# OPTIMAL LOAD FLOW WITH STEADY-STATE SECURITY

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*Abstract*—The Dommel-Tinney approach to the calculation of optimal power-system load flows has proved to be very powerful and general. This paper extends the problem formulation and solution scheme by incorporating exact outage-contingency constraints into the method, to give an optimal steady-state-secure system operating point. The controllable system quantities in the base-case problem (e.g. generated MW, controlled voltage magnitudes, transformer taps) are optimised within their limits according to some defined objective, so that no limit-violations on other quantities (e.g. generator MVAR and current loadings, transmission-circuit loadings, load-bus voltage magnitudes, angular displacements) occur in either the base-case or contingency-case system operating conditions.

## INTRODUCTION

### Optimal load flow

An optimal load-flow solution gives the optimal active- and reactive-power dispatch for a static power-system loading condition. Computationally, it is a very demanding nonlinear programming problem, due to the large number of variables and in particular to the much larger number and types of limit constraints which define the boundaries of technical feasibility.

At the outset of the work reported here, a detailed appraisal of the existing methods for calculating an accurate optimal load-flow solution was carried out. This suggested that the best approach for problems of the size and complexity envisaged is that of Ref. 1. The method is sufficiently general to be applicable to a wide range of objective functions, constraint-types and controllable system elements. The central sparsity-programmed Newton load flow used in it enables large problems to be solved comparatively efficiently. Accordingly, a versatile, efficient and reliable computer program based on this method for "exact" optimal active- and reactive-power dispatching was developed by the authors, and has been used frequently for practical power-system studies.

Static power dispatching is of course only a constituent, but very necessary, part of the overall power-system operational or planning process. Within its context of practical application, the optimal load-flow solution has one primary limitation—that it does not cater for system security constraints.

### Steady-state security

One of the main aspects of system security is so-called steady-state security. This is defined as the ability of the system to operate steady-state-wise within the specified limits of safety and supply quality following a contingency, in the time period after the fast-acting automatic control devices have restored the system load balance, but before the slow-acting controls, e.g. transformer tappings and human decisions, have responded.

Steady-state security monitoring calculations are carried out in system planning and operation, and normally consider a series of contingencies involving "credible" outages of transmission circuits and generating plant[2-5]. Given a base case load-flow condition, the security monitor re-solves the load-flow problem for each outage case in turn, to detect potential overloads and unacceptable voltage levels. Fast AC load-flow methods have recently been developed for this purpose[6-8].

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### Optimal steady-state-secure solutions

Any insecurities detected by the security monitor must be corrected in the base-case operating condition. However, the security monitor gives no automatic indication of the corrections required. Assuming that the insecurities can be removed without changing the system composition, the problem is then to re-schedule the controllable system quantities in such a way that the operating condition becomes steady-state-secure and optimal, usually according to an economic objective.

A number of methods have been proposed for this calculation. In all of them the central process is an optimal load-flow solution of one kind or another, into which only those outage-constraints corresponding to detected insecurities are introduced. Outage security therefore, needs to be checked at intervals during the constrained optimum-seeking process, since the removal of one insecure condition may relieve others or cause new ones. Some methods use an entirely linearised system model, neglecting reactive-power and voltage considerations, and accepting the MW-flow accuracy limitations of the DC load flow[9-13]. Others use a linear model for the outaged system, so that only approximate outage-case MW-overload constraints can be reflected into the optimal load flow[14]. Another approach is to apply security-oriented group import/export constraints[3], which are not easy to define in highly interconnected systems.

### The method of this paper

The method to be described here incorporates the steady-state security constraints into the optimal load-flow solution using an exact formulation, which enables reactive-power and voltage constraints to be fully considered in the outage cases.

The method is simply an extension of the Dommel-Tinney approach, and of the computer program based on it. The principle is outlined by the following steps:

(i) solve the optimal base-case load flow by the Dommel-Tinney method

(ii) monitor outage-security using a fast AC load-flow method

(iii) continue the optimal load-flow solution, introducing the constraints corresponding to the insecure outage cases detected in (ii) at each gradient step, until the insecurities have been removed and the new optimum point is reached

(iv) re-cycle from (ii) until the optimal secure solution is obtained.

One of the main features of the method is the exactness with which the outage constraints can be imposed. The solution thus obtained can be used to evaluate the validity and desirability of approximations made in other approaches, and perhaps to define reliable security limits for the base-case problem. The original optimal load-flow program requires few additions, apart from the fast security-monitor routines, which themselves increase the total computer storage requirements by less than 10%.

The method is described using the general problem-formulation and concise notations of Ref. 1, and its performance is illustrated by a 30-bus example.

## NOTATIONS

$[u]$ = vector of control (independent) variables
$[x]$ = vector of state (dependent) variables
$[g(x,u)]$ = concise vector notation for nodal load-flow equations
$[h(x,u)]$ = concise vector notation for system inequalities
$f(x,u)$ = objective function
$\mathcal{L}(x,u,\lambda)$ = Lagrangian function
$[\lambda]$ = vector of Lagrange multipliers
$w(x,u)$ = penalty function
$p(x,u)$ = sum of penalty functions within a contingency case
$s$ = number of insecure contingency cases

Superscript k denotes constraints or variables associated with contingency case k (k=0 is base-case system). Subscripts x,u or $\lambda$ denote partial derivatives with respect to these variables, e.g. $[g_X]$ = Jacobian matrix of load-flow problem, $[\mathcal{L}_u]$ = gradient vector of Lagrangian function with respect to the control variables.
Superscript T = matrix transpose

## MATHEMATICAL FORMULATION OF PROBLEM

The model characteristics, formulation of system equations and inequalities, different objective functions, and selection of control (independent) and state (dependent) variables that are applicable to the present formulation have been discussed elsewhere[1,15,16]. The optimal load-flow model and steady-state security constraints are summarised here briefly in generalised notation, and the specific functions used in the developed computer program are given in Appendix 1.

### Optimal load-flow model

The problem of optimising the static operation of a power system is expressed as the minimisation of an objective function subject to equality and inequality constraints. The objective is a function of system control and state variables and is written as

$$f = f(x^o,u) \tag{1}$$

The equality constraints are the nodal load-flow equations, represented concisely by the vector equation

$$[g^o(x^o,u)] = 0 \tag{2}$$

and the inequality constraints are the plant and transmission system operating limits, represented by the vector inequality

$$[h^o(x^o,u)] \leq 0 \tag{3}$$

### Security constraints

The additional equality and inequality constraints associated with outage contingencies are termed here the security constraints. Each outage case is characterised by a new set of nodal load-flow equations, in which the control variables are taken to be the same as those in the base-case system operating condition

$$[g^k(x^k,u)] = 0 \tag{4}$$

and the plant and transmission system operating limits give rise to a new set of inequalities

$$[h^k(x^k,u)] \leq 0 \tag{5}$$

which are not necessarily the same in form or limit-values as those specified for the base-case problem.

## METHOD OF SOLUTION

Inequality constraints (3) and (5) are handled by the penalty-function approach except for control variables. In this method a penalty term is added to the objective function for each constraint violation in order to discourage the minimum-seeking process from exceeding the relevant limit.

The new augmented objective function to be minimised becomes

$$\begin{aligned} F &= f(x^o,u) + \sum_{k=0}^{s} \sum_j w_j^k(x^k,u) \\ &= f(x^o,u) + \sum_{k=0}^{s} p^k(x^k,u) \end{aligned} \tag{6}$$

where a penalty $w_j$ is introduced for each violated constraint j in each outage case k. Note that s is the number of outage cases detected as insecure, and not the total number monitored.

The equality constraints (2) and (4) on F are imposed by the method of Lagrange multipliers. By introducing new variables $\lambda$, the problem is transformed into the minimisation with respect to $[u]$ of the unconstrained Lagrangian function

$$\begin{aligned} \mathcal{L}(x^o,\ldots,x^s,u,\lambda^o,\ldots,\lambda^s) = f(x^o,u) &+ \sum_{k=0}^{s} p^k(x^k,u) \\ &+ \sum_{k=0}^{s} [\lambda^k]^T[g^k(x^k,u)] \end{aligned} \tag{7}$$

For a stationary point of this function the following set of necessary conditions must be satisfied:

$$[\mathcal{L}_{x^o}] = [f_{x^o}] + [p^o_{x^o}] + [g^o_{x^o}]^T[\lambda^o] = 0 \tag{8}$$

$$[\mathcal{L}_{x^k}] = [p^k_{x^k}] + [g^k_{x^k}]^T[\lambda^k] = 0 \quad (k=1,\ldots,s) \tag{9}$$

$$[\mathcal{L}_u] = [f_u] + \sum_{k=0}^{s} [p^k_u] + \sum_{k=0}^{s} [g^k_u]^T[\lambda^k] = 0 \tag{10}$$

$$[\mathcal{L}_{\lambda^k}] = [g^k(x^k,u)] = 0 \quad (k=0,\ldots,s) \tag{11}$$

Equations (8) – (11) are little different in form from those derived in Ref. 1 for the base-case optimal load flow, and they may be solved by the same iteration/gradient step technique. For no constraint violations in the contingency cases (s=0, eq. (9) omitted), (8) – (11) reduce to the original optimal load-flow problem.

The algorithm for solving the base-case problem, k=0, is briefly:

(i) assume an initial value for $[u]$
(ii) solve the load-flow problem (11) by Newton's method for $[x^o]$. If there are any constraint violations, calculate the corresponding penalties
(iii) solve the linear set (8) for $[\lambda^o]$
(iv) calculate the gradient vector $[\mathcal{L}_u]$ from (10) and adjust $[u]$ in the negative gradient direction
(v) re-cycle from step (ii) until $[\mathcal{L}_u]$ is sufficiently small.

Once the base-case system operation is optimised, the above algorithm is extended by introducing the insecure contingency cases detected by the security monitor. These cases are connected to the base-case problem only in (10) via their penalty functions and Lagrange multipliers.

The extended algorithmic steps are then:

(a) $k = -1$, $[\mathcal{L}_u] = 0$

(b) $k = k+1$

(c) solve (11) for $[x^k]$ by Newton's method and check for any constraint violations. If there are no violations, go to step (e) if k=0 or (g) if k>0

(d) calculate the penalty terms for this case

(e) solve for $[\lambda^k]$ from (8) if k=0, or from (9) if k>0

(f) calculate the components of $[\mathcal{L}_u]$ from (10) due to the terms for the kth case, and add them to the existing $[\mathcal{L}_u]$

(g) unless k=s, go to step (b)

(h) check the elements of $[\mathcal{L}_u]$ for convergence. If converged, the optimum point has been reached

(i) perform a gradient step using $[\mathcal{L}_u]$ to obtain a new [u]

(j) go to step (a)

**Further considerations**

The types of contingencies represented will typically be the outage of one or more transmission lines, transformers, generators, static or synchronous compensators, or bus loads, or combinations of these. The treatment of outages involving control variables requires further examination.

For instance, for a generator outage, the lost generation will be supplied by the remaining generators, according to some specified redistribution pattern. The set of control variables $[u^k]$ in outage case k is then different in value from that in the base case $[u^o]$. At any given gradient step, a linear relation exists:

$$[u^k] = [M^k][u^o] \tag{12}$$

where $[M^k]$ is normally a diagonal matrix. In (10), the required gradient vector is $[\mathcal{L}_{uo}]$, and therefore in calculating its components due to outage case k, the transformation

$$[\mathcal{L}_{u^o}] = [\partial u^k/\partial u^o][\mathcal{L}_{u^k}] = [M^k][\mathcal{L}_{u^k}] \tag{13}$$

must be invoked.

Separately-represented parallel transformers whose taps are control variables can present difficulties if only one of them is outaged in the study. If possible, they should be represented as a single combined transformer whose impedance is increased to simulate the outage.

## COMPUTATIONAL ASPECTS OF THE METHOD

The purpose of this section is to give some details of the computational experience obtained with the optimal load-flow program, and the steady-state-security extension of it.

### Optimal load flow

A. Jacobian matrix solution

The optimal load-flow process centres around the Newton load-flow method[17], used in solving (11). Each load-flow iteration requires the construction and triangulation of the Jacobian-matrix equation. In the authors' program, the bus power-mismatch vector is calculated concurrently with the compact construction of the Jacobian matrix $[g_x]$. This is efficient, because many of the constituent terms are common to different parts of the mismatch vector and $[g_x]$. Also, it is not necessary to triangulate the latter at the stage when all the mismatches are within their convergence tolerances. The lower-triangular factor of $[g_x]$ is never stored in this scheme. Similarly, (8) is solved for $[\lambda]$ without storing the lower-triangular factor of $[g_x]^T$. Since triangulations are performed many times during the solution, a good "dynamic" bus-ordering scheme is used to minimise fill-up in the elimination process.

B. Penalty functions

The outside penalty function method for inequality constraints, as advocated in Ref. 1, has been satisfactory on all the systems studied. The initial values of the penalty weightings and the factors by which they should be increased during a solution differ for different power power systems. Poor choices of these values lead to excessive oscillation of the solution process between the feasible and infeasible regions, or to very slow convergence. A completely automatic logic for choosing them is very difficult to devise, and various attempts at this were only partially successful. Instead, a simple and satisfactory procedure was evolved where two or three initial experimental runs are made on each new power system studied. The program prints out the information needed to establish good values, which then give satisfactory performance for different system operating conditions. The "soft" limits imposed by outside penalty functions present no problems in enforcing practical limits within specified tolerances. Voltage-magnitude, active-power, and line-current penalty functions behave better than reactive-power and MVA penalty functions. For infeasible problems, the constraints contributing most to the infeasibility are easily identified by the excessive increases in their penalty weightings.

C. Gradient step

A simple gradient-step algorithm using the approximate Hessian-matrix diagonal elements[1] with logic to cater for changes in gradient directions, oscillations and slow convergence has performed satisfactorily for all systems studied. Although the scheme requires storage for the computation of the Hessian-matrix diagonals, it is easy to implement and responds well for sudden changes caused by the penalty terms. If a given system is to be solved on a regular basis, a more sophisticated algorithm may be justified to reduce the total number of gradient steps needed. However, a computational break-even point can be reached when increasing the step size increases the number of Newton load-flow iterations per step, which is one or two for the present program.

Different convergence tolerances are used for the gradient-vector elements corresponding to different types of control variable. The sensitivity of the objective function to the control variables is usually very low in the region of the located optimum. This means that there is little practical advantage to be gained by specifying high-accuracy tolerances at the expense of a large number of gradient steps which produce negligible further improvement in the value of the objective function. It also means that in some cases a near-optimal system operating condition may be noticeably different but not noticeably more costly than a very-accurately calculated optimum. This is particularly true of the tap settings of transformers in network meshes, which is also a possible indication of different local minima.

The reliability of convergence is very high. The overall process is basically a series of transitions from one load-flow solution to another via the gradient steps. Provided that logic is inserted for preventing large oscillations into and out of the feasible region due to the penalties, an optimal solution is obtainable whenever the load flow is soluble by Newton's method, which is almost always.

### Secure optimal load flow

From the algorithmic steps (a) – (j) given previously, it is seen that the equalities and inequalities for each insecure outage case are treated computationally in the same way as those of the base-case problem, and the above comments remain equally valid for them.

One of the main points about the method is that since it is impractical to store the security constraints explicitly (due to the problem nonlinearity), the insecure outage cases are re-solved at each gradient step. This costs very little computer storage additional to that for the base-case optimal load flow, because the base- and outage-cases are solved one at a time in the same storage space, and using the same subroutines for load-flow solution, constraint-checking, [λ] and [$\mathcal{L}_u$]-component calculations. If some constraint-types defined for the outage cases differ from those for the base case, then additional subroutines must be provided.

The amount of computation at each gradient step is directly proportional to the number of cases that need to be included. In a well-designed power system, this number should not be large. Nevertheless, the overall computing time is substantial though not necessarily prohibitive.

It is frequently the case that several outage-insecurities can be corrected by the same control-variable adjustments. The effect of this is to speed up the correction process, since the relevant gradient-vector terms are additive. At the same time, however, the overall computing time is likely to be reduced if these situations can be avoided as far as possible. If those outage cases can be identified whose insecurity-corrections automatically relieve the insecurities of other cases, and are processed first, the total number of cases that have to be brought in as constraints on the base-case problem will be reduced.

When a secure optimal solution does not exist, the constraints in the contingency cases that are most responsible for the infeasibility are indicated by the large values of their penalty weightings.

The complete optimal steady-state-secure load-flow program forms a versatile load-flow package. The optimal load-flow routines can handle alternative objective functions, with or without outage security, and can be used for ordinary load-flow solutions when unusual single- or multiple-criterion controls are to be simulated. The security-monitoring routines provide very fast (faster than Newton) accurate or approximate conventional unadjusted or adjusted load-flow solutions, as well as security checks, as described in Ref. 8. The mode of usage of the program from a common data base is selected by an input parameter.

There are many ways in which the method as programmed can be further developed to improve the computing speed, for application to large systems. For instance, the Newton load flow can be replaced by the fast decoupled method[8], which then fulfils both this and the security-monitor roles. Some improvement in the present programming techniques is possible. As mentioned above, the outage cases can be processed in a suitable order. It may be possible to keep the gradient-vector components due to some or all of the insecure outage cases constant over two or more consecutive gradient steps, thereby avoiding the corresponding load-flow, [λ] and penalty calculations during these steps. Work is in progress to speed up the [λ] calculations by exploiting MW-Θ/MVAR-V decoupling and further simplifications.

## NUMERICAL EXAMPLE

### Sample problem

An adaptation of the IEEE 30-bus standard load-flow test system is used to demonstrate the performance of the method. The data used is given in Appendix 2. The objective function to be minimised is the total system active-power generation cost, allowing all generator MW-outputs and voltage magnitudes, and transformer taps to be controllable. 33 contingency cases are specified for this example, each involving the outage of one of the branches in the network.

### Results

The results are summarised in Table I, showing the progress of the optimisation from the initial point through the optimum without security constraints to the final secure solution. The values of the control variables are given, together with the quantities whose limits are violated in the outage cases. These quantities are shown at the initial point for illustrative purposes, although the security monitor would not normally be applied at this stage. However, it is seen that the process of optimising without the insecurity constraints increases the insecurity of the system. The insecurities are then removed by incorporating the constraints for the 7 relevant outage cases into the solution process, and in this example no further insecurities are created by so doing.

The number of gradient steps required to reach the intermediate "insecure" optimum point is 13. A further 25 steps are needed to obtain the final solution, and the total computing time on the CDC 7600 computer is 14.3 seconds including input/output and with non-optimised FORTRAN compilation. The penalty weightings and convergence tolerances used are given in Appendix 2.

TABLE I. RESULTS FOR SAMPLE PROBLEM

| | | Initial point | Optimum point without outage security | Secure optimum point |
|---|---|---|---|---|
| CONTROL VARIABLES, V (p.u.) | $V_1$ | 1.0500* | 1.0500* | 1.0500* |
| | $V_2$ | 1.0450 | 1.0382 | 1.0338 |
| | $V_5$ | 1.0100 | 1.0114 | 1.0058 |
| | $V_8$ | 1.0100 | 1.0194 | 1.0230 |
| | $V_{11}$ | 1.0500 | 1.0912 | 1.0913 |
| | $V_{13}$ | 1.0500 | 1.0913 | 1.0883 |
| CONTROL VARIABLES, PG (MW) | $P_{G2}$ | 80.00* | 48.84 | 57.56 |
| | $P_{G5}$ | 50.00* | 21.51 | 24.56 |
| | $P_{G8}$ | 20.00 | 22.15 | 35.00* |
| | $P_{G11}$ | 20.00 | 12.14 | 17.93 |
| | $P_{G13}$ | 20.00 | 12.00* | 16.91 |
| CONTROL VARIABLES, TAP (%) | $t_{11}$ | -2.200 | 0.275 | 1.546 |
| | $t_{12}$ | -3.100 | -3.980 | -3.711 |
| | $t_{15}$ | -6.800 | 0.474 | 1.288 |
| | $t_{36}$ | -3.200 | -5.837 | -4.194 |
| INSECURE OUTAGE CASES | Line no. | Violating quantities | Violating quantities | Violating quantities |
| | 1 | | $I_2$ , $I_4$ , $I_7$ | ALL CONSTRAINTS WITHIN SPECIFIED LIMIT TOLERANCES |
| | 2 | $Q_1$ | $Q_1$ , $I_1$ | |
| | 4 | $Q_1$ | $Q_1$ , $I_1$ | |
| | 5 | | $I_6$ , $I_8$ | |
| | 7 | | $Q_1$ , $V_{12}$ | |
| | 33 | | $V_{27}$ | |
| | 35 | | $V_{27}$ | |
| | 37 | $V_{29}$ | | |
| | 38 | $V_{30}$ | | |
| TOTAL GENERATION (MW,MVAR) | | 288.8, 108.3 | 292.9, 124.5 | 290.5, 116.8 |
| TOTAL SYST. LOSSES (MW,MVAR) | | 5.39,-17.95 | 9.48, -1.76 | 7.11, -9.45 |
| TOTAL GEN. COST (£/hr) | | 900.76 | 802.40 | 813.74 |

* values on limits

## CONCLUSIONS

The inclusion of steady-state security constraints makes the optimal load-flow calculation a more powerful and practical tool for system operation and design. The exactness of the method described here enables the calculation of a base-system operating state that ensures satisfactory reactive– as well as active-power conditions in any number of specified contingency cases. In conjunction with a fast security monitor, an existing optimal load-flow program can be extended quite easily and with little extra storage to handle the contingency cases.

The accurate solution provided by the method may be used to evaluate more-approximate treatments of security, and to define operating limits in the base-case problem that give some degree of security. The steady-state-secure solution implies an optimal solution to the MVAR spinning-reserve problem.

For large-scale practical applications, there is scope for further developing the method to suit individual system requirements, and to improve the computing speed.

The technique for incorporating the security constraints into the base-case problem is not restricted to outage contingencies, nor indeed to electrical power systems. It can be applied to other large non-linear system operation or design problems in which a variety of inequalities for different static system states or configurations must be satisfied.

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## APPENDIX 1

### Optimal load-flow problem

#### Equality constraints

The equality constraints of the optimisation problem are the equations defining the ordinary load-flow problem. As selected in Ref. 1, the Newton load-flow polar power-mismatch formulation is particularly suitable for the optimisation study, since the problem variables correspond closely to the quantities in the physical system that require to be controlled and limited.

The relevant equations are

$$P_{Gk} - P_{Lk} - P_k(V,\Theta) = 0, \text{ for all buses except slack}$$
$$Q_{Gk} - Q_{Lk} - Q_k(V,\Theta) = 0, \text{ for all PQ buses} \qquad (14)$$

where

$P_{Gk} + jQ_{Gk}$ = generation at bus k

$P_{Lk} + jQ_{Lk}$ = fixed load at bus k

$P_k + jQ_k$ = net calculated power injection at bus k as a function of the system bus voltage magnitudes V and angles $\Theta$

V and $\Theta$ for each PQ bus, and $\Theta$ for each PV bus become the problem unknowns (dependent variables x). The remaining variables determine the system operating point (independent variables u), and include $P_G$ and V for a PV bus, $\Theta$ and V for the reference bus, and the transformer tap settings t. Adopting the notations x and u for the problem variables, the complete set of equations (14) is expressed concisely by the vector equation

$$[g(x,u)] = 0$$

#### Inequality constraints

Limits for equipment loading and operating requirements comprise the inequality constraints of the problem. Among the various types that may be defined, the following limits are imposed in the developed computer program:

$$V_i^{min} \le V_i \le V_i^{max}, \text{ for } i=1,\ldots,N \qquad (15)$$
$$P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max}, \text{ for } i=1,\ldots,NG \qquad (16)$$
$$Q_{Gi}^{min} \le Q_{Gi}; (P_{Gi}^2+Q_{Gi}^2)^{\frac{1}{2}} \le S_{Gi}^{max}, \text{ for } i=1,\ldots,NG \qquad (17)$$
$$t_i^{min} \le t_i \le t_i^{max}, \text{ for } i=1,\ldots,NT \qquad (18)$$
$$I_i \le I_i^{max}, \text{ for } i=1,\ldots,NB \qquad (19)$$

where N = no. of buses, NG = no. of generating units, NT = no. of tap-changing transformers, and NB = no. of network branches in the system. Using again the notations x and u, the simple and functional inequality constraints (15) – (19) are expressed concisely by the vector inequality

$$[h(x,u)] \leqslant 0$$

All inequality constraints except those on independent (control) variables are imposed using the "outside penalty function" technique, where a penalty of the form

$$w_j = W_j(h_j^2), \quad (W_j > 0 \text{ is penalty weighting})$$

is added to the objective function whenever the inequality $h_j$ is violated. Each control variable is set on its limit when the limit is violated during the course of the gradient steps, but is allowed to back off the limit subsequently, if necessary.

#### Objective function

If both active and reactive power are dispatchable, the usual criterion for optimal operation is the minimisation of generating cost. If only reactive power is dispatchable, then active-power loss minimisation is frequently the desired objective (it is also a convenient dummy objective if the main problem is to determine a feasible reactive-power/voltage solution, or for other purposes). Any other objective may be defined depending on the problems of a particular system, under for instance, light-load conditions.

Assuming third-order generator cost curves, the total-generation-cost objective function is

$$f(x,u) = \sum_{i=1}^{NG} (a_i + b_i P_{Gi} + c_i P_{Gi}^2 + d_i P_{Gi}^3)$$

which reduces to the active-power loss function if all $a_i=b_i=c_i=d_i=0$ except $b_i=1.0$ for the reference bus.

The general optimal load-flow problem may now be expressed as:

$$\begin{aligned} &\text{minimise } f(x,u) \\ &\text{subject to } [g(x,u)] = 0 \\ &\text{and } [h(x,u)] \leqslant 0 \end{aligned}$$

## APPENDIX 2

### Data for sample problem

#### System data

Tables II, III and IV give details of the generator, load, and line/transformer data, respectively.

TABLE II. GENERATOR DATA

| Bus No. | $P_G^{min}$ MW | $P_G^{max}$ MW | $Q_G^{min}$ MVAR | $S_G^{max}$ MVA | Cost coefficients a | b | c |
|---|---|---|---|---|---|---|---|
| 1 | 50 | 200 | -20 | 250 | 0.0 | 2.0 | 0.00375 |
| 2 | 20 | 80 | -20 | 100 | 0.0 | 1.75 | 0.0175 |
| 5 | 15 | 50 | -15 | 80 | 0.0 | 1.0 | 0.0625 |
| 8 | 10 | 35 | -15 | 60 | 0.0 | 3.25 | 0.00834 |
| 11 | 10 | 30 | -10 | 50 | 0.0 | 3.0 | 0.025 |
| 13 | 12 | 40 | -15 | 60 | 0.0 | 3.0 | 0.025 |

Generating cost $f_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2$ £/hr

TABLE III. LOAD DATA

| Bus No. | Load MW | Load MVAR | Bus No. | Load MW | Load MVAR |
|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 16 | 3.5 | 1.8 |
| 2 | 21.7 | 12.7 | 17 | 9.0 | 5.8 |
| 3 | 2.4 | 1.2 | 18 | 3.2 | 0.9 |
| 4 | 7.6 | 1.6 | 19 | 9.5 | 3.4 |
| 5 | 94.2 | 19.0 | 20 | 2.2 | 0.7 |
| 6 | 0.0 | 0.0 | 21 | 17.5 | 11.2 |
| 7 | 22.8 | 10.9 | 22 | 0.0 | 0.0 |
| 8 | 30.0 | 30.0 | 23 | 3.2 | 1.6 |
| 9 | 0.0 | 0.0 | 24 | 8.7 | 6.7 |
| 10 | 5.8 | 2.0 | 25 | 0.0 | 0.0 |
| 11 | 0.0 | 0.0 | 26 | 3.5 | 2.3 |
| 12 | 11.2 | 7.5 | 27 | 0.0 | 0.0 |
| 13 | 0.0 | 0.0 | 28 | 0.0 | 0.0 |
| 14 | 6.2 | 1.6 | 29 | 2.4 | 0.9 |
| 15 | 8.2 | 2.5 | 30 | 10.6 | 1.9 |

Total load = 283.4 MW, 126.2 MVAR

TABLE IV. BRANCH DATA

| Branch No. | Bus No's | R p.u. | X p.u. | B(total) p.u. | Rating MVA |
|---|---|---|---|---|---|
| 1 | 1-2 | 0.0192 | 0.0575 | 0.0264 | 130 |
| 2 | 1-3 | 0.0452 | 0.1852 | 0.0204 | 130 |
| 3 | 2-4 | 0.0570 | 0.1737 | 0.0184 | 65 |
| 4 | 3-4 | 0.0132 | 0.0379 | 0.0042 | 130 |
| 5 | 2-5 | 0.0472 | 0.1983 | 0.0209 | 130 |
| 6 | 2-6 | 0.0581 | 0.1763 | 0.0187 | 65 |
| 7 | 4-6 | 0.0119 | 0.0414 | 0.0045 | 90 |
| 8 | 5-7 | 0.0460 | 0.1160 | 0.0102 | 70 |
| 9 | 6-7 | 0.0267 | 0.0820 | 0.0085 | 130 |
| 10 | 6-8 | 0.0120 | 0.0420 | 0.0045 | 32 |
| 11 | 6-9 | 0.0 | 0.2080 | 0.0 | 65 |
| 12 | 6-10 | 0.0 | 0.5560 | 0.0 | 32 |
| 13 | 9-11 | 0.0 | 0.2080 | 0.0 | 65 |
| 14 | 9-10 | 0.0 | 0.1100 | 0.0 | 65 |
| 15 | 4-12 | 0.0 | 0.2560 | 0.0 | 65 |
| 16 | 12-13 | 0.0 | 0.1400 | 0.0 | 65 |
| 17 | 12-14 | 0.1231 | 0.2559 | 0.0 | 32 |
| 18 | 12-15 | 0.0662 | 0.1304 | 0.0 | 32 |
| 19 | 12-16 | 0.0945 | 0.1987 | 0.0 | 32 |
| 20 | 14-15 | 0.2210 | 0.1997 | 0.0 | 16 |
| 21 | 16-17 | 0.0824 | 0.1932 | 0.0 | 16 |
| 22 | 15-18 | 0.1070 | 0.2185 | 0.0 | 16 |
| 23 | 18-19 | 0.0639 | 0.1292 | 0.0 | 16 |
| 24 | 19-20 | 0.0340 | 0.0680 | 0.0 | 32 |
| 25 | 10-20 | 0.0936 | 0.2090 | 0.0 | 32 |
| 26 | 10-17 | 0.0324 | 0.0845 | 0.0 | 32 |
| 27 | 10-21 | 0.0348 | 0.0749 | 0.0 | 32 |
| 28 | 10-22 | 0.0727 | 0.1499 | 0.0 | 32 |
| 29 | 21-22 | 0.0116 | 0.0236 | 0.0 | 32 |
| 30 | 15-23 | 0.1000 | 0.2020 | 0.0 | 16 |
| 31 | 22-24 | 0.1150 | 0.1790 | 0.0 | 16 |
| 32 | 23-24 | 0.1320 | 0.2700 | 0.0 | 16 |
| 33 | 24-25 | 0.1885 | 0.3292 | 0.0 | 16 |
| 34 | 25-26 | 0.2544 | 0.3800 | 0.0 | 16 |
| 35 | 25-27 | 0.1093 | 0.2087 | 0.0 | 16 |
| 36 | 28-27 | 0.0 | 0.3960 | 0.0 | 65 |
| 37 | 27-29 | 0.2198 | 0.4153 | 0.0 | 16 |
| 38 | 27-30 | 0.3202 | 0.6027 | 0.0 | 16 |
| 39 | 29-30 | 0.2399 | 0.4533 | 0.0 | 16 |
| 40 | 8-28 | 0.0636 | 0.2000 | 0.0214 | 32 |
| 41 | 6-28 | 0.0169 | 0.0599 | 0.0065 | 32 |
| 42 | 10-10 | 0.0 | -5.2600 | | |
| 43 | 24-24 | 0.0 | -25.0000 | | |

Base MVA = 100

Branches numbered 11, 12, 15 and 36 in Table IV are in-phase transformers with assumed tapping ranges of ±10%. The assumed branch loading limits are for convenience taken to be the same in the base and contingency cases, and similarly for bus voltage-magnitude limits.

The lower voltage-magnitude limits at all buses are 0.95 p.u., and the upper limits are 1.1 p.u. for generator buses 2, 5, 8, 11, and 13, and 1.05 p.u. for the remaining buses including the reference bus 1.

The branch MVA ratings are converted by the program to p.u. current limits which are applied to both the sending and receiving ends of each branch.

**Program data**

The bus power-mismatch tolerances used for each solution of the system load flow were 0.01 MW/MVAR for the base case and 0.5 MW/MVAR for the outage cases. These are probably more accurate than required for practical large-system studies, (see Ref. 8). The tolerances on limit-enforcements were 0.5% for bus voltage magnitudes, 0.1 MW/MVAR for generated active and reactive powers, $0.1 \times I^{max}$ for branch currents.

The convergence tolerances for the gradient vector elements, i.e. in testing whether the optimum point is sufficiently close, were 0.03 £/hr for each generator per-unit active power, 0.04 £/hr for each generator per-unit voltage magnitude, and 0.015 £/hr for each transformer tapping.

The initially-specified penalty weightings were 20.0 for voltage-magnitude limits, 0.02 for generator reactive-power limits, 0.15 for branch-current limits, and 10.0 for the slack (reference) bus active power.

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## Discussion

**M. S. Sachdev and S. A. Ibrahim** (University of Saskatchewan, Saskatoon, Canada): We congratulate the authors for presenting an interesting paper. This paper extends the previously presented technique, of optimal load flow, to include steady state security considerations. Would the authors discuss the following comments and questions.

1. In a system, some constraints might be violated after a contingency is experienced. These violations can be corrected by two distinct ways. One consists of taking the necessary control action after the contingency has been experienced. In the second approach control action would be taken before any contingency is experienced which (action) will ensure that in case of contingencies the system constraints are not violated. The later is a philosophy of preventive measures which might be more convenient to apply in many practical cases. However, in a few cases it may not be possible to adopt it and post contingency control actions will have to be relied upon.

2. Every control action affects the entire system operating state. It is, therefore, possible that one such action might correct more than one constraint violations. Also identical violations might be experienced for different contingencies. If control action is taken to correct a serious constraint violation, it might correct some minor outage insecurities too. A suitable control action algorithm is therefore essential to minimize the computing effort.

3. The objective of the proposed procedure is to optamize equation 7. This is a non-linear equation in which many variables will be introduced for each outage insecurity. In a large system a given contingency could result in many insecurities; in which case the variables in equation 7 and the operating constraints would be quite large. Is there a likelyhood of convergence difficulties in such cases?

4. The approach presented in this paper is outlined on page 2. It states that at each gradient step outage-security is monitored using a fast AC load flow method. In the method of solution, the extended algorithm step(c) states that after each gradient step system states are computed by Newton's method. This discrepancy needs to be reconciled.

We once again compliment the authors for a well organized paper.

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**O. Alsac and B. Stott**: We are grateful for the comments and questions raised by Messrs. Sachdev and Ibrahim. In presenting a method of calculation, the paper gives little direct attention to the philosophy and practical details of the method's potential operational application. The discussion gives us the opportunity of examining this aspect more closely.

The definition and calculation of 'secure' optimal dispatch dealt with by the paper are inherently preventive in nature. In the operation of a power system, the relative importances given to preventive and corrective control actions will depend upon the planning/operating policies of the power company, according to the capital and operating costs of providing security, and the probabilities and consequences of each contingency.

The trend for modern control centres with on-line state-estimation/load-flow facilities is to be equipped with steady-state-security monitoring, which for the current or extrapolated system operating point evaluates the effects of postulated outages. As the need and means for greater accuracy increase, AC security monitoring will in many cases supercede the simpler DC approach.

In principle, it is possible to envisage a security-constrained on-line dispatching system that automatically takes preventive control action whenever a dangerous simulated outage case is identified by the security monitor. In practice, there are many factors that mitigate strongly against such an approach at the present time. Instead, the security monitor transmits the relevant information to the control engineer, who must then decide whether to take preventive action, or rely on corrective action should the event take place.

Rapidly-accessed software aids can be provided to assist in these decisions, in cases not resolved clearly by the operating rules or the engineer's experience. These aids can also fulfil an important 'learning' function, by which the rules and experience can be enhanced. One such aid might be a program based on the general method of the paper. The calculation is capable of informing the engineer how, and if, the insecurities can be removed preventively by adjusting the dispatch, perhaps taking into account MW loading rates and transformer tapping rates (where reactive dispatch is included), and what is the resultant increase in operating cost. For preventive calculations of this type the economic objective will normally be chosen.

In the cases where the engineer decides to accept the risk of the relevant contingency-occurence, he may call upon the software to help to formulate a contingency plan. Where the anticipated contingency is not severe enough to require immediate action such as line switching or load shedding, an optimal load flow without the outage constraints might be used, with a non-economic objective function such as minimum or fastest action to correct say, line overloads or poor voltage levels.

Among optimal load-flow methods, the relative generality of the Dommel-Tinney approach is one of its main attractions. It can handle many types of objective and constraints, and is equally applicable to MW and MVAR/voltage control. In an operational implementation, the computing demands of the method's steady-state-secure extension would be vastly less than those suggested by the studies described in the paper, which are essentially for the planning application. In any case, since no outages will have actually occured when the program is called, an instantaneous response is not imperative. Many computational refinements are possible, introducing justifiable approximations, taking cognisance of the particular characteristics of the system, and using practical convergence and limit tolerances.

Point 2 made by the discussers is important in this respect, since it will be computationally economical to identify, and incorporate first, the constraints of those insecure outage cases which may be expected to remove the insecurities (if any) of other outage cases. It would be very unusual in a practical well-interconnected system to have more than a very few insecure outage cases at any one time.

In reply to Messrs. Sachdev and Ibrahim's third point, a large number of constraint violations in a given outage case introduces the same number of penalties, which act to alter the system dispatch. Penalties that attempt to correct the same unsatisfactory operating conditions are additive, and increase the convergence speed of the solution. Any reduction thus obtained in the total number of gradient steps far outweighs the small time taken to compute the penalties. The dimensionality of the optimisation problem is not increased by the introduction of outage constraints, and their penalty terms added to the objective function are no more difficult for the minimisation process to handle than those of the base-case system.

When the method is used in a strictly off-line mode as described in the paper, security monitoring of all outage cases is performed, not at each gradient step as the discussers have supposed in their point 4, but only twice in the whole solution, unless the removal of the originally-detected outage insecurities creates new insecure outage cases. The fast decoupled load flow checks rapidly the list of outages, and thereby identifies the cases that need to be incorporated into the optimisation problem. Once this is done, these particular outage cases are solved at each gradient step by the Newton load-flow method. Shortly after submitting the present paper, we found that it was possible to remove Newton's method entirely from the optimal load flow in favour of the much-faster decoupled method, and to avoid the explicit Jacobian-matrix solution of eqs. (8) and (9) for the Lagrange multipliers.[8,18,19]

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