IL NUOVO CIMENTO VOL. 12 A, N. 1 1 Novembre 1972

Muon Capture in Deuterium.

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(ricevuto il 24 Aprile 1972)

Summary. — The muon capture in deuterium is studied by means of the impulse approximation with relativistic corrections. The deuteron is described by means of the S- and D-wave.

1. – Introduction.

Due to the experimental difficulties of measurement of the muon capture rate in deuterium not very much theoretical effort has been devoted to this problem. There is a paper by WANG (1) where the usual impulse approximation is used. He describes the deuteron as a pure S-wave with hard core, furthermore he takes into account relativistic corrections and the final-state interactions. He obtains for the two hyperfine states Γ_F and for their statistical mixture the values

$$\Gamma_{\frac{1}{2}} = 334\ \text{s}^{-1}\,, \qquad \Gamma_{\frac{3}{2}} = 15\ \text{s}^{-1}\,, \qquad \Gamma = 121\ \text{s}^{-1}\,. \tag{1}$$

In a more recent paper CREMMER (2) carries out a fully relativistic calculation also taking into account the final-state corrections and he obtains

$$\Gamma_{\frac{1}{2}} = 450\ \text{s}^{-1}\,, \qquad \Gamma_{\frac{3}{2}} = 30\ \text{s}^{-1}\,, \qquad \Gamma = 170\ \text{s}^{-1}\,, \tag{2}$$

(1) I. T. WANG: *Phys. Rev.*, **139**, B 1539 (1965).
(2) E. CREMMER: *Nucl. Phys.*, **2** B, 409 (1967).

which are in complete disagreement with the ones obtained by WANG, since only a negligible part of the difference between the quoted values is due to the slightly different values of the coupling constants used.

There are furthermore two experimental results. WANG *et al.* (3) have measured $\Gamma_{\frac{1}{2}}$ by stopping negative muons in liquid hydrogen contamined with $3.2 \cdot 10^{-3}$ parts of deuterium, and they obtained

$$\Gamma_{\frac{1}{2}} = (365 \pm 96)\ \text{s}^{-1}\,. \tag{3}$$

Recently PLACCI *et al.* (4) have measured $\Gamma_{\frac{1}{2}}$ by stopping negative muons in hydrogen gas with 5.0 % of deuterium, with the result

$$\Gamma_{\frac{1}{2}} = (450 \pm 70)\ \text{s}^{-1}\,. \tag{4}$$

The details of both experiments have been recently discussed by ZAVATTINI (5) and the most probable origins of possible errors pointed out.

In this paper we study the muon capture in deuterium in order to clarify the theoretical situation. In Sect. **2** the general formulae are derived and Sect. **3** is devoted to comparing our results with the above given ones.

2. – General equations.

Let us consider the process

$$\mu^- + \text{d} \to \text{n} + \text{n} + \nu_\mu\,, \tag{5}$$

whose capture rate is given by

$$\Gamma = \frac{(\alpha m'_\mu)^3 G^2 C_\text{d} \cos^2\theta}{2(2\pi)^3} \int \text{d}\boldsymbol{\nu}\, \text{d}\boldsymbol{p}_1\, \text{d}\boldsymbol{p}_2\, \delta(E_i - E_f)\, \bar{\Sigma} \sum |M|^2\,, \tag{6}$$

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(3) I. T. WANG, E. W. ANDERSON, E. J. BLESER, L. M. LEDERMAN, S. L. MEYER, J. L. ROSEN and J. E. ROTHBERG: *Phys. Rev.*, **139**, 1528 (1965).

(4) A. PLACCI, E. ZAVATTINI, A. BERTIN and A. VITALE: *Phys. Rev. Lett.*, **25**, 475 (1970).

(5) E. ZAVATTINI: *Analysis of the muon capture experiments done in hydrogen and deuterium: necessity of the (V—A) type interaction and determination of g_P*, Lectures given at the *Academic Training Programme of CERN, 1971.*

where α is the fine-structure constant, m'_μ the muon reduced mass, C_d is a correction factor arising from the effect of the nonpointlike character of the deuteron charge distribution [6], $G\cos\theta$ is the β-decay coupling constant, E_i and E_f are the initial and final energies respectively. Finally $\boldsymbol{\nu}$, $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are the momenta of the neutrino and of the two neutrons. Moreover the transition matrix element is given, in the usual notation, by [7]

$$M = \alpha_\nu^+(1-\hat{\nu}\cdot\boldsymbol{\sigma})\alpha_\mu\left\{G_V\int 1 + G_P\hat{\nu}\int\boldsymbol{\sigma} + \frac{g_A}{M}\int\boldsymbol{p}\cdot\boldsymbol{\sigma}\right\} + \\ + \alpha_\nu^+(1-\hat{\nu}\boldsymbol{\sigma})\boldsymbol{\sigma}\alpha_\mu\left\{G_A\int\boldsymbol{\sigma} + \frac{g_V}{M}\int\boldsymbol{p}\right\}, \tag{7}$$

where M is the nucleon mass and the terms with the momentum operator are the relativistic corrections. In the present case

$$\int B \equiv \langle\Psi_f|\sum_{i=1}^{2}\exp[-i\nu\cdot x_i]\,\tau_-(i)B(i)|\Psi_i\rangle\,, \tag{8}$$

where the space part of Ψ_f is the product of two plane waves with momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, where final-state interactions are neglected. The space part of Ψ_i is given by

$$\begin{cases} \Psi_d(r) = \dfrac{N}{\sqrt{4\pi}}\dfrac{1}{r}\left[u(r) + \dfrac{1}{\sqrt{8}}w(r)S_{12}\right], \\ S_{12} \equiv \dfrac{3(\bar{r}\cdot\bar{\sigma}_1)(\bar{r}\cdot\bar{\sigma}_2)}{r^2} - (\bar{\sigma}_1\bar{\sigma}_2)\,, \end{cases} \tag{9}$$

$u(r)$ and $w(r)$ being the $L=0$ and $L=2$ radial wave functions, while the normalization constant N is given by

$$N^{-2} = \int_0^\infty \mathrm{d}r\,[u(r)^2 + w(r)^2]\,. \tag{10}$$

Then a straightforward calculation for the two hyperfine states as well as for

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[6] C. W. KIM and H. PRIMAKOFF: *Phys. Rev.*, **140**, B 566 (1965).
[7] H. PRIMAKOFF: *Rev. Mod. Phys.*, **31**, 802 (1959).

the statistical mixture gives the following results:

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\begin{aligned}
(11)\qquad \Gamma_{\frac{1}{2}} = {} & \frac{(\alpha m'_\mu)^3 C_\mathrm{d} N^2 G^2 \cos^2\theta}{2\pi^3}\cdot \\
& \cdot\left\{\left[G_V^2 + 7G_A^2 + G_P^2 - 4G_V G_A + \frac{4}{3} G_V G_P - \frac{14}{3} G_A G_P\right] I_1 + \right. \\
& + \left[G_V^2 + G_A^2 + \frac{1}{3} G_P^2 - 4G_V G_A + \frac{4}{3} G_V G_P - \frac{2}{3} G_A G_P\right] I_2 + [G_V + G_A] G_P I_3 + \\
& + \left[(G_V + G_A)^2 + G_P^2 - \frac{2}{3}(G_V + G_A) G_P\right] I_4 + [G_P + G_V - 2G_A] G_P I_5 + \\
& + \frac{1}{2}[G_V + G_A]^2 I_6 + \left[G_V + G_A - \frac{1}{2} G_P\right] G_P I_7 + \\
& + \left[2g_V\left(G_V - 2G_A + \frac{2}{3} G_P\right) + 2g_A\left(-\frac{2}{3} G_V + \frac{7}{3} G_A - G_P\right)\right] I_8 + \\
& \left. + \left[g_V\left(-G_V + 2G_A - \frac{2}{3} G_P\right) + g_A\left(\frac{2}{3} G_V - \frac{1}{3} G_A + \frac{1}{3} G_P\right)\right] I_9\right\},
\end{aligned}
$$

$$
\begin{aligned}
(12)\qquad \Gamma_{\frac{3}{2}} = {} & \frac{(\alpha m'_\mu)^3 C_\mathrm{d} N^2 G^2 \cos^2\theta}{2\pi^3}\left\{\left[(G_V + G_A)^2 + G_P^2 - \frac{2}{3}(G_V + G_A) G_P\right] I_1 + \right. \\
& + \left[(G_V + G_A)^2 + \frac{1}{3} G_P^2 - \frac{2}{3}(G_V + G_A) G_P\right] I_2 - \frac{1}{2}[G_V + G_A] G_P I_3 + \\
& + \left[G_V^2 + 4G_A^2 + G_P^2 - G_V G_A + \frac{1}{3} G_V G_P - \frac{8}{3} G_A G_P\right] I_4 + \left[G_P - \frac{1}{2} G_V - 2G_A\right] G_P I_5 + \\
& + \frac{1}{2}[G_V^2 + G_A^2 - G_V G_A] I_6 + \left[-\frac{1}{2} G_V + G_A - \frac{1}{2} G_P\right] G_P I_7 + \\
& + \left[2g_V\left(G_V + G_A - \frac{1}{3} G_P\right) + 2g_A\left(\frac{1}{3} G_V + \frac{1}{3} G_A - G_P\right)\right] I_8 + \\
& \left. + \left[g_V\left(-G_V - G_A + \frac{1}{3} G_P\right) + g_A\left(-\frac{1}{3} G_V - \frac{1}{3} G_A + \frac{1}{3} G_P\right)\right] I_9\right\},
\end{aligned}
$$

$$
\begin{aligned}
(13)\qquad \Gamma = {} & \frac{(\alpha m'_\mu)^3 C_\mathrm{d} N^2 G^2 \cos^2\theta}{2\pi^3}\left\{G_V^2\left[I_1 + I_2 + I_4 + \frac{1}{2} I_6\right] + \right. \\
& + \Gamma_A^2\left[3I_1 + I_2 + 3I_4 + \frac{1}{2} I_6\right] + G_P(G_P - 2G_A)\left[I_5 - \frac{1}{6} I_6 - \frac{1}{2} I_7\right] + \\
& \left. + 2[g_V G_V + g_A G_A - g_A G_P] I_8 + \left[-g_V G_V - \frac{1}{3} g_A G_A + \frac{1}{3} g_A G_P\right] I_9\right\}.
\end{aligned}
$$

We denote by I_i the integral

$$I_i = \int_M^{(M_d+m_\mu)/2} \mathrm{d}E_1 \int_a^b \mathrm{d}E_2 E_1 E_2 \Omega A_i \,, \tag{14}$$

where

$$a = \frac{(\Delta - p_1)^2 + M^2}{2(\Delta - p_1)} \,, \qquad b = \frac{(\Delta + p_1)^2 + M^2}{2(\Delta + p_1)} \,, \tag{15}$$

where $p_i = \sqrt{E_i^2 - M^2}$, $\Delta = M_d + m_\mu - E_1$ and $\Omega = \Delta - E_2$. Furthermore m_μ, M and M_d are, respectively, the muon, neutron and deuteron mass. Finally

$$\left\{\begin{aligned}
A_1 &= u_0^2(p_1) \,, \\
A_2 &= -u_0(p_1)u_0(p_2) \,, \\
A_3 &= \left[\sqrt{8}\, u_0(p_1) + 2w_2(p_1)\right] w_2(p_1) \left[\frac{(\Omega^2 + p_1^2 - p_2^2)^2}{4\Omega^2 p_1^2} - \frac{1}{3}\right] , \\
A_4 &= w_2^2(p_1) \,, \\
A_5 &= \frac{\sqrt{8}}{3}\, u_0(p_1) w_2(p_2) \left[1 - \frac{3(\Omega^2 - p_1^2 + p_2^2)^2}{4\Omega^2 p_2^2}\right] , \\
A_6 &= w_2(p_1) w_2(p_2) \left[1 - \frac{3(\Omega^2 - p_1^2 - p_2^2)^2}{4p_1^2 p_2^2}\right] , \\
A_7 &= w_2(p_1) w_2(p_2) \left[\frac{1}{3} + \frac{p_2^2}{2\Omega^2} - \frac{\Omega^2}{2p_2^2} + \frac{p_1^2}{p_2^2} - \frac{p_1^4}{2\Omega^2 p_2^2}\right] , \\
A_8 &= -\frac{u_0^2(p_1)}{2M\Omega}\left[\Omega^2 - p_2^2 + p_1^2\right] , \\
A_9 &= -\frac{1}{M}\, \Omega u_0(p_1) u_0(p_2) \,,
\end{aligned}\right. \tag{16}$$

where

$$\left|\begin{aligned}
u_0(p) &= \int_0^\infty \mathrm{d}r\, r\, j_0(pr)\, u(r) \,, \\
w_2(p) &= -\int_0^\infty \mathrm{d}r\, r\, j_2(pr)\, w(r) \,.
\end{aligned}\right. \tag{17}$$

In the above equations the relativistic terms are the ones with I_8 and I_9 where only the deuteron S-wave function has been taken into account.

3. – Results.

In order to describe the deuteron we use the wave function of GOURDIN *et al.* ([8]), which can be written as

$$(18)\qquad \begin{cases} u(r) = \sum\limits_{\lambda=1}^{4} C_\lambda \exp[-\beta_\lambda r]\,, \\ w(r) = \varrho \sum\limits_{\lambda=1}^{4} C_\lambda \dfrac{\beta_\lambda^2}{\beta_1^2} (\beta_\lambda r) h_2(i\beta_\lambda r)\,, \end{cases}$$

where

$$(19)\qquad \begin{cases} \beta_1 = 45.683\ \text{MeV}\,, & \beta_2 = 8.55\beta_1\,, \qquad \beta_3 = 11.13\beta_1\,, \\ \beta_4 = \sqrt{\beta_2^2+\beta_3^2-\beta_1^2}\,, & C_1 = -C_4 = +1\,, \qquad C_2 = -C_3 = -\dfrac{\beta_1^2-\beta_4^2}{\beta_2^2-\beta_3^2}\,, \\ \varrho = 0.0265\,. \end{cases}$$

Then

$$(20)\qquad \begin{cases} u_0(p) = \sum\limits_{\lambda=1}^{4} \dfrac{C_\lambda}{p^2+\beta_\lambda^2}\,, \\ w_2(p) = \varrho \sum\limits_{\lambda=1}^{4} \dfrac{\beta_\lambda^2}{\beta_1^2} \dfrac{C_\lambda}{p^2+\beta_\lambda^2}\,, \end{cases}$$

and the normalization constant turns out to be

$$(21)\qquad N^{-2} = \sum_{\lambda,\mu=1}^{4} \frac{C_\lambda C_\mu}{\beta_\lambda+\beta_\mu} + \frac{\varrho^2}{\beta_1^4} \sum_{\lambda,\mu=1}^{4} \frac{\beta_\lambda^2\beta_\mu^2}{\beta_\lambda+\beta_\mu} C_\lambda C_\mu\,.$$

By means of their wave functions the integrals defined by eq. (14) have been computed using an IBM 360-40 (sometimes the first integral has been done analytically) with the following results:

$$(22)\qquad \begin{cases} I_1 = +145.992\ \text{MeV}\,, & I_2 = -93.863\ \text{MeV}\,, & I_3 = -0.199\ \text{MeV}\,, \\ I_4 = +\ \ 2.655\ \text{MeV}\,, & I_5 = +\ \ 5.808\ \text{MeV}\,, & I_6 = -3.587\ \text{MeV}\,, \\ I_7 = +\ \ 1.688\ \text{MeV}\,, & I_8 = -\ \ 0.966\ \text{MeV}\,, & I_9 = -9.068\ \text{MeV}\,. \end{cases}$$

([8]) M. GOURDIN, M. LE BELLAC, F. M. RENARD and J. TRAN THANH VAN: *Nuovo Cimento*, **37**, 524 (1965).

The values of the coupling constants used are

$$(23)\qquad \begin{cases} G\cos\theta = 1.1484\cdot 10^{-11}\,(\text{MeV})^{-2}\,, \\ g_V = 0.968\,, \qquad g_M = 3.600\,, \qquad g_A = -1.230\,, \qquad g_P = -10.27\,. \end{cases}$$

The correction factor C_d has been calculated by the method outlined in ref. (6) and we have obtained $C_d = 0.977$.

Using all that we obtain the following results immediately:

1) if only the S-wave in the deuteron is taken into account ($\varrho = 0$) and the relativistic corrections are neglected

$$(24)\qquad \Gamma_{\frac{1}{2}} = 331.6\ \text{s}^{-1}\,, \qquad \Gamma_{\frac{3}{2}} = 11.2\ \text{s}^{-1}\,, \qquad \Gamma = 118.0\ \text{s}^{-1}\,;$$

2) if S and D waves are used, but the relativistic corrections are neglected

$$(25)\qquad \Gamma_{\frac{1}{2}} = 308.0\ \text{s}^{-1}\,, \qquad \Gamma_{\frac{3}{2}} = 12.2\ \text{s}^{-1}\,, \qquad \Gamma = 110.8\ \text{s}^{-1};$$

3) if all terms are taken into account

$$(26)\qquad \Gamma_{\frac{1}{2}} = 312.7\ \text{s}^{-1}\,, \qquad \Gamma_{\frac{3}{2}} = 12.0\ \text{s}^{-1}\,, \qquad \Gamma = 112.2\ \text{s}^{-1}\,.$$

In our calculation we have neglected the final-state interaction because, in our case, its influence must be small since in this case the capture rate can be calculated in a good approximation using the closure approximation. CREMMER (2) has taken into account final-state interactions and his results imply that they tend to decrease all capture rates by 4% approximately; this in perfect agreement with the results of ref. (9).

We would like to compare our results with the two previous existing calculations quoted above. WANG (1) carries out a calculation using techniques similar to ours. He describes the deuteron with only an S-wave, with hard core, and simulates the D-waves normalizing to 0.97. Furthermore he uses the values $g_A = -1.19$ and $g_P = -8.12$. If we take all that into account our results are roughly compatible with those of WANG. Nevertheless, there still remains a discrepancy in $\Gamma_{\frac{3}{2}}$ which to our understanding is due to the wrong treatment of the relativistic terms, their contribution being smaller than the one claimed there. If we take into account our results it is clear that there is no reason to consider the relativistic terms and to neglect the D-waves.

CREMMER (2) carries out a fully relativistic calculation using the same wave functions used here. He takes $g_A \simeq -1.15$, $g_P \simeq -7.52$ and $G\cos\theta =$

(9) J. BERNABEU and P. PASCUAL: *Nuovo Cimento*, **10** A, 61 (1972).

$=1.117 \cdot 10^{-5}\,(\mathrm{GeV})^{-2}$. Clearly the discrepancy between his results and ours is not due to the different coupling constants used. A comparison of both methods allows us to establish that his calculation and ours are completely equivalent up to terms of order p/M included. Nevertheless in the calculation of the traces there are some errors. For the direct terms ([10]) there is a mistake in the sign of the $g_V g_A$ term and the $g_A g_M$ term is an order of magnitude too small. Furthermore in the cross terms there is a general minus sign missing and some other small errors. If all that is taken into account both results are in good agreement.

Finally let us say a word on the experimental situation. When muons are stopped in deuterium the muonic atom μd can undergo the following molecular formation process:

$$\mu\mathrm{d}+\mathrm{d} \rightarrow \mathrm{d}\mu\mathrm{d}\,. \tag{27}$$

From the molecule dμd, the following fusion reaction will take place:

$$\mathrm{d}\mu\mathrm{d} \rightarrow {}^{3}\mathrm{He}+\mathrm{n}+\mu^{-}\,. \tag{28}$$

Due to the kinetic energies of this neutron it is practically impossible to single out capture events ([5]) from a dμd molecule if one uses the experimental methods so far used in detecting the outgoing neutron to identify the capture process ([11]).

([10]) E. Cremmer: Thèse de la Faculté des Sciences d'Orsay (1967).
([11]) One of us (P.P.) would like to acknowledge a discussion with Prof. E. Zavattini on this point.

● RIASSUNTO (*)

Si studia la cattura dei muoni nel deuterio usando l'approssimazione degli impulsi con correzioni relativistiche. Si descrive il deutone usando l'onda S e l'onda D.

(*) *Traduzione a cura della Redazione.*

Захват мюона в дейтерии.

Резюме (*). — Используя импульсное приближение с релятивистскими поправками, исследуется захват мюона в дейтерии. Дейтерий описывается с помощью S- и D-волн.

(*) *Переведено редакцией.*