting this garden is \$ 736.505

3) Officer Scheduling

3-1) Decision Variable :

x_i : number of police officers working 8 hours , $i = 1,2,3,4,5$

Period	1	2	3	4	5	6
x_1	1	1				
x_2		1	1			
x_3			1	1		
x_4				1	1	
x_5					1	1
# Required	10	11	9	6	5	4

Objective is to minimize $x_1 + x_2 + x_3 + x_4 + x_5$

∴ LP is

$$\begin{aligned}
\text{Minimize} \quad & x_1 + x_2 + x_3 + x_4 + x_5 \\
\text{subject to} \quad & x_1 \geq 10 \\
& x_1 + x_2 \geq 11 \\
& x_2 + x_3 \geq 9 \\
& x_3 + x_4 \geq 6 \\
& x_4 + x_5 \geq 5 \\
& x_5 \geq 4 \\
& x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}$$

3-2) x_i = number of police officers working 12 hour shifts , $i = 1,2,3,4$

y_i number of police officers working 8 hour shifts , $i = 1,2,3,4,5$

Period	1	2	3	4	5	6	cost
x_1	1	1	1				\$ 650
x_2		1	1	1			\$ 650
x_3			1	1	1		\$ 650
x_4				1	1	1	\$ 650
y_1	1	1					\$ 400
y_2		1	1				\$ 400
y_3			1	1			\$ 400
y_4				1	1		\$ 400
y_5					1	1	\$ 400
Required	10	11	9	6	5	4	

∴ LP

$$\text{minimize } 650(x_1+x_2+x_3+x_4) + 400(y_1+y_2+y_3+y_4+y_5)$$

subject to

$$\begin{aligned}
x_1 + y_1 &\geq 10 \quad (1)\\
x_1 + x_2 + y_1 + y_2 &\geq 11 \quad (2)\\
x_1 + x_2 + x_3 + y_2 + y_3 &\geq 9 \quad (3)\\
x_2 + x_3 + x_4 + y_3 + y_4 &\geq 6 \quad (4)\\
x_3 + x_4 + y_4 + y_5 &\geq 5 \quad (5)\\
x_4 + y_5 &\geq 4 \quad (6)\\
x_1, x_2, x_3, x_4 &\geq 0\\
y_1, y_2, y_3, y_4, y_5 &\geq 0
\end{aligned}$$

4 At the Opera

4-1)

Costs: Weekdays only : \$ 50 (C)
one weekend day : \$ 150 (B)
Two weekend days : \$250 (A, D)

	Sun	Mon	Tue	Wed	Th	Fri	Sat	cost (\$)
A	1					1	1	50(1) + 100(2) = 250
B	1	1	1	1	1			50(4) + 100(1) = 300
C			1	1	1	1		50(4) = 200
D	1				1	1	1	50(2) + 100(2) = 300
#Required	30	8	15	20	25	30	50	

∴ LP is

Objective : Min $250A + 300(B+D) + 200C$

Subject to :

$$\begin{aligned}
A + B + D &\geq 30\\
B &\geq 8\\
B + C &\geq 15\\
B + C &\geq 20\\
B + C + D &\geq 25\\
A + C + D &\geq 30\\
A + D &\geq 50\\
A, B, C, D &\geq 0
\end{aligned}$$

4-2) optimal solution

$A = 45$

$B = 8$

$C = 12$

$D = 5$

optimal objective = \$ 17550

The optimum usher staffing levels for the overture center is 45 for type A, 8 for type B, 12 for type C, 5 for type D, having a minimized salary cost of \$17550