Circumference and Area of Circle

14

Expected Learning Outcomes

At the end of the chapter, learners will be able to

- **Define** the circumference and of the circle.
- **Calculate** the circumference and of the circle.
- **Apply** the circumference and area of a circle to solve real life scenarios.

Warm Up

Complete the following table.

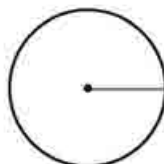| S. No.	Radius	Diameter
1.	_____	20 *cm*
2.	0.75 *cm*	_____
3.	_____	30 *mm*
4.	35 *cm*	_____
5.	_____	12.5 cm
6.	42 *mm*	_____

Circumference of a Circle

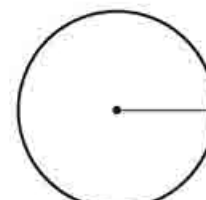*The distance around the circle is known as its circumference.*

Draw three circles of different radii such as 2 cm, 4 cm and 5 cm. With the help of a tape or a thread, measure the circumference and the diameter of each circle.

(i) (ii) (iii)

Fig. 14.1

Note down the observations as under:

	Radius (r)	Diameter (d)	Circumference ©	(C/d)
(I)	....	....	....	....
(ii)	....	....	....	....
(iii)	....	....	....	....

Compute the ratio of the circumference and diameter. Enter the same against each circle in the last column of the table. In each case, we will find:

$$\frac{\text{Circumference}}{\text{Diameter}} = 3.14 \text{ (approx.)}$$

The ratio is denoted by the Greek letter π and is read as *pie* and it is a constant

Its value is 3.14 (approx.) or $\frac{22}{7}$ (approx.)

Now, $\frac{\text{Circumference}}{\text{Diameter}} = \pi$

$\Rightarrow \qquad \frac{C}{2r} = \pi$

$\Rightarrow \qquad C = 2\pi r$

i.e., Circumference $= 2\pi \times$ radius

If d denotes the diameter of the circle, then $d = 2r$.

$\Rightarrow \qquad C = \pi d$

It is proved that π is not a rational number, while calculating, it is used as $\frac{22}{7}$ or 3.14.

Example 1: **Find the circumference of a circle whose radius is 4 cm.**

Solution: Here, radius $= 4$ cm

$\therefore \qquad$ Circumference (C) $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 4$$

$$= \frac{176}{7} \text{ cm}$$

Example 2: **The radius of the wheel of a cart is 63 cm. How many revolutions will it make to travel 7.920 km?**

Solution: Given that, radius of the wheel of the cart, $(r) = 63$ cm.

$\therefore$ Circumference of the wheel of the cart,

$C = 2\pi r$

$$= 2 \times \frac{22}{7} \times 63 \text{ cm}$$

$$= 396 \text{ cm}$$

Note that in one revolution of the wheel, the cart travels a distance equal to the circumference of the wheel.

$\therefore$ Distance travelled by the cart in one revolution of the wheel

$$= 396 \text{ cm}$$

Also, the total distance travelled by the cart $= 7.920$ km

$$= 7,920 \text{ m}$$

$$= 7,92,000 \text{ cm}$$

$\therefore$ Number of revolutions $= \frac{792000}{396} = 2,000.$

Note : Answer may vary slightly depending on the use of value of π.

1. Find the circumference of the following circles having (use $\pi = \frac{22}{7}$)

   (i) radius = 91 cm   (ii) radius = 10.5 cm   (iii) diameter = 24.5 dm   (iv) diameter = 189 cm

   (v) radius = 28 cm

2. Find the diameter of the circles with the following circumferences. (Take $\pi = \frac{22}{7}$)

   (i) 66 cm   (ii) 196 m   (iii) 24 m

3. The ratio of the diameters of the two circles is 3 : 7. Find the ratio of the circumferences.

4. The diameter of a cycle's wheel is 28 cm. Find how many times the wheel will revolve to cover a distance of 704 m?

5. The circumferences of the two circles are 66 cm and 132 cm respectively. Find the difference between their radii.

6. A race track is in the form of a ring whose inner circumference is 220 m and the outer circumference is 880 m. Find the width of the track.

7. A bike's wheel makes 1,500 revolutions in moving 30 km. Find the diameter of the wheel.

8. A piece of wire is bent in the shape of an equilateral triangle of each side 13.2 cm respectively. It is re-bent to form a circular ring. What is the diameter of the ring?

9. A circular field has a 60 cm wide footpath along the edge. A cow walks around the outer edge of the footpath with each step 44 cm long. In 200 steps, It makes a full round. Find the radius of the inner field in metres.

10. An iron wire is round to form a circle of radius 56 cm. If the same wire is converted into a square, find the side of this square.

11. A circular piece of thin wire with a radius of 6.3 cm is bent into an equilateral triangle. What is the side of the triangle?

12. The diameter of a wheel is 56 m. Find the distance travelled by it in 500 revolutions. Write the distance in cm, m and km. (use $\pi = \frac{22}{7}$)

13. The diameter of a wheel is 144 cm. Find how many times the wheel will revolve to cover a distance of 24 km. (use $\pi = \frac{22}{7}$)

    Hint: $\frac{2400000}{5303} = 425.57$ (approx.)

Area of a Circle

In the previous chapter, we have learnt how to find the area of a triangle, rectangle and a parallelogram. Here, we will learn the methods to find the area of a circle.

Activity

Draw a circle, divide it into 16 sectors and shade half of it. Now, cut along the folds.

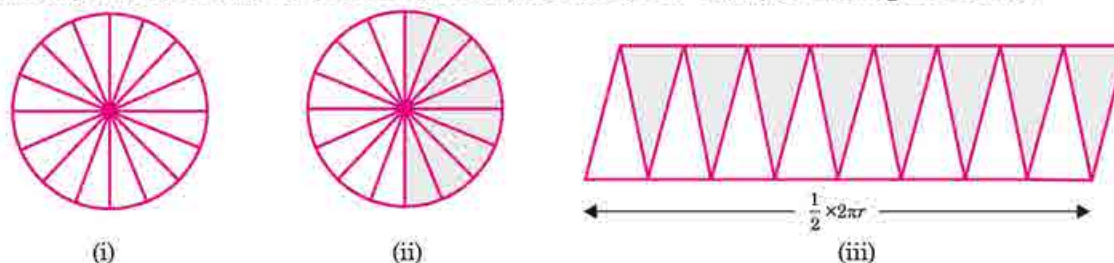(i)   (ii) $\frac{1}{2} \times 2\pi r$ (iii)

Fig. 14.2

Arrange these pieces, as shown in Fig. 14.2 (iii), that looks like a parallelogram. If we divide the circle into very large number of equal sectors and arrange like this, the figure will look like a rectangle where the length is equal to half of the circumference and breadth is equal to the radius of the circle.

Area of a circle = Area of a rectangle with length l and breadth b

$$= \frac{1}{2} \times \text{circumference} \times \text{radius}$$

$$= \frac{2\pi r}{2} \times r = \pi r^2 \text{ sq units}$$

∴ The area A of circle is πr^2 sq units.

Remember
- Circumference $= 2\pi r = \pi d$
- Area $= \pi \times (\text{radius})^2 = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{1}{4}\pi d^2$

Note : Area of a semicircle $= \frac{1}{2} \times$ Area of a circle $= \frac{1}{2} \times \pi r^2 = \frac{\pi r^2}{2}$

Area of a quadrant of a circle $= \frac{1}{4} \times$ Area of a circle $= \frac{1}{4} \pi r^2$.

Let's consider these examples:

Example 3: What is the area of a circle whose radius is 14 cm ?

Solution: We have, $r = 14$ cm

$$\text{Area of a circle} = \pi r^2$$

$$= \frac{22}{7} \times 14 \times 14 \text{ cm}^2$$

$$= 616 \text{ cm}^2.$$

Example 4: The area of a circle is 2464 cm². Find its radius:

Solution: Let radius of the circle be r cm.

$$\text{Its area} = \pi r^2 \text{ cm}^2$$

$$\Rightarrow \qquad r^2 = \frac{2464}{\pi}$$

$$\Rightarrow \qquad r^2 = \frac{2464 \times 7}{22} \qquad (\because \text{area} = 2464 \text{ cm}^2)$$

$$\Rightarrow \qquad r^2 = 784 = 28 \times 28 = (28)^2$$

$$\therefore \qquad r = 28 \text{ cm}$$

Exercise (14.2)

1. What is the area of a circle whose radius is (use $\pi = \frac{22}{7}$)

   (I) 0.35 m   (ii) 21 cm   (iii) 28 cm   (iv) 0.42 dm

2. Find the area of a circle whose diameter is (use $\pi = \frac{22}{7}$)

   (I) 5.4 m   (ii) 56 cm   (iii) 210 mm   (iv) 4.2 km

3. Area of a circle is 182 sq. m. Find the radius of a circle

4. Find the area of a circular ring whose external and internal radii are 42 cm and 35 cm respectively. (use $\pi = \frac{22}{7}$)

5. The circumference of a circular piece of plot is 660 m. Find its area. (use $\pi = \frac{22}{7}$)

6. The areas of two circles are in the ratio 9 : 16. Find the ratio of their circumferences. (use $\pi = \frac{22}{7}$)

7. A rectangular garden is 140 m long and 70 m wide. In the middle of the ground, there is a circular pond of radius 42 m. Find the cost of turfing the remaining portion at the rate of ₹6.50 per square metre.

8. A rectangular cardboard is 70 cm long and 50 cm wide. A circular disc of diameter 42 cm is a cut out from the board. Find the area of the remaining board.

9. Each side of a square iron sheet is 12.5 metre long [Fig. 14.3]. The greatest possible circular sheet is cut out of it. Find the area of the remaining part.
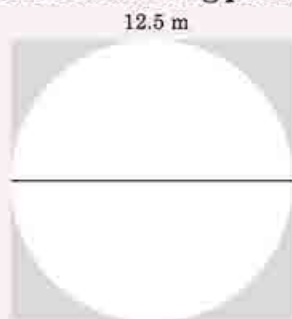
12.5 m

Fig. 14.3

10. A 22 cm long wire is bent to form a square, and then the same wire is bent to form a circle. By how much the area of the circle exceeds by the area of square ?

Multiple Choice Questions

11. Tick (✓) the correct answer.
    (i) If d denotes the diameter of the circle, then circumference C is equal to
        (a) πd ☐   (b) $2\pi d$ ☐   (c) πd^2 ☐   (d) none of them ☐
    (ii) The formula for finding the area of a circle is A =
        (a) $2\pi r$ sq. units   (b) πr^2 sq. units   (c) $2\pi r^2$ sq. units   (d) πr sq. units
    (iii) The circumference of a circle is approximately how many times the radius?
        (a) 3 times   (b) 4 times   (c) 2π times   (d) 10 times

MENTAL MATHS – 14

1. What will be the circumference of a circle whose radius is x^2 ?

2. The ratio of radii of two circles is 2 : 3, what will be the ratio of their areas?

3. Find the diameter of a circle whose circumference is 66 cm.

4. If the radius of a circle is increased by 3 units, what will be the ratio of the new circumference to the old circumference?

5. If the radius of a circle is decreased by 4 units, what will be the ratio of the new circumference to the old circumference?

1. Find the circumference of the circles with radius: (use $\pi = \frac{22}{7}$)

   (a) 7 cm   (b) 3.5 cm   (c) 14 cm   (d) 21 cm

2. A dog is tied to a wooden state in a backyard. It's leash is 3 m long and it runs around in circles pulling the leash as far as it can go. How much area does the dog have to run around.

3. A circular hole of with the diameter 4 cm is cut out from a rectangular plate of measuring 10 cm by 8 cm. Find the area of the plate after the hole has been cut out.

4. A roller is used on a garden with a length of 30 m. If the diameter of the roller is 0.4 m. Find the number of times it has rolled. (use $\pi = \frac{22}{7}$)

5. The circumference of the wheel of a truck is 286 cm. (use $\pi = \frac{22}{7}$)

Key Insights

1. The perimeter of a circle is called its circumference.

2. Circumference 'C' of a circle of radius r is given by
   $C = 2\pi r$, or $C = \pi d$, where $d = 2r$ = diameter.

3. The ratio of the circumference of a circle to its diameter is the same for all circles, regardless of their sizes. The constant ratio is denoted by Greek letter (π) whose approximate value is $\frac{22}{7}$ or 3.14.

   i.e., $\frac{\text{Circumference}}{\text{Diameter}} = \pi$

   $\Rightarrow \quad \frac{C}{2r} = \pi$, where C denotes circumference and r radius of the circle.

   $\Rightarrow \quad C = 2\pi r$

4. Area of the quadrant of a circle = $\frac{1}{4} \times$ Area of a circle = $\frac{1}{4} \pi r^2$.

5. Area 'A' of a circle of radius r is given by $A = \pi r^2$.

KNOWLEDGE OF INDIA

Aryabhatt: The father of Indian Mathematics

Aryabhatta is known as the **father of Indian Mathematics**. His contribution to mathematics is unmatched and cannot be ignored, as he was the one who deduced the approximate value of pi, which he found to be 3.14.
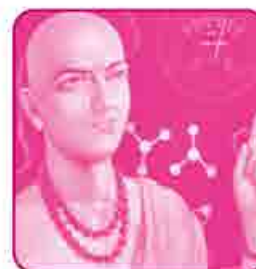
CROSS-CURRICULAR CONNECT

The Konark Temple is dedicated to the Sun God, Surya, and is designed to capture the first rays of the rising sun. The Konark Temple is designed in the shape of a massive chariot drawn by seven spirited horses. It is a significant representation of ancient Indian astronomy and timekeeping. The wheels of the temple served as sundials and timekeepers. The seven horses represent the days of the week, the 12 pairs of wheels symbolise the 12 months of the year, and the 24 wheels represent the 24 hours of a day, with the 8 major spokes denoting three-hour periods (prahars) of a day. The diameter of the wheel is 9 feet 9 inches which is approximately equal to 2.94 m.

1. Calculate the area of a wheel.

2. Find the perimeter of 12 wheels.

Objective: To verify the formula for the area of a circle.

Materials Required: Some thick sheets of paper, scissors, sketch pen, geometry box, pencil, etc.

Procedure:

Step 1: On a thick sheet of paper, draw a circle of any convenient radius (say 3 cm), using compass. Using scissors, cut it out. Circumference of this circle is $2\pi \times 3$ cm $= 6\pi \times$ cm.

Step 2: Divide the circular disc into four equal parts by drawing two perpendicular diameters. Cut out the four sectors as shown.
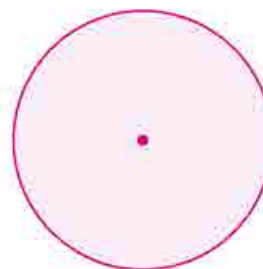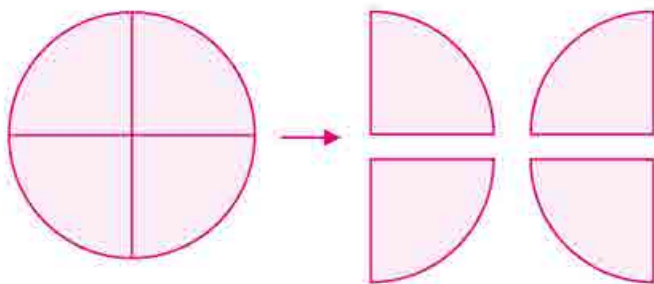
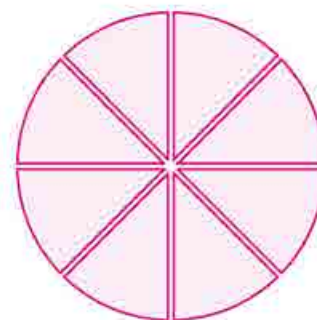Step 3: Now, arrange the four cut outs as shown. AB and CD are line segments equal to the radius of the circle.

The length of the curve between the points A and D is equal to half of the circumference of the circle.

Also, the length between the points B and C is half of the circumference of the circle.
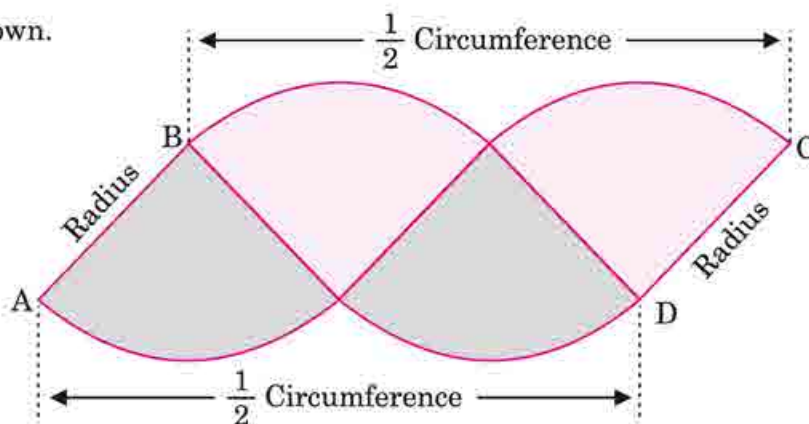
Step 4: Now, on thick sheet of paper, draw a circle of the same radius and divide it into 8 equal parts. Using scissors, cut out the sectors along the radii.

Step 4: Arrange these sectors as shown below. Here also, AB and CD are line segments equal to radius of the circle. Length of the curve between the points B and C is half of the circumference. The shape obtained above is an approximate parallelogram.
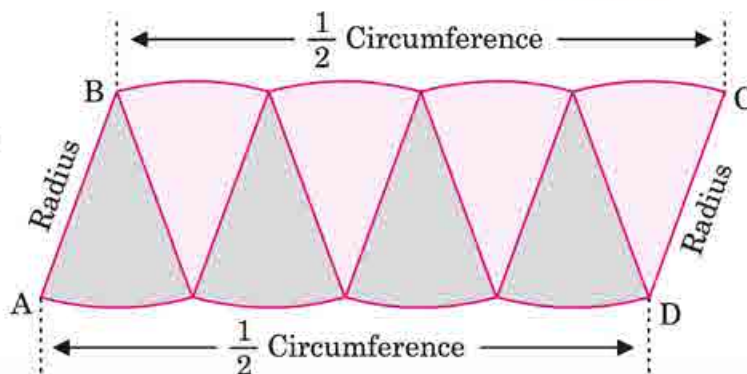
Step 6: Repeat the procedure to divide the circle into equal parts. As you go on increasing the number of sectors, the lengths of the arcs go on decreasing. In this case ABCD will become an approximate rectangle, whose length is equal to half of the circumference of the circle and breadth is its radius.
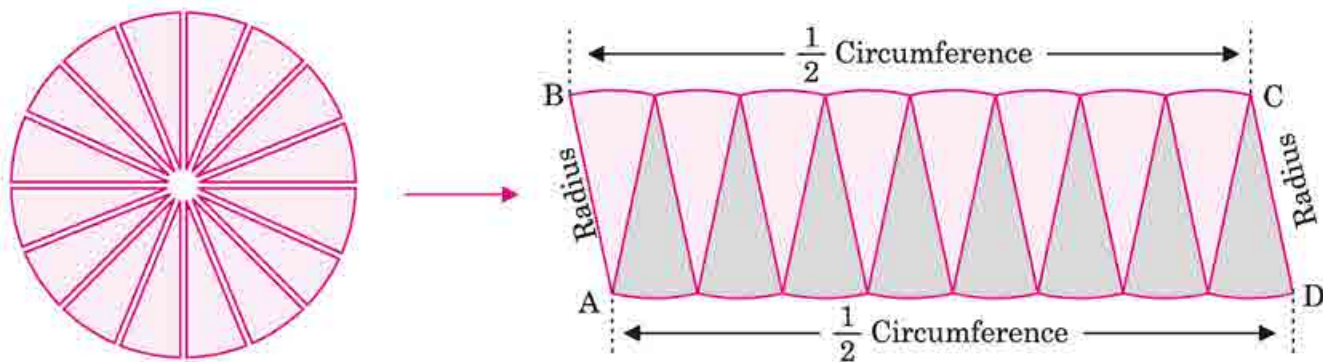
As the whole circle is divided into 32 sectors and on each side we have 16 sectors, the length of the rectangle is the length of the 16 sectors, which is half of the circumference.
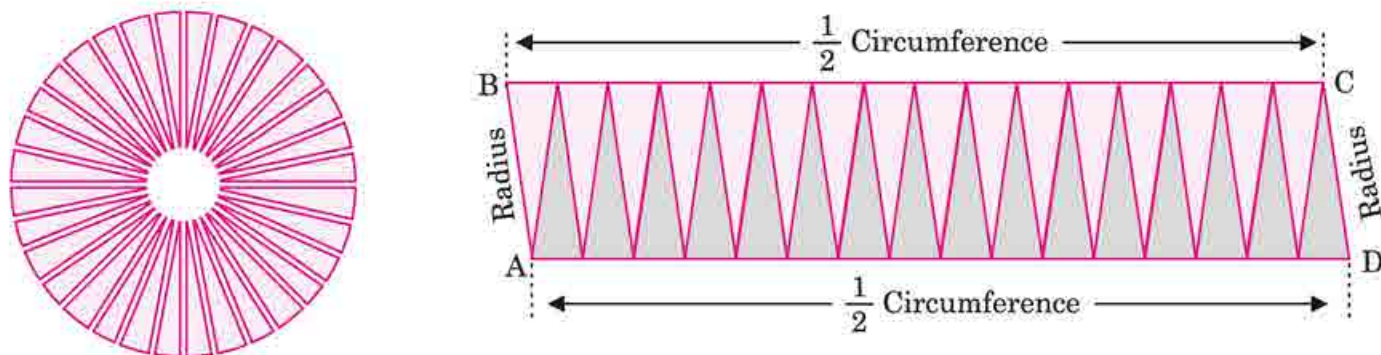
Area of the circle = Area of the rectangle thus formed = length × breadth

$$= \frac{1}{2} \text{ of circumference} \times \text{radius} = \frac{6\pi}{2} \times 3 \text{ cm}^2$$

$$= \pi \times 3^2 \text{ cm}^2$$

$$= \pi \times (\text{radius})^2$$

Thus, area of a circle with radius $r = \pi \times r^2 = \pi r^2$.

Verify the above formula by drawing circles of different radii.

Teacher's Note

Help, students to understand circles, different terms related to circle like radius, diameter circumference, etc. Also explain the relation between the diameter and the circumference.

Project Work

Use glaze paper to make a circle of radius 5 cm using compass. Make another circle of same radius and cut out a circle of radius 3 cm from the second circle keeping centre same. Paste both cut outs in your project copy and write their areas.