6/1/21

Introduction to DIP

midsem - 20% Assignment - 20% Project 20%

End Sem - 40%

Need of DIP

* EMW and sound waves can interact with objects and bring information
* Types of I/p - Image, Sound, text
* 2D Signal Processing
* Devices used to capture and display images are Electronic devices.

Definition And types of Images

→ $f: X \times Y \rightarrow Z$

function domain $(X \times Y)$ codomain (Z)

→ X or Y → non discrete set then image is analog image

X-ray image, film based optical image

→ X and Y → discrete set (1, 2, 3 (nodecimals)) → f discrete

Un-quantified array of values captured by CCD camera.

→ If X, Y, Z is discrete → f is digital

digital camera, CT image, MRI.

→ input of CCD → light intensity

O/p → quantized value Voltage (Z)

Z → output of the X*Y

X-ray → amount of energy absorbed at points on the object when an X-ray is on the object

↳ Z.

Intr IR image → z → temperature at a particular point

Types of Image Processing

↳ Low level image processing: input and output are images.

1. Image enhancement - removing noise, improving contrast
2. Image Reconstruction. (CT, X-ray)
3. Image Restoration
4. ~~Ke~~ Morphological operations → o/p will contain structure of the object
5. Water marking
6. Steganography.

↳ Middle level image processing: input is image, but output is set of ~~fet~~ features of image.

1. Edge detection
2. Segmentation

↳ High level image processing: input is set of features of image and output is information about scene.

1. Object detection
2. Object classification.

HW → Computer Vision (Chpt titles)

Computer vision can't be done without the above two levels. Weightage given is different.

Doesn't edge det come under morphological operation?

edge can rep in 2 ways → show edge image (low middle)
→ show the sequence of points representing edge

The first def for images is not right for RGB. That is right Grey-scale image.

Colour image → 3 X × Y values

Grey Scale image → bounded matrix.

when bounds are removed it is a matrix.

X ↓ row indices, Y ↓ column indices, Z ↓ matrix cell values } assumption ∴ Grey scale image is taken as matrix.

Values need to be finite to be stored. ∴ Can't process analog images.

Image Capturing / Image Acquisition.

* Light
  ↳ Pinhole camera. (very dark image)
    - smaller hole → image is sharp
    - larger hold → blurred.
  ↳ Camera with lens.

ccd camera / Cmos
↳ understand how scanning is done
↳ How film is converted to digital.

RGB → 3 matrices → 1 matrix for red
1 matrix for blue
1 matrix for Green

CMOS Camera → why is it low quality.
CCD

Recap

pixel ⇒ (location, value stored)

↳ Image capturing

- Optical Wave
  - ↳ Analog – film
  - ↳ Digital – CCD, CMOS

Imaging using X-ray

↳ X-ray more freq, more penetrating power.

X-ray detector :- film/memory (RAM)

Bone → $I << I_0$

$I \leq I_0$ Soft tissue → $I < I_0$ (minimal) not very less

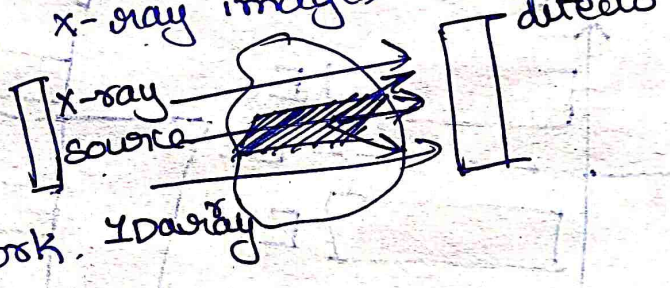
I brings value / info about the object.

I is more ⇒ damage is more ∴ Bone dark

Softer → Bright

∴ Bone – ~~dro~~ ~~Bo~~ dark

When we complement x-ray image, we get bone in white.



How does X-ray work.

CT scan.

↳ x-ray signals various directions.

Assumption in X-ray

↳ No deviation / minimal deviation of x-rays ~~ab~~

Image display

CRT based display Good quality display.

Red ⇒ type 1 phosphate

~~Bluy~~ Blue ⇒ type 2

Green ⇒ type 3

Q primitive operations in algorithm design?

Any operation supported by CPU is primitive operation.

Image processing.

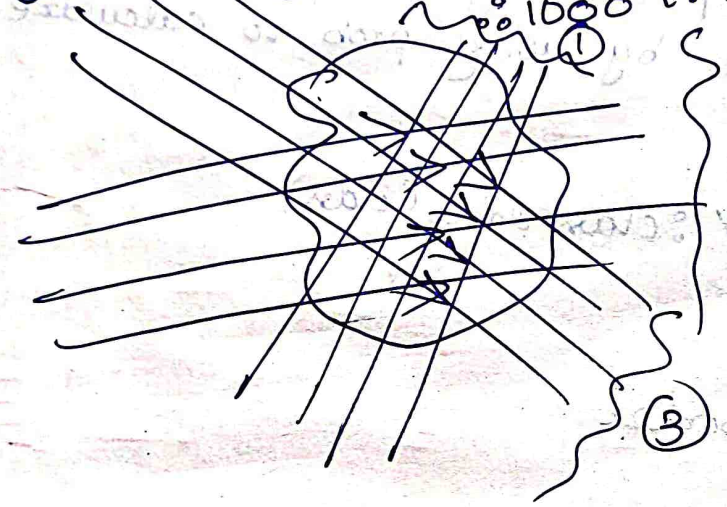
monotonicity property

How to get matrix from image file.

X-ray image → overlapping image (can't check organs properly)

CT scan :- X-ray image in many direction (1000 directions)

∴ 1000 images → reconstruct

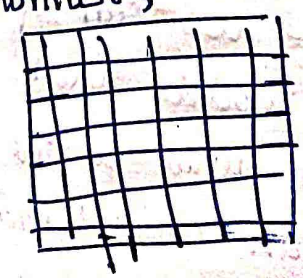


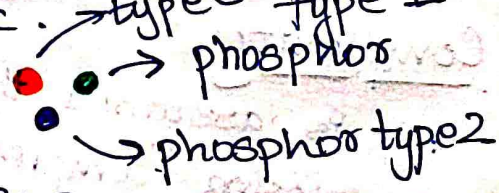
like this multiple images.

X-ray source and detector should have same intensity level. If the isotopic object can give x-rays then Intensity at detector should be more ∴ more range? But we assume won't happen as isotope isn't used as object. Even if some object is there the deviation is very small. This imaging technique has a few assumptions

Image Display :- Monitor, TV etc.

CRT based display :-





RGB.

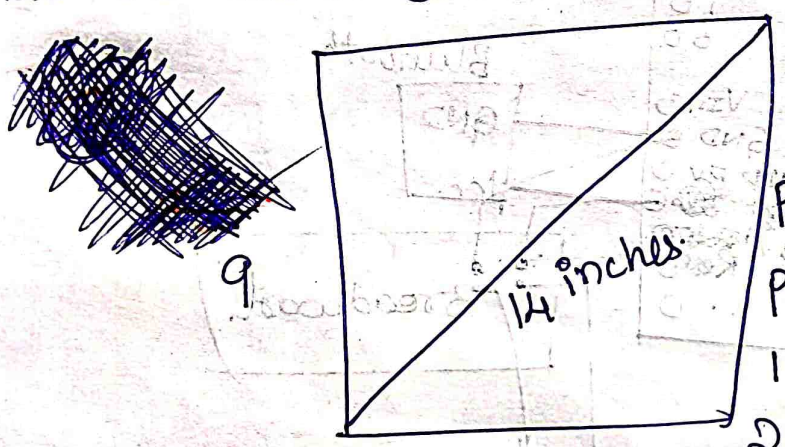
Gray :- R = G = B.

R ≠ G ≠ B ⇒ different colours.

3 electron gun → Bulky image system.

Can control electron guns to get the colours displayed. Can fire the no. of e^- we want.

· How do we know good / bad display? Comparison what we have seen.



9:16 aspect ratio

Pack fixed no. of pixels

pixels pixels

1000 × 1500 } 14 inch

2000 × 3000 } → Better

usually pixels is square!

(Better resolution) quality

Resolution of image :- No. of pixes per inch.

1000 × 2000 → 10 inches × 20 inches → (Best)

→ 100 × 200

→ 1000 × 2000

Print < 180 dots per inch (bad print)

good quality ~360~ .

Image Operations

↳ Image processing :- seq of image operations

↳ Primitive operation :- Operation ~~done~~ supported by CPU.

↳ Arithmetic , Set theoretic (Boolean includes)

Image is represented by function

+
*
—
/

Issue :- Digital image

matrix

$f : \{0, \ldots M-1\} \times \{0 \ldots N-1\}$

$\rightarrow \{0, \ldots L-1\}$ 0 to max

I_1, I_2 (two images)

$f_1 + f_2 : \{0, \ldots M-1\} \times \{0 \ldots N-1\}$

$\rightarrow \{0 \ldots L'-1\}$ $L' > L$

can change

NOT POSSIBLE.

The result need to be within fixed range.

Consider image

$f : \{0, \ldots m-1\} \times \{0, \ldots N-1\} \rightarrow \{0 \ldots (L-1)\}$

convert

$g : \{0, \ldots m-1\} \times \{0 \ldots N-1\} \rightarrow \{0 \ldots (L'-1)\}$

where $L' < L$

So only change codomain (smaller range)

$g \cong f$ in terms of info

repr g with less no. of values.

some display 0, 1 pixel levels may

0 ... 255 change.

0 ... 3.

Algorithm to obtain g from f

g similar info as f → Monotonicity property.

If $f(x,y) \le f(x',y')$
then $g(x,y) \le g(x',y')$ } monotonicity property.

Step 1:- find minimum

$$\min\left\{ f(x,y) \,/\, \begin{matrix} 0 \le x \le m-1 \\ 0 \le y \le n-1 \end{matrix} \right\}$$

Note:- 512 leves $\longrightarrow$ 4 leves

divide 512 by 4 to get 4 levels.

0 128 256 384 512

Step 2:- $g(x,y) = f(x,y) - \text{Min}$

min ~~mum~~ value of $g(x,y) = 0$.

Step 3:-

$$g(x,y) = \frac{g(x,y)}{\text{Max}} \; ; \; \text{Max} = \max \left\{ g(x,y) \,\middle|\, \begin{matrix} 0 \le x \le m-1 \\ 0 \le y \le n-1 \end{matrix} \right\}$$

max of $g = 1$

Step 4:- $g(x,y) = g(x,y) * (L'-1)$

$\hookrightarrow (0, (L'-1))$ range//

Output is g//.

Ceiling and flooring is done in last step to get ~~into last~~ ~~in~~ int

Monotonicity prop of g is true.

Let $f : X \times Y \longrightarrow Z$

$X = \{0, \ldots m-1\}$

$Y = \{0 \ldots n-1\}$

$Z = \{a \ldots b\}$

transfer f into g satisfying monotonicity property with $Z = \{c \ldots d\}$

Step 1 :- find minimum of $f(x,y)$

$$\min \left\{ f(x,y) \,/\, \begin{matrix} 0 \le x \le m-1 \\ 0 \le y \le n-1 \end{matrix} \right\}$$

Step 2 :- $g(x,y) = f(x,y) - \text{Min}$

Step 3 :- $g(x,y) = \dfrac{g(x,y)}{(\text{Max}-\text{Min})}$ | $\text{max} = \max\{f(x,y) \,/\, 0 \le x \le \ldots\}$

$\{0 \ldots 1\}$

Step 4 :- $g(x,y) = g(x,y) \cdot (d-c) + c$

$c \rightarrow$ min range of $g(x,y)$ $\rightarrow$ ceiling

$d \rightarrow$ max range of $g(x,y)$ and flooring to get integer values.

221
222
222

110.5
1111
1111

Monotonicity transformation

Given $f : X \times Y \rightarrow Z$ $\quad Z = \{a, \ldots b\}$ $\quad Z' = \{c \ldots b\}$

To transform f to $g : X \times Y \rightarrow Z'$

001111
00000 1111

st

$f(x,y) \le f(x',y')$

$\Rightarrow g(x,y) \le g(x',y')$

Sum of 2 images

Given $f : X \times Y \rightarrow Z$ $\quad X = \{0 \ldots m-1\}$

$Z = \{0 \ldots z-1\}$ $\quad Y = \{0 \ldots n-1\}$

Step 1 $(f+g)(x,y) \rightarrow$ sum of pixel values. $= f(x,y) + g(x,y)$

Step 2 :- Transform $(f+g)$ using monotonicity transform.

f = matrix of $m \times n$: m_1

g = matrix of $m \times n$ m_2

$f + g = m_1 + m_2$ (matrix addition)

$f_1, f_2 \ldots f_n$ (set of ~~image~~ frames)

$\frac{1}{n}\left(\sum_{i=1}^{n} f_i\right) = f \rightarrow$ storing the image (average of all frames)

Why we do it $\Rightarrow$ To make it error ~~free~~ have less. ~~It~~ No sensor is error free.

$f(x,y)$

$f_1(x,y) = f(x,y) + \epsilon_1$

$f_2(x,y) = f(x,y) + \epsilon_2$

$\vdots$

$f_n(x,y) = f(x,y) + \epsilon_n$

$$f(x,y) = \frac{1}{n}\sum_{i=1}^{n} (f(x,y) + \epsilon_i)$$

$$= f(x,y) + \boxed{\frac{1}{n}\sum_{i=1}^{n} \epsilon_i}$$ average of error.

$\hookrightarrow \approx 0$ As it can have errors in bothe -ve and +ve side.

Difference of two Images

Step 1 :- f_1, f_2 are 2 images

$(f_1 - f_2)(x,y) = f_1(x,y) - f_2(x,y)$

Step 2 :- Transform the resultant using monotonicity transform.

Application :- Motion detection

:- Digital Angiogram.

$f_1(x,y) \rightarrow$ X-ray image of heart

after injecting Iodine in bloodstream, the cholestral absorbs it more. (contrast enhances)

$f_2(x,y) \rightarrow$ X-ray image of heart after injecting I

$(f_2 - f_1)(x,y) \rightarrow$ High value of cholestral portion

low value elsewhere

can search for blocks.

Image multiplication

Pointwise matrix multiplication

S1) $(f_1 * f_2)(x,y) = f_1(x,y) * f_2(x,y)$

Step2 → Transform the result to codomain 0 to L-1 using monotonicity ~~platform~~ transform

↳ Morphing of images ↳ Masking

Template matching

Image Division

S1:- $(f_1/f_2)(x,y) = \dfrac{f_1(x,y)}{f_2(x,y)}$

Step 2:- Transform the result to codomain 0 to L using monotonicity transform

eg:- $I_1 \longrightarrow$ Proj $\longrightarrow I_2$

PC area is more

quality $(I_2) << $ quality (I_1)

Projection model → distortion function

$I_1(x,y) = h(x,y) * I_2(x,y)$ → projected image

↳ Projection model need I_2 in better quality

$I_1(x,y) = 100$

$\Rightarrow I_2(x,y) = \dfrac{100}{h(x,y)}$

$\Rightarrow h(x,y) = \dfrac{100}{I_2(x,y)}$

$I_2 = \dfrac{I_1}{h}$

To project I_1, project $\dfrac{I_1(x,y)}{h(x,y)}$ to get good quality projected

5) Image Complementation

Given $f(x,y)$, the com of f is

$g(x,y) = (L-1) - f(x,y)$.

Moophological operation.

* Binary Image

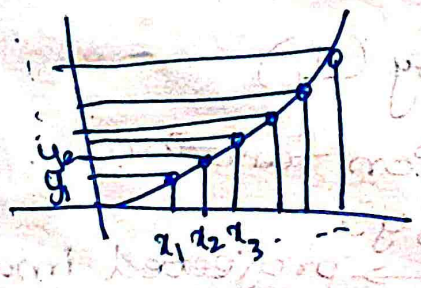
Image processing → Spatial domain → Geometric Transform (pixel location) based IP
→ Pixel values based

→ ~~tranfo~~ transform domain

Interpolation :- Given a set of samples predict an unknown sample. Consider

$\boxed{y = 2x^2}$

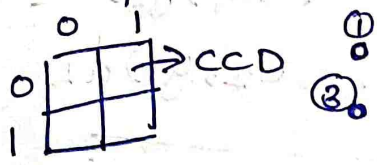
→ unknown



known as (x_i, y_i) $1 \le i \le n$

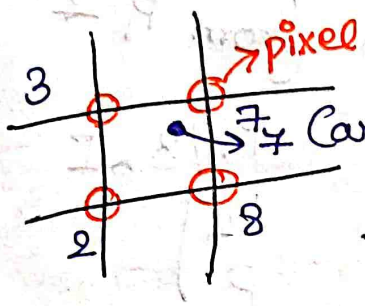
find $f(x)$ for given x.

Samples in Image processing are pixels.



So the pixel value of CCD cells are given. If another CCD cell is placed in between the 4 cells, then can we predict its value.

Nearest Neighbour Interpolation



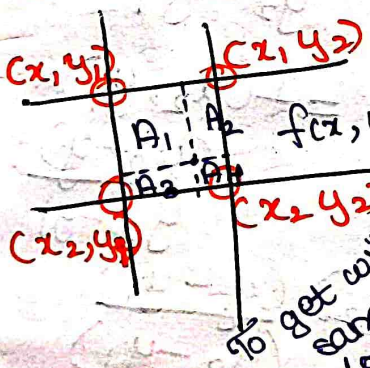
So without placing CCD ~~values~~ cells we can predict the value

gradient :- rate of change of pixel values.

Disadvantage :-

Aliasing can't be handled properly. Only 1 location is considered.

Better ans :- Consult more neighbours and averging but if we do averaging the ~~point~~ value closer and farther will have same influence which is wrong. So we need to do weighted average.



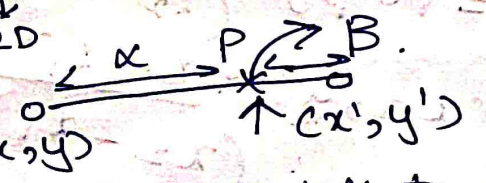
weights $= A_1, A_2, A_3, A_4$

$$f(x,y) = \frac{f(x_1, y_1) \times A_4 + f(x_1, y_2) \times A_3 + f(x_2, y_2) \times A_1 + f(x_2, y_1) \times A_2}{A}$$

To get within same range

Why not use distance :- as for 2D Area is better

The above process ⇒ Bilinear Interpolation.
↓
2D

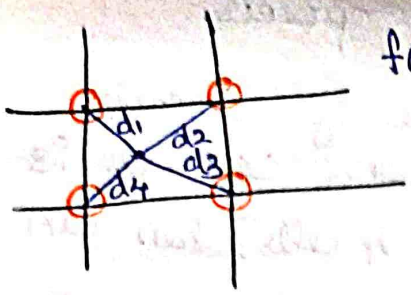


Linear Interpolation

value of $P = 10 * \alpha + 8 * \beta$

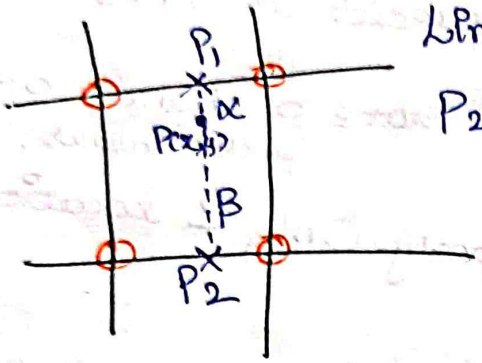
$\alpha + \beta = 1$

Why not use linear rather than Area.



$$f(x,y) = \frac{f(x_1,y_1)d_1 + f(x_1,y_2)d_2 + f(x_2,y_2)d_3 + f(x_2,y_1)d_4}{d_1+d_2+d_3+d_4}$$

$d_1 + d_2 + d_3 + d_4 \neq$ constant.



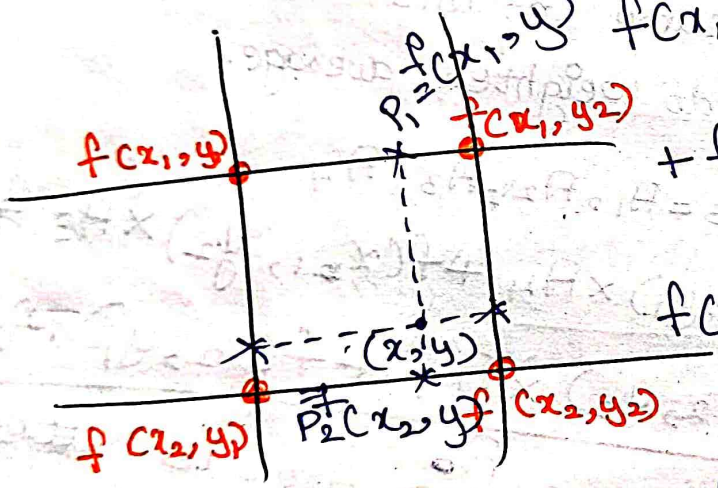
Linear Interpolation from P_1 & P_2 is P.

$P = \beta P_1 + \alpha P_2$

$\alpha + \beta = 1$

$\left.\begin{matrix} P_1 \\ P_2 \\ P \end{matrix}\right\}$ 3 LI

To prove that weighted average is same as taking linear interpolation in x & y direction



$$f(x_1,y) = f(x_1,y_1)\frac{(y_2-y)}{y_2-y_1} + f(x_1,y_2)\frac{(y-y_1)}{y_2-y_1}$$

$$f(x_2,y) = f(x_2,y_1)\times\frac{(y_2-y)}{y_2-y_1} + f(x_2,y_2)\frac{(y-y_1)}{(y_2-y_1)}$$

$$f(x,y) = f(x_1,y)\times\frac{(x_2-x)}{(x_2-x_1)} + f(x_2,y)\times\frac{(x-x_1)}{(x_2-x_1)}$$

After Substitution

$$f(x,y) = \left(f(x_1,y_1)\frac{(y_2-y)}{(y_2-y_1)} + f(x_1,y_2)\frac{(y-y_1)}{(y_2-y_1)}\right)\frac{(x_2-x)}{(x_2-x_1)}$$

$$+ \left(f(x_2, y_1) \times \left(\frac{y_2 - y}{y_2 - y_1}\right) + f(x_2, y_2) \times \left(\frac{y - y_1}{y_2 - y_1}\right) \right)$$

$$* \left(\frac{x - x_1}{x_2 - x_1}\right)$$

Linear Interpolation for 1D.

<u>Image file format</u> → encoding of matrix (image)

* Digital image is 1024 × 1024

pgm → file format
→ Header → All info for decoding
comments → no. of rows, no. of colums.
P_2 → PGM grey valu.
P_5 → Binary values

2 fields
→ Body → Raw data / compressed data
Not compressed at all (simplicity)

jpeg → compressed file format
↳ header
↳ Body <u>Compressed</u> ~~(simpl~~

find a, b, c, d by substituting x_i, y_j in eq 1.

$$f(x, y) = ax + by + cxy + d$$

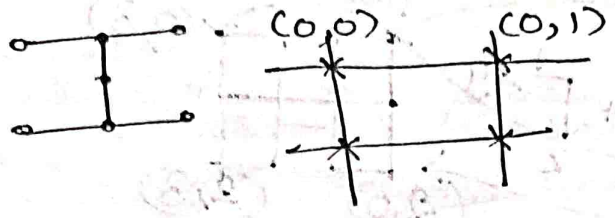
$$= f(x_1, y_1) A_4 + f(x_1, y_2) A_3 +$$

$$f(x_2, y_1) A_2 + f(x_2, y_2) A_1$$

Recall

* Bilinear Interpolation superior to Nearest Neighbour

↳ $f(x,y) = ax + by + cxy + d$



$f(x,y) = ax + by + cxy + cx^2$

Bi-linear.

$f(x,y) = \sum_{j=0}^{1} \sum_{i=0}^{1} a_{ij} x^i y^j =$

00 1 1 1 1
00 1 1
00 1 1 1 1

Bi-~~cubic~~ quadratic

$f(x,y) = \sum_{j=0}^{2} \sum_{i=0}^{2} a_{ij} x^i y^j$

Size (I) ⟶ Size (I')

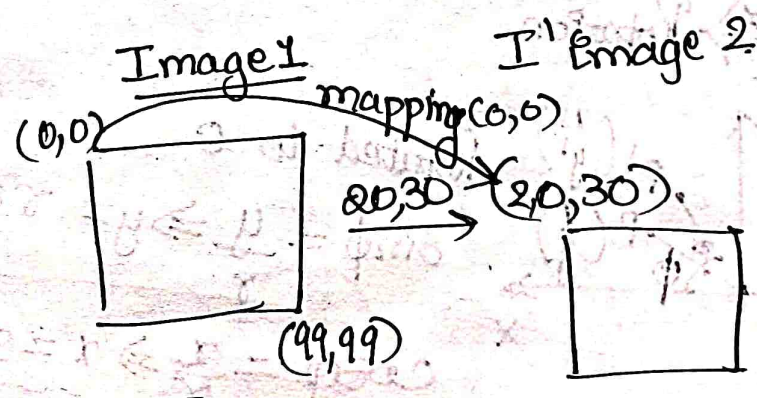
not same size ⇒ Geometric Transformation.

Geometric Transformation

$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix}$

$x' = x + a \qquad y' = y + b$

$T\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$



Translation.

mapping → shifting the pixel values. to another

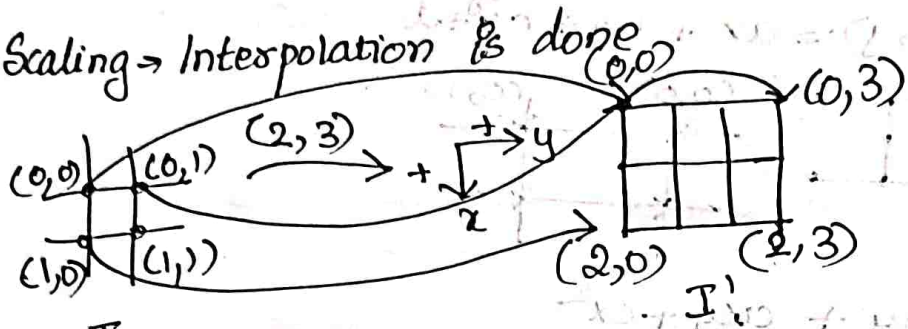
Linear movement → Translation.

① Geometric Transformation → changing the pixel location.

② ~~Sca~~ Scaling

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} ax \\ by \end{pmatrix}$$ Looming and shrinking.

Scaling → Interpolation is done



Inverse mapping $x/2$ $y/3$

Interpolation in Geometric Interpolation.

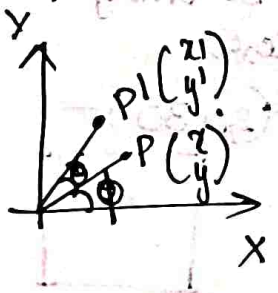
Step 1:- Do Inverse mapping of $\begin{pmatrix} x' \\ y' \end{pmatrix}$ in I', say the I_v map location is $\begin{pmatrix} x \\ y \end{pmatrix}$

Step 2:- Do Bilinear Interpolation at $\begin{pmatrix} x \\ y \end{pmatrix}$ in I, say value is v

Step 3:- Assign the pixel val at $\begin{pmatrix} x' \\ y' \end{pmatrix}$ with v.

floor and ceil of (x', y')

3) Rotation



Rotated to θ

$\sin\phi = \frac{y}{r} \Rightarrow y = r\sin\phi$

$\cos\phi = \frac{x}{r} \Rightarrow x = r\cos\phi$

$\sin(\theta+\phi) = \frac{y'}{r} \Rightarrow y' = r\sin(\phi+\theta)$

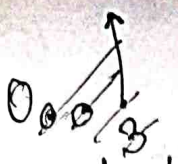
$\cos(\theta+\phi) = \frac{x'}{r} \Rightarrow x' = r\cos(\theta+\phi)$

$= r(\cos\theta\cos\phi - \sin\theta\sin\phi)$

$= x\cos\theta - y\sin\theta$

$y' = r\sin\theta\cos\phi + r\cos\theta\sin\phi$

$= x\sin\theta + y\cos\theta$



Definition of Linear

$$T(ax+by) = aT(x) + bT(y)$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotation is linear

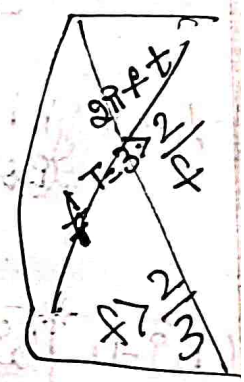
Express as matrix multiplication.

<u>Scaling</u>

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ b \end{bmatrix}$$

∴ Scaling is linear.

Is Translation is linear? ⇒ Not linear.



$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$

$$\text{s.t } T\begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix} \neq T\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + T\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$

eg:- $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ Translation $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Everything should have Bilinear Interpolati

$$T\begin{pmatrix} 0+1 \\ 0+2 \end{pmatrix} = T\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$ LHS

RHS.

$$T\begin{pmatrix} 0 \\ 0 \end{pmatrix} + T\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} =$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} \neq \text{LHS}$$

Shearing in X-direction

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Shearing in y-direction

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Bilinear Transformation

$f(x,y) = ax + by + cxy + d$ $\boxed{\text{Easier}}$

Bi-quadratic

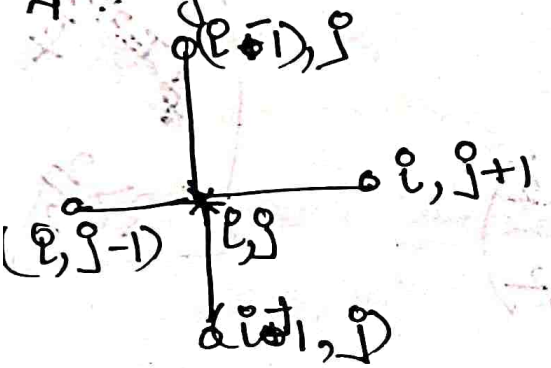
$$f(x,y) = \sum_{j=0}^{2} \sum_{i=0}^{2} x^i y^j a_{ij}$$

Bi-cubic

$$\sum_{j=0}^{3} \sum_{i=0}^{3}$$

→ Bilinear used.

4-neighbourhood



8-neighbour

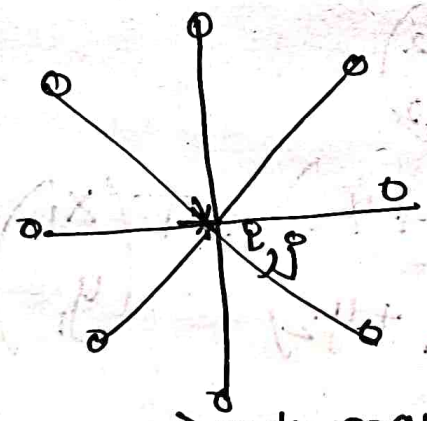


Image Processing → Spacial → only many img
→ single Image

→ Transform

transformation on pixel location (Geometric T) — Image size changes

Transformation on pixel values — Not change

SINGLE IMAGE TRANSFORMATION

① Thresholding:-

Let T be a number, called Threshold. Let $f(x,y)$ be the i/p image. The o/p image $g(x,y) = 0$ if $f(x,y) < T$

$= 1$ if $f(x,y) \geq T$

or 255

Gray scale to Binary Application

- Seperating Foreground from Background.
  - Foreground → 1
  - Background → 0

- To represent object.

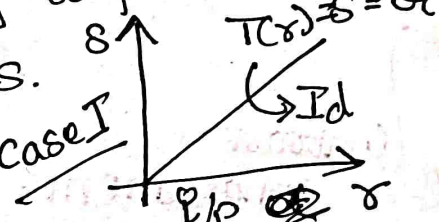
② Image Complementation /negation

$g(x,y) = 255 - f(x,y)$

o/p — i/p

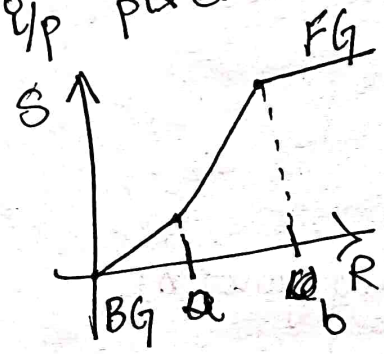
X-ray imag proc from neg to positive

o/p → s.



o/p can be same.

③ i/p pixel val is r



$f(x,y) \rightarrow$ i/p

$\underbrace{g(x,y)}_{s} = T(\underbrace{f(x,y)}_{r})$

measure

Contrast difference b/w Foreground & Background.

④ Power law function

o/p s i/p = r

$s = c r^{\gamma}$ ⇒ power of i/p and multiply by constant

I v

High Intensity → [device] → voltage

$v = c I^{\gamma}$ (2.8)

$I \propto v$

But we want image only so instead of 2.8.

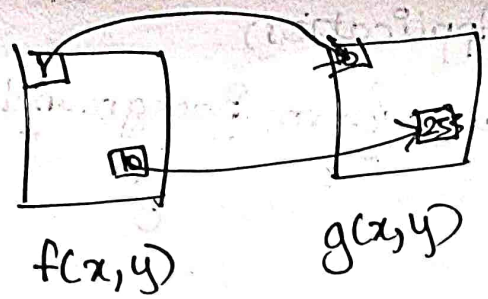
$(i)^{0.4} \rightarrow$ [device] $\xrightarrow{c((i)^{0.4})}$ 2.8. Gamma correction.

i/p → $f(x,y)$ o/p → $g(x,y) = T(f(x,y))$

$= c(f(x,y))^{\gamma}$

Q.) When we use Power law function f, g; will it have same size? Yes.

Q>



True → Yes same.

Intensity Transformation

$s = T(r)$

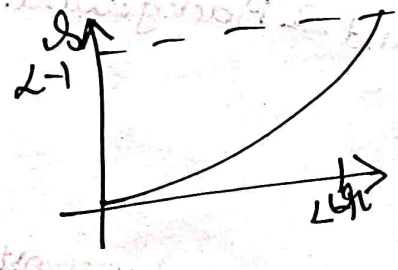
$$\begin{cases} T(r) = r \;/\; T(r) = \frac{r}{2} & r \in [0, a] \\ \quad\quad\quad\quad\quad\quad = 2r & r \in [a+1, b] \\ \quad\quad\quad\quad\quad\quad = 4r & r \in [b+1, L-1] \end{cases}$$

Contrast enhancement.

⑤ Logarithmic Transformation

$s = T(r) = C \log r$

mistake (not log function).



[0, 255]

$f * g \rightarrow [0, 10{,}000]$

↓

$[0, L-1]$

51:50

~ ~ ~ ——— ~ ~ ~ ——— ~ ~ ~ ———

$$f(x,y) = \left[f(x_1, y_1)\left(\frac{y_2 - y}{y_2 - y_1}\right) + f(x_1, y_2)\left(\frac{y - y_1}{y_2 - y_1}\right) \right]\left(\frac{x_2 - x}{x_2 - x_1}\right) +$$

$$\left[f(x_2, y_1)\left(\frac{y_2 - y}{y_2 - y_1}\right) + f(x_2, y_2)\left(\frac{y - y_1}{y_2 - y_1}\right) \right]\left(\frac{x - x_1}{x_2 - x_1}\right)$$

$$= \frac{f(x_1, y_1)\, y_2 x_2}{(y_2 - y_1)} \frac{1}{(y_2 - y_1)(x_2 - x_1)} \Big[f(x_1, y_1)\, y_2 x_2 - f(x_1, y_2)\, x_2 y_1 + f(x_1, y_1)\, xy - f(x_1, y_2)\, xy - f(x_1, y_1)\, y_2 x + f(x_1, y_2)\, y_1 x - f(x_1, y_1)\, x_2 y + f(x_1, y_2)\, x_2 y \Big]$$

$- f(x_2,y_1)\,xy + f(x_2,y_2)\,xy - f(x_2,y_1)\,x_1y_2$
$+ f(x_2,y_2)\,x_1y_1 + f(x_2,y_1)\,y_2x - f(x_2,y_2)\,y_1x$
$+ f(x_2,y_1)\,x_1y - f(x_2,y_2)\,x_1y.$

$\Rightarrow Ax + By + Cxy + D$

$A = f(x_1,y_2)\,y_1 - f(x_1,y_1)\,y_2 + f(x_2,y_1)\,y_2$
$- f(x_2,y_2)\,y_1$

$B = -f(x_1,y_1)\,x_2 + f(x_1,y_2)\,x_2 + f(x_2,y_1)\,x_1 -$
$f(x_2,y_2)\,x_1$

$C = f(x_2,y_2) - f(x_2,y_1) + f(x_1,y_1) - f(x_1,y_2)$

$D = f(x_1,y_1)\,y_2x_2 - f(x_1,y_2)\,x_2y_1 - f(x_2,y_1)\,x_1y_2 +$
$f(x_2,y_2)\,x_1y_1.$

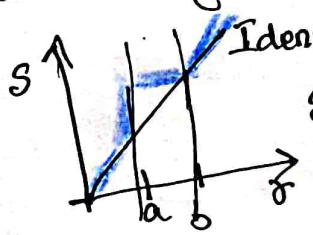
Back to Tan3decture.

Growth is less than linear function

log reduces the range.

Galaxy images (stars) → IR range to visible range
(range is high)

⑥ Intensity Slicing Transformation.



Slice a, b are highlighted.

⑦ Bit-plane Slicing

Assume that pix values takes 8bit.

Image can be thought of 8 binary matrices.

MSB $m \times n$; $m \times n$ 2nd MSB ... LSB $m \times n$

$1 \le i \le 8$ $\quad$ $3 \le i \le 5$

MSB $\quad$ LSB $\quad$ $g(x,y) = 3^{rd}, 4^{th}, 5^{th}$ bit of pix value in $f(x,y)$

Used for bit level modification (watermarking)

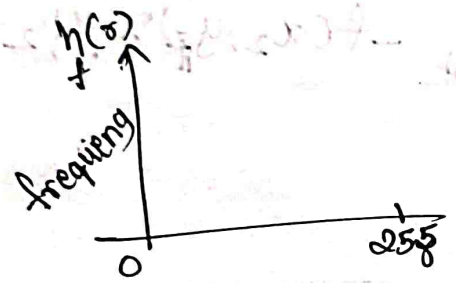
(8) Histogram Processing

↳ Histogram equalization

↳ Histogram matching.

Let $f(x,y)$ with value 0 to 255. The histogram of f is denoted by $h_f(r) = \frac{\#\text{ of pixels with value } r}{\cancel{\#\text{ of pixels in } f}}$

$0 \le r \le 255 \; (L-1)$



frequencies of pixel values.

$I \rightarrow h_I = h_{I'}$

I' Yes

Image $\quad$ B ↔ W Histogram won't change

Probability of $r = \frac{\#\text{ of pixels with value } r}{\#\text{ of pixels in } f}$

↓ normalized histogram

↓ feature

Lecture – 3/feb/21

3/feb/21 Lecture

Recap.

I/P r		O/P s
0	map →	5
1	→	6
2	→	9
⋮		⋮

$s = T(r)$ ⇒ Point processing.

eg:- Image negation

Thresholding.

HISTOGRAM EQUALIZATION:-

↳ Improving contrast / Enhancement Constant

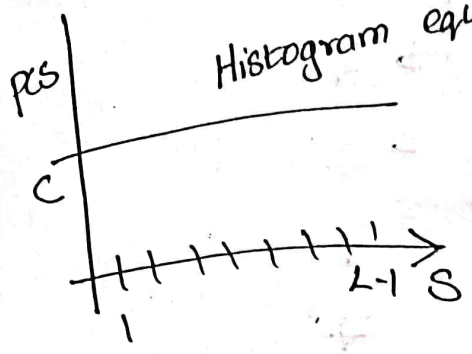
$T(r) = \left(\int_0^r p(r)\,dr\right)(L-1)$ ~~Pdf~~ Probability is constant

$r \xrightarrow{T} \left(\int_0^r p(r)\,dr\right)(L-1)$

$\downarrow$

histogram is equalised.

$p(s) = c \quad \forall\ 0 \le s \le 1$



To prove. $p(s) = c$

where $s = \left(\int_0^r p(r)\,dr\right)(L-1)$

Proof

Let $s = T(r) = \left(\int_0^r p(r)\,dr\right)(L-1)$

We know that when $s = T(r)$

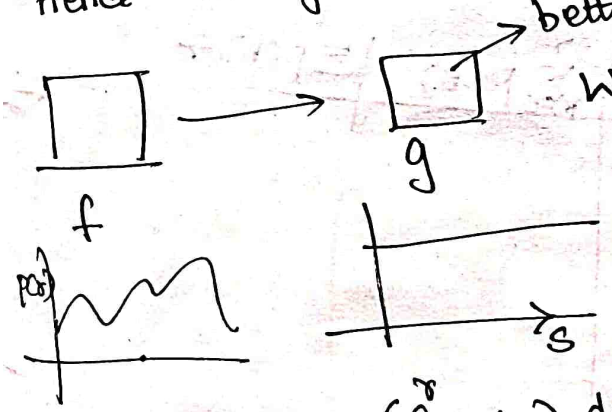
$$\boxed{p(s) = p(r)\frac{dr}{ds}} \qquad s = \int_0^r p(r)\,dr(L-1)$$

$$\frac{ds}{dr} = p(r)(L-1)$$

$$\frac{dr}{ds} = \frac{1}{p(r)(L-1)}$$

$$p(s) = p(r) * \frac{1}{p(r)(L-1)} = \frac{1}{L-1}$$ which is constant

Hence Histogram equalization has uniform Pdf.



better contrast

When pixels are distributed

$\therefore$ more contrast.

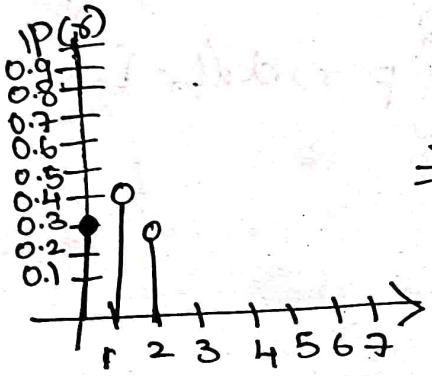
$$s = T(r) = \left(\int_0^r p(r)\,dr(L-1)\right)$$

$$= \left(\sum_{k=0} p(k)\right)(L-1)$$ $\rightarrow$ Hist Equalisation transform in discrete form.

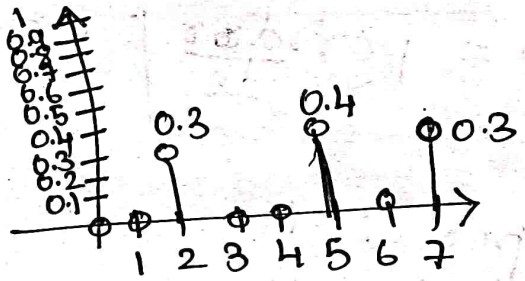
r	$p(r)$
0	0.3
1	0.4
2	0.3
3	0
4	0
5	0
6	0
7	0

$f \rightarrow$ i/p (dark image :- contrast is bad)

r	$p(r)$	$\sum_{k=0}^{r} p(k)$	$\sum p(k)(L-1)$
0	0.3	0.3	2
1	0.4	0.7	5
2	0.3	1	7
3	0	1	7
4	0	1	7
5	0	1	7
6	0	1	7
7	0	1	7



$\Rightarrow$



More contrast

It basically stretches the contrast

Disadvantage :- for discrete $\rightarrow$ not constant. But still contrast is present. Convert to continuous domain and discretization of the output with some approximation

Discrete $\rightarrow$ continuous $\rightarrow$ Discrete

$L-1$.

H.W.

r	$f_r(r)$ (×1000)	$p(r)$	$\sum_{k=0}^{r} p(k)$	$\sum p(k)7$
0	0.3	3/165		
1	0.2	2/165		
2	1	10/165		
3	1	10/165		
4	2	20/165		
5	3	30/165		
6	4	40/165		
7	5	50/165		
	16.5			

HISTOGRAM MATCHING TRANSFORMATION

→ Why?

Given histogram of an image of a scene, to improve the quality of any image of the scene, using h.

① f (bad) → □ → g → Improved 2 I/P

② h (good) → □

Allignment not an issue.

Do hist equal of h

$h \quad r \xrightarrow{T_1} s$

Do hist equal of f

$f \quad u \xrightarrow{T_2} v$ All has range 0 to L-1.

$T_1^{-1}(T_2(u)) \quad u \to r$

p(r) is of h → good img

$T(u) = T_1^{-1}(T_2(u))$ p(r) is histogram of good img

to hist of g is p(r) g is good.

Match histogram of g with histogram of h.

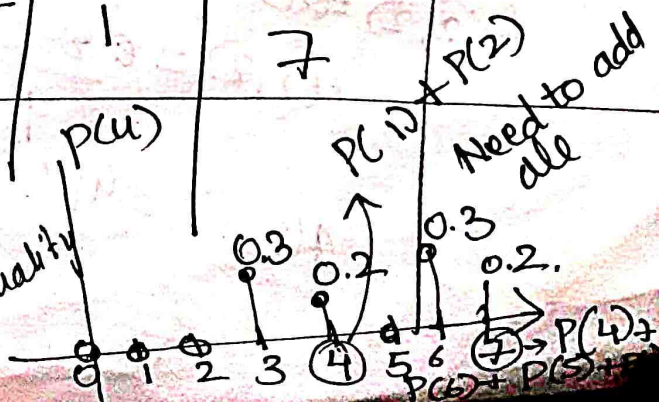
r	P(r)	P(u)	Σp(r)	Σp(r)7	Σp(u)	Σp(u)7
0	0	0.3	0	0	0.3	2
1	0	0.2	0	0	0.5	4
2	0	0	0	0	0.5	4
3	0.3	0.3	0.3	2	0.8	6
4	0.2	0.2	0.5	4	1	7
5	0	0	0.5	4	1	7
6	0.3	0	0.8	6	1	7
7	0.2	0	1	7	1	7

u	r
0	3
1	4
2	4
3	6

u	r
4	7
5	7
6	7
7	7

good quality

p(u)



Need to add

Feb-4th-21 → Lecture

Type of Spatial Domain Image Processing.

↳ Point → Img Com, Histogram processing, Slicing(Bit)

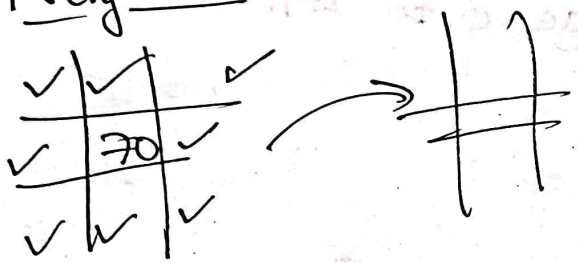
↳ Neighbour hood

had to look into the pixel value of the pixel that needs to be transformed

Builds look up table

r	s

Neighbour



any two adjacent pixels are very similar.

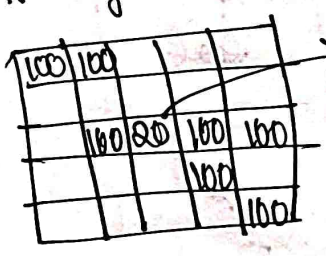
* Any two neighbours will be similar with v.high probability

$$f(x,y) = f(x,y) * r(x,y)$$

* 10% of pixels will be edges.

Eg:-

Average filter.



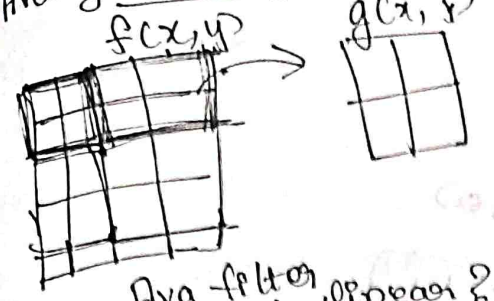
$$g(x,y) = \frac{\sum_{(i,j) \in \text{Neighbour } x,y} f(i,j)}{|N_{xy}|}$$

$f(2,2) = 20$

$$g(2,2) = \frac{820}{9} = 91.11.$$ | 8 Neighbour hood.

$g(2,1) = 91.$

Average Pooling



Is this Avg-filter process linear?

$f \rightarrow \boxed{} \rightarrow g$

$$AF(f_1 + f_2) = AF(f_1) + AF(f_2)$$

$$AF(cf) = cAF(f)$$

It is linear.

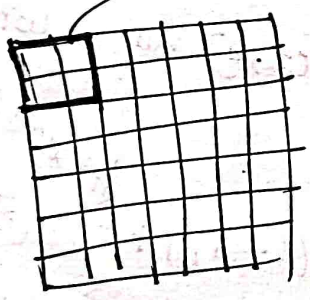
$$LHS = AF(f_1 + f_2) = \left(\sum_{i,j \in N_{x,y}} (f_1 + f_2)(i,j) \right) / N_{xy}$$

$$= \left(\sum f_1 + \sum f_2 \right) / N_{xy}$$

$$= \frac{\sum f_1}{N_{xy}} + \frac{\sum f_2}{N_{xy}} = AF(f_1) + AF(f_2)$$

② Median filter.

f



find median of this set of pixels — same size

removes outliers.

- Salt and Pepper Noise
  - 255
  - 0

Noisy image → median filter → Noise is reduced.

Is it Linear → Non-Linear.

Hello

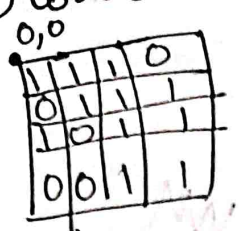
$$MF(f_1 + f_2) \neq MF(f_1) + MF(f_2)$$

12	56
34	78

→ 4.5 not equal.

2.5 6.5 → 9

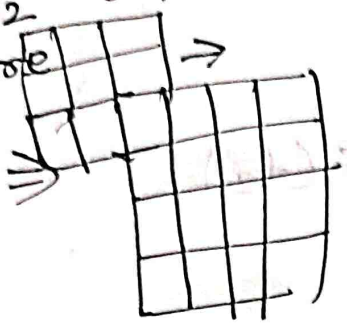
③ Correlation



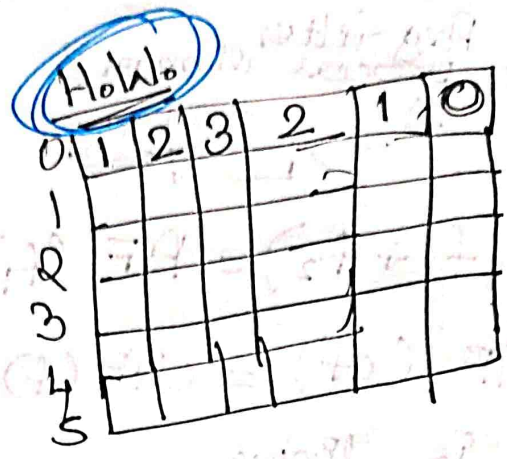
3x3

$f(4\times 4$ $\quad$ $W \rightarrow$ centre $(0,0)$

$m/2$ $n/2$ → centre



move.



Size f is $m\times n$

W $a\times b$

What is the size of $f \odot \omega$

$(m-a-1,\ n+b-1)$

Definition of Correlation

Similarity of a signal with another signal

– Weighted sum

Let $f(x,y)$ be an image & $W(x,y)$ be a window

$0\le x\le a-1,\ 0\le y\le b-1$

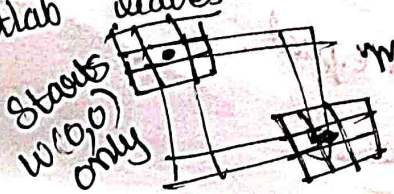
Assume $(0,0)$ centre

$$g(x,y)=(f\odot\omega)(x,y)=\sum_{t=-\frac{b}{2}}^{b/2}\sum_{s=-\lfloor\frac{a}{2}\rfloor}^{a/2} f(x+s,y+t)\,w(s,t)$$

$$g(0,0)=\sum_{t=-1}^{1}\sum_{s=-1}^{1} f(0+s,0+t)\,w(s+1,t+1)$$

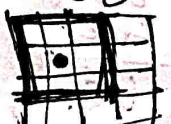
Varient 1 $= f(-1,-1)w(0,0)+f(0,-1)\,w(1,0)+f(1,-1)w($

Matlab leaves into $m\times n$.



Varient 2 (No padding)

size of $g < f$



window want go out

Used in CNN

HoWo

Find Correlation of f and W with (i) full
(ii) Invarian 1
(iii) varian 2.

f:

0	0	0	0	0
0	1	1	1	0
0	1	0	1	0
0	1	1	1	0
0	0	0	0	0

W:

1	1	1
1	1	1
1	1	1

Feb/10th/21 Lecture

Correlation

$$g(x,y) = \sum_{t=-\frac{b}{2}}^{b/2} \sum_{t=-\frac{a}{2}}^{a/2} f(x+s, y+t)\, W(s,t)$$

"Assume that centre of the window is (0,0)"

CNN → correlation without padding.
i/p and o/p → same

Use of Correlation (weighted sum)

* To do template matching
high value when window and set of pixels is same.

* filtering (smoothing, sharpening, finding edge)

Normalised Cross Correlation

$$C(x,y) = \frac{\sum\sum (f(x+s, y+t) - \bar{f})(w(s,t) - \bar{w})}{\left[\left(\sum\sum (f(x+s, y+t - \bar{f})^2\right)\left(\sum\sum (w(s,t) - \bar{w})^2\right)\right]^{1/2}}$$

This is better for matching.

Convolution

Natural process where convolution is applied?

1D :- f : | 1 | 2 | 3 | 4 | 5 | $\boxed{5}$

W : | -1 | 0 | 1 | $\boxed{3}$

$Cor(f, w) =$

| 1 | 2 | 2 | 2 | 2 | -4 | -5 | $\boxed{5+3-1}$

$Conv(f, w) = Cor(f, w_{180})$

W_{180}

| 1 | 2 | 3 | 4 | 5 | $*$ | 1 | 0 | -1 |

$=$ | -1 | -2 | -2 | -2 | -2 | 4 | 5 |

Polynomial multiplication

$(a_0 + a_1 x + a_2 x^2 \ldots a_m x^m) . (b_0 + b_1 x + \ldots b_n x^n)$

$= (a_0 b_0 + a_0 b_1 x + a_1 b_0 x \ldots \ldots) \quad ( ) x^{m+n})$

$(a_0, a_1, a_2 \ldots)$ $\Downarrow$ f

$(b_0, b_1, b_2 \ldots$ $\Downarrow$ w

$Conv(f, w) = Cor(f, w_{180}) = [a_0 b, (a_0 b_1 + a_1 b_0), \ldots$

direct computation $O(n^2) \rightarrow$ Bruteforce

Using Convolution $O(n^2)$ time domain

Using Convolution in freq-domain $O(n \log n)$

~~Write a program to make conv~~

2-D convolution

$g(x,y) = Cor(f, W_{180})$

$$g(x,y) = \sum_{t=-b/2}^{b/2} \sum_{t=-a/2}^{a/2} f(x-s, y-t)\, w(s,t)$$

Uses of Convolution

"CNN" → convolution.

Conv = Corr for Symmetric filters.

→ Fourier Transform.

$F(f * w) = F(f).F(w)$

Conv
- ↳ Smoothing
- ↳ Sharpening
- ↳ find edges.

can find conv faster.

Is Correlation Linear? – Yes

① $Cor(f_1 + f_2, w) = Cor(f_1, w) + Cor(f_2, w)$

② $Cor(cf, w) = c\,Cor(f, w)$

LHS $Cor(f_1 + f_2, w) = \sum\sum (f_1 + f_2).(x+s, y+t).w(s,t))$

$= \sum\sum f_1(x+s, y+t) . w(s,t) + \sum f_2(x+s, y+t) w(s,t).$

$= Cor(f_1, w) + Cor(f_2, w) = RHS$

Let f be an image of size $m \times n$

$w = \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$

$f \odot w \Rightarrow$ Blurring → Mean filter / Average filter.

(↗ correlation.)

Allong boundary perfect averaging won't happen.

Filter 2

$w = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$f \odot w$ → horizontal edge extractor

Filter 3

$w = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

$f \odot w$ → Vertical edge extractor

Can you imp median filtering using Conv / Cor? No.

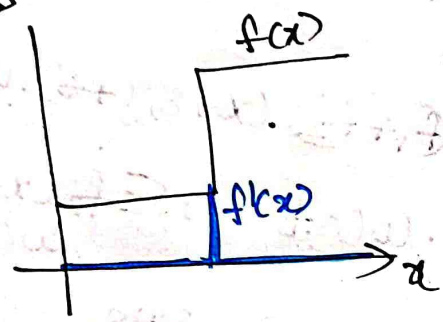
↓

Non-linear.

feb 11/~~2020~~ 21 Lecture

Derivative filer
↳ to find edges in image.

→ Types of Derivative filters
* first Order
* Second Order.



Scan f'(x)
very high value indicates the edge point.

Derivative filter design.

1-D Derivative

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

Let $h = 1$

$$f'(x) = f(x+1) - f(x)$$

Let $h = -1$

$$= \frac{f(x-1) - f(x)}{-1} = f(x) - f(x-1)$$

Let f be an image of size $m \times n$ and W be the filter

$W = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ Consider $g = f \odot \omega$.
↳ derivative of f'

Again → boundary can lead an issue.

$f(1,1)(-1) + f(2,1) \times 1 = f(2,1) - f(1,1)$
in x direction.

$g = f \circledast \omega_1$, $W_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

① Robert filter (Not symmetric

$w_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ $w_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$g = f \odot w_1$ $g = f \odot w_2$

② Prewitt filter

$w_1 = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ $g_x = f \odot w_1$

$w_y = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ derivating in y direction $g_y = f \odot w_2$

Used for edge detection. use multiple filters and overlap.

Problem

find the cor(f, w)

$f = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$f \odot w = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = w_1 = f \odot (T_1 \odot T_2)$

$T_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $T_2 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$\underbrace{(f \odot T_1)}_{\text{derivative}} \odot \underbrace{T_2}_{\text{smoothing}}$

↳ reduces (~~removing~~ noise)

Note:- Prewitt filter
↳ reduces noise before taking derivative.

③ Sobel filter.
Observation:- finds sum of current and neighbouring pixe before derivative.

Replaced Sum with weighted sum

$(f \odot w_1) \odot w_2 = g$.

$w_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $\quad w_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix}$

$f \odot \underbrace{(w_1 \odot w_2)}$

$w = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

$l_i \quad l_o = l_i - l_s + 1$

HoWo:- find Cor, Conv for f and w given as follow.

(i) full form

(ii) size of i/p = size of o/p

(iii) no padding

a) $f = \begin{matrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$ $\quad w = \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{matrix}$

b) $f = \begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{matrix}$ $\quad w = \begin{matrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{matrix}$

c) $f = \begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$ $\quad w = \begin{matrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{matrix}$

d $\quad w = \begin{matrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{matrix}$

17/feb/21 Lecture

Filter design
- ↳ Average
- ↳ Median
- ↳ Derivative

Spatial Domain
- ↳ Manipulating pixels directly

Second Order Derivative filter Design.

Given $f(x,y)$

$$\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \qquad \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \nabla^2 f$$ Laplacian operation.

$$f'(x) = f(x+1) - f(x) \rightarrow (1)$$

Let $f(x,y)$ be an Image.

$$\frac{\partial f}{\partial x} = f(x+1,y) - f(x,y) \quad (\text{By } (1))$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}(f(x+1,y)) - \frac{\partial}{\partial x}(f(x,y))$$

$$= f(x+1,y) - f(x,y) - (f(x,y) - f(x-1,y))$$

$$= f(x+1,y) - 2f(x,y) + f(x-1,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) - 2f(x,y) + f(x,y-1)$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial f^2}{\partial y^2} = f(x+1,y) + f(x,y+1) - 4f(x,y) + f(x-1,y) + f(x,y-1) \rightarrow (3)$$

$\nabla^2 f =$ "

$g = f \odot w$, where $w =$

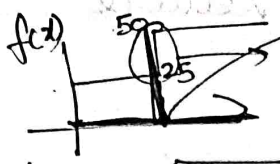
0	1	0
1	-4	1
0	1	0

↳ $\nabla^2 f$

Laplacian operation can be rep by conv/corr.

Edge Model

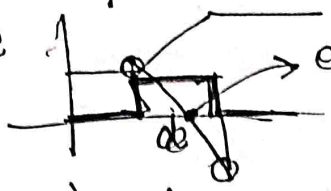
↳ step edge



→ 1st derivative → derivative → Single peak

1D image

↳ Ramp edge



→ edge point

2 peaks

1D → point
2D → line

2nd derivative is required

↳ root edge

Small.

H.W.
Which derivative is require

Image Sharpening

↳ Highlighting the edges.

Given $f(x,y)$

$$g(x,y) = f(x,y) + c|\nabla^2 f|$$

↳ edge points with gray values.

g is sharpened image

How to find edge points?

$$\frac{\partial f}{\partial x} = f \odot w \qquad w = \begin{matrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{matrix} \qquad \nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$

$$\frac{\partial f}{\partial y} = f \odot w \qquad w = \begin{matrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{matrix}$$

R1 If $|\nabla f(x,y)| > T$ Then (x) is an edge pixel.

or

R2 If $T_1 \le |\nabla f(x,y)| < T_2$ then (x,y) is and edge pixel

Fourier Transform

Converting time domain to frequency domain

* It gives representation for many function using $\sin x$ & $\cos x$.

What is frequency? $1/T$

# of times "event" is repeated.
cycle

What does freq refer in non-periodic function?

To define freq of non-periodic function $\rightarrow$ FT

<u>FT for 1D</u>

Drichlet condition. to be true.

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j\frac{2\pi u x}{M}}$$

$\downarrow$ O/P $\quad$ $\downarrow$ I/P

$$f(x) = \sum_{u=0}^{M-1} F(u) e^{j\frac{2\pi}{m} u x} = \frac{1}{2} \sum_{n=0}^{M-1} F(u) \left[\cos\left(\frac{2\pi}{m} n x\right) + j \sin\left(\frac{2\pi}{m} n x\right)\right]$$

$\downarrow$

Inverse FT

What are the freq of $f(x)$

Periodic $\rightarrow$ 1 freq

Non-periodic $\rightarrow$ Many frequencies.

H.W

find the no. of freq of $f(x) = 1 \quad -\frac{n}{2} \le x \le \frac{n}{2}$
$= 0$

∞ freq. = no. of freq.

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j 2\pi u x} dx$$

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j 2\pi u x} du.$$

2D → DFT of Image

Given the Image $f(x,y)$ $\quad 0 \le x \le m-1$, $\quad 0 \le y \le n-1$

$$DFT(f) = F(u,v) = \sum_{y=0}^{n-1} \sum_{x=0}^{m-1} f(x,y)\, e^{-j2\pi \frac{ux}{m}}\, e^{-j\frac{2\pi vy}{N}}$$

$$= \frac{1}{M} \sum_{y=0}^{N-1} \sum_{x=0}^{m-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{m} + \frac{vy}{N}\right)}$$

$$IDFT(F(u,v)) = \frac{1}{N} \sum_{v=0}^{N-1} \sum_{u=0}^{m-1} F(u,v)\, e^{j2\pi\left(\frac{ux}{m} + \frac{vy}{N}\right)}$$

Is DFT periodic? Yes.

Is DFT Linear? Yes.

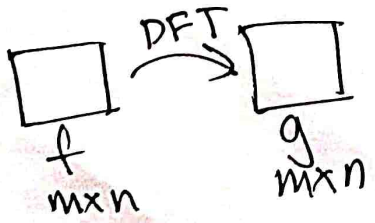
To prove

$$F(u+m, v+N) = F(u,v)$$

$$LHS = F(u+m, v+N) = \frac{1}{m} \sum_{y=0}^{N-1} \sum_{x=0}^{m-1} f(x,y)\, e^{-j2\pi\left(\frac{(u+m)x}{M} + \frac{(v+n)}{N}\right)y}$$

$$= \frac{1}{m} \sum\sum f(x,y)\, e^{-\frac{j2\pi nx}{m}} * \underbrace{e^{-j2\pi mx}}_{1} * e^{-\frac{j2\pi ny}{N}} * \underbrace{e^{-j2\pi ny}}_{1}$$

$$= F(u,v)$$



Circular extension.

$$F(u+m, v+N) = F(u,v)$$

Convolution theorem

$$DFT(f(x) * g(x)) = \underset{O(m^2)}{DFT(f(x))} \cdot \underset{O(m^2)}{DFT(g(x))}$$

- Time complexity

$$DFT(f(x) \circledast g(x)) \to O(m^2)$$

$$O(m^2) + O(m^2) \to 2(O(m^2)) \to O(m^2)$$

$$O(2m^2) \to O(m^2)$$

Fast fourier Transform (FFT)

Algoritm to implement DFT

Time complexity $O(m \log m)$

Derivation of FFT scheme

$$DFT(f(x)) = \sum_{x=0}^{m-1} f(x) e^{-\frac{2\pi i n x}{m}}$$

How to compute $F(u)$ $0 \le u \le m-1$

$$F(u) = \sum_{x=0}^{m-1} f(x) e^{-\frac{j2\pi}{m} ux}$$

$$= \underbrace{\sum_{l=0}^{\frac{m}{2}-1} f(2i) e^{-\frac{j2\pi}{m} n(2i)}}_{\text{even}} + \underbrace{\sum_{l=0}^{\frac{m}{2}-1} f(2l+1) e^{-\frac{j2\pi}{m} u(2l+1)}}_{\frac{m}{2}\text{-odd}}$$

$$= \sum_{l=0}^{\frac{m}{2}-1} g(l) e^{-\frac{j2\pi}{\frac{m}{2}} ul} + \sum_{l=0}^{\frac{m}{2}-1} h(l) e^{-\frac{j2\pi}{m} n(2l+1)}$$

$$= DFT(g(l)) + \sum_{l=0}^{\frac{m}{2}-1} h(l) e^{-\frac{j2\pi}{m/2} ul} e^{-\frac{j2\pi}{m}}$$

can take it out

$$= DFT(g(l)) + e^{-j\frac{2\pi}{m}u} \sum_{l=0}^{m/2-1} h(l)\, e^{-j\frac{2\pi}{\frac{m}{2}}ul}$$

$$= DFT(g(l)) + e^{-j\frac{2\pi}{m}n} \, DFT(h(l))$$

$$F(u) = G(u) + e^{-j\frac{2\pi u}{m}} H(u)$$

$DFT(g(u))$ $\quad 0 \le u \le \frac{m}{2}-1$ $\qquad$ $DFT(H(u))$

To find $F(u)$ where $\frac{m}{2} \le u \le m-1$

$v = u - \frac{m}{2}$, $0 \le v \le \frac{m}{2}-1$

$$F(u) = \sum_{l=0}^{m/2-1} f(2l)\, e^{-j\frac{2\pi}{m}u(2l)} + \sum_{l=0}^{m/2-1} f(2l+1)\, e^{-j\frac{2\pi}{m}(2l+1)}$$

$$= \sum_{l=0}^{m/2-1} f(2l)\, e^{-j\frac{2\pi}{m}(v+\frac{m}{2})(2l)} + \sum_{l=0}^{m/2-1} f(2l+1)\, e^{-j\frac{2\pi}{m}(2l+1)(v+\frac{m}{2})}$$

$$= \sum_{l=0}^{m/2-1} g(l)\, e^{-j\frac{2\pi}{m}v} \underbrace{e^{-j\frac{2\pi}{m}\times\frac{m}{2}(2l)}}_{1} + \sum_{l=0}^{m/2-1} h(l)\, e^{-j\frac{2\pi}{m}(2l+1)} \underbrace{e^{-j\frac{2\pi}{m}(l+1)\frac{m}{2}}}_{-1}$$

$$= \underbrace{\sum_{l=0}^{m/2-1} g(l)\, e^{-j\frac{2\pi}{m/2}vl}} - \sum_{l=0}^{m/2-1} h(l)\, e^{-j\frac{2\pi}{m}2lv}\, e^{-j\frac{2\pi}{m}v}$$

$$= G(v) - e^{-j\frac{2\pi}{m}v} H(v) \quad \text{— (2)}$$

$0 \le v \le m/2 - 1$

$F(u)$

$0 \le u \le \frac{m}{2}-1$

$F(u)$

$\frac{m}{2} \le u \le m-1$

$F(u) = F_1 \Delta F_2$ (concatination)

Algorithm to find DFT using FFT scheme

FFT (f[0 . . . . m-1]

{ If (m==1)

return (DFT(f(0))) // constant time.

else

{ $F1 = G(u) + e^{-\frac{i2\pi u}{m}} H(u)\ T(m/2)$

$F2 = G(u) - e^{-\frac{i2\pi u}{m}} H(u)\ T(m/2)$

return (F1 Δ f2)}

$O(m)$

}

$T(m) = 2T(\frac{m}{2}) + O(n)$

Master's theorem:- If $T(m) = aT(\frac{m}{b}) + f(m)$

then (ii) if $f(m) = \Theta(n^{\log_b a})$

Then $T(m) = \Theta(n^{\log_b a} \log m)$

Hence $a = b = 1$ $f(n) = \Theta(n)$

~~$T(m) = \Theta(m^{\log_b a} \log m)$~~

$T(m) = \Theta(m \log m)$ $\Theta(m) = \Theta(m')$ is time.

Consider DFT $(f(x) * g(x)) = DFT(f(x)).DFT(g(x))$

$f(x) * g(x) =$ IDFT $(\underbrace{DFT(f(x)}_{FFT}.\underbrace{DFT(g(x))}_{FFT})$

$O(m^2)$. $m\log m$

$m\log m + m\log m + m$

How to use FFT for Images?

$$F(u,v) = DFT_x(DFT_y(f(x,y)))$$

RHS

$$DFT'_x(DFT'_y(f(x,y)))$$

$$= DFT'_x\left(\sum_{y=0}^{N-1} f(x,y)\, e^{-\frac{2\pi}{N} vy}\right)$$

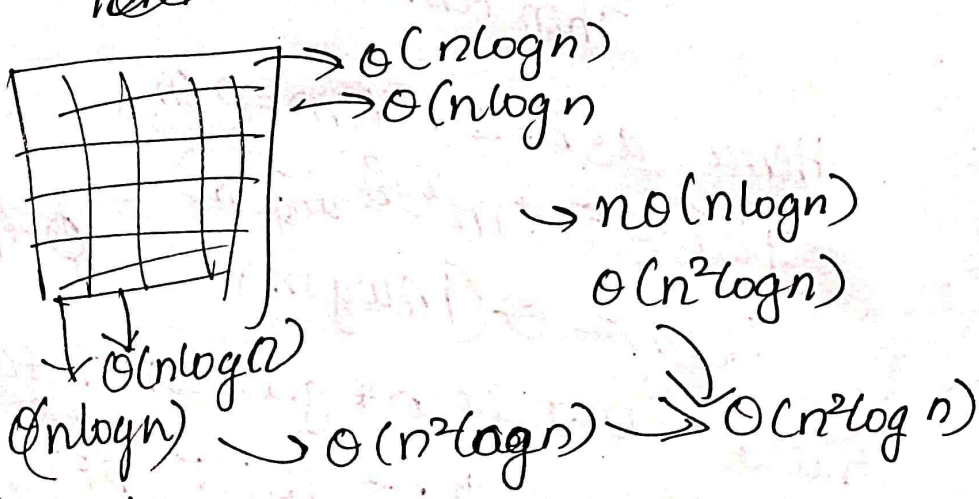
$$= \sum_{x=0}^{m-1}\left[\sum_{y=0}^{N-1} f(x,y)\, e^{-j\frac{2\pi}{N} vy}\right] e^{-j\frac{2\pi}{m} ux}$$

$$= \sum_{x=0}^{m-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{vy}{N}+\frac{ux}{m}\right)}$$

$$= F(u,v)$$

TC of finding DFT of f with size $n \times n$ using formul is $O(n^4)$

TC of finding DFT of f with size $n \times n$ using FFT



$$F(u,v) = \sum_{y=0}^{N-1}\sum_{x=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{N}+\frac{vx}{N}\right)}$$

DIGITAL IMAGE PROCESSING

feb 24

$$FT(\underbrace{f(x,y) \circledast g(x,y)}_{O(n^4)}) = \underbrace{FT(f(x,y)).FT(g(x))}_{O(n^2 \log n)}$$

filtering In Spatial Domain

↳ Conv/Corr

↳ Median

filtering in freq Domain

↳ Conv/Cor

↳ $INFT(FT(f).FT(g))$ → Linear.

filter design

In spatial domain, window w → filter

filtering → $f \odot w$

filter :- $IFT(FT(f).\underbrace{FT(w)}_{\text{filter}})$

In spatial domain Avg filter → low pass filter

$f_{m\times n}$ $\quad h_{a\times b}$ $\rightarrow F_{m\times n}$

$\rightarrow H_{a\times b} \Rightarrow$

can be extended.

$f * h \rightarrow$ ~~m+n~~ $(m+a-1)(n+b-1)$ size. $\quad m\times n$

$$f(x,y) = \frac{1}{MN}\sum_{x=0}^{n-1}\sum_{y=0}^{m-1} F(u,v)\, e^{+j2\pi\left(\frac{ux}{m}+\frac{vy}{n}\right)}$$

filter frequencies

filtering in freq Domain

If $FT(f(x,y)) = F(u,v)$

Then $FT\left(e^{j2\pi\left(\frac{ux}{m}+\frac{vy}{n}\right)} f(x,y)\right)$

$= F(u-u_0, v-v_0)$ Shifting (u,v)

$LHS = FT\left(e^{j2\pi\left(\frac{u_0x}{m}+\frac{v_0y}{N}\right)} f(x,y)\right)$

$$= \sum_{y=0}^{N-1}\sum_{x=0}^{m-1}\left[e^{j2\pi\left(\frac{u_0x}{m}+\frac{v_0y}{n}\right)} f(x,y)\right] e^{-j2\pi\left(\frac{ux}{m}+\frac{vy}{N}\right)}$$

$$= \sum\sum f(x,y)\, e^{-j2\pi\left[\left(\frac{u-u_0}{m}\right)x + \left(\frac{v-v_0}{N}\right)y\right]}$$

$= F(u-u_0, v-v_0) = RHS$

Corrolary:

$FT\left(f(x,y)\, e^{+j2\pi\left(\frac{u_0x}{M}+\frac{v_0y}{N}\right)}\right) = F(u-u_0, v-v_0)$

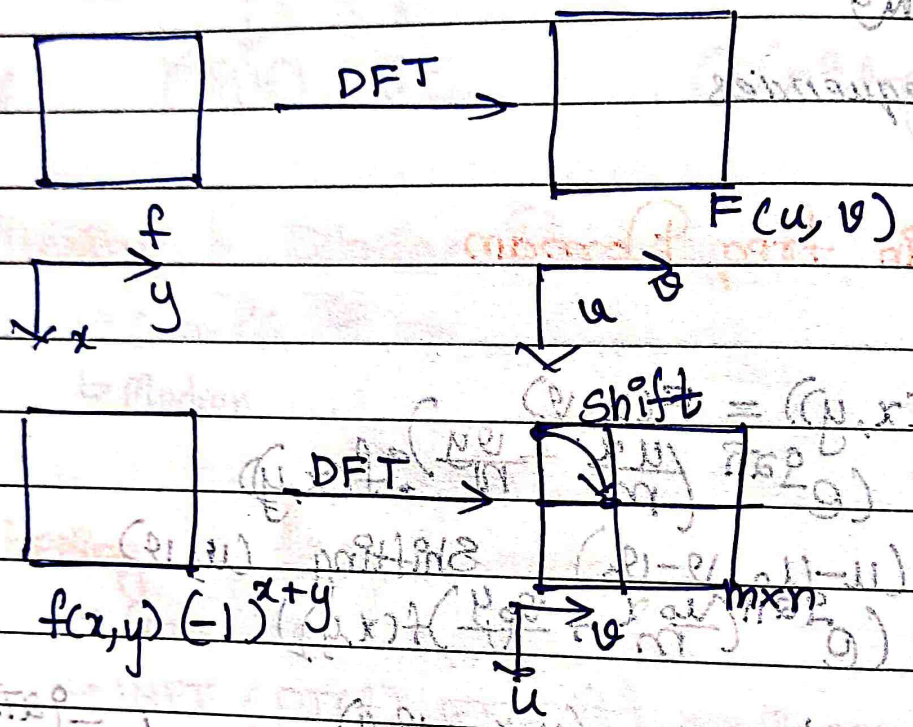
$u_0 = \frac{m}{2}$ & $v_0 = \frac{N}{2}$

Teacher's Signature .....

$$FT\left(f(x,y)\left(e^{j\pi}\right)^{(x+y)}\right) = F\left(u-\frac{m}{2}, v-\frac{n}{2}\right)$$

$$FT\left((-1)^{x+y} f(x,y)\right) = F\left(u-\frac{m}{2}, v-\frac{n}{2}\right)$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$



When we design filter in freq domain

1.) Ideal LPF

$H(u,v) = 1$ when $d((u,v),(0,0)) \le r$

$= 0$ o.w.

$u, v \rightarrow$ freq variables

$u \uparrow$ freq $\uparrow$

<u>Proc for freq domain filtering</u>

i/p $f(x,y)$ $H(u,v)$

spatial Freq

o/p → $f(x,y)$ which is the filtered version of $f(x,y)$ using filter $H(u,v)$

Step 1) Find $F_1 = DFT(f(x,y)(-1)^{x+y})$

2) Fing $G = F_1 \cdot H$

3) find $g = IFT(G) \rightarrow f(x,y)(-1)^{x+y}$

4) find $f(x,y) = g(x,y)(-1)^{x+y}$

The origin will stay in middle.

Ideal High Pass filter

$H(u,v) = 0$ when $d(u,v)(0,0) \leq r$

$= 1$ o.w.

To remove noise.

Band Pass filter

$H(u,v) = 1$ $\quad r_3 \leq d(u,v)(0,0) \leq r_2$

$= 0$ otherwise

Average filter ⇒ Low pass filter.

In spatial domain

$$Avg = \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} \quad f(x,y) \circledast h(x,y)$$

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Design of Gaussian LPF filter in spatial dom.

$$h(x,y) = e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$\hat{h}(x) = e^{\frac{-x^2}{2\sigma^2}}$$ (scaled version of $h(x)$)

$$h(x,y) = e^{\frac{-x^2}{2\sigma^2}} \cdot e^{\frac{-y^2}{2\sigma^2}}$$

$$h(x,y) = 0 \text{ for } 0 \le x \le m-1$$
$$0 \le y \le N-1$$

Why the sum of coefficients in the filter ≤ 1

To keep the values in range

$$h(x,y) = \frac{h(x,y)}{\sum_{y=0}^{N-1}\sum_{x=0}^{m-1} h(x,y)}$$

$$0 \le h(x,y) \le 1$$

$$\sum_{y=0}^{N-1}\sum_{x=0}^{m-1} h(x,y) = 1$$

eg:- G. filter

$h(x,y) =$

1	2	2	2	1
2	3	4	3	2
2	4	8	4	2
2	3	4	3	2
1	2	2	2	1

$\hat{h}(x,y) \rightarrow$ filter

$$= \frac{h(x,y)}{\sum\sum h(x,y)}$$

March 10th

Recap

DIP
- Spatial
  - Point
  - Avg (Cor/Con)
- Transform
  - FFT
  - WT
  - SVD

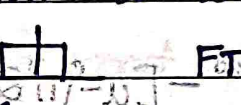
Application
- Sharpen
- Noise removal
- Increase contrast

Edge finding Algo

↳ Step, roof, ramp (types of edgemodel)

why avg filter is LPF?

Filtering frequencies. fourier transform of rectangular function is sinc function.

~~for rectangular function~~ $f(x) \Rightarrow$ FT(f(x)) 

Amp high near origin. So highlights lower freq

In spatial Domain IDFT freq domain

$f(x,y) \circledast h(x,y) \Rightarrow (F(x,y) \cdot H(x,y))$

Why is Gaussian filter

$f(x,y) = 1$ when $d(C_0,(x,y)) < r$

$= 0$ o.w.

Average filter is just scaled version of above function
↳ LPF

Why Gaussian filter is LPF

In spatial domain

middle high weight.

FT(Gaussian) = Gaussian

Teacher's Signature .....

proof ⇒ TP → IFT (Gaussian) = Gaussian

Let $F(u) = A e^{-u^2/2\sigma^2}$

$$IFT(F(u)) = \int_{-\infty}^{\infty} A e^{-u^2/2\sigma^2} e^{j2\pi ux} du$$

$$= A \int_{-\infty}^{\infty} e^{-\left[\frac{u^2}{2\sigma^2} - j2\pi ux\right]} du$$

$$= \int_{-\infty}^{\infty} e^{-\frac{[(u-j\pi x)^2 - (u-2j\pi x\sigma^2)^2]}{2\sigma^2}} du$$

$$= \int_{-\infty}^{\infty} e^{-\frac{[(u-(j\pi\sigma^2 x\times 2)^2 - (j\pi\sigma^2 x\times 2)^2)]}{2\sigma^2}}$$

$$= A \int_{-\infty}^{\infty} e^{-[u-j\pi 2\sigma^2 x)^2} e^{(j\pi 2\sigma^2 x)^2} du$$

WKT $\int_{-\infty}^{\infty} e^{-\frac{(u-\mu)^2}{2\sigma^2}} du = \sqrt{2\pi\sigma^2}$ 1.

$$= A e^{\frac{(j2\pi\sigma^2 x)^2}{2\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{[u-j2\pi\sigma^2 x]^2}{2\sigma^2}} du$$

$$= A e^{-\frac{2\pi\sigma^2 x^2}{2\sigma^2}} \times \sqrt{2\pi}\,\sigma$$

$$= A\sqrt{2\pi}\,\sigma\, e^{-\frac{x^2}{2(\frac{1}{2\pi\sigma})^2}}$$

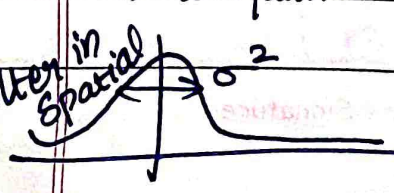
$$Var = \left(\frac{1}{2\pi\sigma}\right)^2 \propto \frac{1}{\sigma^2} \quad \mu = 0$$

larger window more blurring

∴ It is Gaussian.

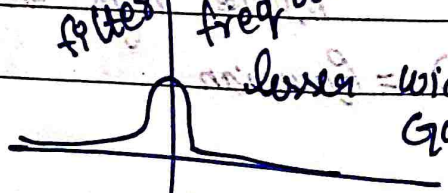
filtering to narrow domain.

Filter in Spatial



FT

filter in freq domain



lesser width but still Gaussian

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$f(x,y) \rightarrow F(u,v) = R_1(u,v) + I_1(u,v) = r_1(u,v)\, e^{i\theta_1(u,v)}$

$g(x,y) \rightarrow G(u,v) = R_2(u,v) + I_2(u,v) = r_2(u,v)\, e^{i\theta_2(u,v)}$

$F'(u,v) = \cancel{R_1(u,v) + I_2(u,v)}\; r_1(u,v)\, e^{i\theta_1(u,v)}$

$IFT(F') = g_1 \cong dog$

phase information is more.

$IFT(FT(f(x,y)) \cdot FT(g(x,y))) \Rightarrow$ blurred image of $f(x,y)$.

March 11th

Edge detection

↳ Edge mode → step, Ramp, roof

↳ 1st order Filter → Robert, Prewitt, Sobel

↳ 2nd → Laplacian.

Edge detection using Sobel

correlation

S1:- $g_x = f \circledast w_x$ $\quad w_x = \begin{matrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{matrix}$

$g_y = f \odot w_y$ $\quad w_y = \begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix}$

S2:- $\nabla f(x,y) = \begin{pmatrix} g_x(x,y) \\ g_y(x,y) \end{pmatrix}$

Teacher's Signature .....

S3: $E(x,y) = 1$ if $|\nabla f(x,y)| > T$ (threshold)
$= 0$ O.W.

Can use 1/2 thresholds.

Canny edge detection Algorithm

Input:- Gray scale Image

S1:- find $g(x,y) = f(x,y) * h(x,y)$
where h is Gaussian filter. It blurs the image
reason:- To remove some noise as noise of edge would have high freq.

S2:- $g_x = g \odot w_x$ $\quad w_x, w_y \rightarrow$ Sobel filter
$g_y = g \odot w_y$

S3:- $\nabla g = \begin{bmatrix} g_x \\ g_y \end{bmatrix}$

S4:- We also look into phase.
Non-maximal supression
If $|\nabla g(x,y)| < |\nabla g(x',y')|$ $(x',y') \in$ Neighbourhood (x,y)
then $M(x,y) = 0$
$M(x,y) = |\nabla g(x,y)|$
So if the pixel has highest value amongst its neighbours then it is considered edge.

It is for keeping only strong edges.

S5:- Choose two thresholds T_1 & T_2

$E(x,y) = 1$ if $m(x,y) \geq T_2$ → strong edge point

weak edge point is $E(x,y) = 1$ if $m(x,y) < T_2$ & $m(x,y) \geq T_1$ &

~~Including~~ ~~x,y~~ ~~in the edge image~~ neighbour is edge point.

Can we do just 2 cor instead of 3

$f \rightarrow$ Gaussian filter $\rightarrow$ Derivative in x $\rightarrow g_x$

$\hookrightarrow$ Derivaty in y $\rightarrow g_y$

How to reduce computation

Derivative of Gaussian $\rightarrow \underbrace{(D \circ G)}(f) \Rightarrow$

can be pre computed and use as single filter

Derivative of Gaussian

$$D * G = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} h(x,y)$$

Marr-Hildreth Edge Detection Algorithm

↳ 2nd derivative

$f \rightarrow \boxed{\text{Gaussian filter}} \xrightarrow{g} \boxed{\text{Laplacian filter}} \rightarrow \nabla^2 g$ → resonable val for high change.

If $|\nabla^2 g(x,y)| > T$

$E(x,y) = 1$

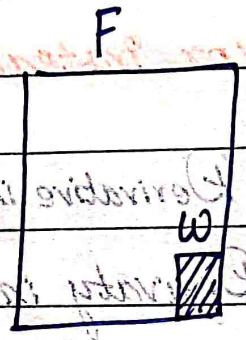
else $E(x,y) = 0$

Teacher's Signature .....

Canny → Low Contrast Images.

WATERMARKING

Input :- F, W



fw.

watermarks
- ↳ visible
- ↳ barely visible
- ↳ Invisible

Watermarking → Spatial
Watermarking → Transform

Watermarking → Visible, Invisible

Visible → fully visible (opaque), Partial Visible (Semi-transparent)

Goal of Watermarking
- ↳ Authentrate owner
- ↳ " content
- ↳ monitor content

note :- If variance is higher the weaker edges are not seen.

March 17th

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Application of watermarking → Owner Autentication, Copy prot, Usage monitoring, User Authentication

Image security → Steganography, watermarking
↳ Illegal copies.

Steganography

hiding info without revealing the existence of info.

Goal of watermarking is to provide authenticity.

A WaterMark

↳ data hidden is for authentication not secrecy.

imag → Watermark algo → watermarked image.
↑ watermark

Pair of algo :- Insertion, Extraction.

A visible Watermarking Algorithm

Step 1 :- Select axb size area in the image f, with (x_0, y_0) as left top most position

Step 2 :- $f_w(x,y) = f(x,y)$ for all (x,y)

St 3 :- Choose an α in $(0,1)$

4 :- for $(i = x_0 + a; q < a; ++i)$
for $(j = y_0 + b; j < b; j++)$
$f_w(i,j) = (1-\alpha) f(i,j) + \alpha (i - x_0, j - y_0)$

5 :- Output f_w.

Teacher's Signature

Invisible Watermarking Scheme

→ Random loc

→ $f_w(x,y) = f(x,y)$ for all (x,y)

→ for each locat (x,y) in K

find $f_w(x,y) = \underbrace{4\,(f(x,y)/4)}_{6\,LSB\ retaind} + \underbrace{w(x,y)}_{2\ MSB\ of\ w\ retaind}$ (64) ← Point of

loss of quality in watermark

Shifting bits to left so LSB is reduced.

Extraction

→ for each location (x,y) in K

find $w(x,y) = f_w(x,y) - 4\,(f_w(x,y)/4)$

→ Output $w'(x,y)$

location can be known by manufacturer

Attacks on Watermarked Image

↳ Inadvertent attack

↳ Compression

↳ filtering

↳ Image Transformation

↳ Cropping

↳ Resampling

↳ Intentional Attack

↳ To change Info

↳ remove

Encryption

Encryption :- can't be inverted without key.

Authentication → data, Owner, User

Time stamps.

One-One mapping

↳ Symmetric

$C = E(P, K_{enc})$

$P = D(C, K_{dec})$

$K_{ENC} = K_{DEC}$ Example is DES

↳ Assymetric

$C = E(P, K_{PUB})$

$P = D(C, K_{PRI})$

Hash function → extra info.

Input is of arbitrary length binary vector P, and we will obtain a fixed length binary o/p P'

$P' = H(P)$

prop :- Computationally trivial to produce P', given P

:- Extremely difficult to reproduce P, given only P'

:- Computational hard to get P1 and P2 that produce same P'

Teacher's Signature .....

March 19th

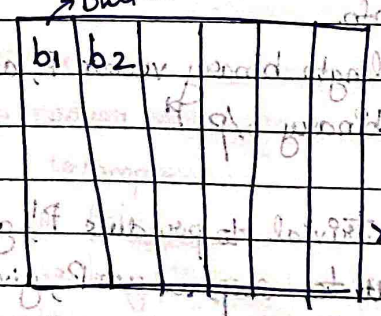
Checksum Based Watermarking algo

* checksum: modulo2 addition of a sequence of fixed length. Binary co

Algorithm.

i/p : Image →

- Divide the image f into 1x8 block
- form 1 binary word w_i for each block b_i
  - Concatenate 7 MSB's for each pixel in block b_i
- find checksum for words w_i → 56 bits
- Randomly choose loc 56, last bit store the water
- The set of Pixel loc where check sum is stored is the key; checksum is watermark



b_1 → 1x8 pixels.

7x8 bits → 56 $\forall i, b_i - b_i$

XOR of words

$1 \oplus 0 = 1$ $\quad w_i$ → length 56

$0 \oplus 0 = 0$ $\quad w_1$

$1 \oplus 1 = 0$ $\quad w_2$

w_3

XOR of all bits

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checksum → watermark (56 bits)

So random location and check LSB

the LSB is only changed to the checksum.

The location is the key

A reciever's side → location is known.

divide image to blocks.

↳ reciever will extract w_i same.

* checksum by the reciever is the same

So we can see the authenticity of data.

w is correlated with w' $> T$ → clean data.

why? as the image can go through changes

Discrete Cosine Transform

2-D ~~DFT~~ DCT

$$C(u,v) = a(u)\,a(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{(2x+1)u\pi}{2N}\right] \cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

$$f(x,y) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} a(u)\,a(v)\,C(u,v) \cos\left[\frac{(2x+1)u\pi}{2N}\right]$$

W is a sequence of normally distributed, zero-mean unit variance random no.

X original img, X_d → row concatenated DCT

$X_d(i)$ i^{th} DCT coeffic

$$Y_D(i) = X_D(i)(1 + aW)$$

Teacher's Signature .....

Image

$Y = IDCT(Y_d)$

⇒ Authentication:

↳ Let $\mathcal{Y}$ be the recieved version of watermarked Image Y

↳ ~~Apon~~ Obtain a copy of W

$$W^*(i) = \frac{1}{a}\left[\frac{\mathcal{Y}_D(i)}{X_D(i)} - 1\right]$$

measure correlation

~~Haar~~ Wavelet Transform

$f \rightarrow 1024 * 1024$

Consider LPF 1D HPF 1D

~~1D~~ x-dir y-dire LH

converted $m/2 \times n/2$

Similarly LPF y-dir HPF x-dir

$m/2 \times n/2$ $m/2 \times n/2$

$$f_{m \times n} \begin{bmatrix} LL & LH \\ HL & HH \end{bmatrix}$$

$[LL, LH, HL, HH] = dwt2(f, 'haar', 'd')$

$[L_L, L_H, H_L, H_H] = dwt2(w, '', '')$

$g = LL + (sf * L_L)$; scaling factor

$fw = idwt2(g, LH, HL, HH, 'haar')$

Extract:

$[a\ b\ c\ d] = dwt2(fw)$

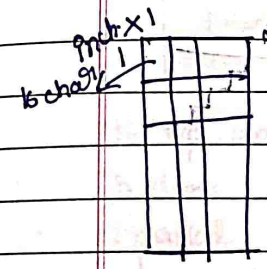
$w = [a - LL]/sf$;

Rotation attack cannot be handled → SVD.

March 21

Image Compression

to reduce the size ~~occupied~~ of the Image.



25×11 inch

The size $= 25 \times 11 \times 2^4$ bytes.

$= 2^2 2$ Bytes

$= 4kB$.

Mat of size 1024×1024

header format for compression

Types of Image Compression

↳ Losless png

↳ Lossy jpeg

comp $f \rightarrow f'$ → Losless

$f \xrightarrow{compress} f' \rightarrow$ Lossy

almost same $f \leftrightarrow f''$ ← decompress

↳ Psydo visual redundance

↳ Coding

↳ Interpixel

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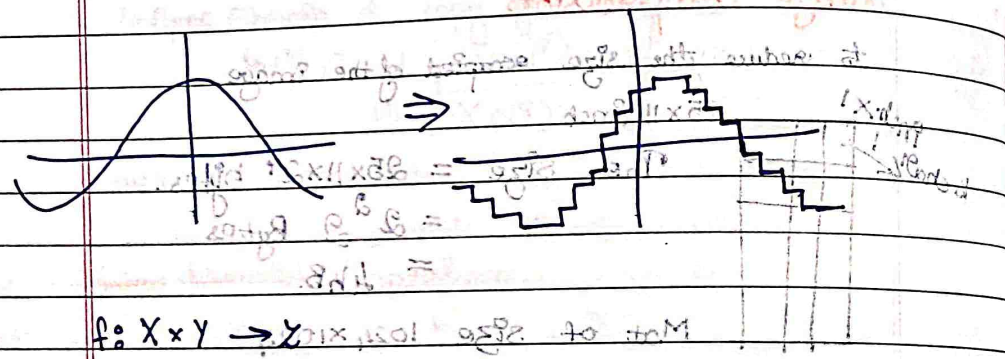
Psycho Visual Redundancy

Consider f with values 0 to 255

f_1 = quantised (f) with 128 levels.

The look is the same but the matrix is different.

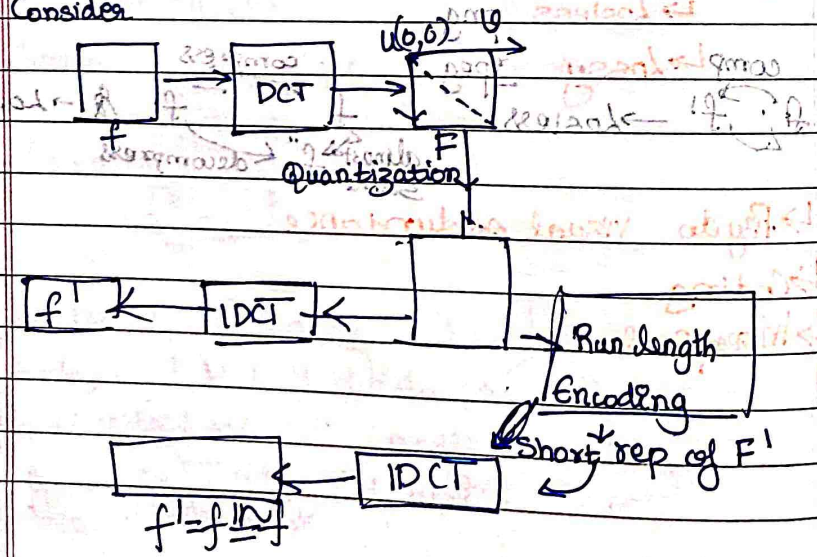


$f: X \times Y \rightarrow Z$

$Z \rightarrow Z'$ with specific proc.

Encoding in freq domain.

Consider

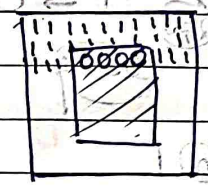


Run Length Encoding (Go through it

↳ for Binary.

↳ for Grayscal.

for Binary

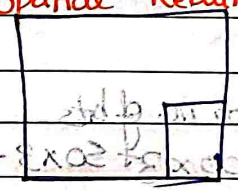


Compression (1, 1000, 50, 200 .....)

Decomp

If i/p image has good property then run length coding is done.

Spatial Redundancy



→ large size

↓

run length encodi

↓

(0, 10000)

Coding Redundancy

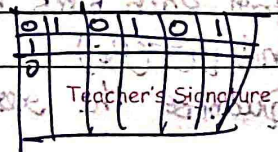
f → 0 to 255

pixel → 8 bits

size = 1MB.

Chess - Board - Image.

no spatial redundancy



Teacher's Signature .....

Huffman Encoding

0 → 100 time

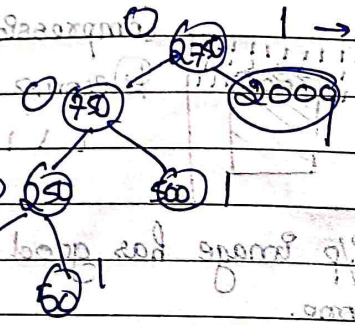
1 → 1000 time

	encoding
0	0
1	1

If not 0 → 000000

1 → 000000

Large	freq
0	200
1	50
2	500
3	2000



Symbol

	code	freq
0	000	200
1	001	50
2	01	500
3	1	2000

Total no. of bits

$= 200 \times 3 + 50 \times 3 + 500 \times 2 + 2000 = 3750$

The Brute force method

$(200 + 50 \times 500 + 2000) \times 8 \gg 3750$

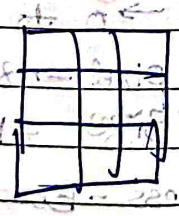
JPEG

S1:- Divide the img into blocks

S2:- for each block b_i

Find DCT(B_i)

S3:- Quantise the result of S2



S4:- Run length Encoding algo

S5) Huffman encoding on result of S_4

JPEG 2000 Instead of DCT → WT

March 25

Image Restoration And Reconstruction

Image quality metrics → PSNR

Original Image $\xrightarrow{\text{degeneration}}$ g

restoration

Image Enhancement

noisy Image ⟹ Image with less noisy.

make the image better

while restoration is showing the object as it is.

Image Restoration Problem

Given a degraded image g and also the degradation function T, find f S.T

$g = T(f)$

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Prob

Suppose the original img f is degraded by "Linear Shift Invariant" transform, followed by additive random noise, find f.

Soln

$$f \longrightarrow \boxed{T} \longrightarrow T(f)$$

$$g = T(f) + e$$

$$g(x,y) = T(f(x,y)) + e(x,y) \quad \text{—①}$$

where T is Linear and Shift Invariant

How to find $T(f)$, when T is Linear and Shift Invariant

w.k.T. $$f(x_0,y_0) = \int_{y=-\infty}^{\infty}\int_{x=-\infty}^{\infty} f(x,y)\,\delta(x-x_0, y-y_0)\,dx\,dy.$$

$$f(x,y) = \iint f(a,b)\,\delta(a-x, b-y)\,da\,db$$

$x_0 \to x \quad y_0 \to y \quad x \to a \quad y \to b$

$$T(f(x,y)) = T\left[\iint f(a,b)\,\delta(a-x, b-y)\,da\,db\right]$$

$$= \iint T\left[f(a,b)\,\delta(a-x, b-y)\,da\,db\right]$$

T is Linear

$$= \iint f(a,b)\left[T(\delta(a-x, b-y))\right]da\,db$$

$\because T$ is linear

Suppose $T(\delta(x,y)) = h(x,y)$

Then $T(\delta(a-x, b-y)) = h(a-x, b-y)$

$\because T$ is SI.

$$T[f(x,y)] = \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)\,h(a-x, b-y)\,da\,db}_{\text{convolution}}$$

$$= f(x,y) \circledast h(x,y)$$

$\therefore$ eq. ① becomes

$$g(x,y) = f(x,y) \circledast h(x,y) + e(x,y) \longrightarrow ②$$

We know T

$\Rightarrow h(x,y) = T(\delta(x,y))$

Apply F.T. for ②

$$F.T(g(x,y)) = FT[f(x,y) \circledast h(x,y) + e(x,y)]$$

$$= FT[f(x,y) \circledast h(x,y)] + FT(e(x,y))$$

$$= FT[f(x,y)].FT(h(x,y)) + FT(e(x,y))$$

$$\frac{FT(g) - FT(e)}{FT(h)} = FT(f)$$

$$f = IFT\left[\frac{FT(g) - FT(e)}{FT(h)}\right]$$

$$\cong IFT\left[\frac{FT(g)}{FT(h)}\right]$$

$$\cong IFT\left[\frac{FT(g)}{H'}\right] \quad \text{where } H'(u,v) = FT(h(u,v))$$

If $FT(h(u,v)) \neq 0$

$\hookrightarrow$ pseudo inverse of H.

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finding H'

↳ Find $H = FT(h)$

↳ $h = T(\delta)$

camera

Reconstruction Problem

* The restoration problem will be called as Reconstruction prob if the degradation Process is "Projection".

Define Projection of Images

Cont-Domain

The project of img $f(x,y)$ along a line L is

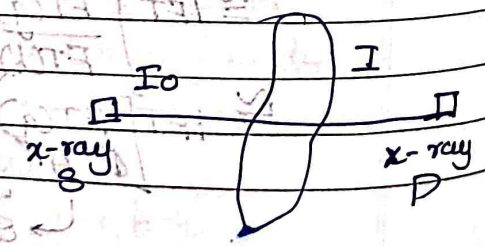
$$\int_{(x,y)\in L} f(x,y)\,dx\,dy$$

In discreat domain

Projection of f along a line L is $\sum_{(x,y)\in L} f(x,y)$

How to find Projection.

$I < I_0$



Rotate Source and Detector synchronously we can get various projection.

$$I = I_0\, e^{-\int_{(x,y)\in L} f(x,y)\,dx\,dy}$$ for all EMW.

$$e^{-\int_{(x,y)\in L} f(x,y)\,dx\,dy} = \frac{I}{I_0}$$

$$\Rightarrow -\int_{x,y} f(x,y)\,dx\,dy = \log\left(\frac{I}{I_0}\right)$$

$$\Rightarrow \int_{x,y} f(x,y)\,dx\,dy = \log\left(\frac{I_0}{I}\right)$$

Discrete Domain Reconstruction Problem

H.W.

1	0	1	1	3
1	0	1	0	2
0	1	1	1	3
1	0	1	1	3
3	1	4	3	

Can you reconstruct f from R & C

Multiple solutions are possible

Can you construct in such a way that row sum of f is R and column sum of f is C in polynomial time.

31/March To solve the above proble combinatorics by Ryser.

					R
0	1	1	0		2
1	1	1	0		3
1	1	0	0		2
0	1	1	1		3
2	4	3	1		C

Given R, C, n.

1	1			2 ✓
1	1	1	1	3 ✓
1	1			2 ✓
1	1	1		3 ✓
2	4	3	1	

Now check C

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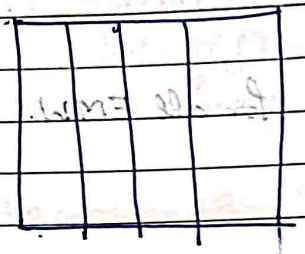
Next projection along diagonal



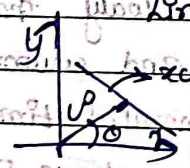
To get the original → more projection.

Solve reconstruction problem in continuous domain and convert into discrete domain

Back Projection Method

$$\text{Proj}(f(x,y)) = \int_{(x,y)\in L} f(x,y)\,dx\,dy$$

$$= \int_{y=-\infty}^{\infty}\int_{x=-\infty}^{\infty} f(x,y)\,\underbrace{\delta(x\cos\theta + y\sin\theta - \rho)}_{\text{Line equation}}\,dx\,dy$$



$x\cos\theta + y\sin\theta = \rho$

$\delta(0)=1 \quad \delta(x)=0 \;\forall\, x \neq 0$

$$g(\rho,\theta) = \text{Proj}\, g(f(x,y)) = \iint_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy$$

$\text{Proj}(f) \to g$

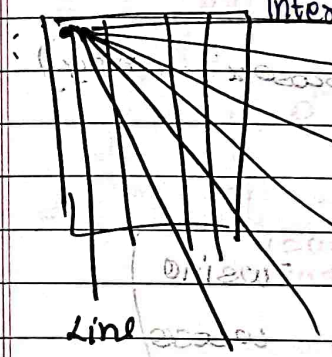
We define back Proj of $g(\rho,\theta)$ as

$$b(x,y) = \int_0^{\pi} g(\rho,\theta)\,d\theta = \int_0^{\pi} g(x\cos\theta + y\sin\theta, \theta)\,d\theta$$

$$\boxed{b(x,y) = \int_0^{\pi} g(x\cos\theta + y\sin\theta, \theta)\,d\theta}$$

$\Downarrow$

Interpretation



Line

So all the sum from various ~~examples~~ lines covers entire plane

Consider

0	2	2	0
2	4	4	2
2	4	4	2
0	2	2	0

2 projections

0	0	0	0	0
0	1	0	0	2
0	1	1	0	2
0	0	0	0	0
0	2	2	0	

Some noise is present

$$b(x,y) = \sum_{\theta=0}^{\pi} g(\rho,\theta)$$

Hold

→ 4x4 → Rsum, Csum, Dsum → Back propagation

Rsum, Csum → Back propagat

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Filtered Back Projection Algorithm

freq domain

WKT IFT is given by

$$f(x,y) = \iint_{-\infty}^{\infty} F(u,v)\, e^{2\pi i(ux+vy)}\, du\, dv$$

put $u = w\cos\theta$

$v = w\sin\theta$

$$f(x,y) = \iint |F(wu,v)|\, e^{2\pi i(w\cos\theta\, x + w\sin\theta\, y)} \quad u=w\cos\theta,\ v=w\sin\theta$$

$$|J| = \begin{vmatrix} \frac{\partial u}{\partial w} & \frac{\partial w}{\partial \theta} \\ \frac{\partial v}{\partial w} & \frac{\partial v}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -w\sin\theta \\ \sin\theta & w\cos\theta \end{vmatrix}$$

$$= w[\cos^2\theta + \sin^2\theta]$$

$$= w$$

$$f(x,y) = \iint [F(u,v)]_{u=w\cos\theta,\ v=w\sin\theta}\, e^{2\pi i[w\cos\theta\, x + w\sin\theta\, y]} \rightarrow (1)$$

To prove: Fourier Slice Theorem

$$[F(u,v)]_{u=w\cos\theta,\ v=w\sin\theta} = FT_\rho(g(\rho,\theta))$$

$$RHS = FT_\rho(g(\rho,\theta)) = \int_{-\infty}^{\infty} g(\rho,\theta)\, e^{-2\pi i\rho w}\, d\rho$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, \delta(x\cos\theta + y\sin\theta - \rho)\, dx\, dy\, e^{-2\pi i\rho w}\, d\rho$$

$$= \int_{y=-\infty}^{\infty}\int_{x=-\infty}^{\infty} f(x,y) \left[\int_{-\infty}^{\infty} \delta(x\cos\theta + y\sin\theta - \rho)\, e^{-2\pi i\rho w}\, d\rho\right] dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{x=-\infty}^{\infty} e^{-2\pi i w[x\cos\theta + y\sin\theta]}\, dx\, dy \times f(x,y)$$

$$= \iint f(x,y)\, e^{-2\pi i[ux+vy]}\, dx\, dy \quad u=w\cos\theta,\ v=w\sin\theta$$

$$= [F(u,v)]_{u=w\cos\theta,\ v=w\sin\theta}$$

Applying

from (1)

$$f(x,y) = \int_0^{2\pi}\int_0^{\infty} FT_\rho(g(\rho,\theta))\, e^{2\pi i(w\cos\theta\, x + w\sin\theta\, y)}\, w\, dw\, d\theta$$

[∵ fourier slice theorem]

$$= \int_0^{2\pi}\int_0^{\infty} G(w,\theta)\, e^{2\pi i(w\cos\theta\, x + w\sin\theta\, y)}\, w\, dw\, d\theta$$

$$= \int_0^{\pi}\int_0^{\infty} G(w,\theta)\, e^{2\pi i(w\cos\theta\, x + w\sin\theta\, y)}\, w\, dw\, d\theta + \int_0^{\pi}\int_0^{\infty} G(w,\theta)\, e^{2\pi i(w\cos(\theta+\pi)\, x + w\sin(\theta+\pi)\, y)}\, w\, dw\, d\theta$$

$$= \text{Term I} + \int_0^{\pi}\int_0^{\infty} G(w,\theta+\pi)\, e^{2\pi i(w\cos(\theta+\pi)\, x + w\sin(\theta+\pi)\, y)}\, w\, dw\, d\theta$$

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$$= \text{Term I} + \int_0^{\pi}\int_0^{\infty} G(-\omega,\theta)\, e^{j2\pi(\omega(-\cos\theta\, x)+\omega(-\sin\theta\, y))}$$

$$[\because G(\omega,\theta+\pi) = G(-\omega,\theta)]$$

$$= \text{Term I} + \int_0^{\pi}\int_0^{\infty} G(-\omega,\theta)\, e^{j2\pi(\omega\cos\theta\, x+\omega\sin\theta\, y)}\, \omega\, d\omega\, d\theta$$

change $\omega \to -\omega$

$$= \text{Term I} + \int_0^{\pi}\int_0^{-\infty} G(-\omega,\theta)\, e^{j2\pi(-\omega\cos\theta\, x+\omega\sin\theta\, y)}\, (-\omega)(-d\omega)\, d\theta$$

$$= \text{Term I} + \int_0^{\pi}\int_{-\infty}^{0} G(\omega,\theta)\, e^{j2\pi(\omega\cos\theta+\omega\sin\theta\, y)}\, (-\omega)\, d\omega\, d\theta$$

$$= \int_0^{\pi}\int_0^{\infty} G(\omega,\theta)\, e^{j2\pi(\omega\cos\theta+\omega\sin\theta\, y)}\, \omega\, d\omega\, d\theta + \int_0^{\pi}\int_{-\infty}^{0} G(\omega,\theta)\, e^{j2\pi(\omega\cos\theta+\omega\sin\theta\, y)}\, |\omega|\, d\omega\, d\theta$$

$$= \int_0^{\pi}\int_{-\infty}^{\infty} G(\omega,\theta)\, e^{j2\pi(\omega\cos\theta\, x+\omega\sin\theta\, y)}\, |\omega|\, d\omega\, d\theta$$

$$= \int_0^{\pi}\int_{-\infty}^{\infty} G(\omega,\theta)\, e^{j2\pi\omega\rho}\, |\omega|\, d\omega\, d\theta \qquad \rho = x\cos\theta + y\sin\theta$$

$$= \int_0^{\pi}\left[\int_{-\infty}^{\infty} G(\omega,\theta)\, |\omega|\, e^{j2\pi\omega\rho}\, d\omega\right] d\theta$$

$f(x) \to F(\omega)$

$h(x) \to H(\omega)$

$f \circledast h \cong IFT(F(\omega)\cdot H(\omega))$

Date
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$$= \int_0^{\pi} \left[\text{filtering of projection of } g(\rho,\theta) \text{ wrt } \rho\right] d\theta$$

$f(x,y) = \text{Backprop}\left[\text{Filtering of the proj } g(\rho,\theta) \text{ wrt } \rho\right]$

Filtering Back Proj Algorithm

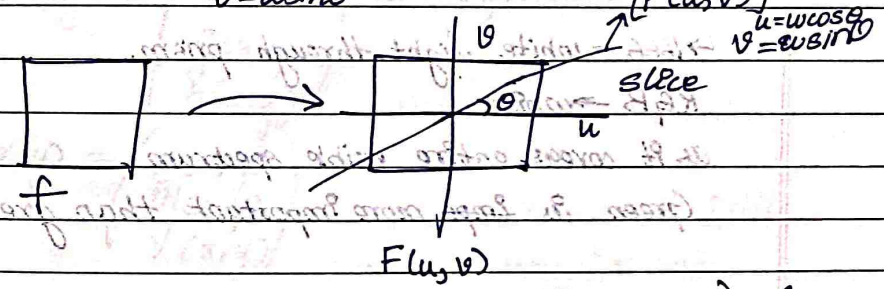
S1) Filter $g(\rho,\theta)$ wrt ρ in freq domain

S2) Back proj the result in S1

Fourier Slice Theorem Based Algo. (Go through once)

Recall $[F(u,v)] = FTP(g(\rho,\theta))$

$u = \omega\cos\theta$
$v = \omega\sin\theta$



We can get $F(u,v)\ \forall\ u,v$ by $FTP(g(\rho,\theta))$ for each $0 \le \theta \le \pi$

To find $f(x,y)$
Do $IFT(F(u,v))$

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Convolution Basic Projection Algorithm

Doing equivalent process in spaction domain to filtered Back propagation.

$$\underbrace{f \circledast h}_{G} = IF(F.H)$$

In terms of time → filtered Back Propagation algo is faster
Both are same based on performance

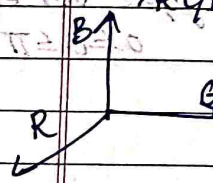
Look into CT Scan working.

Colour Image Process.

Colour Science

↳ 1666 → white light through prism.
RGB → white
as it covers entire visible spectrum
Green is ~~Imp~~ more important than green

↳ RGB colour space model
R: 0 to 255
G: 0 to 255 256^3
B: 0 to 255



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CMYK or CMYK Color Space $C = 1-R$ $M = 1-G$ $Y = 1-B$
RGB ∈ [0, 1]
R: 0 to 255 → [0, 1] printer White [1, 1, 1]
G → "
B: "

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

C M Y
1 1 1 → Black. quality of B not good
CMYK is used in printer
Black is sprayed in various ways to get various shades of Black

April 7th

Colour Image

$$f(x,y) = \begin{pmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{pmatrix}$$ in RGB space

$$f(x,y) = \begin{pmatrix} c(x,y) \\ m(x,y) \\ y(x,y) \end{pmatrix}$$

How to use IP methods designed for grayscale.
Colour Image Proce:
↳ Plane proc: In each mat, do the pro
↳ Vector proc: Do pr on $\begin{pmatrix} r \\ g \\ b \end{pmatrix}$

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Gray Scale $\longrightarrow$ Colour Image

$f(x,y)$: 0 to 255

Pseudo colour image processing.

0-25 $\rightarrow$ 1 colour (20, 30, 50)

25-50 $\rightarrow$ 2 colour C

Collect on r value to get R matrix

Given $f = \begin{pmatrix} r \\ g \\ b \end{pmatrix}$, to get the grayscale image

(i) $f(x,y) = (r(x,y) + g(x,y) + b(x,y))/3$

(ii) $f(x,y) = \max(r,g,b)$

(iii) $f(x,y) = \text{median}(r,g,b)$

(iv) weighted

$f(x,y) = 0.25 \times r + 0.5 \times g + 0.25 \times b$

weights can be different.

HSI colour Space

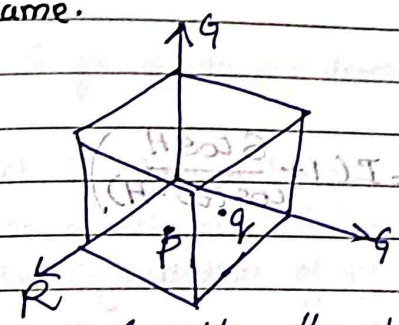
Hue $\rightarrow$ "name" of colour

Colour info of P1 $\rightarrow$ $\underbrace{\text{dis}(\text{grayscale}, P)}_{\text{Saturation}}$

quantum of colour $\rightarrow$ Saturation

Consider any point q on the triangle enclosed by (0,0,0), (1,1,1) and a fixed point P.

What is colour of q wrt to p in terms of name.



q can be linear comb of (0,0,0), (1,1,1), P

has info

$\therefore$ colour of q is the same as colour of p.

— Consider the plane containing the triangle

All points on the plane are given the same name of colour.

Hence: The angle of plane containing triangle with vertices (0,0,0) (1,1,1), and P, and Red axis

Intensity $\rightarrow I(x,y) = (r(x,y) + b(x,y) + g(x,y))/3$

$$S(x,y) = 1 - \frac{3\min\{r,g,b\}}{r+g+b}$$

S for (20,20,20) $\rightarrow 1 - \frac{3 \times 20}{3 \times 20} = 0$

$H_0 = \theta$ when $B < G$

$= 360 - \theta$ o.w.

$$\theta = \cos^{-1}\left(\frac{\frac{1}{2}[(R-G)+(R-B)]}{[(R-G)^2+(R-B)(G-B)]^{1/2}+\epsilon}\right)$$

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8/Apr/21

Given HSI find RGB

Case 1:- $120 \le H \le 240$

S1) $H = H - 120$

S2) $R = I(1-S)$ $\quad G = I\left(1 + \frac{S\cos H}{\cos(60-H)}\right)$

$B = 3I - (R+G)$

Case 2:- $240 \le H < 360$

S1) $H = H - 240$

S2) $G = I(1-S)$ $\quad B = I\left(1 + \frac{S\cos H}{\cos(60-H)}\right)$

$R = 3I - (B+G)$

Case 3:- when $0 \le H \le 120$

$B = (I(1-S))$

$R = I\left(1 + \frac{S\cos H}{\cos 60-H}\right)$

$G = 3I - (B+R)$

Image Enhancement

- Smoothing
- Sharpening
- Noise removal

RGB → Individual matrix do the operation.

$E(x,y) = r(x,y)$ or $g(x,y)$ or $b(x,y)$ → weak edges.

$= r(x,y)$ and $g(x,y)$ and → many edges might

Image Segmentation

To get objects and denote it. (boundary)

Let I be an image of size mxn. Let R be the set of all pixel location (i.e. $|R| = mxn$)

The segmentation of I is a partition of R in $R_1, R_2, \ldots$ such that

(i) $R = \bigcup_{i=1} R_i$

(ii) $R_i \cap R_j = \phi \ \forall i \ne j$

(iii) $Q(R_i)$ is true $\forall i$ (property)

(iv) R_i is connected $\forall i$

(v) If $Q(R_i)$ & $Q(R_j)$ are true when R_i and R_j are adj then $Q(R_i \cup R_j)$ is false.

Segmentation using Thresholding.

Threshold $T = 100$ (eg)

$g(x,y) = 1$ if $f(x,y) \ge T$

$= 0$ o.w.

How do we know the threshold

By histogram → find local nodes in histogram, m_1, m_2

find $T = (m_1 + m_2)/2$

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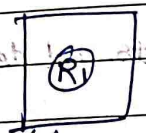
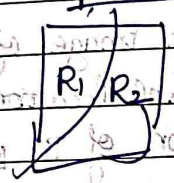
Otsu's Thresholding Algorithm

Trial & Error: T_1 do thresholding on f



for T_2 do thresholding



Otsu's is used to check if segmentation is good.

$P_1^{(k)} P_2^{(k)} (m_1^{(k)} - m_2^{(k)})^2$ = measure of quality of segmentation

$$\sigma_B^2(k) = P_1(k) P_2(k) (M_1(k) - M_2(k))^2$$

Optimal Threshold $T = \text{Arg Max}\{\sigma_B^2(k) / 0 \le k \le 255\}$

Region Growing Segmentation Algorithm

$R_i = \{(x,y) \mid L(x,y) = i\}$

Step 1) Select k seed points

S2) Define $R_i = \{S_i\}$

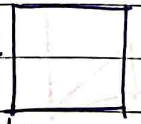
S3) $R_i = R_i \cup \{x,y\}$ when (x,y) is similar to S_i

S4) Repeat S_3 until no more update is possible

Histograms can be used for finding k.

Split And Merge Segmentation algorith.

Split



$f \to R$ $Q(R)$ is true?

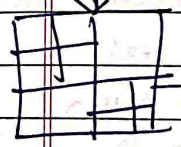
$Q(R)$ is false

R_1	R_2
R_3	R_4

$Q(R_1)$ = true or not check

$Q(R_1) \to$ false

$Q(R_3) \to$ false

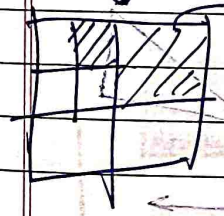


at the end $Q(R_a) = T$

$R(R_1) = F$

merge adjoining segments

Merge



$Q(R_{1+2} \cup R_{12}) = T$

whichever unions are true

$\to$ merge segments

Homogenity based (Region based) Image seg.

Discontinuity base

$\to$ edge

How is ~~se~~ edge dire segment.

Teacher's Signature .....

March 15th

Image Processing

↳ Low level → Spacial, Transformed.

↳ middle " → edge, segment, morphological.

↳ High level

Morphological Image Processing

Study of structures

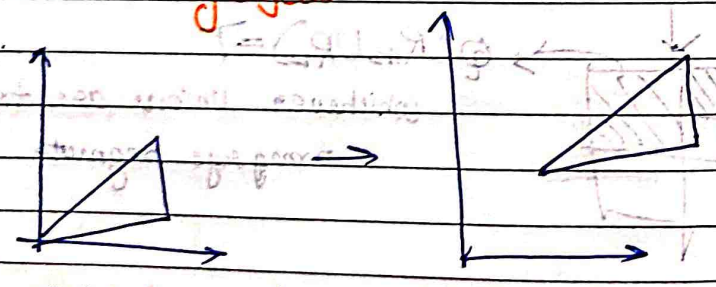
↳ Boundary

↳ Connected components

To get structure → Binary can be used.

Morphological Operations

1) Translation of object

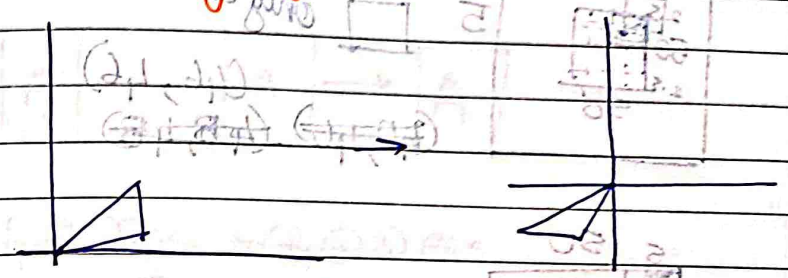


Let A be an obj (A is set of location where 1 is present)

Translation of A by ~~(x,y)~~ (a,b);

$$A_{(a,b)} = \{(x,y)+(a,b) / (x,y) \in A\}$$

2) Reflection of object



Let A be an object

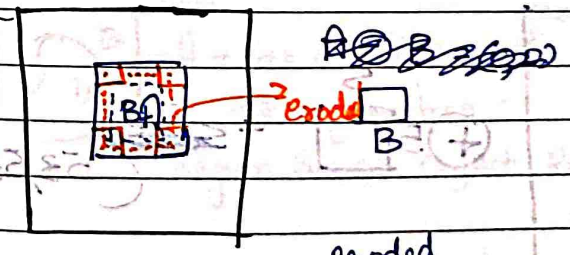
$$Ref(A) = \hat{A} = \{(-a,-b) / (a,b) \in A\}$$

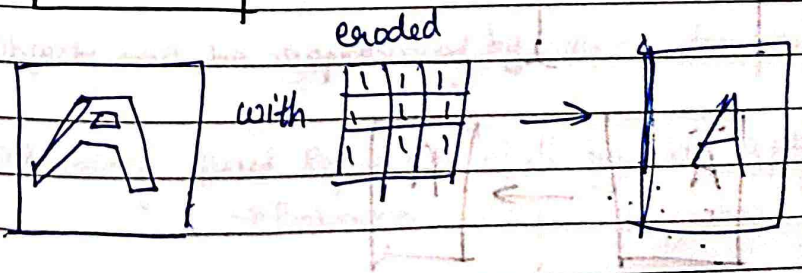
3) Erosion -

Let A be an obj and B be another image, called structuring element.

$$A \ominus B = \{z = (x,y) / (B)_z \subseteq A\}$$

eroded — Trans B by z.





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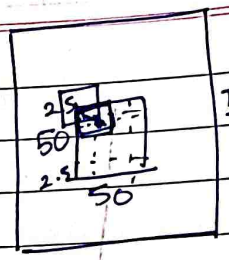
April 21

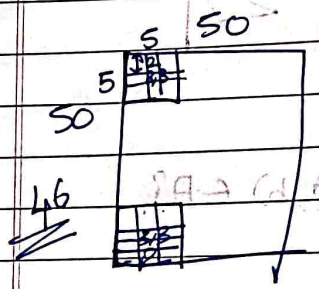


5

5 only consider top

(46, 46)

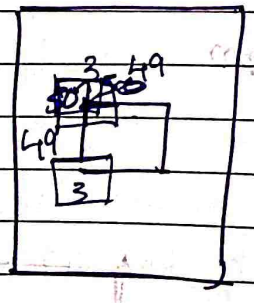
~~(40, 40)~~ ~~(48, 48)~~



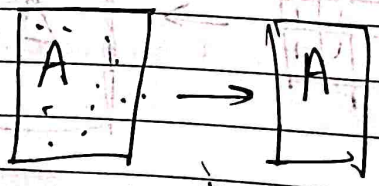
4.) Dilation

$$A \oplus B = \{z = (x,y) \mid (\hat{B})_z \cap A \neq \phi\}$$

so if it has any component then the centre is taken

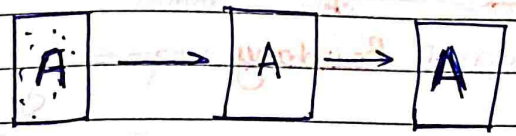


$\oplus$ 5 5 = ~~56x56~~ (53, 53)



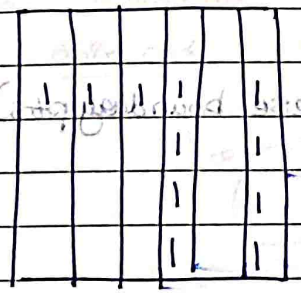
Eroding

Erosion → Dilation



$(f \ominus s) \oplus s \neq (f \oplus s) \ominus s$

opening op. closing op.



Each segment is object.

Object Representation & Object Description

A → set of all pixel loc in the obj.

not a good rep

Region Based Object Rep

Object can be represented by area, description, shapes

↳ Boundary Based Rep → set of all boundary points

↳ Perimeter

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Object

Region — Boundary

Region Based

↳ DFT (pixel val at those loc of the obj)

↳ WT (")

Boundary Based

↳ DFT (pixel val at these boundary pts)

↳ WT

Desc based on FFT

Avg of FFT coefficient, median, mode, moments

moments $= \{x_i / 1 \le x_i \le n\}$ → multiset

$$\sum_{i=1}^{k} p(v_i)(v - v_i)^n$$

$\{v_i / 1 \le i \le k\}$ is the set of all value in A

$k \le n$

If we take 7 moments

$FV(A) = (m_1, m_2, m_3, \ldots m_7)$

Discriminating obj is difficult.

Boundary box length, width

length major axis → b

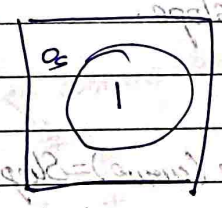
minor → a

$\frac{a}{b}$ ⇒ aspect ratio $AR(O_1) \ge AR(O_2)$

$\frac{\overline{M_A} + \overline{M_B}}{2} = T$ If $AR(obj) > T$ then say A else say B.

RST Invariant

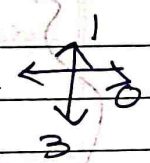
chain code



topmost left point.

perimeter.

sequence of no.



not a good descriptor

0 3 3 3 0 3 2 2 2 1 1 0

Cyclic difference chain code

seer the change with its adjacent val

3 0 0 1 3 0 0 3 0 0 3

It is a cycle.

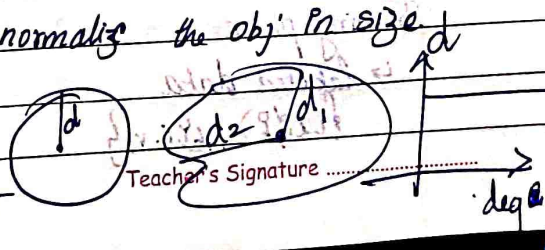
Shape number: The min value cyclic diff chain code

normalising all the cyclic perm

To make scaling inv, normalise the obj in size.

Signature Descriptor

Translation invariant



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Compactness

Area of the object. $\frac{P^2}{A}$

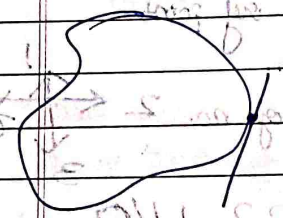
Circularity Rotation

~~Area of obje~~

Area of circle with perimeter p where P is the perimeter of the object.

Curvature of the object

↳ Rate of change of slope.



derivative (curve) = Slope.

Feature vectors.

22/April

Classification

I_i $m \times n$ → Img Enhancement → Seg → Obj Rep → F_i of size k

Training phase

↳ labeled data

$\{(x_i, y_i) \mid 1 \le i \le n\}$

2) Construct model using the DS

Testing phase.

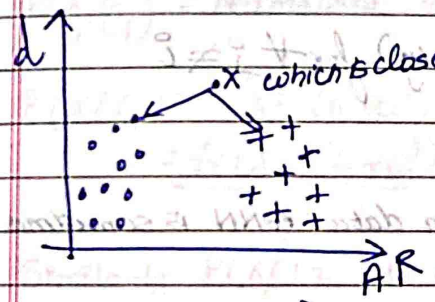
$x \rightarrow \boxed{M} \rightarrow L$ $\quad L \in \{i \mid 1 \le i \le k\}$

$A \to 0$, $B \to 1$

If K is finite, the problem is classifiable

countable

K-Nearest Neighbour



FR = (AR, diameter)

diameter (object) = longest distance

Assign x to C_i if majority of k-neighbours of x belong to C_i.

Hence $C_1, C_2 \dots C_k$ are classes.

$O(nk)$ need to find n distances.

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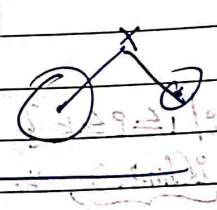
2) Minimum Distance Classifier

X distance from centre of the class.



Training phase

① $\{(x_i, L_i) \mid 1 \le i \le n\}$

② $\mu_i = \left(\sum_{f \in C_i} f\right) / |C_i|$

↳ learning

Test phase

If $d(x, \mu_i) \ge d(x, \mu_j)$ for $\forall j \ne i$
assign x to C_i.

Accuracy depends on data. K-NN is sometimes better.

3) Bayes Classifier

Let $C_1, C_2 \dots C_k$ be the classes and X be the FV to be classified

if $P(C_i/X) \ge P(C_j/X)$ $\forall j \ne i$ then x to C_i

Bayes theorem:

$$P(C_i/X) = P(X/C_i) \frac{P(C_i)}{P(X)}$$

$$P(C_j/X) = P(X/C_j) \frac{P(C_j)}{P(X)}$$

$$P(X/C_i) \frac{P(C_i)}{\cancel{P(X)}} \ge P(X/C_j) \frac{P(C_j)}{\cancel{P(X)}}$$

Assign x to C_i if

$$P(X/C_i)\, P(C_i) \ge P(X/C_j)\, P(C_j)$$

likelihood prior

① = ③

$P(X/C_1)$ = Normalised histogram for C_1

$$P(X/C_1) = P(X = (6,70)/C_1) = \frac{fr(X = (6,70))}{|C_1|}$$

Similarly $P(X/C_2)$ can be computed.

Prob Suppose C_1 C_2 $P(C_1) = P(C_2)$

$$P(X/C_1) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu_1)}{2\sigma^2}}$$

$$P(X/C_2) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu_2)}{2\sigma^2}}$$

Assign x to C_i if

$$P(X/C_i)\, \cancel{P(C_i)} \ge P(X/C_j)\, \cancel{P(C_j)}$$

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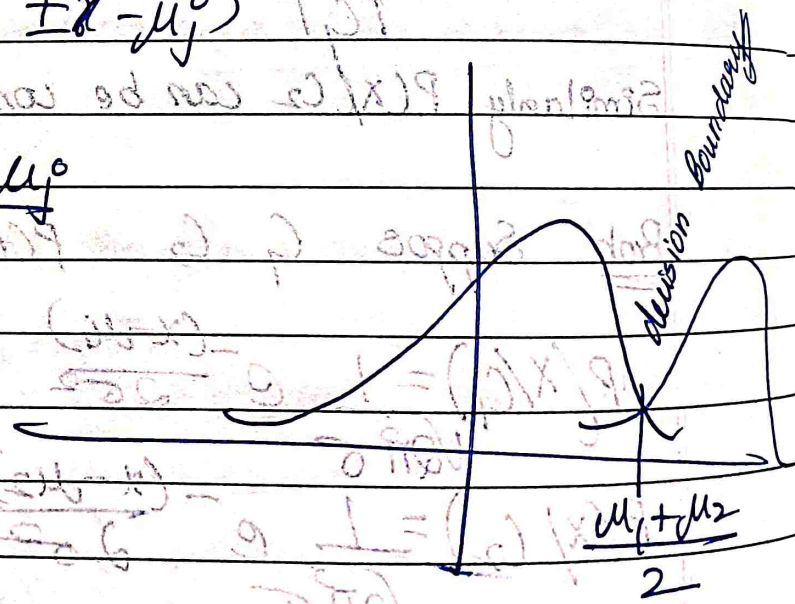
$$P(x/C_i) \geq P(x/C_j)$$

Assign x to C_i if

$$\cancel{\frac{1}{\sqrt{2\pi}\sigma}} e^{-\frac{(x-\mu_i)^2}{2\sigma^2}} \geq \cancel{\frac{1}{\sqrt{2\pi}\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\frac{-(x-\mu)^2}{\cancel{2\sigma^2}} \geq \frac{-(x-\mu_j)^2}{\cancel{2\sigma^2}}$$

$$(x-\mu_i)^2 \leq (x-\mu_j)^2$$

$$(x-\mu_i) \leq \pm(x-\mu_j)$$

$$x \leq \frac{\mu_i+\mu_j}{2}$$