B_2 Integrals from Urqmd and comparison to sum rule.

- This presentation shows Urqmd test of balance function Integration, comparing to the 'sum rule'.
- The code tested here is also used to analyze data.
- Since this simulation has no tracking inefficiency, centrality is exactly known, no particle misidentification, no track crossing effects etc. these clinical comparisons lead to very precise tests.

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RHIG Group Meeting
01/24/2022

Balance Function ($B_{2\pm}$) Definition and Integral of $B_{2\pm}$

We calculate differential $B_2(dy,d\phi)$ using prefactors and R_2 and then integrate the resulting histogram

Balance Function ($B_{2\pm}$) Definition

BF of positive charged particle 1 (a^+):

$$\mathbf{B_2^{a^+b}(\Delta y, \Delta\varphi)} = N^{b^-} \cdot R_2^{a^+b^-}(\Delta y, \Delta\varphi) - N^{b^+} \cdot R_2^{a^+b^+}(\Delta y, \Delta\varphi)$$

BF of negative charged particle 1 (a^-):

$$\mathbf{B_2^{a^-b}(\Delta y, \Delta\varphi)} = N^{b^+} \cdot R_2^{a^-b^+}(\Delta y, \Delta\varphi) - N^{b^-} \cdot R_2^{a^-b^-}(\Delta y, \Delta\varphi)$$

$$\boxed{\begin{aligned}\Delta y &= y_1 - y_2 \\ \Delta\varphi &= \varphi_1 - \varphi_2\end{aligned}}$$

a,b – two particle species

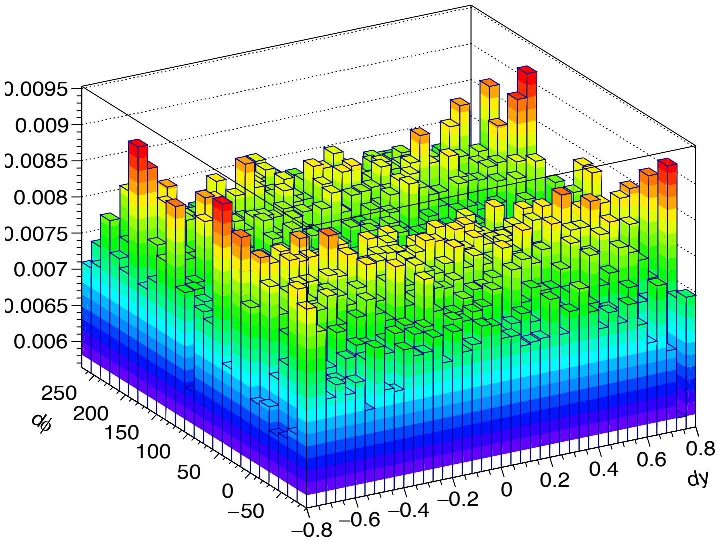
2-particle correlation function:

$$\boxed{R_2^{ab}(\Delta y, \Delta\varphi) = \frac{\rho_2^{ab}(\Delta y, \Delta\varphi)}{\rho_1^{a} . \rho_1^{b}(\Delta y, \Delta\varphi)} - 1}$$

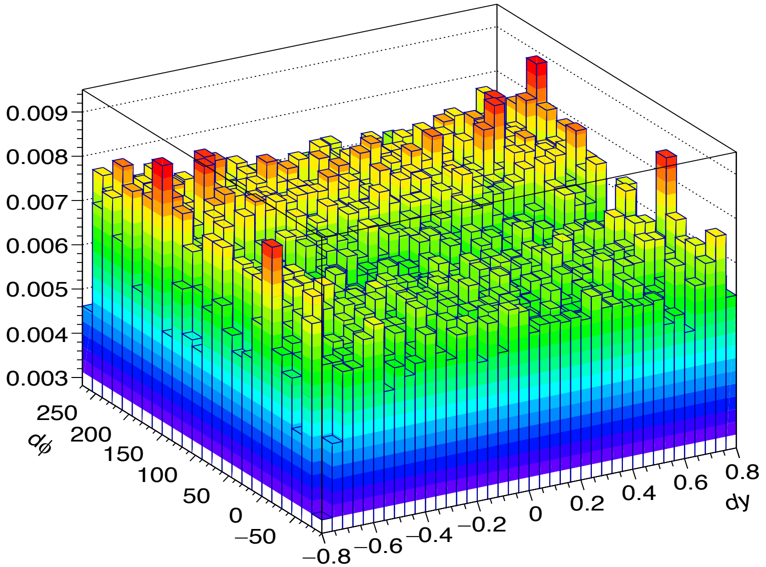
Integral of $B_{2\pm}$

Integral of $B_{2\pm}$ evaluated as sum of bin contents divided by the number of bins $B_2(dy,d\phi)$ distribution.

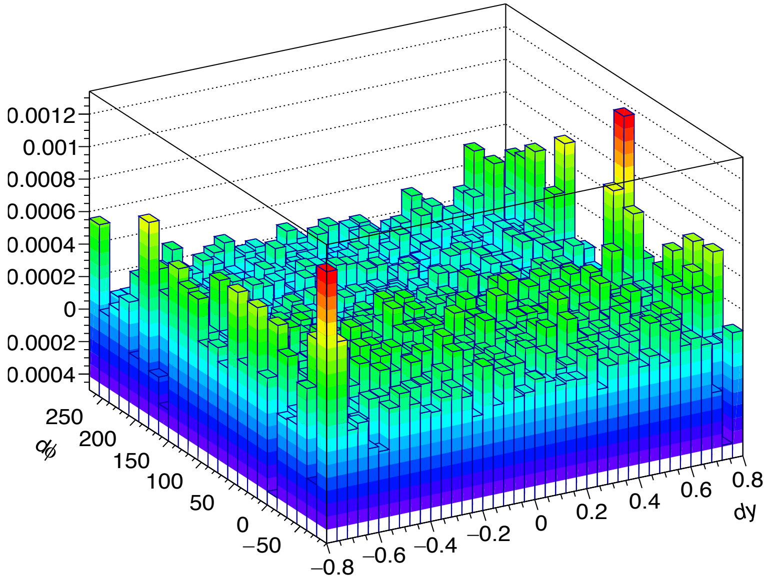
π+π-, R_2 vs. (dy,dϕ), 0-5%



π+π+, R_2 vs. (dy,dϕ), 0-5%



$\pi\pi$ B_2+ vs. (dy,dϕ), 0-5%



Au Au collisions at 200 GeV for 0-5% centrality using urqmd

Integral of $B_{2\pm}$ from sum rule

We also calculate the balance function integral using only the multiplicity distribution of the event ensemble.

From the definition of B_2+, we have

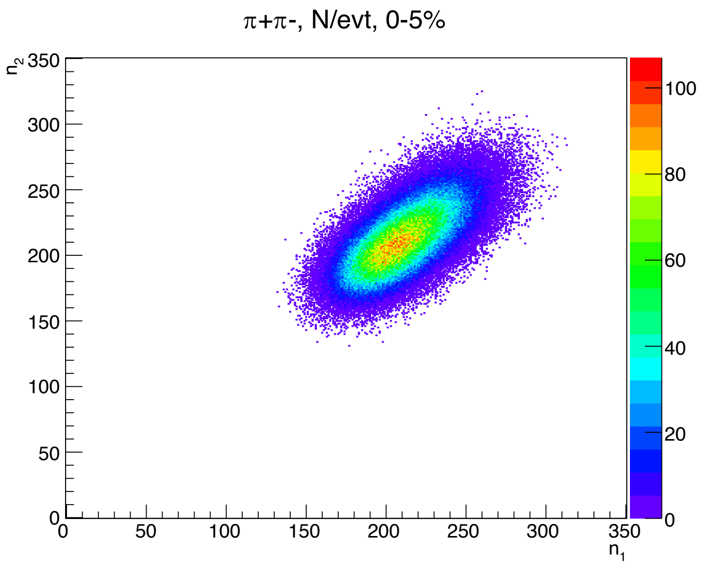
$$\mathbf{B_2^{a^+b}(\Delta y, \Delta\varphi) = N^{b^-} \cdot R_2^{a^+b^-}(\Delta y, \Delta\varphi) - N^{b^+} \cdot R_2^{a^+b^+}(\Delta y, \Delta\varphi)}$$

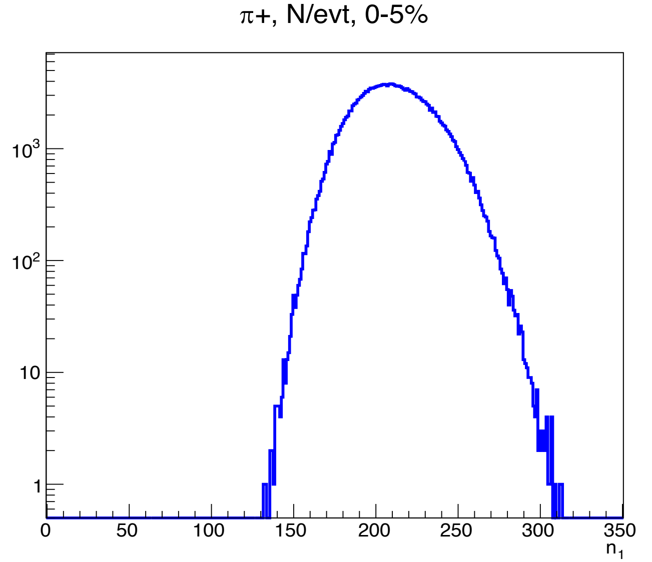
Assuming the integral of sum (or difference) = the sum (or difference) of integral , we have

$$S_+ = \int B_2^{\pi^+\pi} d\Omega = \int N^{\pi^-} . R_2^{\pi^+\pi^-} d\Omega \; - \int N^{\pi^+} . R_2^{\pi^+\pi^+} d\Omega$$

$$\int R_2 \, d\Omega = \frac{\int [\rho_2 - \rho_1\rho_1] d\Omega}{\int \rho_1\rho_1 d\Omega} \qquad \int \rho_2 \, d\Omega = \begin{cases} \langle n(n-1) \rangle & \text{Identical part.} \\ \langle n_{\pi^+} \, n_{\pi^-} \rangle & \text{Distinguishable part.} \end{cases}$$

$$\int \rho_1 \, d\Omega = \langle n_{\pi^+} \rangle$$





$$C_{+-} = \langle n_+ n_- \rangle - \langle n_+ \rangle \langle n_- \rangle \qquad C_{-+} = \langle n_- n_+ \rangle - \langle n_- \rangle \langle n_+ \rangle$$

$$C_{++} = \langle n_+(n_+ - 1) \rangle - \langle n_+ \rangle^2 \qquad C_{--} = \langle n_-(n_- - 1) \rangle - \langle n_- \rangle^2$$

B_2 +

$$\int S_+ = \frac{1}{n_+} [C_{+-} - C_{++}]$$

B_2 -

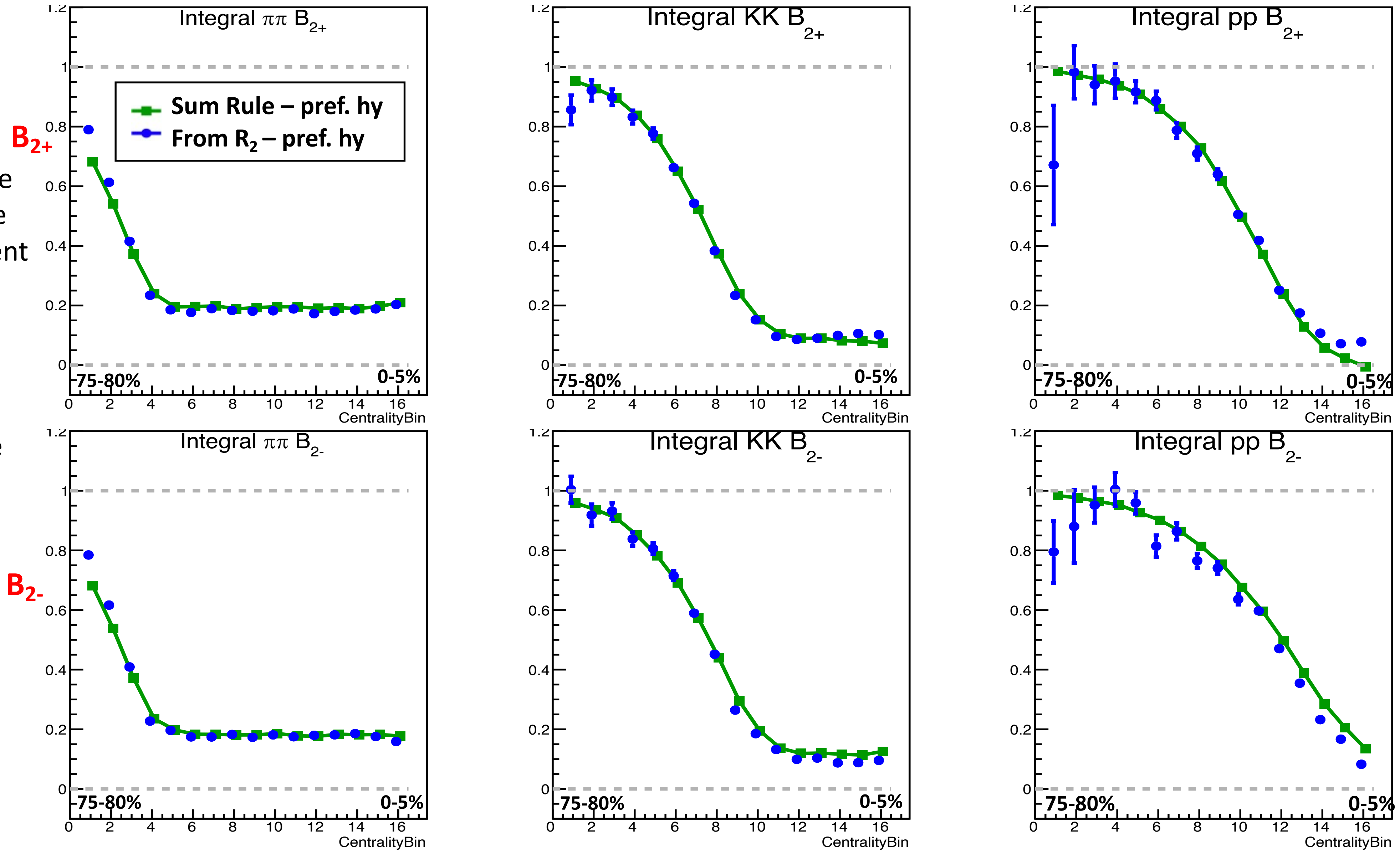
$$\int S_- = \frac{1}{n_-} [C_{-+} - C_{--}]$$

Detailed steps in the back-up.

Comparing B_2 Integrals from simulations with Sum Rule – 200 GeV

The urqmd test indicates that the analysis code produces the same BF integral from two very different approaches

- it shows that we are integrating our 2D BFs built from R_2s correctly
- we are using the same code for the STAR data



Summary

- Urqmd test with balance function integration compares very well to the ‘Sum Rule’.
- This precisely tests that we are integrating our balance functions correctly.

- **In progress:** We are working on applying this for STAR data.

Back-Up

Bill's Derivation:

B_2 is some function of R_2's which are a function of $\rho_1\rho_1$'s & ρ_2's …

$$\int_{all} \rho_{1a}\, d\Omega = \langle n_a \rangle$$

$$\int_{all} \rho_{1b}\, d\Omega = \langle n_b \rangle$$

$\langle\ \rangle$ evt. ensemble average

$$\int_{all} \rho_2\, d\Omega = \begin{cases} \langle n(n-1) \rangle & \text{identical particle pairs} \\ \langle n_a n_b \rangle & \text{distinguishable particle pairs} \end{cases}$$

from correlations class — from mult. dist

units: # pairs/bin/event (counts) (poisson uncertainties)

ρ_1: avg # of single particles per bin per event

ρ_2: actual avg # of pairs per bin per event

$\rho_1\rho_1$: avg # of pairs per bin in the "reference event" [same acceptance & efficiency, but no correlations]

from these, form C_2 & R_2

(capital-C) here $C_2 = C_2(\Delta y, \Delta\phi)$ (a density)

$C_2 = \rho_2 - \rho_1\rho_1$ "correlator" (or "correlation")

$R_2 = \frac{C_2}{\rho_1\rho_1} = \frac{\rho_2}{\rho_1\rho_1} - 1$ "normalized correlation" (correlation per pair)

- BF of positive charged particle 1 (π^+):

$B_2^{\pi^+\pi}(\Delta y, \Delta\varphi) = N^{\pi^-} \cdot R_2^{\pi^+\pi^-}(\Delta y, \Delta\varphi) - N^{\pi^+} \cdot R_2^{\pi^+\pi^+}(\Delta y, \Delta\varphi)$

- BF of negative charged particle 1 (π^-):

$B_2^{\pi^-\pi}(\Delta y, \Delta\varphi) = N^{\pi^+} \cdot R_2^{\pi^-\pi^+}(\Delta y, \Delta\varphi) - N^{\pi^-} \cdot R_2^{\pi^-\pi^-}(\Delta y, \Delta\varphi)$

$$B_2^{a^+b} = n_{b^-} R_2^{a^+b^-} - n_{b^+} R_2^{a^+b^+}$$
$$B_2^{\pi^+\pi} = N_{\pi^-} R_2^{+-} - N_{\pi^+} R_2^{++}$$

$$\int B_2^{\pi^+\pi} = n_-\left[\frac{\langle n_+ n_- \rangle}{\langle n_+\rangle\langle n_-\rangle} - 1\right] - n_+\left[\frac{\langle n_+(n_+-1)\rangle}{\langle n_+\rangle^2} - 1\right]$$
$$= \frac{\langle n_+ n_- \rangle}{\langle n_+ \rangle} - n_- - \frac{\langle n_+(n_+-1)\rangle}{\langle n_+ \rangle} + n_+$$

⊗ here we assumed $\int R_2\, d\Omega = \frac{\int [\rho_2 - \rho_1\rho_1]\, d\Omega}{\int [\rho_1\rho_1]\, d\Omega}$!?

$$\boxed{\int B_2^{\pi^+\pi} = (n_+ - n_-) + \frac{1}{n_+}\left[\langle n_+ n_- \rangle - \langle n_+(n_+-1)\rangle\right]}$$

integral of $B_2^{\pi^+\pi}$ — diff avg. chgs (>0: $n_+ > n_-$; <0: $n_- > n_+$) — actual # of ULS pairs — actual # of LS+ pairs

if no correlations: $C_2 = 0$ (i.e. $C_{+-} = C_{++} = 0$)

$\langle n_+ n_- \rangle = \langle n_+ \rangle \langle n_- \rangle = n_+ n_-$

$\langle n_+(n_+-1) \rangle = \langle n_+ \rangle^2 = n_+^2$

$$\int B_2^{\pi^+\pi}\Big|_{C_2=0} = (n_+ - n_-) + \frac{n_+ n_-}{n_+} - \frac{n_+^2}{n_+}$$

$$\int B_2^{\pi^+\pi}\Big|_{C_2=0} = n_+ - n_- + n_- - n_+$$

$$\Rightarrow B_2^{\pi^+\pi}\Big|_{C_2=0} = 0 \ \checkmark$$

$$\int B_2^{\pi^+\pi} = \left[n_+ + \frac{\langle n_+ n_- \rangle}{n_+}\right] - \left[n_- + \frac{\langle n_+(n_+-1)\rangle}{n_+}\right]$$

correlator integral

$C_{+-} = \langle n_+ n_- \rangle - \langle n_+ \rangle \langle n_- \rangle$ — $\langle n_+ n_- \rangle = C_{+-} + \langle n_+ \rangle \langle n_- \rangle$

$C_{++} = \langle n_+(n_+-1) \rangle - \langle n_+ \rangle^2$ — $\langle n_+(n_+-1) \rangle = C_{++} + \langle n_+ \rangle^2$

C_{+-} or C_{++} is an "integral correlation" (little-c) $C_{ab} = \langle ab \rangle - \langle a \rangle \langle b \rangle$

$$\int B_2^{\pi^+\pi} = \left[n_+ + \frac{[C_{+-} + n_+ n_-]}{n_+}\right] - \left[n_- + \frac{[C_{++} + n_+^2]}{n_+}\right]$$

$$\int B_2^{\pi^+\pi} = \left[n_+ + n_- + \frac{C_{+-}}{n_+}\right] - \left[n_- + n_+ + \frac{C_{++}}{n_+}\right] \Rightarrow \boxed{\int B_2^{\pi^+\pi} = \frac{1}{n_+}\left[C_{+-} - C_{++}\right]}$$

& presumably $\int B_2^{\pi^-\pi} = \frac{1}{n_-}\left[C_{-+} - C_{--}\right]$

$n_+ \int B_2^{\pi^+\pi} = C_{+-} - C_{++}$

C_{+-}: # of excess pairs of a w/ b^-

C_{++}: # of excess pairs of a w/ b^+

P1

P3