Homework 8

1) Routing Beer

1.1 Sets

Set of possible trips : $S = \{(D,B1), (D,B2), (D,B3), (D,B4), (D,B1,B2), (D,B1,B3), (D,B2,B3), (D,B1,B4), (D,B3,B4)\}$

Set of distances : $R = [10, 24, 14, 30, 21, 33, 27, 30]$

Decision Variable:

x_j { 1, if trip j is selected, $j = 1,2 \ldots 8$ [8 trips in S]
0, otherwise

Objective : minimize the total distance

Min $\sum R_j \times x_j$ for j in S [$j = 1,2,3,4,5,6,7,8$]

Constraints:

Bar B_1 $x_1 + x_5 + x_7 \geq 1$ [needs 7]

Bar B_2 $x_2 + x_5 + x_6 \geq 1$ [needs 2]

Bar B_3 $x_3 + x_6 + x_8 \geq 1$ [needs 6]

Bar B_4 $x_4 + x_7 + x_8 \geq 1$ [needs 3]

$x_j \in \{0,1\}$ for $j = 1,2 \ldots 9$

(1.2) Minimum distance = 251 miles

Routes selected : $D \rightarrow B_1 \rightarrow B_2 \rightarrow D$

$D \rightarrow B_3 \rightarrow B_4 \rightarrow D$

2 Basis

$$\max z = 3x_1 + 2x_2$$

$$\text{s.t} \quad x_1 + 2x_2 \leq 2$$

$$3x_1 + 4x_2 \leq 6$$

$$x_1 \geq 0 \; , x_2 \geq 0$$

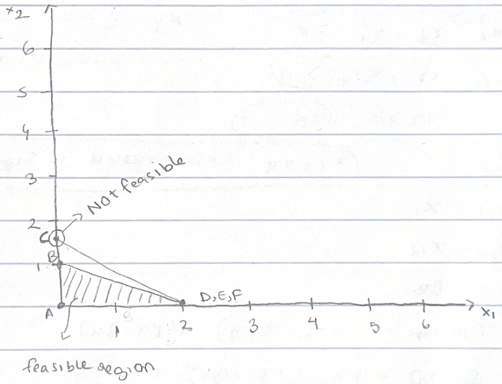
$$\Rightarrow \quad \max z = 3x_1 + 2x_2$$

$$\text{st} \quad x_1 + 2x_2 + s_1 = 2$$

$$3x_1 + 4x_2 + s_2 = 6$$

$$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0 \; , s_2 \geq 0$$

2-1



	Basic Solution #	NBV	BV	x_1, x_2, s_1, s_2	Feasible ?	Objective V
2-2)	A	(x_1, x_2)	(s_1, s_2)	0, 0, 2, 6	Yes	0
	B	(x_1, s_1)	(x_2, s_2)	0, 1, 0 2	yes	2
	C	(x_1, s_2)	(x_2, s_1)	0, 1.5, -1, 0	NO	3
	D	(s_1, s_2)	(x_1, x_2)	2, 0, 0, 0	Yes	6
	E	(x_2, s_2)	(x_1, s_1)	2, 0, 0, 0	Yes	6
	F	(x_2, s_1)	(x_1, s_2)	2, 0, 0, 0	Yes	6

3 LP Standard Format

$$\min \; x_1 - 2x_2 - 3x_3 - 2x_4$$

$$\text{st} \quad x_1 + 3x_2 + x_4 \leq 5$$

$$3x_2 - x_3 - x_4 = 8$$

$$3x_1 - x_2 + 3x_4 \geq 4$$

$$x_2 \geq 0, \; x_3 \leq 0$$

x_1, x_4 Unrestricted in Sign(Free)

$$\Rightarrow \; p_1 - q_1 = x_1$$

$$p_4 - q_4 = x_4$$

$$x_3 = -y_3$$

$$\min \; [p_1 - q_1] - 2x_2 - 3(-y_3) - 2[p_4 - q_4]$$

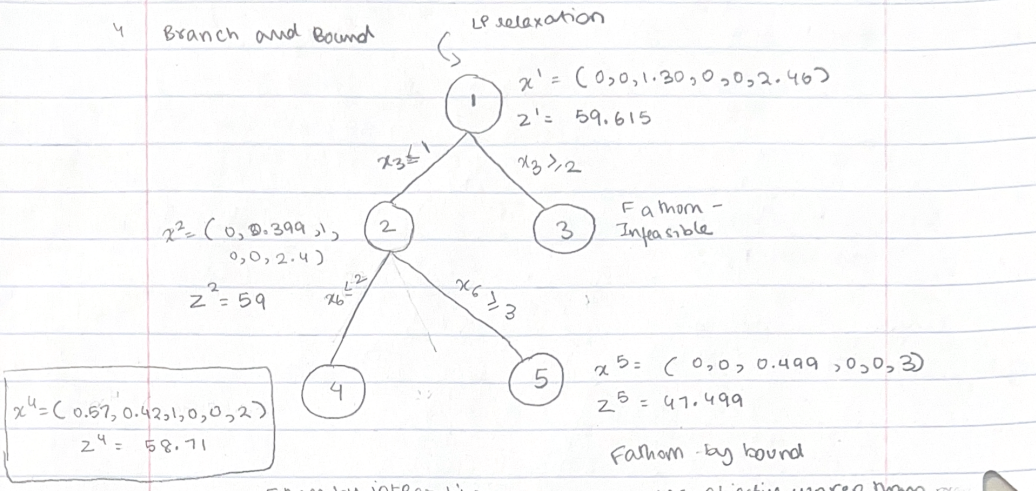
$$\text{st} \quad [p_1 - q_1] + 3x_2 + [p_4 - q_4] + s_1 = 5$$

$$2x_2 - (-y_3) - [p_4 - q_4] = 8$$

$$3[p_1 - q_1] - x_2 + 3[p_4 - q_4] - s_2 = 4$$

$$\{x_2, -y_3, p_1, p_4, q_1, q_4, s_1, s_2\} \geq 0$$

4 Branch and Bound



Optimal!

(No more nodes to explore)

Fathom by integrality of x^4

Fathom by bound b/c objective worse than previous branches