1 Now Hiring!!

P : The set of time periods for which staffing req. must be met

b_i : the min. number of baristas that must be working during period $i \in P$

S : the set of all possible schedules a barista can work

a_{si} : defines the periods covered by schedule $s \in S$. $a_{si} = 1$ means the schedule $s \in S$ covers period $i \in P$, and $a_{si} = 0$ means the schedule $s \in S$ does not cover period $i \in P$

c_s : the cost to assign a barista to schedule $s \in S$

f_s : indicates if schedule $s \in S$ is full time. $f_s = 1$ means schedule s is full time, $f = 0$ means schedule s is part-time.

m : maximum number of baristas that can be assigned to a part-time schedule.

1-1

Decision Variables

x_s : Number of baristas assigned to schedule $s \in S$

Objective

minimize $$\sum_{s \in S} c_s \cdot x_s$$

Constraints

1) minimum number of baristas working during period $i \in P$

$$\sum_{s \in S} a_{si} \cdot x_s \geq b_i \qquad \forall i \in P$$

2) Part time schedule limitation

$$\sum_{s \in S \mid f_s = 0} x_s \leq m$$

3) $$x_s \geq 0 \qquad \forall s = S$$

2 Venture Capital

→ 5 different startup companies

→ 4 year expenditure for each startup

	1	2	3	4	5
Years 1 Cost (\$million)	6	2	8	10	2
Years 2 Cost (\$million)	2	5	9	11	9
Years 3 Cost (\$million)	4	3	10	9	6
Years 4 Cost (\$million)	2	3	4	8	3
NPV of investment (\$million)	6	5	12	13	9

→ Budget \$20 million

→ 1, 2, 5 low risk

→ 3, 4 high risk

→ amt spent high risk = 40% total amount

Sum of Total amount in fraction = 1

2-1

Decision Variables

x_i → fraction spent on each company $i = 1,2,3,4,5$

Objective

maximize $6x_1 + 5x_2 + 12x_3 + 13x_4 + 9x_5$

Subject to

$$6x_1 + 2x_2 + 8x_3 + 10x_4 + 2x_5 \le 20$$
$$2x_1 + 6x_2 + 9x_3 + 11x_4 + 9x_5 \le 20$$
$$4x_1 + 3x_2 + 10x_3 + 9x_4 + 6x_5 \le 20$$
$$2x_1 + 3x_2 + 4x_3 + 8x_4 + 3x_5 \le 20$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

$$8x_3 + 10x_4 \le 0.4\ (x_1 + x_2 + x_3 + x_4 + x_5)$$
$$9x_3 + 11x_4 \le 0.4\ (x_1 + x_2 + x_3 + x_4 + x_5)$$
$$10x_3 + 9x_4 \le 0.4\ (x_1 + x_2 + x_3 + x_4 + x_5)$$
$$4x_3 + 8x_4 \le 0.4\ (x_1 + x_2 + x_3 + x_4 + x_5)$$

$$0 \le x_i \le 1$$

In General format

S = set of start-up companies $[1,2,3,4,5]$

T : Time period [Year 1,2,3,4]

parameters P_i = NPV of cash flow where $i \in S$

B_j : available budget for $j \in T$

c_{ij} : cash expenditures for company $i \in S$ in year $j \in T$

DV : x_i → fraction spent on each company $i = 1,2,3,4,5$

Objective max $\sum_{i \in S} p_i \cdot x_i$

Subject to $\sum_{i \in S} c_{ij} \cdot x_i \le B_j \quad \forall j \in T$

$$\sum_{i \in \{3,4\}} c_{ij}\, x_i \le 0.4 \sum_{i \in S} c_{ij}\, x_i \quad \text{for } j = 1,2,\dots 4$$

$$\sum_{i \in S} x_i = 1$$

$$0 \le x_i \le 1 \quad \forall i \in S$$

$x_1 = 0$ $x_2 = 0$

$x_3 = 0.1428$

$x_4 = 0$

$x_5 = 0.857142$

Max NPV = \$ 9.42857 million dollars

3 Optimal Bus Strategy

Area	Fraction Minority	Number Students
ES	0.2	1200
D	0.1	900
NS	0.85	1700
SW	0.6	2000
FW	0.4	2500

→ no school can have more than 70% minority enrollment nor less than 30% minority enrollment.

→ 3 schools → CE, WCE, CC

School	Capacity	ES	D	NS	SW	FW
				Area		
CE	3900	2.7	1.4	2.4	1.1	0.5
WCE	3100	0.5	0.7	2.9	0.8	1.9
CC	2100	1.6	2.0	0.1	1.3	2.2

Objective is to minimize total distance

Sets

Area, $A = \{ES, D, NS, SW, FW\}$

School, $S = \{CE, WCE, CC\}$

Parameters

d_{ij} = distance travelled from area $i \in A$ to school $j \in S$

n_i = number of students in area $i \in A$

m_i = fraction of minority students in area $i \in A$

c_j = capacity of each school $j \in S$

Decision variables

x_{ij} = number of students that travel from area $i \in A$ to school $j \in S$

Objective

Minimize $\sum_{i \in A} \sum_{j \in S} (d_{ij} \cdot x_{ij})$

1) Capacity

$$\sum_{i \in A} x_{ij} \leq c_j \qquad \forall j \in S$$

2) Fraction of Minority Students

$$0.3 \sum_{i \in A} x_{ij} \leq \sum_{i \in A} (m_i \cdot x_{ij}) \leq 0.7 \sum_{i \in A} x_{ij} \qquad \forall j \in S$$

3) Student Assignment Constraints

$$\sum_{j \in S} x_{ij} = \sum_{i \in A} n_i$$

4) $x_{ij} \geq 0 \qquad \forall i \in A, \forall j \in S$

4) You're a Big Success

Sets A = Set of district areas in each city

S = set of schools

Parameters

r_a = percentage of minorities living in that area

n_a = students living in that area $a \in A$

b_s = capacity of school $s \in S$

d_{as} = distance between area a & school s

l = low percentage

h = high percentage

k = min students enrolled in school $s \in S$

DV : x_{as} → number of students from each area $a \in A$, assigned to school $s \in S$.

y_s → the total number of students that go to school s

Objective : $\min \sum_{a \in A} \sum_{s \in S} d_{as} x_{as}$ Distance Travelled

st

$$\sum_{a \in A} x_{as} = y_s \quad \forall s \in S$$

$$\sum_{s \in S} x_{as} = n_a \quad \forall a \in A$$

$$y_s \leq b_s \quad \forall s \in S$$

$$\sum_{a \in A} r_a x_{as} - h y_s \leq 0 \quad \forall s \in S$$

$$\sum_{a \in A} r_a x_{as} - l y_s \geq 0 \quad \forall s \in S$$

$$y_s \geq k \quad \forall s \in S$$

$$x_{as} \geq 0 \quad \forall a \in A, \forall s \in S$$

4-2 i. Optimal objective value

= 4810