/ MuSun / 262 : **kinetics**

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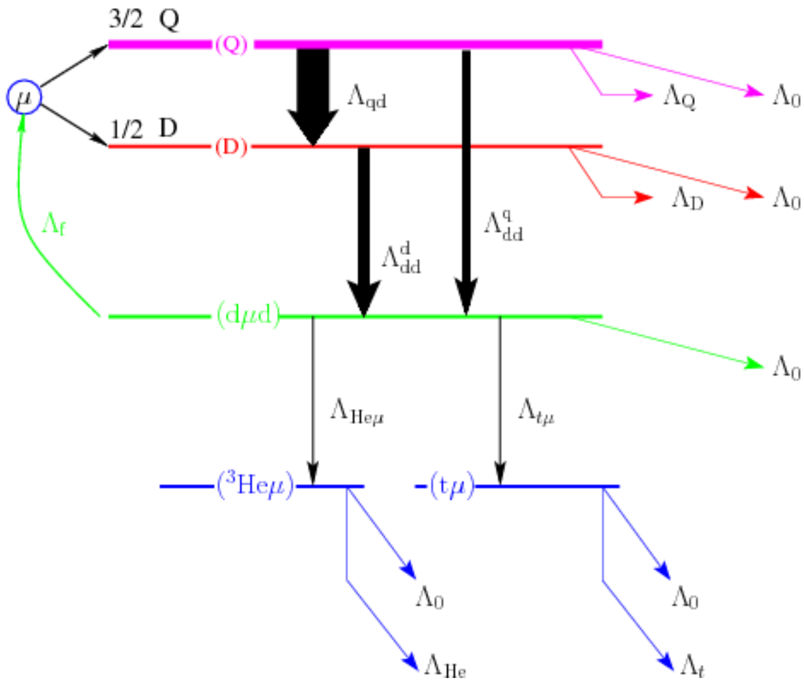
keywords	
tags	

kinetics



rates

parameter	value (s^{-1})
Λ_{qd}	$\varphi \times 3.7 \times 10^7$
Λ_{Q}	8
Λ_{D}	400
$\Lambda^{\mathrm{q}}_{\mathrm{dd}}$	$\varphi \times 3.75 \times 10^6$
$\Lambda^{\mathrm{d}}_{\mathrm{dd}}$	$\varphi \times 2.549 \times 10^6$
Λ_{f}	0.26×10^9
$\Lambda_{\mathrm{He}\mu}$	0.036×10^9
$\Lambda_{\mathrm{t}\mu}$	0.003×10^9
Λ_{He}	2216
Λ_{t}	10 (?)
Λ_0	455162

kinetic equations

$$\begin{cases} dN_{\mathrm{Q}} &= -N_{\mathrm{Q}}(\Lambda_{\mathrm{qd}} + \Lambda^{\mathrm{q}}_{\mathrm{dd}} + \Lambda_{\mathrm{Q}} + \Lambda_0)dt + \frac{2}{3}N_{\mathrm{d}\mu\mathrm{d}}\Lambda_{\mathrm{f}}dt & (1a) \\ dN_{\mathrm{D}} &= -N_{\mathrm{D}}(\Lambda^{\mathrm{d}}_{\mathrm{dd}} + \Lambda_{\mathrm{D}} + \Lambda_0)dt + N_{\mathrm{Q}}\Lambda_{\mathrm{qd}}dt + \frac{1}{3}N_{\mathrm{d}\mu\mathrm{d}}\Lambda_{\mathrm{f}}dt & (1b) \\ dN_{\mathrm{d}\mu\mathrm{d}} &= -N_{\mathrm{d}\mu\mathrm{d}}(\Lambda_{\mathrm{He}\mu} + \Lambda_{\mathrm{t}\mu} + \Lambda_{\mathrm{f}} + \Lambda_0)dt + N_{\mathrm{D}}\Lambda^{\mathrm{d}}_{\mathrm{dd}}dt + N_{\mathrm{Q}}\Lambda^{\mathrm{q}}_{\mathrm{dd}}dt & (1c) \end{cases}$$

or

$$\begin{cases} dN_{\mathrm{Q}} &= -N_{\mathrm{Q}}\lambda_{\mathrm{Q}}dt + \frac{2}{3}N_{\mathrm{d}\mu\mathrm{d}}\Lambda_{\mathrm{f}}dt & (2a) \\ dN_{\mathrm{D}} &= -N_{\mathrm{D}}\lambda_{\mathrm{D}}dt + N_{\mathrm{Q}}\Lambda_{\mathrm{qd}}dt + \frac{1}{3}N_{\mathrm{d}\mu\mathrm{d}}\Lambda_{\mathrm{f}}dt & (2b) \\ dN_{\mathrm{d}\mu\mathrm{d}} &= -N_{\mathrm{d}\mu\mathrm{d}}\lambda_{\mathrm{d}\mu\mathrm{d}}dt + N_{\mathrm{D}}\Lambda^{\mathrm{d}}_{\mathrm{dd}}dt + N_{\mathrm{Q}}\Lambda^{\mathrm{q}}_{\mathrm{dd}}dt & (2c) \end{cases}$$

where the following notations for disappearance rates were introduced:

$$
\begin{aligned}
\lambda_{\mathrm{Q}} &= \Lambda_{\mathrm{qd}} + \Lambda_{\mathrm{dd}}^{\mathrm{q}} + \Lambda_{\mathrm{Q}} + \Lambda_0 \\
\lambda_{\mathrm{D}} &= \Lambda_{\mathrm{dd}}^{\mathrm{d}} + \Lambda_{\mathrm{D}} + \Lambda_0 \\
\lambda_{\mathrm{d}\mu\mathrm{d}} &= \Lambda_{\mathrm{He}\mu} + \Lambda_{\mathrm{t}\mu} + \Lambda_{\mathrm{f}} + \Lambda_0
\end{aligned}
$$

solution of kinetic equations

Let's search the solution of the system of differential equations (2) in the form:

$$
\begin{cases}
N_{\mathrm{Q}}(t) &= Q(t)e^{-\lambda_{\mathrm{Q}}t} \qquad (3a) \\
N_{\mathrm{D}}(t) &= D(t)e^{-\lambda_{\mathrm{D}}t} \qquad (3b) \\
N_{\mathrm{d}\mu\mathrm{d}}(t) &= M(t)e^{-\lambda_{\mathrm{d}\mu\mathrm{d}}t} \qquad (3c)
\end{cases}
$$

Substitution of expressions (3) into the system (2) gives:

$$
\begin{cases}
\dot{Q} = \frac{2}{3}\Lambda_{\mathrm{f}} M e^{-(\lambda_{\mathrm{d}\mu\mathrm{d}}-\lambda_{\mathrm{Q}})t} & (4a) \\
\dot{D} = \Lambda_{\mathrm{qd}} Q e^{-(\lambda_{\mathrm{Q}}-\lambda_{\mathrm{D}})t} + \frac{1}{3}\Lambda_{\mathrm{f}} M e^{-(\lambda_{\mathrm{d}\mu\mathrm{d}}-\lambda_{\mathrm{D}})t} & (4b) \\
\dot{M} = \Lambda_{\mathrm{dd}}^{\mathrm{d}} D e^{-(\lambda_{\mathrm{D}}-\lambda_{\mathrm{d}\mu\mathrm{d}})t} + \Lambda_{\mathrm{dd}}^{\mathrm{q}} Q e^{-(\lambda_{\mathrm{Q}}-\lambda_{\mathrm{d}\mu\mathrm{d}})t} & (4c)
\end{cases}
$$

The system of differential equations of first order (4) can be reduced to a single differential equation of third order (5)

$$\dddot{Q} + a_2\ddot{Q} + a_1\dot{Q} + a_0 Q = 0 \qquad (5)$$

where

$$
\begin{aligned}
a_2 &= \lambda_{\mathrm{D}} - \lambda_{\mathrm{d}\mu\mathrm{d}} - 2(\lambda_{\mathrm{Q}} - \lambda_{\mathrm{d}\mu\mathrm{d}}) \\
a_1 &= (\lambda_{\mathrm{d}\mu\mathrm{d}} - \lambda_{\mathrm{Q}})^2 - (\lambda_{\mathrm{d}\mu\mathrm{d}} - \lambda_{\mathrm{Q}})(\lambda_{\mathrm{d}\mu\mathrm{d}} - \lambda_{\mathrm{D}}) - \tfrac{2}{3}\Lambda_{\mathrm{f}}(\Lambda_{\mathrm{dd}}^{\mathrm{q}} + \tfrac{1}{2}\Lambda_{\mathrm{dd}}^{\mathrm{d}}) \\
a_0 &= \tfrac{2}{3}\Lambda_{\mathrm{f}}(\Lambda_{\mathrm{dd}}^{\mathrm{q}}(\lambda_{\mathrm{Q}} - \lambda_{\mathrm{D}}) - \Lambda_{\mathrm{dd}}^{\mathrm{d}}\Lambda_{\mathrm{qd}})
\end{aligned}
$$

Let's search the solution of equation (5) in the form (6):

$$Q(t) = ce^{\alpha t} \qquad (6)$$

Substitution of expression (6) into equation (5) results in following cubic equation for α:

$$\alpha^3 + a_2\alpha^2 + a_1\alpha + a_0 = 0 \qquad (7)$$

The roots α_1, α_2, α_3 of equation (7) can be found using the [Wiet's trigonometric formula]:

$$
\begin{aligned}
\alpha_1 &= -2\sqrt{q}\cos(\phi) - \frac{a_2}{3} \\
\alpha_2 &= -2\sqrt{q}\cos(\phi + \tfrac{2}{3}\pi) - \frac{a_2}{3} \\
\alpha_3 &= -2\sqrt{q}\cos(\phi - \tfrac{2}{3}\pi) - \frac{a_2}{3}
\end{aligned}
$$

where

$$
\begin{aligned}
q &= \frac{a_2^2 - 3a_1}{9} \\
r &= \frac{2a_2^3 - 9a_2a_1 + 27a_0}{54} \\
\phi &= \frac{\arccos\left(\frac{r}{\sqrt{q^3}}\right)}{3}
\end{aligned}
$$

Note that equation (7) has three different roots if $S = q^3 - r^2$ is positive, which should always be the case in our problem (I guess). Otherwise, one should use other formulas for finding the roots of cubic equation.

Thus, the solution of eq. (5) has the following form:

$$Q(t) = c_1e^{\alpha_1 t} + c_2e^{\alpha_2 t} + c_3e^{\alpha_3 t}$$

Which gives the following expressions for temporal evolution of the number of muons in Q or D states of μd atom or in dμd molecules:

$$\begin{aligned} N_{\mathrm{Q}}(t) &= \left(c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t} + c_3 e^{\alpha_3 t}\right) e^{-\lambda_{\mathrm{Q}} t} \\ N_{\mathrm{D}}(t) &= \left(c_1 r_1 e^{\alpha_1 t} + c_2 r_2 e^{\alpha_2 t} + c_3 r_3 e^{\alpha_3 t}\right) e^{-\lambda_{\mathrm{Q}} t} \\ N_{\mathrm{d}\mu\mathrm{d}}(t) &= \frac{3}{2}\frac{1}{\Lambda_{\mathrm{f}}}\left(\alpha_1 c_1 e^{\alpha_1 t} + \alpha_2 c_2 e^{\alpha_2 t} + \alpha_3 c_3 e^{\alpha_3 t}\right) e^{-\lambda_{\mathrm{Q}} t} \end{aligned} \tag{8}$$

where

$$\begin{aligned} r_1 &= \frac{1}{\Lambda^{\mathrm{d}}_{\mathrm{dd}}}\left(\frac{3}{2}\frac{1}{\Lambda_{\mathrm{f}}}\alpha_1(\alpha_1 + \lambda_{\mathrm{d}\mu\mathrm{d}} - \lambda_{\mathrm{Q}}) - \Lambda^{q}_{\mathrm{dd}}\right) \\ r_2 &= \frac{1}{\Lambda^{\mathrm{d}}_{\mathrm{dd}}}\left(\frac{3}{2}\frac{1}{\Lambda_{\mathrm{f}}}\alpha_2(\alpha_2 + \lambda_{\mathrm{d}\mu\mathrm{d}} - \lambda_{\mathrm{Q}}) - \Lambda^{q}_{\mathrm{dd}}\right) \\ r_3 &= \frac{1}{\Lambda^{\mathrm{d}}_{\mathrm{dd}}}\left(\frac{3}{2}\frac{1}{\Lambda_{\mathrm{f}}}\alpha_3(\alpha_3 + \lambda_{\mathrm{d}\mu\mathrm{d}} - \lambda_{\mathrm{Q}}) - \Lambda^{q}_{\mathrm{dd}}\right) \end{aligned}$$

The coefficients *c_1, c_2, c_3* can be found from initial conditions (9) of the problem (1):

$$\begin{cases} N_{\mathrm{Q}}(t=0) &= \frac{2}{3}N_0 \\ N_{\mathrm{D}}(t=0) &= \frac{1}{3}N_0 \\ N_{\mathrm{d}\mu\mathrm{d}}(t=0) &= 0 \end{cases} \tag{9}$$

This gives the following expressions:

$$\begin{aligned} c_3 &= \frac{1}{3}N_0\frac{(\alpha_1-\alpha_2) + 2(\alpha_2 r_1 - \alpha_1 r_2)}{(\alpha_2-\alpha_3)r_1 - (\alpha_1-\alpha_3)r_2 + (\alpha_1-\alpha_2)r_3} \\ c_2 &= \frac{\frac{2}{3}\alpha_1 N_0 - c_3(\alpha_1-\alpha_3)}{\alpha_1-\alpha_2} \\ c_1 &= \frac{2}{3}N_0 - c_2 - c_3 \end{aligned} \tag{10}$$

The differential equation (11) describes the number of muons in Heμ molecule:

$$dN_{\mathrm{He}\mu} = -N_{\mathrm{He}\mu}(\Lambda_0 + \Lambda_{\mathrm{He}})dt + N_{\mathrm{d}\mu\mathrm{d}}\Lambda_{\mathrm{He}\mu}dt \tag{11}$$

with initial condition $N_{\mathrm{He}\mu}(t=0)=0$. It has the following solution:

$$\begin{aligned} N_{\mathrm{He}\mu}(t) &= \frac{3}{2}\frac{\Lambda_{\mathrm{He}\mu}}{\Lambda_{\mathrm{f}}}\left[\frac{\alpha_1 c_1}{\Lambda_0+\Lambda_{\mathrm{He}}+\alpha_1-\lambda_{\mathrm{Q}}}\left(e^{(\alpha_1-\lambda_{\mathrm{Q}})t} - e^{-(\Lambda_0+\Lambda_{\mathrm{He}})t}\right) + \right. \\ &\left. \frac{\alpha_2 c_2}{\Lambda_0+\Lambda_{\mathrm{He}}+\alpha_2-\lambda_{\mathrm{Q}}}\left(e^{(\alpha_2-\lambda_{\mathrm{Q}})t} - e^{-(\Lambda_0+\Lambda_{\mathrm{He}})t}\right) + \frac{\alpha_3 c_3}{\Lambda_0+\Lambda_{\mathrm{He}}+\alpha_3-\lambda_{\mathrm{Q}}}\left(e^{(\alpha_3-\lambda_{\mathrm{Q}})t} - e^{-(\Lambda_0+\Lambda_{\mathrm{He}})t}\right)\right] \end{aligned} \tag{12}$$

Attachments:

	file	size
	.root	3703
	c2.png	8440
	fig.png	16539
L	fig.tex	49
	killme.ps	20080
	kinetics.C	12385
	kinetics.eps	11143
	kinetics.fig	3774
	kinetics_test.C	1272

	simul.C	3084
	simul_e_0_01_1e16.root	3745
	solve.C	9838
	solve.C.01	3808

/eLogs/MuSun/000262/attachments