NEAR-FIELD/FAR-FIELD MODEL MISMATCH ANALYSIS FOR RIS-ASSISTED LOCALISATION

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In the rapidly evolving environment of wireless communications, the precision and dependability of user equipment (UE) positioning play a major role in improving the performance of various applications, including navigation, location-based services, and emergency response. Accurate localization begins with channel estimation and structural feature extraction, typically based on the assumption of a sparse channel with a limited number of paths. The position of the UE can be inferred by estimating the angle and delay relative to known reference points, such as base stations. With the substantial bandwidth and array size offered by millimeter-wave (mmWave) and terahertz (THz) systems, high angular and delay resolution is achievable, leading to significantly improved localization performance. Reconfigurable intelligent surfaces (RISs) are poised to revolutionize wireless systems by providing smart radio environments where the propagation channel can be dynamically programmed to enhance the Quality of Service (QoS). [1,2] A RIS can be described as a planar array of sub-wavelength component elements, each capable of scattering the incoming signal by varying its phase in a controlled manner [2]. These elements can be electronically manipulated to modify the electromagnetic properties of incident signals, including phase, amplitude, or polarization. By optimizing the RIS phase profile, it is possible to achieve improved beamforming, localization, and interference management. Consequently, RISs are essential for signal modelling in advanced wireless systems, offering significant potential to enhance signal quality and positioning accuracy.

System Geometry

The system and signal model are for single-antenna user equipment, with an emphasis on an uplink scenario in which a base station (BS) with a single reconfigurable intelligent surface (RIS) predicts the location of a single-antenna UE. The model proposed here assumes a synchronised system without a line-of-sight (LOS) path, providing a simple but instructive framework for analysing the interactions between the BS, RIS, and UE. The user's position is mathematically represented, considering the signal's time of arrival (TOA) and angle of arrival (AOA), both of which are crucial parameters for precise localisation. Here we begin with the signal model and the considered MM and TM are described. Consider an uplink system with a Base Station with a reflective ULA RIS with N elements, centered at the origin $\mathbf{p}_r = [0,0]^{\top}$ and each RIS element is located at $\mathbf{p}_n = [x_n, y_x]^{\top}$ ($y_n = 0$). The goal is to analyse the estimation bounds of the azimuth angle of the UE. For Base Station, the antenna is located at $\mathbf{b} = \begin{bmatrix} x_b, & y_b \end{bmatrix}^T$ where λ_c is the wavelength of the carrier frequency f_c. We analyse a simple scenario by assuming the system is synchronized and no LOS path exists. The position of User Equipment $\mathbf{p}$ can be expressed as $\mathbf{p} = \tau c \begin{bmatrix} \cos(\theta) & \sin(\theta) \end{bmatrix}^T$ where θ is the angle-of-arrival (AOA), τ is the time-of-arrival (TOA), and c is the speed of light.

Signal Model

A mismatched channel model has been provided to investigate the differences between ideal and practical channel conditions. This model includes the complex channel gain, steering vector, and delay components, which together make up the channel vector for each subcarrier and emphasise the effects of phase shifts and variations in frequency on the received signal.

Considering x_{uv} ($|x_{uv}|^2 = P$ is the average transmission power) as the transmitted orthogonal frequency-division multiplexing (OFDM) symbol at u-th transmission (1 ≤

$u \leq U$) and v-th subcarrier ($1 \leq v \leq V$), the observation at the Base Station can be given as

$$y_{uv} = h_{uv} x_{uv} + n_{uv} \tag{1}$$

where $h_{uv} \in \mathbf{C}^{1\times1}$ is the channel vector at the v-th subcarrier which is assumed to be constant during U transmissions and $n_{uv} \in \mathbf{C}^{1\times1}$ denotes the noise component following a complex normal distribution.

Mismatched Channel Model

We consider a widely used channel model as mismatched model. The channel vector for the v-th subcarrier $h_v^{MM} \in \mathbb{C}^{1\times1}$ can be formulated using a complex channel gain $\alpha \in \mathrm{C}^{1\times1}$, steering vectors, one from the base station $\mathbf{a}_{pb} \in \mathrm{C}^{N\times1}$ and the other from the UE $\mathbf{a}_p \in \mathrm{C}^{N\times1}$, RIS phase profile vector $\phi \in \mathrm{C}^{N\times1}$ and is the phase shift applied by RIS and delay component $D_v(\tau)$ as

$$h_v^{MM} = \alpha \mathbf{a}_{pb}^{\top} \mathbf{\Phi} \mathbf{a}_p D_v(\tau) \tag{2}$$

where $\mathbf{\Phi} = \mathrm{diag}(\phi)$.

$$\alpha = \frac{(\lambda_c)^2}{(4\pi)^2 \|\mathbf{p}\| \|\mathbf{p}_b\|} e^{-j\xi}$$

$$\mathbf{a}_{pb} = \left[e^{-j\pi\left(\frac{N-1}{2}\right)\sin\zeta} \quad \ldots \quad 1 \quad \ldots \quad e^{-j\pi\left(\frac{1-N}{2}\right)\sin\zeta} \right]^T \tag{4}$$

$$\mathbf{a}_{p} = \left[e^{-j\pi\left(\frac{N-1}{2}\right)\sin\theta} \quad \ldots \quad 1 \quad \ldots \quad e^{-j\pi\left(\frac{1-N}{2}\right)\sin\theta} \right]^T \tag{5}$$

$$D_v(\tau) = e^{-j2\pi(f_c + v\Delta f)\tau} = D_v(\mathbf{p}) = e^{-j\frac{2\pi}{\lambda_v}\|\mathbf{p}\|} \tag{3}$$

where θ is the angle of arrival from the UE, ζ is the angle of departure to the BS and α is the complex channel gain. Therefore,

$$\tilde{y}_{uv} = \alpha \, \mathbf{a}^{\top}_{pb} \, \boldsymbol{\Phi} \mathbf{a}_{p}(\theta) \, D_{v}(\tau) \, x_{uv} + n_{uv}. \tag{4}$$

Variable Descriptions

- N: Number of elements of RIS.
- α: Complex channel gain, representing the strength and phase shift of the signal.
- θ: Angle of Arrival (AOA).
- ζ: Angle of Departure (AOD).
- τ: Time of Arrival (TOA).
- c: Speed of light (approx. 3×10^{8} m/s)
- λ_c: Wavelength of the carrier frequency f_c.
- $\|\mathbf{p}\|$: Norm (magnitude) of the position vector $\mathbf{p}$, indicating the distance between the transmitter and receiver, located at the origin of the cartesian system.
- j: Imaginary unit, where $j^2 = -1$.
- ϕ: Phase component, representing the phase shift in the channel.

True Model

When comparing the mismatched model with the true model, the channel vector for the true model can be written as:

$$h_v^{\mathrm{TM}} = \alpha_v(\mathbf{p}) \odot \boldsymbol{a}_{pb,v}^{\top} \boldsymbol{\Phi} \boldsymbol{a}_{p,v}(\mathbf{p}) D_v(\mathbf{p}) \tag{5}$$

$$\alpha_{v,n} = \alpha c_{v,n} \tag{6}$$

$$c_{v,n}(\mathbf{p}) = \frac{\lambda_v \|\mathbf{p}\|}{\lambda_c \|\mathbf{p} - \mathbf{b}_n\|} \tag{7}$$

$$[\mathbf{a}_{p,v}(\mathbf{p})]_n = e^{-j\frac{2\pi}{\lambda_v}(\|\mathbf{p}-\mathbf{p}_n\| - \|\mathbf{p}\|)} \tag{8}$$

Here $\alpha_{v,n}$ is the n-th element of the vector α_v which indicates the channel non-stationarity band $d_{v,n}$ is the n-th element of the steering vector d_v.

$$\bar{y}_{uv} = \alpha_v(\mathbf{p}) \odot \boldsymbol{a}_{pb,v}^{\top} \boldsymbol{\Phi} \boldsymbol{a}_{p,v}(\mathbf{p}) D_v(\mathbf{p}) x_{uv} + n_{uv} \tag{9}$$

CRLB (Cramer-Rao Lower Bound)

Let the channel parameter vector $\mathbf{\Gamma} = [\mathrm{Re}(\alpha), \mathrm{Im}(\alpha), \theta]^T$ and introduce the noiseless part of the received signal as

$$\mu_{uv} = \alpha_v(\mathbf{p}) \odot a_{\top pb,v} \mathbf{\Phi} a_{p,v}(\mathbf{p}) D_v(\mathbf{p}) x_{uv} \tag{10}$$

The FIM can be computed as

$$\mathrm{FIM} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re\left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Gamma} \right)^H \frac{\partial \mu_{u,v}}{\partial \Gamma} \right\}, \tag{11}$$

then the element of the FIM at the i-th row and the j-th column can be written as

$$\mathrm{FIM}_{i,j} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re\left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Gamma_i} \right)^H \frac{\partial \mu_{u,v}}{\partial \Gamma_j} \right\}. \tag{12}$$

Then the angle error bound (AEB) is given by:

$$\text{AEB} = \sqrt{[\text{FIM}^{-1}]_{(3,3)}} \tag{13}$$

Deriving FIM Elements

To compute the FIM, we need to compute the partial derivatives of the received signal $y_{u,v}$ with respect to the parameters.

Derivative w.r.t. real α

$$\text{FIM}_{1,1} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Re(\alpha)} \right)^H \frac{\partial \mu_{u,v}}{\partial \Re(\alpha)} \right\}. \tag{14}$$

$$\text{FIM}_{1,2} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Re(\alpha)} \right)^H \frac{\partial \mu_{u,v}}{\partial \Im(\alpha)} \right\}. \tag{15}$$

$$\text{FIM}_{1,3} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Re(\alpha)} \right)^H \frac{\partial \mu_{u,v}}{\partial \theta} \right\}. \tag{16}$$

Derivative w.r.t. imaginary α

$$\text{FIM}_{2,1} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Gamma_2} \right)^H \frac{\partial \mu_{u,v}}{\partial \Gamma_1} \right\}. \tag{17}$$

$$\text{FIM}_{2,2} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Gamma_2} \right)^H \frac{\partial \mu_{u,v}}{\partial \Gamma_2} \right\}. \tag{18}$$

$$\text{FIM}_{2,3} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Gamma_2} \right)^H \frac{\partial \mu_{u,v}}{\partial \Gamma_3} \right\}. \tag{19}$$

Derivative w.r.t. AoA θ

$$\text{FIM}_{3,1} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Gamma_3} \right)^H \frac{\partial \mu_{u,v}}{\partial \Gamma_1} \right\}. \tag{20}$$

$$\text{FIM}_{3,2} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Gamma_3} \right)^H \frac{\partial \mu_{u,v}}{\partial \Gamma_2} \right\}. \tag{21}$$

$$\text{FIM}_{3,3} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Gamma_3} \right)^H \frac{\partial \mu_{u,v}}{\partial \Gamma_3} \right\}. \tag{22}$$

Derivatives

$$\frac{\partial \mu_{uv}}{\partial \Re(\alpha)} = \boldsymbol{a}_{pb}^{\top} \cdot \boldsymbol{\Phi} \cdot \boldsymbol{a}_p(\theta) \cdot D_v(\tau) \cdot x_{uv} \tag{23}$$

$$\frac{\partial \mu_{uv}}{\partial \Im(\alpha)} = j \cdot \boldsymbol{a}_{pb}^{\top} \cdot \boldsymbol{\Phi} \cdot \boldsymbol{a}_p(\theta) \cdot D_v(\tau) \cdot x_{uv}$$

$$\frac{\partial \mu_{uv}}{\partial \theta} = \alpha \cdot \boldsymbol{a}_{pb}^{\top} \cdot \boldsymbol{\Phi} \cdot \frac{\partial \boldsymbol{a}_p(\theta)}{\partial \theta} \cdot D_v(\tau) \cdot x_{uv}$$

Now the derivative $\frac{\partial \boldsymbol{a}_p(\theta)}{\partial \theta}$ can be computed as

$$\frac{\partial}{\partial \theta} \mathbf{a}_p = \begin{bmatrix} -j\pi \left(\frac{N-1}{2}\right) \cos\theta e^{-j\pi\left(\frac{N-1}{2}\right)\sin\theta} \\ \vdots \\ 0 \\ \vdots \\ -j\pi \left(\frac{1-N}{2}\right) \cos\theta e^{-j\pi\left(\frac{1-N}{2}\right)\sin\theta} \end{bmatrix}$$

$$\frac{\partial}{\partial \theta} \mathbf{a}_p = [-j\pi \left(\frac{N-1}{2}\right) \cos\theta e^{-j\pi\left(\frac{N-1}{2}\right)\sin\theta} \ldots 0 \ldots -j\pi \left(\frac{1-N}{2}\right) \cos\theta e^{-j\pi\left(\frac{1-N}{2}\right)\sin\theta}]^T \tag{24}$$

Mismatched CRLB

The MCRLB provides a lower bound on the variance of any unbiased estimator when there is a mismatch between the assumed and true models. The MCRLB is useful when the assumed model differs from the true model, which can happen due to model uncertainties, approximations, or incorrect assumptions. In many systems, we assume a simplified model (mismatched model, MM) while the actual system follows a more complex true model (TM). This mismatch introduces bias in the estimator. Let's define the true model as:

$$\mu_{TM}(\mathbf{\Gamma}) = \mu(\mathbf{\Gamma},\text{True})$$

$$\mu_{TM} = \alpha_v(\mathbf{p}) \odot a^{\top}_{pb,v}\mathbf{\Phi}a_{p,v}(\mathbf{p})D_v(\mathbf{p})x_{uv} \tag{25}$$

$$\mu_{TM} = \bar{\mu}_{u,v} \tag{26}$$

And the mismatched model as:

$$\mu_{MM}(\mathbf{\Gamma}) = \mu(\mathbf{\Gamma},\text{Mismatched}) \tag{27}$$

$$\mu_{\text{MM}} = \alpha \mathbf{a}_{pb}^{\top} \mathbf{\Phi} \mathbf{a}_p(\theta) D_v(\tau) x_{uv} \tag{28}$$

$$\mu_{MM} = \tilde{\mu}_{u,v} \tag{29}$$

where Γ is the parameter vector that we are estimating

$$\mathbf{\Gamma} = [\Re(\alpha), \Im(\alpha), \theta]$$

where $\Re(\alpha)$ and $\Im(\alpha)$ represent the real and imaginary parts of the complex channel gain α, and θ is the angle of arrival (AoA).

When estimating Γ, if we use the mismatched model μ_{MM}, there will be a bias in the estimator, since the true signal is μ_{TM}.

The Fisher Information Matrix (FIM) quantifies the amount of information that the observed data carries about the unknown parameter vector Γ.

The FIM for the *mismatched model* is given by:

$$FIM_{MM} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{\tilde{u},v}}{\partial \Gamma} \right)^H \frac{\partial \mu_{\tilde{u},v}}{\partial \Gamma} \right\}, \tag{30}$$

The FIM for the *True model* is given by:

$$FIM_{TM} = \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{\bar{u},v}}{\partial \Gamma} \right)^H \frac{\partial \mu_{\bar{u},v}}{\partial \Gamma} \right\}, \tag{31}$$

where N_0 is the noise variance, and U and V are the number of transmissions and subcarriers, respectively.

The bias introduced by using the mismatched model instead of the true model can be calculated as the difference between the signals generated by the true and mismatched models:

$$\text{bias}(\Gamma) = \mu_{\text{TM}}(\Gamma) - \mu_{\text{MM}}(\Gamma)$$

This bias directly impacts the estimator’s variance, leading to a degradation in performance.

The MCRLB accounts for the bias in the estimator due to the model mismatch. It combines the Fisher Information Matrix for the mismatched model and the bias term to compute the lower bound on the estimation error. The MCRLB is given by:

$$\text{MCRLB} = A_{\theta_0}^{-1} B_{\theta_0} A_{\theta_0}^{-1} + (\theta - \theta_0)(\theta - \theta_0)^T$$

Where:

- $A_{\theta 0}$ is the FIM for the mismatched model FIM_{MM},
- $B_{\theta 0}$ is the bias term derived from the true model and mismatched model.

$$MCRLB = MCRLB\ (\bar{\theta},\theta_0) = \underbrace{A_{\theta_0}^{-1} B_{\theta_0} A_{\theta_0}^{-1}}_{\text{MCRLB}(\theta_0)} + \underbrace{(\bar{\theta} - \theta_0)(\bar{\theta} - \theta_0)^T}_{\text{Bias}(\theta_0)}$$

where $\bar{\theta}$ is the parameter vector for the true model (TM) and θ_0 is the pseudo-true parameter vector The pseudo-true parameter vector θ_0 is given by:

$$\theta_0 = \arg\min \|\mu_{TM}(\theta) - \mu_{MM}(\theta)\|^2 \theta$$

The generalization of $A_{\theta 0}$ is given by:

$$[A_{\theta_0}]_{i,j} == \frac{2}{N_0} \sum_{u=1}^{U} \sum_{v=1}^{V} \Re \left\{ \left(\frac{\partial \mu_{u,v}}{\partial \Gamma_i} \right)^H \frac{\partial \mu_{u,v}}{\partial \Gamma_j} \right\}. \quad (32)$$

The generalization of $B_{\theta 0}$ is given by:

$$[B_{\theta_0}]_{i,j} = \frac{4}{\sigma_n^4} \Re \left[\frac{\partial^2 \mu_{u,v}(\theta)}{\partial \Gamma_i} \epsilon(\theta) \right] \Re \left[\frac{\partial^2 \mu_{u,v}(\theta)}{\partial \Gamma_j} \epsilon(\theta) \right] + \frac{2}{\sigma_n^2} \Re \left[\frac{\partial \mu_{u,v}(\theta)}{\partial \Gamma_j} \left(\frac{\partial \mu_{u,v}(\theta)}{\partial \Gamma_i} \right)^H \right]_{\theta=\theta_0}$$

where

$$\epsilon(\theta) = \mu_{TM} - \mu_{MM}$$

Derivatives for Mismatched Model and True Model

Derivative with Respect to $\Re(\alpha)$

For True Model (μ_{TM}): The model can be written as:

$$\mu_{\mathrm{TM}} = \alpha_v(p) \cdot a_{Tpb,v} \mathbf{\Phi} a_{p,v}(\theta) D_v(\tau)$$

The derivative with respect to $\Re(\alpha)$ is:

$$\frac{\partial \mu_{\mathrm{TM}}}{\partial \Re(\alpha)} = a_{pb,v}^T \mathbf{\Phi} a_{p,v}(\theta) D_v(\tau) \quad (33)$$

For Mismatched Model (μ_{MM}):

For the mismatched model:

$$\mu_{\text{MM}} = \alpha \cdot \boldsymbol{a}_{\boldsymbol{pb}}^{T} \boldsymbol{\Phi} \boldsymbol{a}_{\boldsymbol{p}}(\boldsymbol{\theta}) D_v(\tau)$$

The derivative with respect to $\Re(\alpha)$ is:

$$\frac{\partial \mu_{\text{MM}}}{\partial \Re(\alpha)} = \boldsymbol{a}_{\boldsymbol{pb}}^{T} \boldsymbol{\Phi} \boldsymbol{a}_{\boldsymbol{p}}(\theta) D_v(\tau)$$

This is similar to the true model because $\Re(\alpha)$ directly affects the entire signal.

Derivative with Respect to $\Im(\alpha)$

For True Model (μ_{TM}):

$$\mu_{\text{TM}} = \alpha \cdot \boldsymbol{a}_{\boldsymbol{pb,v}}^{T} \boldsymbol{\Phi} a_{p,v}(\theta) D_v(\tau)$$

Taking the derivative with respect to $\Im$ (α), we get:

$$\frac{\partial \mu_{\text{TM}}}{\partial \Im(\alpha)} = j \cdot \boldsymbol{a}_{\boldsymbol{pb,v}}^{T} \boldsymbol{\Phi} a_{p,v}(\theta) D_v(\tau)$$

The factor j appears because of the imaginary component

$\Im(\alpha)$.

For Mismatched Model (μ_{MM}):

Similarly, for the mismatched model: $\mu_{\text{MM}} = \alpha \boldsymbol{a}_{\boldsymbol{pb}}^{T} \boldsymbol{\Phi} \boldsymbol{a}_{\boldsymbol{p}}(\theta) D_v(\tau)$

Taking the derivative with respect to $\Im(\alpha)$:

$$\frac{\partial \mu_{\text{MM}}}{\partial \Im(\alpha)} = j \cdot \boldsymbol{a}_{\boldsymbol{pb}}^T \boldsymbol{\Phi} \boldsymbol{a}_{\boldsymbol{p}}(\theta) D_v(\tau)$$

Again, j multiplies the derivative since we are differentiating the imaginary part of α.

Derivative with Respect to θ (AoA)

The derivative with respect to θ is more complex, as it affects the steering vector $a_p(\theta)$.

Steering Vector Definition:

For a typical Uniform Linear Array (ULA), the steering vector is given by:

$$a_p(\theta) = \left[e^{-j\frac{2\pi}{\lambda}\frac{N-1}{2}\sin(\theta)}, e^{-j\frac{2\pi}{\lambda}\frac{N-2}{2}\sin(\theta)}, \ldots, e^{-j\frac{2\pi}{\lambda}\frac{1-N}{2}\sin(\theta)} \right]^T$$

The derivative of $a_p(\theta)$ with respect to θ is:

$$\frac{\partial a_p(\theta)}{\partial \theta} = -j\frac{2\pi}{\lambda} \left[\frac{N-1}{2}\cos(\theta) e^{-j\frac{2\pi}{\lambda}\frac{N-1}{2}\sin(\theta)}, \ldots, \frac{1-N}{2}\cos(\theta) e^{-j\frac{2\pi}{\lambda}\frac{1-N}{2}\sin(\theta)} \right]^T$$

For True Model (μ_{TM}):

$$\frac{\partial \mu_{\text{TM}}}{\partial \theta} = \alpha \cdot \boldsymbol{a}_{pb,v}^T \boldsymbol{\Phi} \cdot \left(-j\frac{2\pi}{\lambda} \cdot \frac{N-1}{2} \cdot \cos(\theta) \cdot \boldsymbol{a}_{p,v}(\theta) \right) D_v(\tau)$$

For Mismatched Model (μ_{MM}):

Similarly:

$$\frac{\partial \mu_{\text{MM}}}{\partial \theta} = \alpha \cdot \boldsymbol{a}_{pb}^T \boldsymbol{\Phi} \cdot \left(-j\frac{2\pi}{\lambda} \cdot \frac{N-1}{2} \cdot \cos(\theta) \cdot \boldsymbol{a}_p(\theta) \right) D_v(\tau)$$