Simulation of Positron Shower Leakage

Abstract: One of the goals of this study is to understand and quantify the leakage of the positron showers in the passage through our calorimeters and then add it back to the observed energy. This study has been done as a function of the positron energy, impinging point on the calorimeter face, angle of incidence, ... We need to understand how the radiation is lost in the passage through PbF_2 crystals. Leakage effects at boundaries between crystals can be larger than when the positron hits the center of a crystal. The positrons emanating from muons decaying at the magic momentum on the ideal orbit would have a different angular distribution compared to the off-track ones. Thus we expect a different rate in the 9×6 grid structure of our calorimeter. There could also be other effects due to beam oscillations etc. All this needs to be quantified with simulations. The simulations in this note have been done using Geant4.

1. Introduction

The Muon g-2 experiment uses 24 calorimeters surrounding a circular storage ring. Each calorimeter is made up of a matrix of 9×6 segmented PbF_2 crystals ($2.5\times2.5\times14$ cm^3) read by Silicon Photo Multipliers (SiPMs). We use PbF_2 crystals since they have a high density and high refractive index. Its high density allows the containment of the positron shower in a small area thereby preventing losses or leakages. The Moliere radius of PbF_2 is 2.1 cm which is greater than 2.5 cm thereby allowing 90% of the shower to be contained within the crystals [1]. The high refractive index of PbF_2 produces Cherenkov radiation corresponding to an energy threshold of $\approx$ 100 keV for positrons. The energy threshold of positrons for measuring ω_a is above 1 GeV, which is favourable for our experiment. Finally, the light yield of Cherenkov detectors are proportional to incident particle energy that will enable an accurate and easy energy calibration. In the passage of muons through these detectors, we include all physical processes it undergoes before decaying to positrons in our simulation. Some of these processes include Cherenkov radiation, transition radiation, bremsstrahlung and pair production [2]. The positrons incident at varies positions in a crystal have been studied to check its dependence on the energy. The leakage of the positron shower with varying energy is also studied here.

2. Positrons through Lead–Fluoride Crystals

A visualization of the simulation of the positron shower using Geant4 for 10000 events incident perpendicular on the XY–plane of crystal 22 of the 9×6 segmented PbF_2 crystals ($2.5\times2.5\times14$ cm^3) is shown in figure 1.

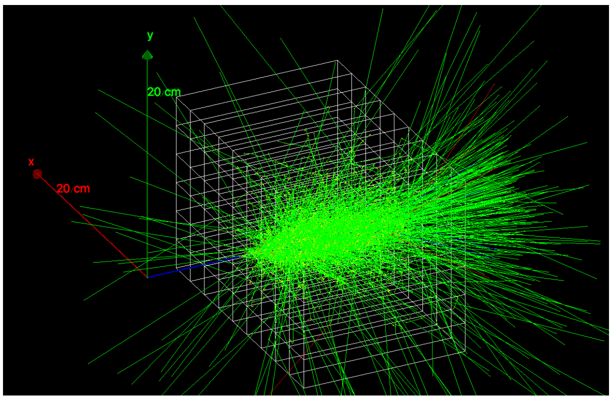


Figure 1. Visualization of 10000 events passing through the center of crystal 22.

The beam is along the Z-direction with an energy of 3.1 GeV hitting the center of the matrix (no lateral leakage). Crystal 22 (counting from zero) was selected since it is almost centrally located. The default color scheme has been used showing the electrons in red, positrons in blue and gamma rays in green. The blue line at the origin along the z-axis denotes the original positron beam.

Figure 2 below shows the sum of the energies of all the particles in the positron shower that pass through the 9×6 segmented PbF_2 crystals for the above-mentioned configuration. Note that the mean energy collected for a positron beam of initial energy 3100 MeV is 2913 MeV which is about 93.9%, indicating a leakage of about 6.1% of the initial energy of 3.1 GeV.

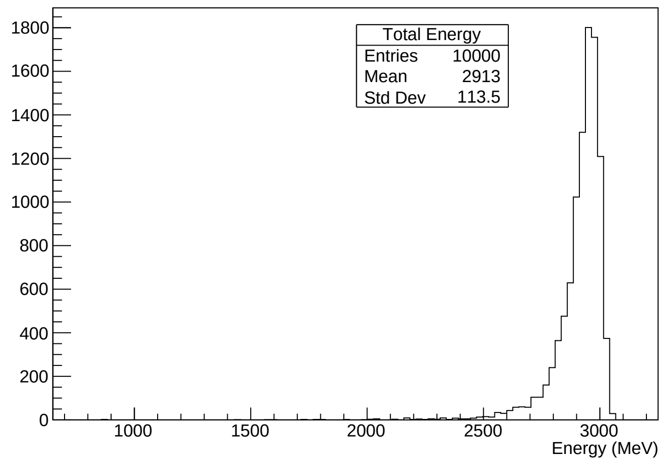


Figure 2. .Total positron energy spectrum of all particles of energy 3.1 GeV.

To check the dependence of the energy distribution on the beam position we considered the beam incident normally on at three positions of crystal 22 - center, top (1 mm below the upper edge) and bottom (1 mm above the lower edge) of this crystal. The energy spectrum of the positron shower through the 9×6 grid system of the experimental setup for these three positions is shown in 3. This is the sum of energies of all crystals. Note that the distribution about the center is almost symmetrical. Thus, the left and right plots of figure 3 have a similar distribution.

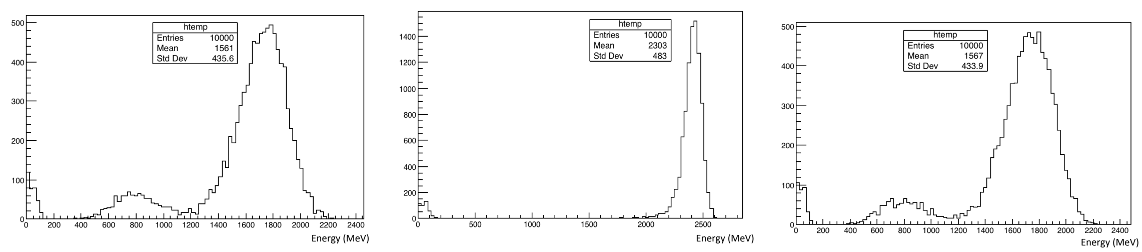


Figure 3. The positron energy distribution through a 9×6 segmented PbF_2 crystals of a calorimeter when the moun beam is incident at the top (left panel - 9.5 mm above the center) center (middle panel) and bottom (right panel - 9.5 mm below the center) of crystal 22 respectively. This depicts the positron shower leakage in a calorimeter.

The number of positrons hitting the crystals for these three cases is shown in figure 4. We used a log scale to visualize better the distribution in each crystal. Notice the distribution is almost symmetrical about crystal 22 when the beam hits the center of crystal 22 (middle panel of figure 4). The number of hits are more above crystal 22 when the beam moves above the center (left panel of figure 4). Similarly, the number of hits are more below crystal 22 when the beam moves below the center (right panel of figure 4) which is obvious.

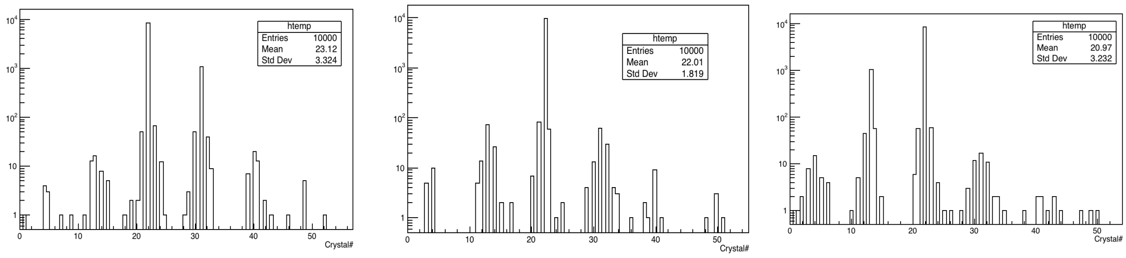


Figure 4. The positron distribution (in log scale) through a 9×6 PbF_2 segmented crystals of a calorimeter with exact same conditions as 3.

3. Energy spectrum of each crystal

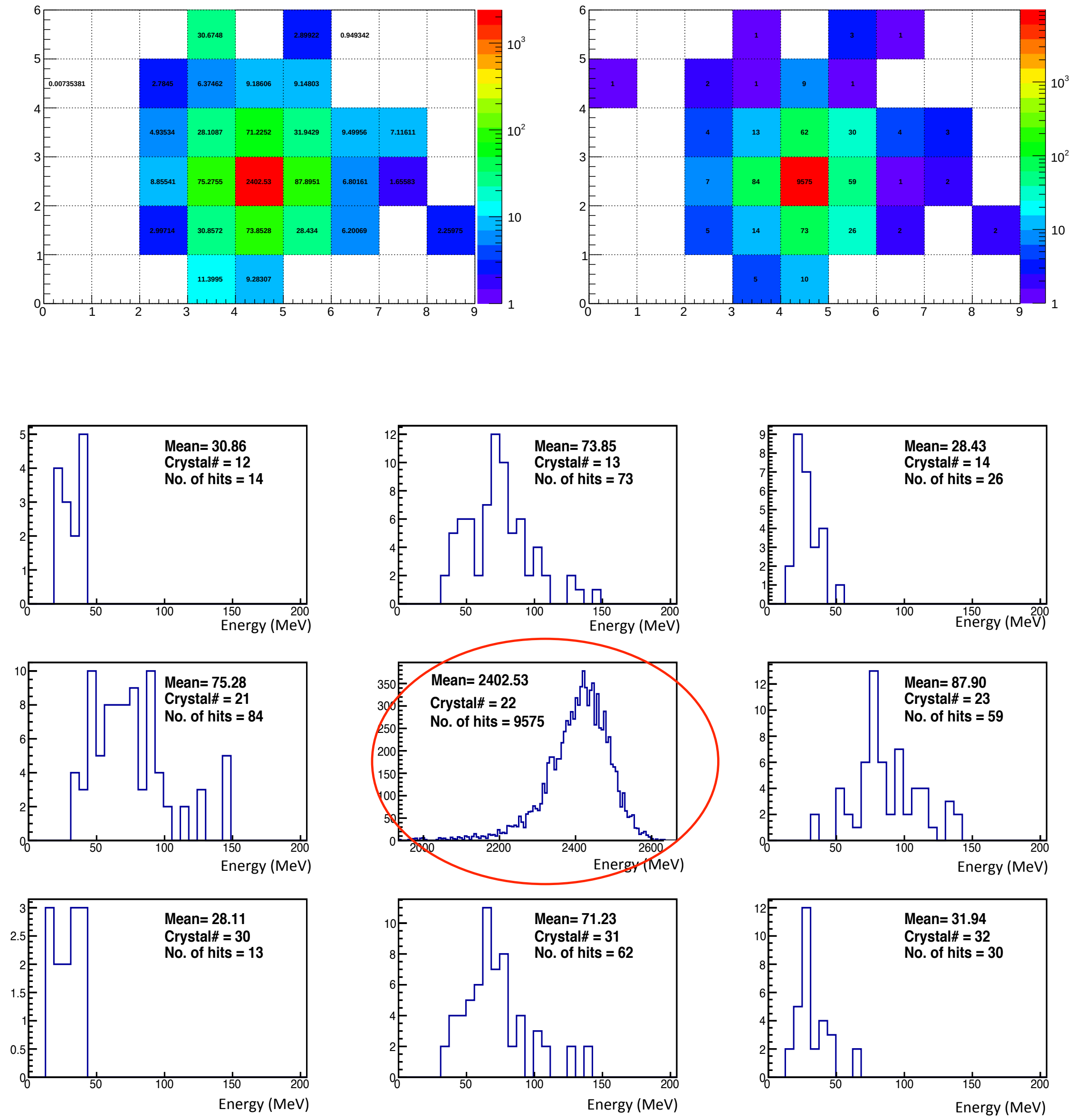


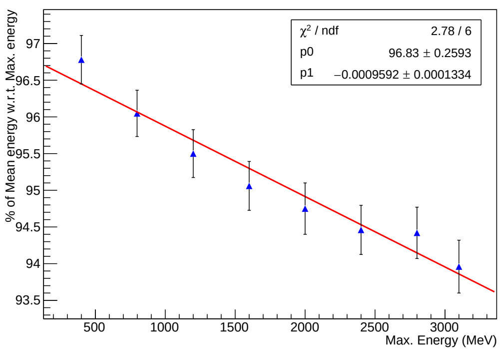
Figure 5. Top left: Distribution of mean positron energy for each crystal when the positron beam is incident on the center of crystal 22. Top right: Same for occupancy of each crystal. Bottom: Crystals around the crystal hit (22) showing the actual total energy distribution in these crystals.

A study of the energy distribution on each crystal for 3.1 GeV positron beam incident normally on crystal 22 is done in this section. It is obvious that the maximum energy will be deposited in crystal 22 and then decrease as we move away from this crystal. This is clearly seen in figure 5. The left panel of this figure shows the energy distribution of each crystal. The number on the crystals here is the mean energy deposited on the crystal. The percentage of energy deposited in the central crystal 22 i.e. 2402.53 MeV compared to the total energy is 3100 MeV is 77.5% which matches the value 77.46% of the simulations of TRD (figure 17.3 left). The same holds true for all crystals in the calorimeter. The right panel in this figure shows the same for number distribution in each crystal.

The lower panel shows the detailed energy spectrum of each crystal i.e. crystal 22 and the crystals surrounding it. The rest of the crystals with really low counts are ignored. Note, the fraction of energy deposited in the individual crystals shown in the top left of figure 5 matches the values of the simulations of TDR (figure 17.3 left) [3].

4. Variation of incident energy of positron.

The total energy of all particles of incident energy of 3.1 GeV is shown in figure 2. We varied this energy from 400 MeV to 3100 MeV (in steps of 200 MeV) and investigated the effect. The fraction of the mean energy i.e. 2913 MeV wrt the incident energy of 3.1 GeV positrons is 0.9396 or 93.96%. This fraction in % is plotted for incident energies started from 400 MeV is shown in the left plot figure 6.



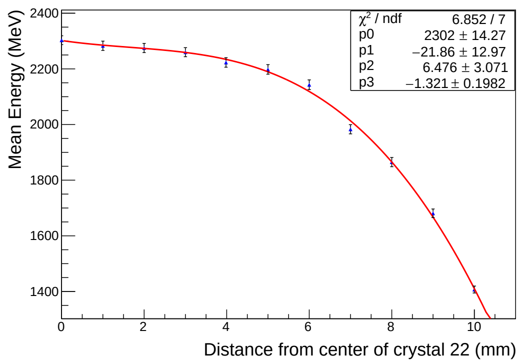


Figure 6. Left: A plot of % of mean energy compared to the total energy of positrons with the total energy varying in steps of 400 MeV, through the 9×6 segmented PbF2 crystals of a calorimeter (depicts leakage of positron shower). Right: Plot of mean energy of the positron as a function of beam position.

The right plot in figure 6 shows the mean energy of all the particles as a function beam position. This plot again was done using a simulation with GEANT4. These results match the simulation for the shower leakage into the neighbouring crystals as a function of beam position shown in the TDR (figure 17.3 right)[3].

References

1. A.T. Fienberg *Measuring the Precession Frequency in the E989 Muon g − 2 Experiment*. PhD thesis, University of Washington, Seatle, 2019.
2. A.A. Savchenko et al. *Geant4 simulations of the lead fluoride calorimeter. Nucl.Instrum.Meth. B402 (2017) 256-262*
3. J. Grange et al. *Muon (g-2) Technical Design Report, 2015.*