A proposal for the g-2 calorimeter gain calibration

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1 Introduction

The muon g-2 experiment at Fermilab aims to improve by at least a factor four the previously available precision on the measurement of the muon anomalous magnetic moment. This requirement poses extremely tight constraints on the maximum acceptable systematic error. One of the main sources of systematics is the gain stability of the calorimeters, which detect the positrons from muon decays and measure their energy. Indeed, since the measurement is based on fitting the positron rate variation as a function of time for positrons above a given energy threshold, any gain change can vary this threshold and affect the final result.

In this documents we describe:

Section 1: models of gain variation due to the high rate of positrons during a muon fill, and how gain variations affect the measurement of the muon spin precession frequency;

Section 2: the calibration system and its characteristic performance as measured in several tests with electron beams;

Section 3: our proposed calibration procedures in different moments of the detector operation;

Section 4: hardware configurations used to operate the above procedures;

Section 5: software implementations to operate the calibration procedures and to use the calibration results during data reconstruction and analysis.

2 Goals of the calibration system

The systematic error budget on the fit of ω_a due to gain variation has been quantified at 20 ppb in the g-2 TDR [1]. This is a very ambitious goal as it is six times smaller than in the previous BNL g-2 experiment [2]. In the following we study how gain variations can affect the measurement and show that variations much slower than the typical time of a muon fill, $\sim$700 μs, have no effect on ω_a, but modify the other fit parameters thus making the interpretation of the results harder.

Faster gain variations must be corrected. To study how precisely this correction needs to be understood and to guide us in the calibration process we make a model of the gain variation based on previous studies of the SiPM response [3]. This model is then calibrated on test data to derive the needed calibration accuracy.

2.1 Effect of gain variations on ω_a fit

We recall that the muon precession frequency about its momentum vector, ω_a, is obtained by fitting a finely binned histogram of the detection time of positrons above a given energy threshold relative to the start of the fill. In order to reduce ring variations effects the bin are usually taken as wide as the time it takes for the muons to travel around the ring, about 140 nsec. In the following this histogram, which is obtained by adding data from a large number of muon fills, will be called "wiggle histogram" and the function describing it simply the "wiggle function", $W(\bar{E}, t)$, where we have emphasized in the notation the dependence on the threshold energy $\bar{E}$. We also define the energy derivative of W: $w(E, t) = \partial W/\partial E$.

In appendix A we derive the functional form of the functions W and w starting from first principles and then applying detector acceptance and resolution effects. We note that both functions are a linear combination of three functions of time $exp(-t/\tau_\mu)$, $exp(-t/\tau_\mu)cos(\omega_a t)$ and $exp(-t/\tau_\mu)sin(\omega_a t)$ with energy dependent coefficients, therefore any linear combination of W and w will be a linear combination of those three time functions with time independent coefficients. We will say that any such combination has the same time structure. An important conclusion of appendix A is the effect of gain variations on the wiggle function, W. If we let W^* be the modified wiggle function due to gain variations Δg_i in fill i, we have shown that:

$$W^*(t) = W(\bar{E}, t) + w(\bar{E}, t) \sum_{i=1}^{N_{fill}} f_i \, \bar{E} \Delta g_i \tag{1}$$

where f_i is the fraction of events contained in fill i and W, w are the unperturbed wiggle functions.

This result has two relevant implications:

a. if Δg_i can be considered constant during the time scale of a fill, then the modified wiggle has the same time structure of the unperturbed one. This implies that if we fit for ω_a, using the unperturbed wiggle form leaving as free parameters the energy dependent coefficients, we will still obtain the correct ω_a, but the coefficient will be shifted by an amount proportional to the average energy shift.

b. if Δg_i is instead varying within the fill, it introduces an additional time dependence that will in general affect also the measurement of ω_a. However eqs. (1) tells us also that what matters to correct for this effect is not the knowledge of the individual Δg_i time functions in every fill, but just their average over all fills used for our analysis. This has important implications for the calibration procedure.

The conclusion of point b. above tell us that we could correct for the gain variation if we could determine the average gain variation function. One option is the direct measurement of this function with the calibration system during a muon fill, while another is to use calibration system to measure this function when no muons are present in the ring. A comparison of the two approaches can give us a very good control on the quality of the calibration process, however we need to develop a model of the gain variation function. This model is described in detail in appendix B and C.

It is shown that if the recovery function for a single pulse is given by the weighted sum of exponentials and is proportional to the energy, E, of the pulse as shown in the following equation:

$$G(t) = 1 - \alpha E \theta(t - t_i) \sum_{k=1}^{p} f_k e^{-(t-t_i)/\tau_k} \tag{2}$$

(where the weights f_k indicate the relative strength of a given component ($\sum f_k = 1$) and θ is the step function) then the average gain function over a muon fill is given by:

$$< G(t) >= 1 - n_0 \alpha \sum_{k=1}^{p} f_k(\tilde{N} S_{N_k}(t) + \tilde{a} S_{a_k}(t) + \tilde{b} S_{b_k}(t)) \tag{3}$$

where the S functions of time are reported in appendix C and the parameters with a tilde are related to the wiggle energy dependent parameters and can in principle be determined from a detailed simulation of the energy distribution in the calorimeter cells. If we trust this model using the calibration system we could determine the single pulse response function with a scan of the time difference between two pulses, the tilde parameters with simulation and data, and n_0, the average number of pulses per fill, directly from the data.

A direct emulation of the fill can be done using the flight simulator embedded in the laser calibration system. In this case laser pulses of constant energy are flashed on the calorimeter cells with a falling exponential distribution with the same time constant of the muons in the ring. Since the oscillating terms are missing and the pulse energy is constant eqa. (3) simplifies considerably. Assuming only one recovery exponential with time constant τ_r:

$$< G(t) >= 1 - n_0 \alpha E \frac{\tau_\mu \tau_r}{\tau_\mu - \tau_r} \left(e^{-t/\tau_\mu} - e^{-t/\tau_r} \right) \tag{4}$$

A fit of the average of several flight simulation runs allows the independent measurement of τ_r and the combination $n_0 \alpha E \tau_\mu \tau_r / (\tau_\mu - \tau_r)$; since n_0 is known repeating the procedure with different values of the laser intensity E allows the determination of α. An example of a fit with this procedure is shown in fig. 1. Typical recovery times are found to be in the order of 10-20 μs depending on the specific features of SiPM powering hardware configurations, while α is in the range of a few $\times\ 10^{-5}$ GeV^{-1}.

3 Calibration system characterization

Brief system description

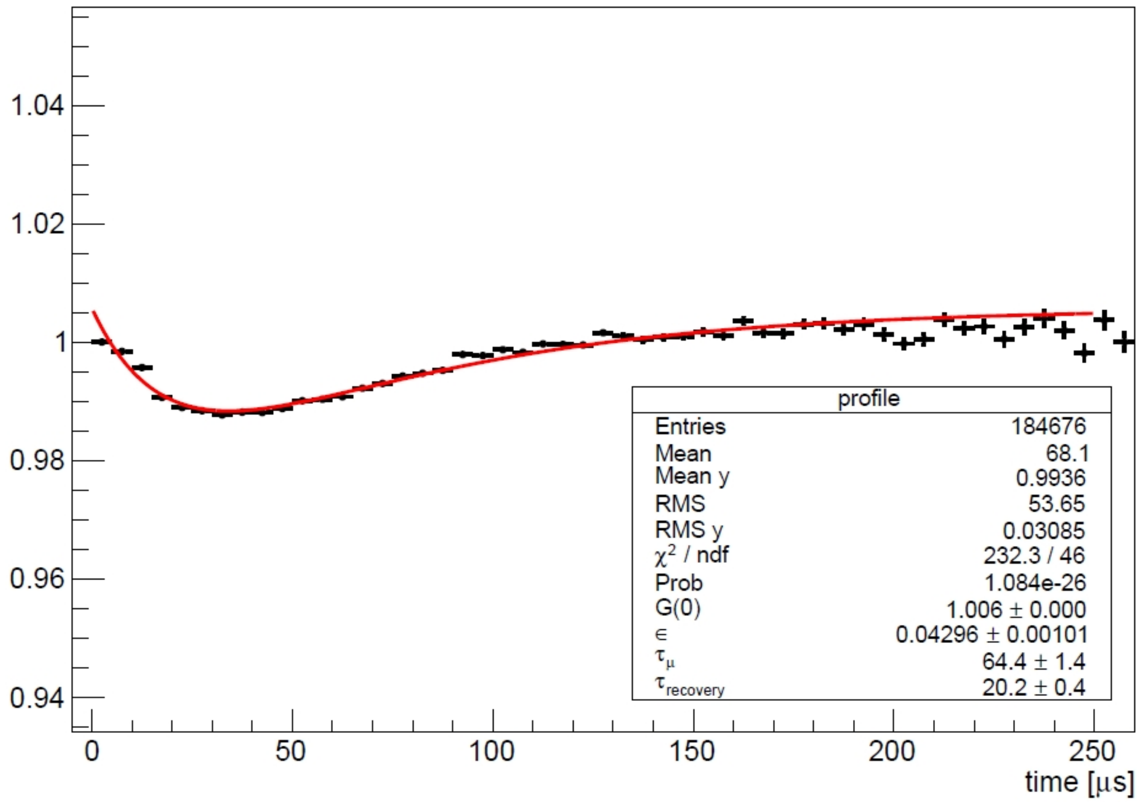

Figure 1: Gain function fit from flight simulator runs.

3.1 Performance benchmarks

4 Calibration proposal

4.1 Calibration with muons in ring

4.2 Calibration during data taking

5 Hardware implementation

5.1 HW configuration for double pulse time scan

5.2 HW configuration for flight simulator/pulse trains

5.3 HW configuration for calibration and monitor cycles

6 Software implementation

6.1 Data streams and data formats

6.2 Pre-processing for event reconstruction

6.3 Processing for ω_a fit

Appendices

A Detector and gain variation effects on wiggle shape

We study the effect of energy resolution and potential miscalibrations on the basic wiggle equation. Such equation can be determined from first principles to be in the form:

$$\frac{\partial^2 P}{\partial e \partial t} = N e^{-\frac{t}{\tau}} (1 + A\, cos(\omega t + \varphi)) \tag{5}$$

where $N,\ A$ and φ are known functions of the true energy e. An alternative formulation that is linear with respect to the parameters which depend on energy is:

$$\frac{\partial^2 P}{\partial e \partial t} = e^{-\frac{t}{\tau}} (N + a\, cos\, \omega t + b\, sin\, \omega t) \tag{6}$$

where $a = NA\, cos\, \varphi$ and $b = NA\, sin\, \varphi$. Given a, b the parameters A and φ are given by: $A = \sqrt{a^2 + b^2}/N$ and $\varphi = Atan2(b/(NA),\, a/(NA))$, where $-\pi < \varphi < \pi$.

The energy distribution integrated over time is given by:

$$\frac{dP}{de} = \int_0^\infty \frac{\partial^2 P}{\partial e \partial t}\, dt = \tau \left(N + \frac{a + b\omega\tau}{1 + \omega^2 \tau^2} \right) \tag{7}$$

A.1 Detector effects: acceptance and resolution

The inclusion of energy acceptance effects is described by a function $\epsilon(e)$ which multiplies the wiggle equation. In its form (6) the effect on the wiggle equation is just a redefinition of the energy dependent parameters N, a, , b with ϵN, ϵa , ϵb. The normalization is also affected and this should be taken into account.

We then assume a gaussian resolution function with energy dependent standard variation $\sigma(E) = \alpha E^2 + \beta\sqrt{E} + \gamma$. Given the linearity of eqs. (6) with respect to the energy dependent parameters, the effect of the resolution function is just a convolution of the original parameters with the resolution function, e. g.:

$$N_1 = \int_0^\infty \epsilon(e) N(e)\, e^{-(e-E)^2/(2\sigma(E)^2)} / (\sigma(E)\sqrt{2\pi})\, de \tag{8}$$

similarly one can get a_1 and b_1.

We note that this procedure can be generalized to any resolution function and that the energy resolution does not affect in any way the structure of the wiggle equation that becomes:

$$\frac{\partial^2 P}{\partial e \partial t} = w(E, t) = e^{-\frac{t}{\tau}} (N_1 + a_1\, cos\, \omega t + b_1\, sin\, \omega t) \tag{9}$$

The energy dependence of the wiggle parameters as obtained by this procedure are shown in fig. 2.

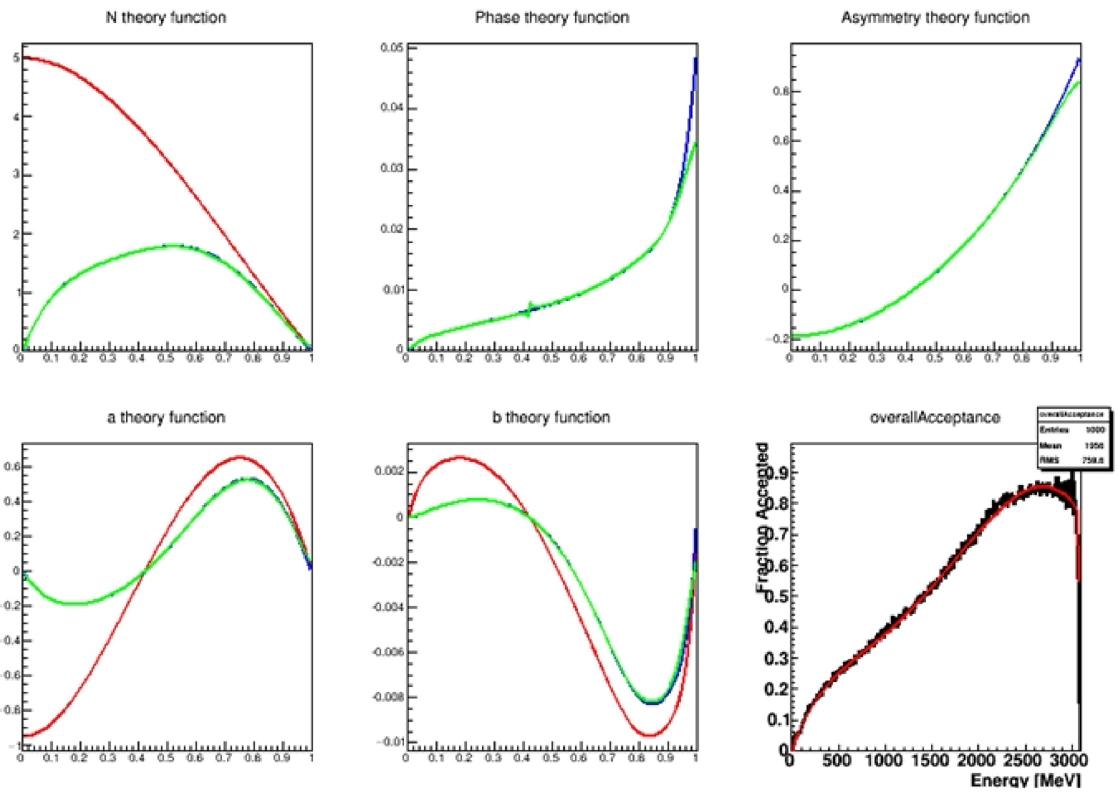

Figure 2: Wiggle parameter dependence on energy: theory (red), acceptance correccted (blue) and energy resolution corrected (green). In the right bottom corner we show the acceptance function.

A.2 Fitting equation

In the end we want to fit a time distribution so we integrate eqs. (5) or (6) with respect to the measured energy E from a given threshold $\bar{E}$. Also in this case the structure of the wiggle equation is unchanged; we only have to integrate the parameters N_1, a_1, b_1 as for instance:

$$N_2 = \int_{\bar{E}}^{\infty} N_1(E)\, dE \tag{10}$$

and similarly to obtain a_2 and b_2. In conclusion the fitting equation is given by:

$$\frac{dP}{dt} = W(\bar{E}, t) = e^{-\frac{t}{\tau}}(N_2 + a_2\, cos\, \omega t + b_2\, sin\, \omega t) \tag{11}$$

where the parameters N_2, a_2, b_2 can be extracted by the fit and compared to those obtained from first principles and the prescription given in the previous sections. We also note that they are functions of the cutoff energy $\bar{E}$. For small variations of the cutoff energy, $\Delta\bar{E}$, considering their definition we have:

$$\Delta N_2 = -N_1(\bar{E})\, \Delta\bar{E} \tag{12}$$

A.3 Effect of calibration errors

We consider two types of calibration effects: slow variations, which are defined as those which are not noticeable on the time scale of a fill, and fast variations, which affect every fill. We discuss them separately in the following.

A.4 Slow variations

Since only fill to fill variations are relevant in this case we define the actual constant relative gain in fill i as g_i. Assuming the nominal gain to be 1, the measured energy is E^* corresponding to a true energy of $E = E^*/g_i \sim E^*(1 - \Delta g_i)$. This corresponds to an error on the cutoff energy used for fill i: $\Delta\bar{E}_i = (\Delta g_i/g_i)\bar{E} \sim -\bar{E}\Delta g_i$.

The final fitted function is given by the average over all fills used by the fit so the fitted functions is:

$$W^*(t) = W(\bar{E}, t) + w(\bar{E}, t) \sum_{i=1}^{N_{fill}} f_i\, \bar{E}\Delta g_i \tag{13}$$

where f_i is the fraction of events contained in fill i.

As expected slow variations therefore do not affect the determination of the frequency ω as both functions W and w have the same structure for ω, however the interpretation of the coefficients is now biased by a quantity that is proportional to the mean variation of the cutoff energy: $< \Delta\bar{E} >= - < \Delta g > \bar{E}$.

A.5 Fast variations

In case of fast variations the quantity Δg_i is a function of time so it can affect the structure of the ω dependent terms in the wiggle equation. We note however that what really matters here is the average variation over all fills: $< \Delta g >= \epsilon\, h(t)$. This function can be determined by the calibration system because we do not need the individual $\Delta\, g_i$, but only the average over all fills. This requires however constant pulsing of the calibration during fills or at least for a statistically significant fraction of them. Alternatively if we assume that the gain variations during fills have always the same shape and are controlled by just one intensity parameter ϵ_i, i.e. $\Delta g_i = \epsilon_i\, h(t)$ we can determine $h(t)$ with dedicated calibration runs and then include an extra parameter in the wiggle fit to account for the unknown mean value of ϵ_i. We note however that if every crystal responds in a different way the number of additional fit parameters can easily become quite large.

B Average gain variations

In this appendix we derive the dependence of the of the average gain variation over many fills to the gain variation due to a single pulse.

B.1 Normalized wiggle

As we shall see the energy dependence gets integrated out so energy smearing effects become irrelevant. We can therefore use the wiggle representation (6) multiplied by the acceptance function:

$$\frac{\partial^2 P}{\partial e \partial t} = \frac{1}{N_0} \epsilon(e)\, e^{-\frac{t}{\tau}} (N(e) + a(e)\, cos\, \omega t + b(e)\, sin\, \omega t) = w(e,t) \tag{14}$$

where the normalization N_0 is given by:

$$N_0 = \tau \left(\bar{N} + \frac{\bar{a} + \bar{b}\omega\tau}{1 + \omega^2\tau^2} \right) \tag{15}$$

where the bar over a parameter indicates it's integral multiplied by the acceptance function:

$$\bar{x} = \int_0^\infty \epsilon(e) x(e)\, de \tag{16}$$

These formulas define clearly the normalized pdf of time and energy of every positron entering the detector. We will use this pdf later to average the individual gain functions.

B.2 Single pulse gain function

Given a pulse of energy E at time t_i, we describe the resulting gain variation with a sum of p exponentials of different time constants τ_k:

$$G(t) = 1 - \alpha e \theta(t - t_i) \sum_{k=1}^{p} f_k e^{-(t-t_i)/\tau_k} \tag{17}$$

where the weights f_k indicate the relative strength of a given component ($\sum f_k = 1$) and θ is the step function.

In the case of a sequence of many pulses the gain function can be generalized as follows:

$$G(t) = 1 - \alpha \sum_{i=1}^{n_0} e_i \sum_{k=1}^{p} f_k \theta(t - t_i) e^{-(t-t_i)/\tau_k} \tag{18}$$

where n_0 is the total number of pulses.

B.3 Average gain function determination

The average gain function after many fills will the same number of pulses is obtained by averaging the gain for a single fill as described in eq. (18) over the energy and time distribution of the single pulses as described by the normalized wiggle equation (14). After eliminating the step functions we obtain:

$$< G(t) >= 1 - n_0\alpha \sum_{k=1}^{p} f_k \int_0^\infty de \int_0^t ew(e,t')e^{-(t-t')/\tau_k}dt' \tag{19}$$

This expression can be determined in a semi-analytic way by integrating numerically the energy dependent terms and analytically the time dependent terms. Here is how it goes: the double integrals in time and energy that appear in eq. (19) decouple, given the structure of $w(e,t)$, as products of energy and time integrals. For every function of the energy, $x(e)$, we define the energy integral weighted with the energy and acceptance as:

$$\tilde{x} = \int_0^\infty e\epsilon(e)x(e)\,de \tag{20}$$

In the case of the wiggle quantities N, a, b these integrals can be obtained numerically. For the time dependent part we define the following quantities whose analytic form is shown in the Appendix C:

$$S_{N_k}(t) = \int_0^t e^{-(t-t')/\tau_k}e^{-t'/\tau_\mu}dt' \tag{21}$$

$$S_{a_k}(t) = \int_0^t e^{-(t-t')/\tau_k}e^{-t'/\tau_\mu}cos\omega t' dt' \tag{22}$$

$$S_{b_k}(t) = \int_0^t e^{-(t-t')/\tau_k}e^{-t'/\tau_\mu}sin\omega t' dt' \tag{23}$$

With these definitions the average gain function is given by:

$$< G(t) >= 1 - n_0\alpha \sum_{k=1}^{p} f_k(\tilde{N}S_{N_k}(t) + \tilde{a}S_{a_k}(t) + \tilde{b}S_{b_k}(t)) \tag{24}$$

Values obtained with the presently available information are:

$$\begin{aligned} \tilde{N} &= \quad 0.420484 \\ \tilde{a} &= \quad 0.0457639 \\ \tilde{b} &= -0.000865762 \end{aligned} \tag{25}$$

indicating that the smooth, non oscillating term, is dominant and that the phase contribution is minimal. Examples of gain functions are shown below in fig. 3.

We note that in practice one can extract the single pulse response function pulsing the calorimeters with two separate lasers and sweeping the time difference between the pulses and energy dependence. This should provide for each calorimeter cell: α, f_k, τ_k. The average number of pulses, n_0, can be obtained directly from the data, while the energy distributions and the acceptances can be obtained with the simulation and verified on the data. So in principle we should have a model of the fast gain variations that could be compared to that obtained with the calibration system. This model also

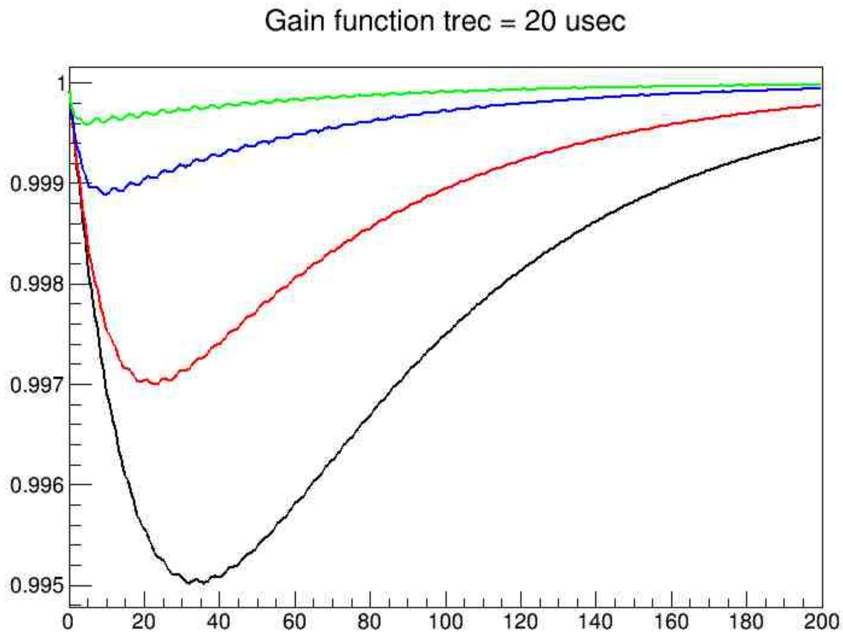

Figure 3: Example gain functions with single exxponential for 1, 3, 10, 20 μsec recovery time.

suggests a parameterization of a fit function that we could choose to use in the analysis of the data with additional correction parameters, unfortunately several assumptions on the uniformity of these parameters across the calorimeter are needed to reduce the number of parameters to a manageable number.

C Analitic integrals

$$S_{N_k}(t) = \int_0^t e^{-(t-t')/\tau_k} e^{-t'/\tau_\mu} dt' = \frac{\tau_\mu \tau_k}{\tau_\mu - \tau_k} \left(e^{-t/\tau_\mu} - e^{-t/\tau_k} \right) \tag{26}$$

$$S_{a_k}(t) = \int_0^t e^{-(t-t')/\tau_k} e^{-t'/\tau_\mu} cos\, \omega t' dt' = \frac{\tau_\mu \tau_k}{(1+\omega^2\tau_\mu^2)\tau_k^2 - 2\tau_\mu\tau_k + \tau_\mu^2)} \left[(\tau_\mu - \tau_k) \left(e^{-t/\tau_\mu} cos\, \omega t - e^{-t/\tau_k} \right) + \tau_k \omega \tau_\mu e^{-t/\tau_\mu} sin\, \omega t \right] \tag{27}$$

$$S_{b_k}(t) = \int_0^t e^{-(t-t')/\tau_k} e^{-t'/\tau_\mu} sin\, \omega t' dt' = \frac{\tau_\mu \tau_k}{(1+\omega^2\tau_\mu^2)\tau_k^2 - 2\tau_\mu\tau_k + \tau_\mu^2)} \left[(\tau_\mu - \tau_k) e^{-t/\tau_\mu} sin\, \omega t - \tau_k \tau_\mu \omega \left(e^{-t/\tau_\mu} cos\, \omega t - e^{-t/\tau_k} \right) \right] \tag{28}$$

It is interesting to note that when $\tau_\mu >> \tau_k$ all contributions scale roughly with τ_k.

References

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