1 Patterns in Mathematics

Expected Learning Outcomes

At the end of the chapter, learners will be able to:

- **Remember** key number sequences and shape patterns.
- **Understand** the relationship between mathematics and pattern recognition.
- **Apply** visualisation techniques to represent number sequences pictorially.
- **Analyse** relationships between different number sequences and shape patterns.
- **Evaluate** the connections between mathematical patterns and real-world applications.
- **Create** new patterns or relationships among number sequences or shapes.

Warm Up

1. What number comes next?
   5, 10, 15, 20, ___, ___
2. Fill in the missing numbers:
   3, 6, ___, 12, ___, 18, ___
3. Identify the pattern and write the next three numbers:
   1, 3, 6, 10, 15, ___, ___, ___
4. Which shape comes next?
   □ ○ □ ○ □ ○ _____
5. Complete the pattern:
   △ △ ○ ○ △ △ ○ ○ _____

The Role of Patterns in Mathematics

In mathematics, a pattern is a repeated or predictable arrangement of numbers, shapes, or objects that follow a specific rule or formula. Patterns help in recognising regularities and making predictions about what comes next in a sequence.

For example:

Number pattern: 2, 4, 6, 8, ... (Rule: Add 2 each time)

Shape pattern: △, ■, △, ■, ... (Rule: Alternating triangle and square)

Mathematics not only finds patterns but also explains them, leading to important advancements. For example, studying the patterns of weather helps us create better forecasts, which can prepare us for storms, while understanding patterns in animal behavior can improve conservation efforts to protect endangered species.

Exploring Number Patterns and Their Visual Representations

A pattern is a repeated or recurring design, structure, or motif observed in various contexts. It becomes a sequence when it follows a specific order or rule that defines how the elements relate to one another.

A whole number sequence is represented by (0, 1, 2, 3, 4, ...), where each term follows the rule that the next member is greater than the previous member by 1.

A natural number sequence is represented by (1, 2, 3, 4, ...), following the same rule: each term is greater than the previous term by 1.

Let us explore some special patterns using numbers.

Remember

Patterns are present in nature, such as the stripes on a zebra and the spirals in shells, but these do not form sequences with specific rules. Other examples, like the Fibonacci sequence in plants and the regular intervals in tree rings, illustrate sequences that follow defined patterns.

(i) Sequence of All 1's

This pattern is made up entirely of the number 1, repeating endlessly

1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, ...

All 1's

It shows a constant value with no change.

Visual Representation of the Pattern for All 1's

To visually represent the pattern of all 1's, we can arrange dots from left to right or side by side, with each dot representing a 1.

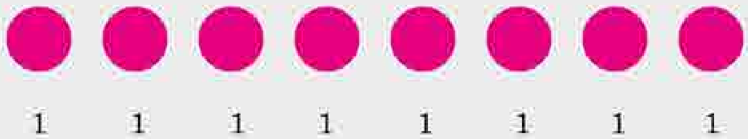


Fig. 1.1

Here, the number of dots remains 1 for every member

(ii) Sequence of Counting numbers

This is a sequence of numbers that begins at 1 and increases by 1 with each step.

1, 2, 3, 4, 5, 6, 7, 8, ...

Counting numbers

These are the numbers we use for counting things in everyday life.

Visual Representation of the Pattern for Counting Numbers

Fig. 1.2

Note: Counting numbers are also called natural numbers.

(iii) Sequence of Odd Numbers

This pattern consists of all whole numbers that are not divisible by 2, starting with 1.

1, 3, 5, 7, 9, 11, 13, 15, ...

Odd Numbers

Each odd number is formed by adding 2 to the previous odd number.

Visual Representation of the Pattern for Odd Numbers

To visually represent the odd number pattern, we can arrange dots from left to right, increasing by 2 for each odd number, then move upward and arrange more dots from left to right again.

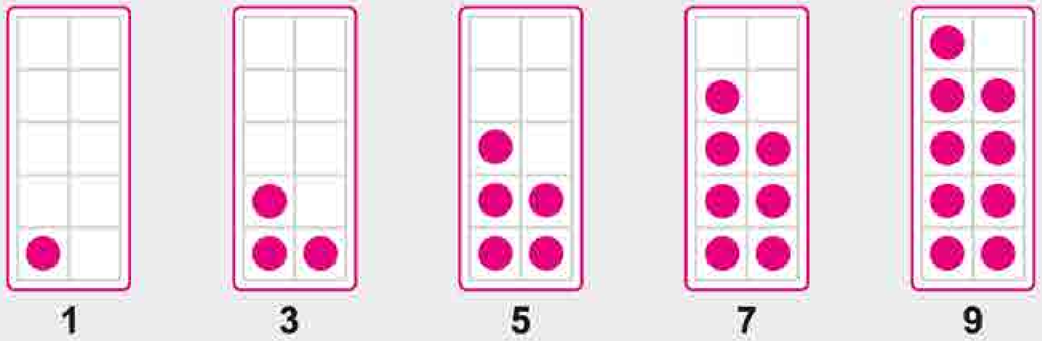


Fig. 1.3

(iv) Sequence of Even Numbers

This pattern consists of all whole numbers that can be divided by 2, starting from 2.

2, 4, 6, 8, 10, 12, 14, 16, ...

Even Numbers

Each even number is made by adding 2 to the previous even number.

Visual Representation of the Pattern for Even Numbers

For a visual representation of the pattern, we can arrange the dots from left to right, then go up and arrange more dots from left to right again.

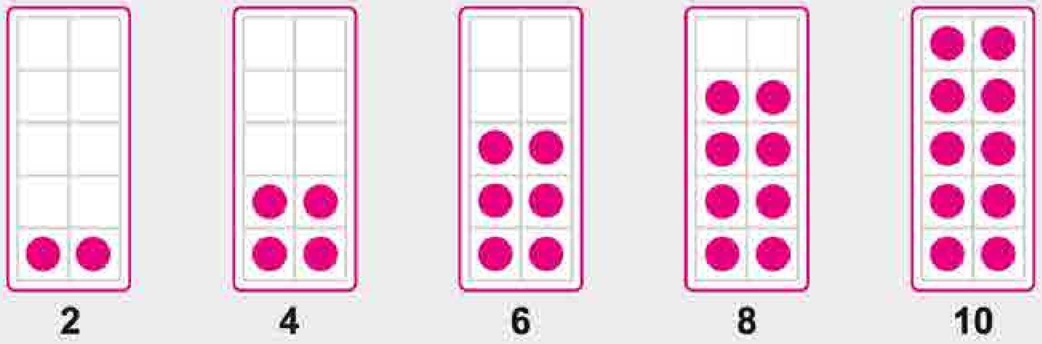


Fig. 1.4

(v) Sequence of Triangular Numbers

These numbers represent the total number of dots that can form a triangle.

1, 3, 6, 10, 15, 21, 28, 36 ...

Triangular Numbers

Each triangular number is the sum of all counting numbers up to a certain point as shown below.

1	=	1
1 + 2	=	3
1 + 2 + 3	=	6
1 + 2 + 3 + 4	=	10
1 + 2 + 3 + 4 + 5	=	15

Visual Representation of the Pattern for Triangular Numbers

For a visual representation of the pattern, start with one dot at the top. Then, go down and arrange the next set of dots based on the value of each term, placing additional dots below, to represent the increasing numbers.

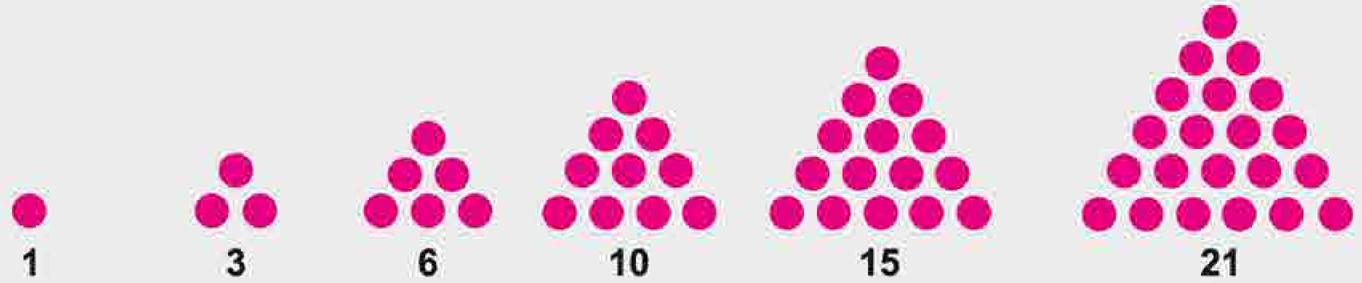


Fig. 1.5

Teacher's Note

Tell students that a sequence is an ordered collection of elements that follows a particular rule. Every sequence is a type of pattern, but not all patterns qualify as sequences. Additionally, explain that powers (e.g., an) refer to multiplying the number a by itself n times, and any number raised to the power of 0 equals 1. In this context, a is known as the base, and n is the exponent; together, they are expressed as a raised to the power of n.

Example 1: Write the next three numbers in the sequence 1, 3, 6, 10, 14, 21, 28, 36,...

Solution: The numbers 1, 3, 6, 10, 21, 28, 36 represent triangular numbers. Triangular numbers are the sum of the first n natural numbers.

$1 = 1$

$1 + 2 = 3$

$1 + 2 + 3 = 6$

$1 + 2 + 3 + 4 = 10$

$1 + 2 + 3 + 4 + 5 = 15$

$1 + 2 + 3 + 4 + 5 + 6 = 21$

$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$

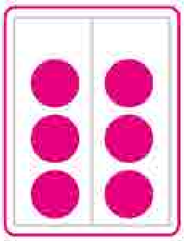
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 66$

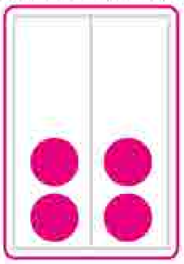
Hence, the next three numbers are 45, 55, and 66.

Example 2: Identify and draw the previous and the next numbers of the even number series for the given figure.

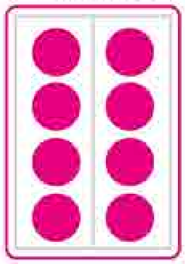


Solution: The given figure has 6 dots, which represent the number 6, the third number of the even number sequence. Hence, the previous numbers will be 4, and the next numbers will be 8.

Previous Member



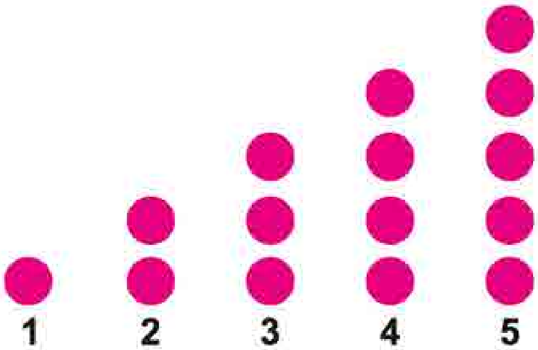
Next Member



Example 3: Give two different visual or pictorial representations for counting numbers.

Solution: Counting numbers are 1, 2, 3, 4, 5, 6... We can represent them in two ways as follows:

Horizontal Orientation



Vertical Orientation

(vi) Sequence of Square Numbers

This pattern includes numbers that are the squares of whole numbers, starting from 1.

1, 4, 9, 16, 25, 36, 49, ...

Square Numbers

Each number is found by multiplying a number by itself as shown.

1×1	=	1
2×2	=	2
3×3	=	9
4×4	=	16
5×5	=	25

Remember

Square number is a number that is the result of multiplying an integer by itself. For example, 4 is a square number because it can be expressed as 2×2 (or 2^2). Similarly, 9 is a square number because it equals 3×3 (or 3^2).

Visual Representation of the Pattern for Square Numbers

For a visual representation of square numbers, begin with a single dot. Next, create equal numbers of rows and columns of dots, with the number of dots in each row and column matching the value of the term. The arrangement of dots looks like a square every time.

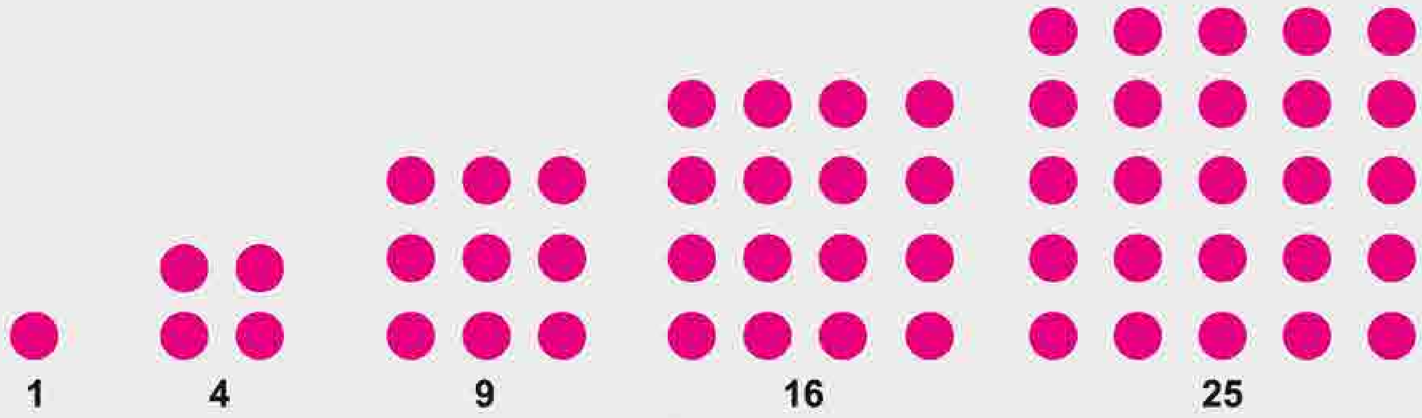


Fig. 1.6

(vii) Sequence of Cube Numbers

This pattern includes numbers that are the cubes of whole numbers, starting from 1.

1, 8, 27, 64, 125, 216, 343, ...

Cube Numbers

Each number is found by multiplying a number by itself twice, as shown.

$1 \times 1 \times 1 = 1$

$2 \times 2 \times 2 = 8$

$3 \times 3 \times 3 = 27$

$4 \times 4 \times 4 = 64$

$5 \times 5 \times 5 = 125$

Remember

- A cube number is a number that results from multiplying an integer by itself twice. For example, 8 is a cube number because it can be expressed as 2 x 2 x 2 (or 2^3). Similarly, 27 is a cube number because it equals 3 x 3 x 3 (or 3^3).

Visual Representation of the Pattern for Cube Numbers

For a visual representation of cube numbers, start with a single dot. Then, create layers of dots where each layer contains an equal number of rows and columns, matching the value of the term. Each layer forms a square, and stacking these layers creates a three-dimensional cube. This arrangement illustrates how cube numbers represent the volume of a cube with side lengths equal to the term's value.

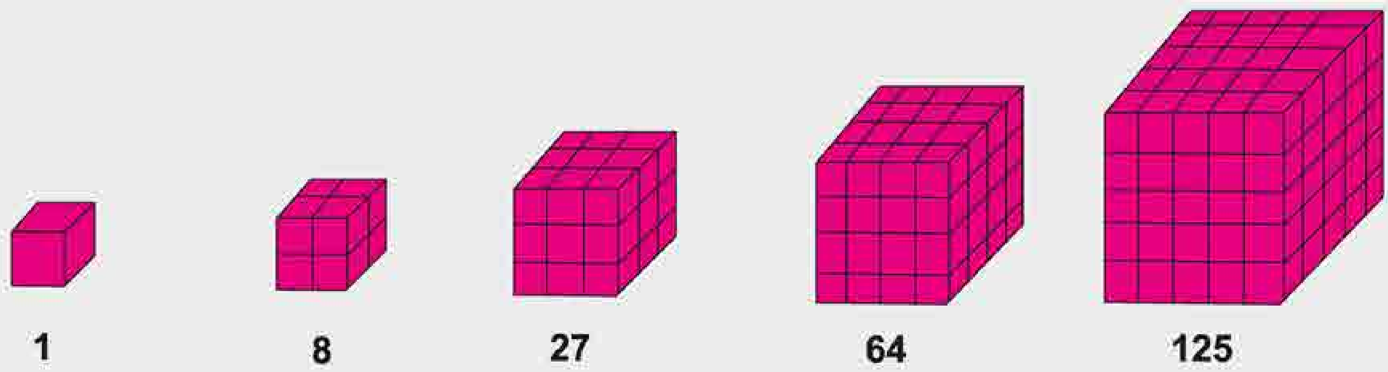


Fig. 1.7

(viii) Sequence of Virahānka Numbers (Fibonacci Sequence)

The Virahānka Numbers, or Fibonacci sequence, typically starts with 1, with each following number being the sum of the two previous ones.

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Virahānka Numbers

Visual Representation of the Pattern for Virahānka Numbers

Number patterns such as odd, even, triangular, square, and cube numbers can all be represented using shapes or models. For example, cube numbers can be illustrated with stacks of cubes, while square numbers can be depicted with square grids. However, the Fibonacci sequence is different and does not fit easily into simple shapes or three-dimensional models.

Some possible visualisation of Virahānka numbers or fibonacci sequence

(a) Fibonacci Spiral

You can show the Fibonacci sequence by drawing squares with side lengths from the sequence (1, 1, 2, 3, 5, and so on). Placing these squares together and drawing quarter-circles creates a spiral that grows with each number.

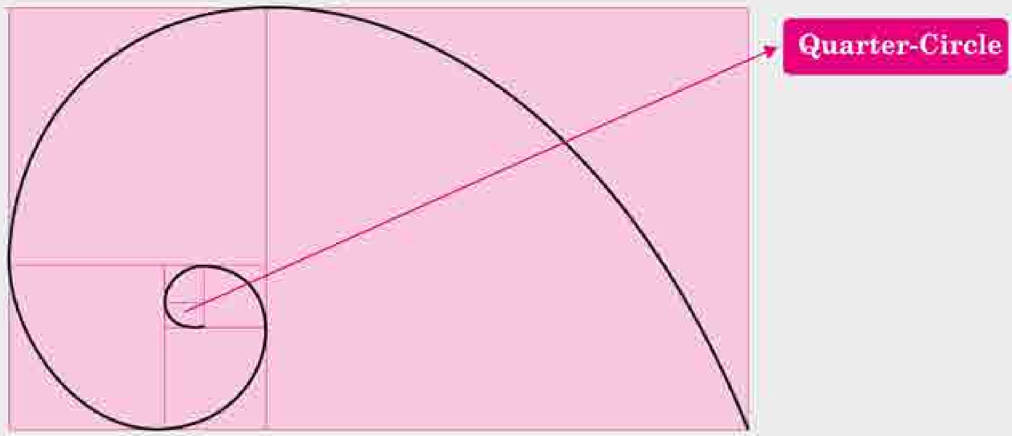


Fig. 1.8

(b) Fibonacci Rectangles

Another way is by arranging rectangles with sides equal to Fibonacci numbers. Adding larger rectangles (1×1, 2×1, 3×2, 5×3, 8×5 etc.) shows how the sequence grows.

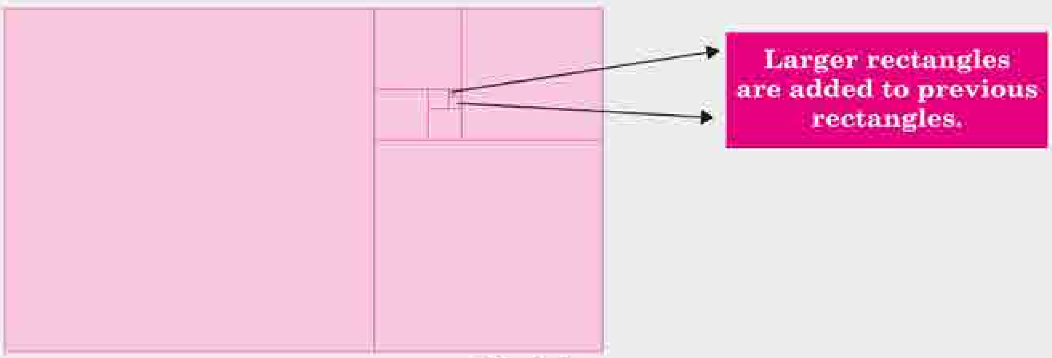


Fig. 1.9

Note: These visual methods help to understand the Fibonacci sequence, even though it is not as easy to visualise as patterns like square or triangular numbers.

(ix) Sequence of Powers of 2

This sequence consists of numbers created by raising 2 to different powers or multiplying 2 n times where n is the nth member, starting from 0 and the sequence starts from 1.

1	=	1	=	2^0
2	=	2	=	2^1
4	=	2×2	=	2^2
8	=	$2 \times 2 \times 2$	=	2^3
16	=	$2 \times 2 \times 2 \times 2$	=	2^4
125	=	$2 \times 2 \times 2 \times 2 \times 2$	=	2^5

Remember

- The power of 2, expressed as 2^n, represents 2 multiplied by itself n times, such as $2^3 = 8$. The power of 3, written as 3^n, signifies 3 multiplied by itself n times, like $3^2 = 9$. Both concepts introduce the idea of exponents in mathematics.

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, ...

Powers of 2

Visual Representation of the Pattern for Powers of 2

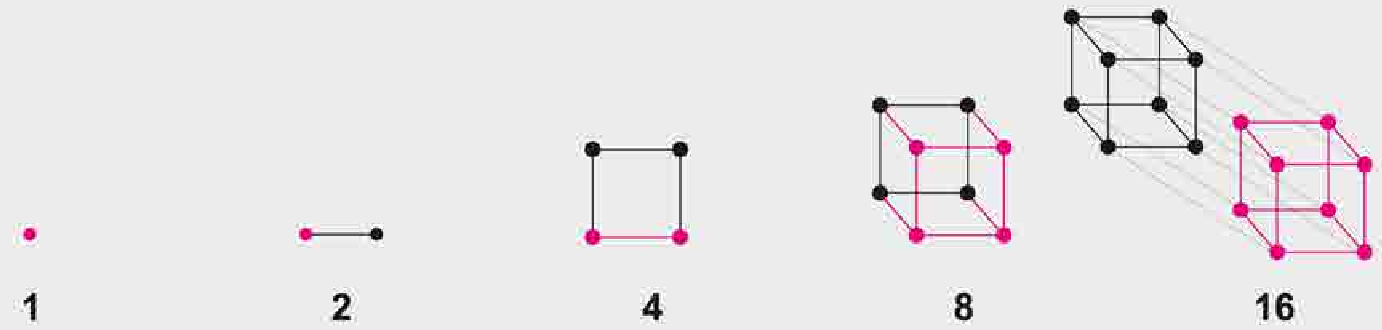


Fig. 1.10

(x) Sequence of Powers of 3

This sequence consists of numbers created by raising 3 to different powers, starting from 0, and the sequence begins with 1.

1	=	1	=	3^0
3	=	3	=	3^1
9	=	3×3	=	3^2
27	=	$3 \times 3 \times 3$	=	3^3
81	=	$3 \times 3 \times 3 \times 3$	=	3^4
243	=	$3 \times 3 \times 3 \times 3 \times 3$	=	3^5

1, 3, 9, 27, 81, 243, 729, 2187, 6561, ...

Powers of 3

Visual Representation of the Pattern for Powers of 3

To show cube numbers visually, start with $1^3 = 1$, and use one dot to represent a single cube. Next, for $2^3 = 8$, arrange 8 dots in a $2 \times 2 \times 2$ cube to show volume in three dimensions. For $3^3 = 27$, position 27 dots in a $3 \times 3 \times 3$

cube, showing how volume increases. For $4^3 = 64$, create a $4 \times 4 \times 4$ cube with 64 dots to illustrate further growth. Finally, for $5^3 = 125$, make a $5 \times 5 \times 5$ cube to highlight how volume expands as the base number increases.

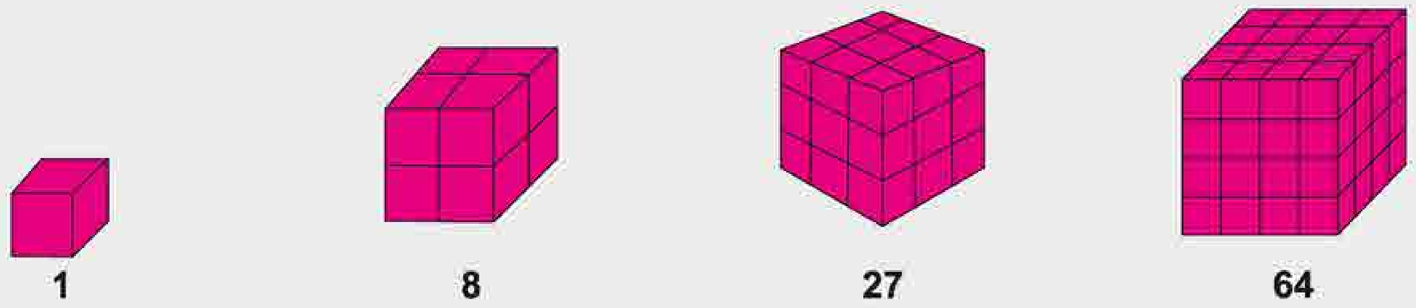
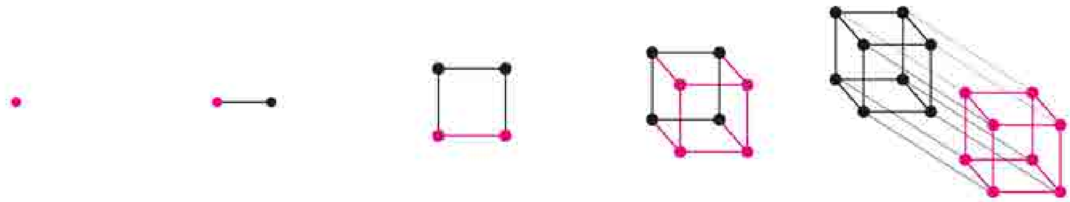


Fig. 1.11

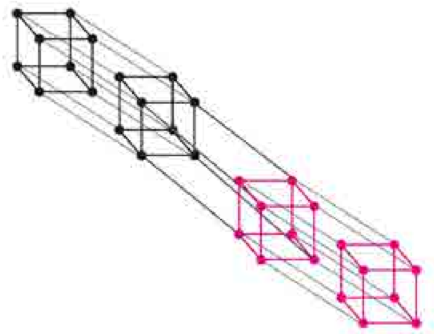
Teacher's Note

Explain to students that patterns using numbers involve identifying sequences based on specific rules, and each type can be visually represented in various ways. The given visual representations are one example of this concept. Encourage students to explore different arrangements, as these visualisations enhance their understanding of the underlying mathematical principles.

Example 4: Identify the type of number sequence shown in the given picture and draw the next figure in the sequence.



Solution: The number of dots used is 1, 2, 4, 8, 16, etc., which forms a sequence of powers of 2. Therefore, the next figure, which will represent 32, will look like this:



Example 5: Given the powers of 3 sequence, if you have 81 dots, which term of the sequence does this represent? Explain how you determined this.

Solution: The powers of 3 sequence is given by:

Powers of 3: $3^0, 3^1, 3^2, 3^3, 3^4, \ldots$

This corresponds to the sequence: 1, 3, 9, 27, 81, ...

To determine which member of the sequence 81 represents:

Since 81 is equal to 3^4, it represents the 5th member of the powers of 3 sequence.

Example 6: If the ninth member of the Virahanka Numbers is represented as F(9), what is the value of F(7) + F(8)?

Solution: We know that in the Virahanka numbers, each member is the sum of its two previous members.

F(1) = 1; F(2) = 1; F(3) = 2; F(4) = 3; F(5) = 5; F(6) = 8; F(7) = 13; F(8) = 21; F(9) = 34

So, F(7) + F(8) = F(9) = 34.

Example 5: List two numbers that are both square numbers and cube numbers.

Solution: $1^2 = 1$ and $1^3 = 1$

$8^2 = 64$ and $4^3 = 64$

Hence, the two numbers that are both square numbers and cube numbers are 1 and 64.

Exercise 1.1

1. **Answer the following questions.**
   (i) What is the 8th triangular number?
   (ii) Identify the triangular number represented by the sum of the first 12 natural numbers.
   (iii) If the 4th even number is represented with dots, how many dots would be in total?
2. **What is the closest square number to 200, and which member of the square number sequence is it?**
3. **What is the closest square number to 200, and which member of the square number sequence is it?**
   (i) 5th member of the triangular number sequence.
   (ii) 4th member of the powers of 2 sequence.
   (iii) 3rd member of the powers of 3 sequence.
   (iv) 3rd member of the cube number sequence.
4. **If the sixth member of the Fibonacci sequence is represented as F(6), what is the value of F(4) + F(5)?**
5. **What is the difference between C(11) and C(5), where C(n)represents the nth member of the sequence of cube numbers?**
6. **How many cubes will interconnected in the sequence of power of 2 in case when the 2 is multiplied 5 times with itself?**
7. **Identify a number that is both a triangular number and a square number, and is greater than 1.**
8. **Consider the rule given below:**

   Rule: F(n) = F(n – 1) + F(n – 2), where n is the nth term.

   Find the first five members of the sequence and identify the type of number sequence displayed by these numbers.
9. **If you have 80 dots, how can you arrange all or some of them to form a triangle with the maximum number of dots?**

Investigating Patterns in Number Sequences

Number sequences can be connected in surprising ways that makes learning even more enjoyable. These connections often reveal patterns that help us understand numbers better. By exploring these sequences, we can see how they relate to one another. In the following examples, you will discover some interesting patterns and relationships. Let us dive in and see what we can learn.

Example 8: What happens when we start adding up odd numbers?

Solution: Let us try to add the odd numbers

$1 = 1$

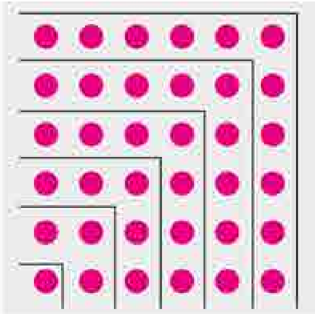
$1 + 3 = 4$

$1 + 3 + 5 = 9$

$1 + 3 + 5 + 7 = 16$

$1 + 3 + 5 + 7 + 9 = 25$

$1 + 3 + 5 + 7 + 9 + 11 = 36$



Visual representation

Explanation: When you add up the first few odd numbers (like 1, 3, 5, etc.), you get square numbers. For example, adding the first five odd numbers gives you 25, which is 5×5. This pattern continues forever, and it always works.

Example 9: What happens when we start adding counting numbers up to a certain number and then back down?

Solution: Let us try to add the odd numbers up to a certain number and then back down.

$1 = 1$

$1 + 2 + 1 = 4$

$1 + 2 + 3 + 2 + 1 = 9$

$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$

$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25$

$1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36$

Explanation: This pattern shows that when you add counting numbers up to a certain number and then back down, you get square numbers.

For example, adding 1 to 2, then back to 1 gives you 4, which is 2×2.

Example 10: What are triangular numbers and how are they calculated?

Solution:

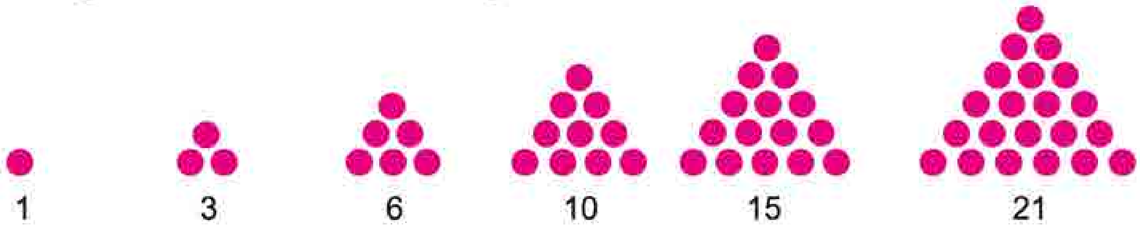
$1 = 1$

$1 + 2 = 3$

$1 + 2 + 3 = 6$

$1 + 2 + 3 + 4 = 15$

$1 + 2 + 3 + 4 + 5 = 15$



Explanation: Triangular numbers are special numbers that you can arrange in a triangle shape. For example, the third triangular number is 6 because you can make a triangle with three rows of dots. The formula to find the nth triangular number is n(n+1)2, which means you add up the first n numbers together.

Example 11: What happens when we sum even numbers?

Solution:

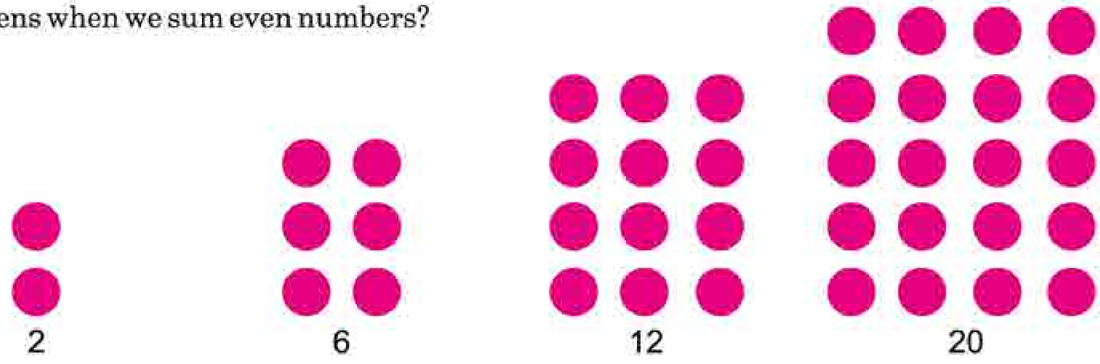
$2 = 2$

$2 + 4 = 6$

$2 + 4 + 6 = 12$

$2 + 4 + 6 + 8 = 20$

$2 + 4 + 6 + 8 + 10 = 30$



Explanation: When you add up even numbers (like 2, 4, 6, etc.), you get a pattern. The sum of the first few even numbers gives you numbers like 6 and 12.

For any number n, the sum of the first n even numbers is always accurately given by $n(n + 1)$.

This formula means that if you want to find the sum of the first n even numbers, you can simply multiply n by $(n + 1)$.

Example 12: What happens when you add up pairs of consecutive triangular numbers? That is, take 1 + 3, 3 + 6, 6 + 10, 10 + 15, ... Which sequence do you get? Why? Can you explain it with a picture?

Solution: Let us add consecutive triangular numbers

$1 + 3 = 4 = 2^2$

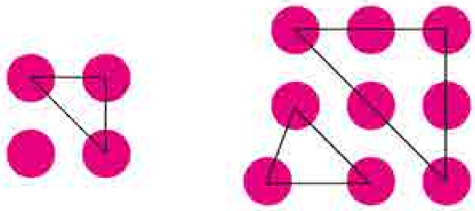
$3 + 6 = 9 = 3^2$

$6 + 10 = 16 = 4^2$

$10 + 15 = 25 = 5^2$

Explanation: When you add two consecutive triangular numbers, the result is a square number.

A triangular number represents dots arranged in an equilateral triangle. When you combine two consecutive triangular numbers, they fill out to form a perfect square.



Visual representation

Exercise 1.2

1. **Can you find a visual representation for adding counting numbers up and down, i.e., 1, 1 + 2 + 1, 1 + 2 + 3 + 2 + 1, ...?**

2. **What happens when you start to add up powers of 2 starting with 1, i.e., take 1, 1 + 2, 1 + 2 + 4, 1 + 2 + 4 + 8, ... ? Now add 1 to each of these numbers — what numbers do you get? Why does this happen?**

3. **What happens when you add up pairs of consecutive triangular numbers? That is, take 1 + 3, 3 + 6, 6 + 10, 10 + 15, ... Which sequence do you get? Why? Can you explain it with a picture?**

4. **State any two different ways out of presenting a pictorial representation of a square using number sequence.**

5. **What do you notice when you add consecutive multiples of 3? How it is related to triangular numbers?**

6. **Observe the pattern, find the next three members, and identify how the sequence progresses.**

   $1 + 2 - 3 = 0$

   $1 + 2 + 3 - 4 = 2$

   $1 + 2 + 3 + 4 - 5 = 5$

   $1 + 2 + 3 + 4 + 5 - 6 = 9$

   $1 + 2 + 3 + 4 + 5 + 6 - 7 = 14$

7. **Observe the sequence given below, write the next three terms, and determine what sequence it is following.**

   $1 = 1$

   $1 + 8 = 9$

   $1 + 8 + 27 = 36$

   $1 + 8 + 27 + 64 = 100.$

Shape Patterns and Their Relationship with Numbers

Patterns are a fundamental aspect of mathematics, and one of the key areas where patterns emerge is in shapes. These patterns can be found in various dimensions – one-dimensional (1D), two-dimensional (2D), and three-dimensional (3D). The study of these patterns falls under the branch of mathematics known as **geometry**.

Shape patterns help us understand the structure and relationships between different geometrical figures. By analysing these patterns, we can observe how shapes evolve, identify their properties, and make predictions about future shapes in a sequence.

Shape Sequences

Shape sequences are one of the most important types of patterns in geometry. These sequences involve shapes following a specific rule or arrangement, which mathematicians study to understand their geometric properties.

Types of shape sequences

Below are some important shape sequences. These sequences show how shapes can evolve or repeat according to a specific rule or pattern, starting from simple forms like lines to more complex geometric shapes.

(i) Sequence of Line

A sequence where points are arranged in a straight line, extending in one dimension, is the most basic form of sequence, involving points connected in a single direction. These line sequences are directly related to number patterns, as they represent points placed at equal intervals along a straight line, often corresponding to a sequence of counting numbers such as 1, 2, 3, 4, 5, and so on.

Visual representation of Line Sequences
Fig. 1.12

(ii) Sequence of Stacked Triangles

A sequence where triangles are arranged in layers, each building upon the previous one, is known as a stacked triangle sequence.

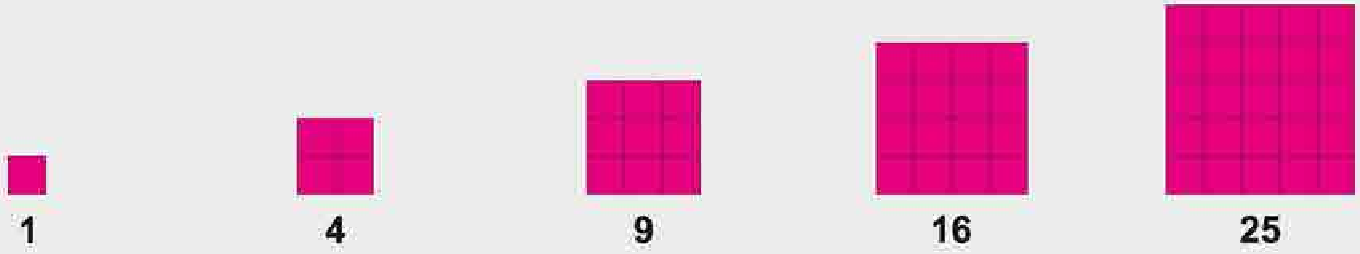
This pattern begins with one small triangle, and in each subsequent term, two more triangles are added in each row, forming a complete triangle. The total number of small triangles in this sequence corresponds to square numbers, such as 1, 4, 9, 16, 25, and so on.

Visual representation of Line Sequences
Fig. 1.13

(iii) Sequence of Stacked Squares

A sequence where squares are arranged in layers, with each square relating to the previous one according to a consistent rule, is known as a stacked square sequence.

This pattern begins with one small square, and in each subsequent term, more squares are stacked below in such a way that the overall figure resembles a square. The total number of small squares in this sequence forms perfect square numbers, such as 1, 4, 9, 16, 25, and so on. The number of squares in each term follows the sequence: $1, 1+3, 1+3+5, 1+3+5+7+9$, and so on.



Visual Representation of stacked Squares
Fig. 1.14

(iv) Sequence of Regular Polygons

A **polygon** is a two-dimensional (2D) geometric figure formed by a finite number of straight line segments that connect to create a closed shape, consisting of vertices and edges. They can be classified based on the number of sides and can be either convex or concave.

Fig. 1.15

A regular polygon is a polygon with equal sides and equal interior angles, creating a symmetrical shape. Examples include the equilateral triangle (3 sides), square (4 sides), and regular pentagon (5 sides). The number pattern indicating the sides of these polygons is 3, 4, 5, 6, 7, and so on.

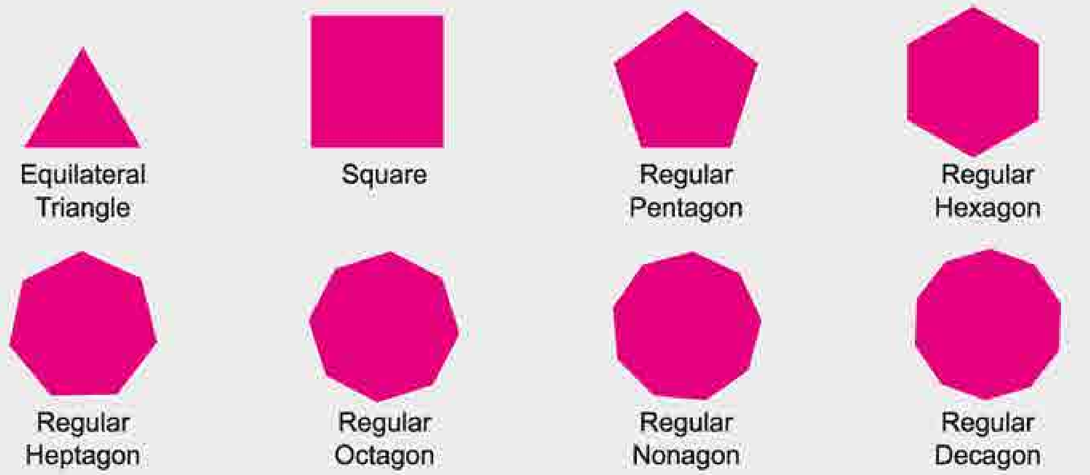


Fig. 1.16

(v) Sequence of Complete Graphs

In a complete graph, every dot (or node) is connected to every other dot with a line. This means that if there are n nodes in the graph, each node will have edges (lines) connecting it to n – 1 other nodes.

For example: In K2 (a complete graph with 2 nodes), there is 1 edge connecting the two nodes. In K3 (3 nodes), each node is connected to the other two, resulting in 3 edges. In K4 (4 nodes), each node is connected to the other three, resulting in 6 edges.

Fig. 1.17

Try to find the sequence of the number of edges in a complete graph.

(vi) Koch Snowflake

The Koch snowflake, also known as the Koch curve, is a captivating fractal shape introduced by Swedish mathematician Helge von Koch in 1904. It begins with an equilateral triangle, which has 3 sides. The shape evolves through a repeated process of adding smaller triangles to each side.

Remember

- A fractal is a geometric shape consisting of a repeating pattern that looks similar at various scales.

In the first iteration, the initial triangle develops into a figure with 12 sides. This is achieved by dividing each of the 3 sides into 3 equal segments, creating a new equilateral triangle on the middle segment and replacing that segment. In the second iteration, each of the 12 sides is similarly divided into 3 equal parts, resulting in 48 sides as new triangles are added to the middle segments. By the third iteration, this process continues: each of the 48 sides is divided into 3 equal segments, leading to 192 sides.

This progression illustrates how the number of sides increases in the sequence of 3, 12, 48, 192, and so on. The Koch snowflake is not only an intriguing geometric transformation but also exemplifies the unique properties of fractals, showcasing an infinitely intricate boundary while enclosing a finite area.

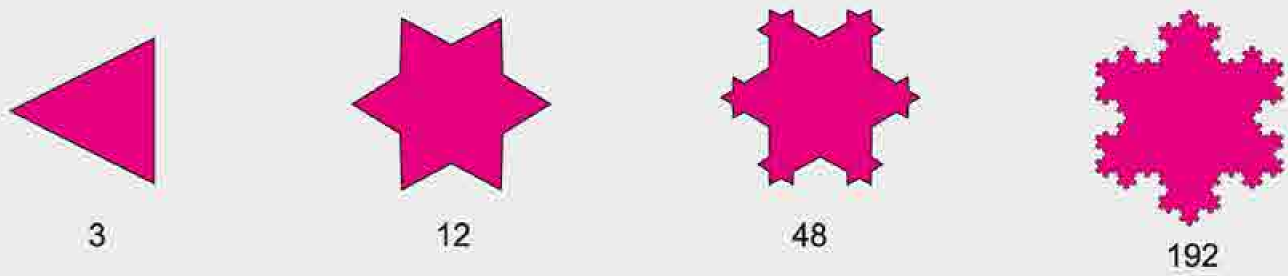
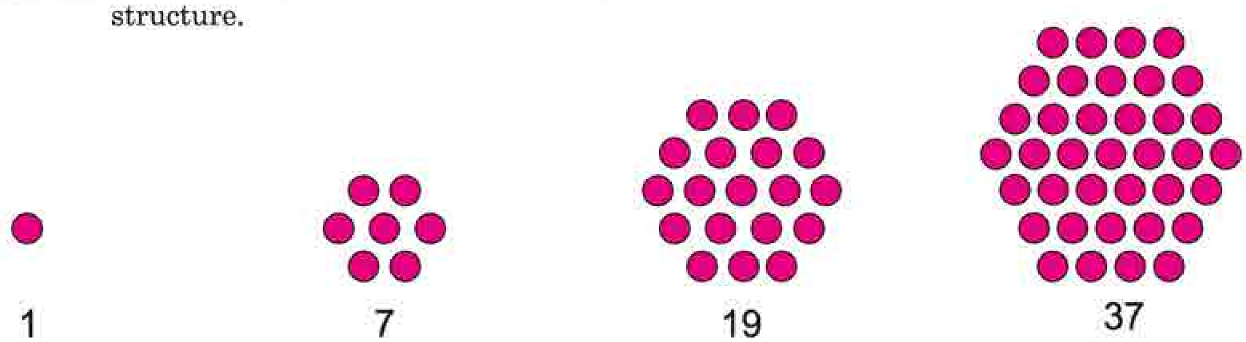


Fig. 1.18

Example 13: What would you call the following sequence of numbers? Write the Next member and draw its structure.



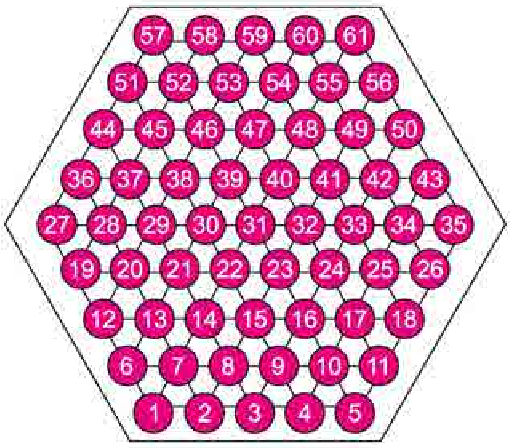
Solution: The numbers 1, 7, 19, and 37 can be referred to as a hexagonal number sequence, and when arranged properly, these dots seem to form a hexagon. Observing the sequence, we see that the differences between consecutive terms are 6, 12, and 18, which are multiples of 6. We can express the difference between the n-th term and the previous term as $6(n-1)$.

To find the next term in the sequence:

The last term is 37.

The next difference, for $n = 5$, is $6(5-1) = 6 \times 4 = 24$.

Thus, the next term will be $37 + 24 = 61$.

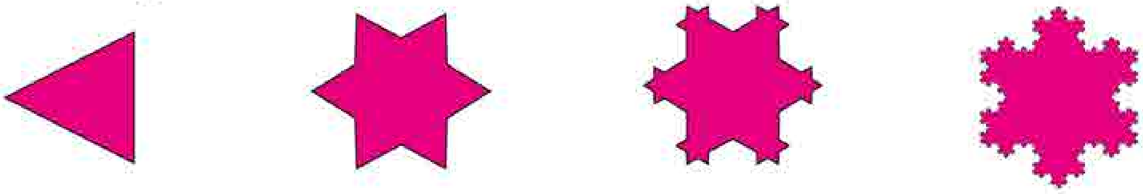


Example 14: How many little triangles are there in each shape of the sequence of Stacked Triangles? Which number sequence does this give? Can you explain why?

Solution: In the Stacked Triangles sequence, the number of little triangles in each shape corresponds to square numbers: 1, 4, 9, 16, 25, and so on. The sequence starts with 1 triangle, and each subsequent layer adds 2 more triangles than the previous layer, leading to the totals of 1, 4 (1 + 3), 9 (4 + 5), 16 (9 + 7), and 25 (16 + 9). This pattern arises because each new layer builds upon the previous one, forming a complete triangle, resulting in the total being equal to the square of the layer number (n^2).

Example 15: To move from one shape to the next in the Koch Snowflake, we change each straight line into a 'speed bump (__∧__).' As we keep doing this, the changes become smaller and smaller. How many line segments are there in each shape of the Koch Snowflake? What is the pattern in the numbers?

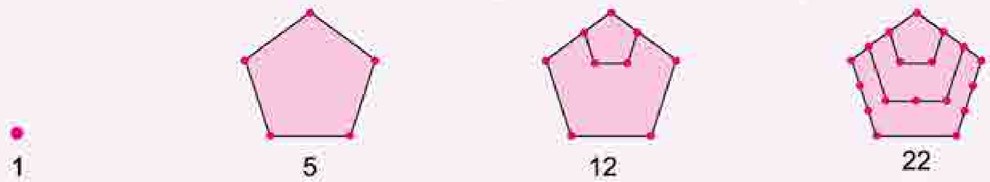
Solution: The Koch snowflake starts with an equilateral triangle, which has 3 line segments. As you repeat the process of adding smaller triangles to each side, the number of line segments increases: 12 in the second shape and 48 in the third.



The pattern follows the sequence **3, 12, 48**, ..., where each time the number of segments is multiplied by **4**. This growth highlights the fractal nature of the Koch snowflake.

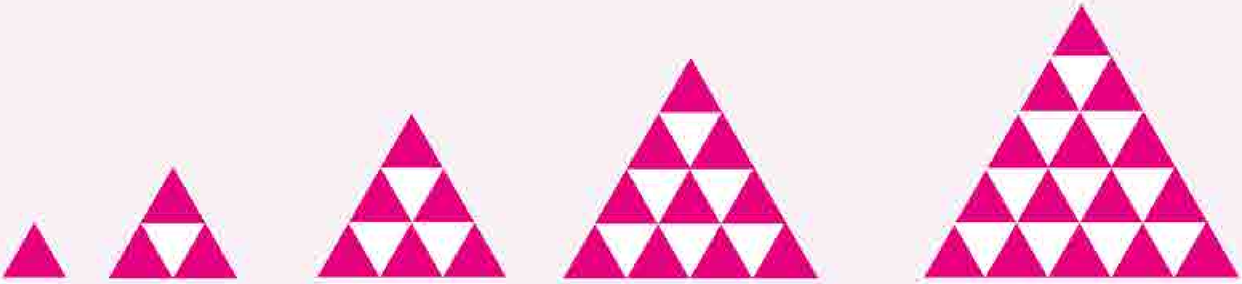
Exercise 1.3

1. **If we visually represent stacked squares, how many small squares will be in the 15th member of it?**
2. **How many little triangles are there in each shape of the sequence of Stacked Triangles? Which number sequence does this give? Can you explain why?**
3. **Examine the figure provided below. What conclusions can you make, and what might these figures be referred to as? Create the next figure in the same sequence.**



4. **What mathematical properties make regular polygons unique compared to irregular polygons?**
5. **Examine the Sierpiński triangle displayed below. Try to determine the pattern in the number of magenta triangles present in each figure.**

6.



7. **Consider a complete graph Kn, where n represents the number of nodes. If you were to remove one node from the complete graph K5, what would be the effect on the number of edges in the graph?**

MENTAL MATHS – 1

1. **Identify the next three members of the sequence: 25, 49, 81, ...**
2. **In a star configuration, one central node is connected to seven other nodes. What is the total number of nodes in this configuration?**
3. **Provide the first five numbers in the hexagonal number sequence, starting with 1. Each subsequent number is calculated by adding increasing multiples of 6 to the previous number (6, 12, etc.).**
4. **How many small squares will be there in the sixth member of the stacked squares sequence?**

Enrichment Exercise

1. Write the first five numbers of the given patterns.
   (a) Odd Numbers (b) Even Numbers (c) Square Numbers (d) Triangular Number
   (e) Virahānka Numbers (f) Powers of 3
2. Identify and draw the previous and next numbers of the odd number sequence given in the Fig. 1.19.

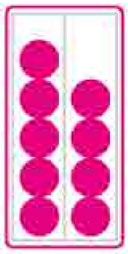
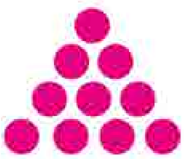


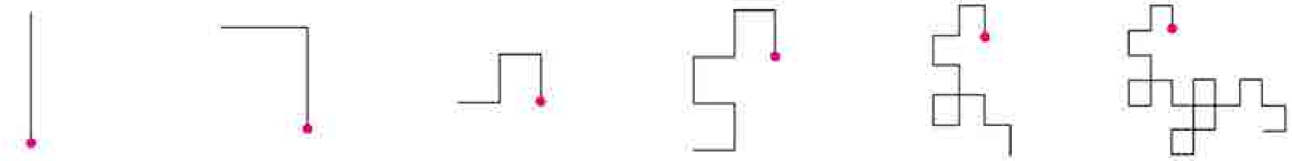
Fig. 1.19

3. Identify the member of the triangular number sequence and draw the figure of the next and previous member.



4. Write the fifth member of the following number sequence.
   (a) Square Number (b) Cube Number (c) Power of 2 (d) Power of 3
5. In a complete graph, every node is connected to every other node with an edge (line). If a complete graph has 10 edges, how many nodes are present in the graph?
6. How many edges are in a complete graph with 4 nodes (K4)?
7. Take any odd number and start adding consecutive odd numbers backwards down to 1. What patterns do you observe, and how is the result related to the number of odd numbers you have added?

9. What occurs when you multiply the powers of 2 by 3 and add 1, and which sequence is generated from this operation? Additionally, how is the resulting sequence related to the number 3?
10. Observe the figure of the dragon curve below and identify the sequence the figure possesses in terms of the number of lines used.



Key Insights

1. Mathematics explores patterns and seeks to understand their underlying reasons.
2. A key type of pattern is found in number sequences.
3. Important examples of number sequences include counting numbers, odd numbers, even numbers, square numbers, triangular numbers, cube numbers, Virahānka numbers, and powers of 2.
4. These sequences can be linked in intriguing ways, such as summing odd numbers from 1 to produce square numbers.
5. Visualising number sequences with pictures helps in understanding their relationships.
6. Another type of pattern in mathematics is shape sequences.
7. Examples of shape sequences include regular polygons, complete graphs, stacked triangles and squares, and Koch snowflakes, which also connect to number sequences in fascinating ways.

HOTS

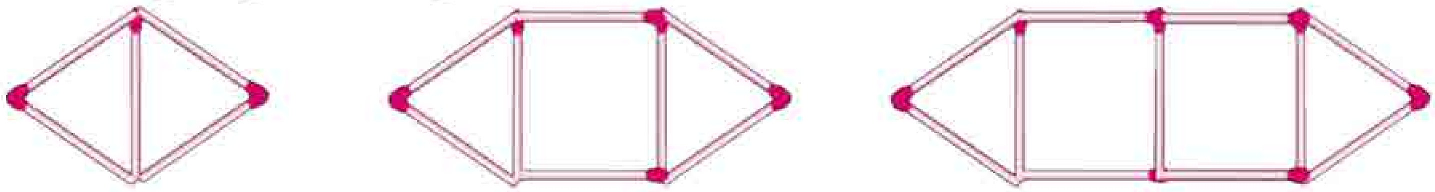
1. What happens when you add pairs of consecutive perfect squares? For instance, consider $1^2 + 2^2$, $2^2 + 3^2$, $3^2 + 4^2$, ... What sequence do you discover?
2. Consider the sequence S = n(n-1) + 1, n(n-1) + 3, n(n-1) + 5, ... (n terms)
   (i) Write down the first four terms of the sequence for n = 4
   (ii) What can you conclude about the relationship between the sum of these terms and n^3?

KNOWLEDGE OF INDIA

The sequence 1, 1, 2, 3, 5, 8 is known as the Fibonacci sequence, named after the Italian mathematician Fibonacci, who popularised it in the 13th century. However, it was originally developed by Indian mathematicians such as Virahanka in the 7th century CE, where it was referred to as 'Virahanka numbers.'

CROSS-CURRICULAR CONNECT

Create the given pattern using matchsticks.



What sequence do you observe for the number of matchsticks in the pattern?

Maths Lab Activity

Objective: To Construct a K5 complete graph using ice cream sticks as edges and paper balls as nodes to visualise the structure of a complete graph.

Materials Required: 5 paper balls (for nodes), 10 ice cream sticks (for edges), Glue or tape, and Marker (optional, for labelling nodes)

Procedure:

Step 1: Create 5 small paper balls to represent the nodes. Label each paper ball (A, B, C, D, E) using a marker, if desired.

Step 2: Place the 5 paper balls in a circular arrangement on a flat surface, with some space between them to allow connections with the ice cream sticks.

Step 3: Using the 10 ice cream sticks, connect each paper ball (node) to every other node.

Start by connecting paper ball A to paper balls B, C, D, and E using ice cream sticks.

Then, connect paper ball B to C, D, and E.

Repeat similarly for paper balls C, D, and E, ensuring that every node is connected to every other node.

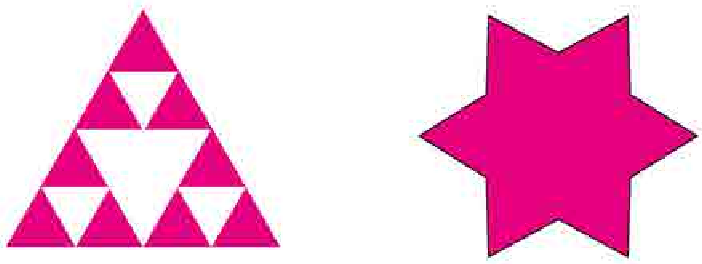
Step 4: Secure the Connections:

Use glue or tape to attach the ends of the ice cream sticks to the paper balls. Ensure that all connections (edges) are stable and clearly visible.

Step 5: After connecting all the nodes, you will have a K5 complete graph, where each node is connected to every other node using a total of 10 edges.

Project Work

A fractal is a geometric figure characterised by a repeating pattern that looks similar at every scale. Design a PowerPoint presentation to explain fractals, including multiple examples of fractals.



Fractals