PROBABILISTIC GRAPHICAL MODELS
ASSIGNMENT-1

GROUP NO. 11

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Q1.

I 1) Income ⊥ Security : False

* As there is flow path from Income to Security through Payment which is not observed.

2) Income ⊥ Security | Payment : True

* As Income is a non-descendant of Security given its parent Payment, which is Local Semantic

$(X_i \perp \text{Non-descendant} \mid \text{Parent}(X_i)) = \text{True}$

* d-Separation exists and so Income and Security are conditionally independent.

3) Income ⊥ Payment : False

* As Income and Payment are directly connected by an edge.

4) Income ⊥ Security | Payment, Deposit : True

* Income is a non-descendant of Security given its parent Payment, which is Local Semantic. But addition of deposit as an evidence does not add a flow path from Income to Security

* d-Separation exists and so Income and Security are conditionally independent.

5) Deposit $\perp$ Payment : False

* As Deposit and Payment are directly connected by an edge.

6) Income $\perp$ Payment | Deposit : False

* Income and Payment are directly connected by an edge. So, Deposit as an evidence does not separate Income and Payment.

* d-separation does not exist and so Income and Payment are not conditionally independent.

II Factorized form of Joint Distribution over all the variables: By Bayes's chain Rule,

$$P(A,B,C,D,E) = P(A)\,P(B|A)\,P(C|A,B)\,P(D|C,E)\,P(E)$$

Using the Directed model and by P(node | parents), the above factorized form is obtained.

III P(Payment = False)

$$P(-c) = P(-c, b, a)$$

$$= P(-c|b,a)\,P(b|a)\,P(a) + P(-c|-b,a)\,P(-b|a)\,P(a) + P(-c|-a,b)\,P(b|\bar{a})\,P(\bar{a}) + P(-c|-b,-a)\,P(-b|-a)\,P(-a)$$

$$= (0.95 \times 0.1 \times 0.3) + (0.5 \times 0.9 \times 0.3) + (0.55 \times 0.6 \times 0.7) + (0.4 \times 0.4 \times 0.7)$$

$$= 0.0285 + 0.135 + 0.231 + 0.112$$

$$\boxed{P(-c) = 0.5065}$$

IV $P(C = \text{Payback} \mid A = \text{Low})$

$= P(-c \mid B, -A)\ P(B \mid -A) + P(-c \mid -B, -A)\ P(-B \mid -A)$

$= (0.55 \times 0.6) + (0.4 \times 0.4)$

$\boxed{P(-c \mid -A) = 0.49}$

V $P(C = \text{Payback} \mid A = \text{Low},\ B = \text{Large})$

From Joint Probability Distribution table,

$\boxed{P(-c \mid -A, B) = 0.55}$

VI $P(C = \text{Payback} \mid A = \text{High},\ D = \text{No})$

$$P(-c \mid A, -D) = \frac{P(-c, A, -D)}{P(A, D)}$$

$$P(-c, A, -D) = \sum_{B,E} P(-c, A, -D, B, E)$$

$$= \sum_{B,E} P(-c \mid A, B)\ P(A)\ P(B)\ P(-D \mid -c, E)\ P(E)$$

$$= \sum_{B} P(A) \left\{ P(-c \mid A, B)\ P(B) \left[P(-D \mid -c, E)\ P(E) + P(-D \mid -c, -E)\ P(-E) \right] \right\}$$

$$= P(A) \left\{ \left[P(-c \mid A, B)\ P(B) \left[P(-D \mid -c, E)\ P(E) + P(-D \mid -c, -E)\ P(-E) \right] \right] + \left[P(-c \mid A, -B)\ P(-B) \left[P(-D \mid -c, E)\ P(E) + P(-D \mid -c, -B)\ P(-E) \right] \right] \right\}$$

$$= P(A)\left\{\left[0.95\times0.1\times0.3\left[(0.25\times0.35)+(0.69\times0.65)\right]\right.\right.$$
$$\left.\left.+\left[0.5\times0.9\times0.3\left[(0.25\times0.35)+(0.69\times0.65)\right]\right]\right]\right\}$$

$$= P(A)\ [0.015276 + 0.07236]$$

$$= P(A)\ [\cancel{0.0876}][0.292]$$

$$P(A,-D) = \sum_{B,C,E} P(A,-D,B,C,E)$$

$$= \sum_{B,C,E} P(A)\,P(-D|E,C)\,P(B|A)\,P(C|A,B)\,P(E)$$

$$= \sum_{B,C} P(A)\,P(B|A)\,P(C|A,B)\left[P(-D|E,C)P(E)+P(-D|-E,C)P(-E)\right]$$

$$= \sum_{B} P(A)\,P(B|A)\left\{\left[P(C|A,B)\left[P(-D|E,C)P(E)+P(-D|-E,C)P(-E)\right]\right]\right.$$
$$\left.+\left[P(-C|A,B)\left[P(-D|E,-C)P(E)+P(-D|-E,-C)P(-E)\right]\right]\right\}$$

$$= P(A)\left\{\left[P(B|A)\left[P(C|A,B)\left[P(-D|E,C)P(E)+P(-D|-E,C)P(-E)\right]\right]\right.\right.$$
$$\left.+\left[P(-C|A,B)\left[P(-D|E,-C)P(E)+P(-D|-E,-C)P(-E)\right]\right]\right]$$
$$+\left[P(-B|A)\left[P(C|A,-B)\left[P(-D|E,C)P(E)+P(-D|-E,C)P(-E)\right]\right]\right.$$
$$\left.\left.+\left[P(-C|A,-B)\left[P(-D|E,-C)P(E)+P(-D|-E,-C)P(-E)\right]\right]\right]\right\}$$

$$= P(A)\left\{0.1\left\{0.05\left[(0.99\times0.35)+(0.5\times0.65)\right]+\right.\right.$$
$$\left.0.95\left[(0.25\times0.35)+(0.69\times0.65)\right]\right\}$$
$$+0.9\left\{0.5\left[(0.99\times0.35)+(0.5\times0.65)\right]+\right.$$
$$\left.\left.0.5\left[(0.25\times0.35)+(0.69\times0.65)\right]\right\}\right\}$$

$= P(A)[0.1\,(0.033575 + 0.5092) + 0.9\,(0.33575 + 0.268)]$

$= P(A)\,[0.05427 + 0.543375]$

$= P(A) \times 0.5976$

$$P(-c \mid A, -D) = \frac{P(A) \times 0.0292}{P(A) \times 0.5976}$$

$$\boxed{P(-c \mid A, -D) = 0.4888}$$

Q2.

I $\quad P(B \mid L) = \dfrac{P(B \cap L)}{P(L)}$

$P(B \cap L) = P(B \cap L \mid S)\,P(S) + P(B \cap L \mid -S)\,P(-S)$

$= P(B \mid S)\,P(L \mid S)\,P(S) + P(B \mid -S)\,P(L \mid -S)\,P(-S)$

$= (0.6 \times 0.1 \times 0.5) + (0.3 \times 0.01 \times 0.5)$

$= 0.0315$

$P(L) = P(L \mid S)\,P(S) + P(L \mid -S) \cdot P(-S)$

$= (0.1 \times 0.5) + (0.01 \times 0.5)$

$= 0.055$

$$P(B \mid L) = \frac{0.0315}{0.055}$$

$$\boxed{P(B \mid L) = 0.5727}$$

$P(B) = P(B \mid S)\,P(S) + P(B \mid -S)\,P(-S)$

$= (0.6 \times 0.5) + (0.3 \times 0.5)$

$$\boxed{P(B) = 0.45}$$

$\therefore$ Knowing that you have lung cancer increases the likelihood of having bronchitis makes sense intuitively

$\underline{\text{II}}$ $P(T|A,L,x) = \dfrac{P(T,A,L,x)}{P(A,L,x)}$

$$P(T,A,L,x) = \sum_{S,E} P(T|A)\,P(A)\,P(L|S)\,P(S)\,P(x|E)\,P(E|L,T)$$

$$= \sum_{S} P(T|A)\,P(A)\,P(L|S)\,P(S)\left[P(x|E)P(E|L,T) + P(x|-E)\,P(-E|L,T)\right]$$

$$= P(T|A)\,P(A)\left\{P(L|S)\,P(S)\left[P(x|E)P(E|L,T) + P(x|-E)\,P(-E|L,T)\right] + P(L|-S)P(-S)\left[P(x|E)\,P(E|L,T) + P(x|-E)\,P(-E|L,T)\right]\right\}$$

$$= P(T|A)\,P(A)\left\{P(L|S)\,P(S)\left[P(x|E)P(E|L,T)\right] + P(L|-S)\,P(-S)\left[P(x|E)\,P(E|L,T)\right]\right\}$$

$$= P(T|A) \times 0.1\left[(0.1\times 0.5\times 0.98\times 1) + (0.01\times 0.5\times 0.98\times 1)\right]$$

$$= P(T|A) \times 0.1\left[0.0539\right]$$

$$P(T,A,L,x) = P(T|A) \times 0.00539$$

$$P(A,L,x) = \sum_{E,T,S} P(A)\,P(L|S)\,P(S)\,P(x|E)\,P(E|L,T)\,P(T|A)$$

$$= \sum_{T,S} P(A)\,P(L|S)P(S)\,P(T|A)\left[P(x|E)P(E|L,T) + P(x|-E)\,P(-E|L,T)\right]$$

$$= \sum_{T} P(A)\,P(T|A)\left[P(L|S)\,P(S)\left[P(x|E)\,P(E|L,T) + P(x|-E)\,P(-E|L,T)\right] + P(L|-S)\,P(-S)\left[P(x|E)P(E|L,T) + P(x|-E)\,P(-E|L,T)\right]\right]$$

$$= P(A)\Big\{P(T|A)\Big[P(L|S)P(S)\big[P(x|E)\,P(E|L,T) + P(x|-E)\,P(-E|L,T)\big]$$

$$+P(L|-S)P(-S)\big[P(x|E)P(E|L,T)+P(x|-E)\,P(-E|L,T)\big]\Big]$$

$$+P(-T|A)\Big[P(L|S)P(S)\big[P(x|E)\,P(E|L,T)+P(x|-E)\,P(-E|L,T)\big]$$

$$+P(L|-S)P(-S)\big[P(x|E)\,P(E|L,T)+P(x|-E)\,P(-E|L,T)\big]\Big]\Big\}$$

$$= P(A)\Big\{P(T|A)\Big[0.1\times 0.5\big[(0.98\times 1)+(0.05\times 0)\big]$$

$$+0.01\times 0.5\big[(0.98\times 1)+(0.05)\times 0)\big]$$

$$+P(-T|A)\Big[0.1\times 0.5\big[(0.98\times 1)+(0.05\times 0)\big]$$

$$+0.01\times 0.5\big[(0.98\times 1)+(0.05\times 0)\big]\Big]\Big\}$$

$$= P(A)\Big\{P(T|A)\big[0.049+0.0049\big]$$

$$+P(-T|A)\big[0.049+0.0049\big]\Big\}$$

$$= P(A)\times 0.0539\times\big[P(T|A)+P(-T|A)\big]$$

$$= P(A)\times 0.0539 \qquad \because P(T|A)+P(-T|A)=1$$

$$= 0.1\times 0.0539$$

$$= 0.00539$$

$$P(T|A,L,x) = \frac{P(T|A)\times 0.00539}{0.00539}$$

$$= P(T|A) = 0.05$$

$$\boxed{P(T|A,L,x) = 0.05}$$

III $P(D|x) = \dfrac{P(D,x)}{P(x)}$

$$P(D,x) = \sum_{B,E} P(D|E,B)\, P(x|E)\, P(E)\, P(B)$$

$$= \sum_{B} P(B)\left[P(D|E,B)\, P(x|E)\, P(E) + P(D|-E,B)\, P(x|-E)\, P(-E)\right]$$

$$= P(B)\left[P(D|E,B)\, P(x|E)\, P(E) + P(D|-E,B)\, P(x|-E)\, P(-E)\right]$$

$$+ P(-B)\left[P(D|E,B)\, P(x|E)\, P(E) + P(D|-E,B)\, P(x|-E)\, P(-E)\right]$$

Since $P(E)$, $P(B)$, $P(T)$ and $P(L)$ are not directly available, we can find those.

$P(B) = P(B|S)\, P(S) + P(B|-S)\, P(-S)$

$= (0.6 \times 0.5) + (0.3 \times 0.5) = 0.3 + 0.15 = 0.45$

$P(B) = 0.45$

$P(T) = P(T|A)\, P(A) + P(T|-A)\, P(-A)$

$= (0.05 \times 0.01) + (0.01 \times 0.99) = 0.0005 + 0.0099$

$P(T) = 0.0104$

$P(L) = P(L|S)\, P(S) + P(L|-S)\, P(-S)$

$= (0.1 \times 0.5) + (0.01 \times 0.5) = 0.05 + 0.005$

$P(L) = 0.055$

$P(E) = P(E|L,T)\, P(L)\, P(T) + P(E|L,-T)\, P(L)\, P(-T) +$
$P(E|-L,T)\, P(-L)\, P(T) + P(E|-L,-T)\, P(-L)\, P(-T)$

$= (1 \times 0.055 \times 0.0104) + (1 \times 0.055 \times 0.9896)$
$+ (1 \times 0.945 \times 0.0104) + (0)$

$= \cancel{0.04255 + 0.02702}$

$= 0.000572 + 0.054428 + 0.009828$

$P(E) = 0.064828$

Applying the values in $P(D,x)$ and $P(x)$

$$P(D,x) = 0.45\left[(0.9 \times 0.98 \times 0.0648) + (0.8 \times 0.05 \times 0.9352)\right]$$

$$+ 0.55\left[(0.7 \times 0.98 \times 0.0648) + (0.1 \times 0.05 \times 0.9352)\right]$$

$$= 0.45\left[0.05715 + 0.037408\right] + 0.55\left[0.04445 + 0.00467\right]$$

$$= 0.04255 + 0.027$$

$$P(D,x) = 0.0695 = 0.07$$

$$P(x) = \sum_{E} P(x|E)\, P(E)$$

$$= P(x|E)\,P(E) + P(x|\neg E)\,P(\neg E)$$

$$= (0.98 \times 0.0648) + (0.05 \times 0.9352)$$

$$= 0.0635 + 0.0467$$

$$P(x) = 0.11026$$

$$P(D|x) = \frac{0.07}{0.11026}$$

$$= 0.635$$

$$\boxed{P(D|x) = 0.64}$$

IV Independencies in the graph

$E \perp S \mid L$

$E \perp A \mid T$

~~$A \perp B \mid T$~~

$A \perp E \mid T$

$A \perp D \mid E, B, X$

$A \perp E \mid X, T, L$

$A \perp B \mid X, L, S$

$A \perp S \mid X, L, B$

$X \perp A, T, L, S, B, D \mid E$

$X \perp A \mid T$

$X \perp A \mid E, T$

$X \perp T \mid E, L$

$X \perp L \mid E, T, A$

$X \perp S \mid L, B, D$

$X \perp B \mid E, A, T, D$

$X \perp D \mid E, T, L, S$

$X \perp S \mid A, L, T, B$

$B \perp A$

$B \perp X \mid E$

$B \perp A \mid T$

$B \perp X \mid E, T$

$B \perp A \mid A, X, T, L, S, E$

$D \perp X \mid E$

$D \perp A \mid T, E$

$D \perp A \mid T, S, B$

$D \perp L \mid E, A, X, T, S$

$T \perp L$

$T \perp S$

$T \perp B$

$T \perp S \mid L, B$

$T \perp L \mid S, B$

$T \perp X \mid E, A, L$

$L \perp A$

$L \perp T$

$L \perp T \mid A$

$L \perp X \mid E$

$L \perp D \mid E, B, X$

$L \perp B \mid X, A, S$

$L \perp A \mid X, T, D, X$

$S \perp A$

$S \perp T$

$S \perp X \mid E, L$

$S \perp E \mid L$

$S \perp D \mid B, L, E, X, A, T$

$S \perp E \mid A, X, B, D, T, L$

These are some of the independencies that exists.

Q. 2.4

Following independencies from the Graph:-

A = Visit to Asia
D = Dispnea
X = Positive X Ray
E = Either TB or Lung Cancer
T = Tuberculosis
L = Lung cancer
S = Smoker
B = Bronchitis

Independencies:-

~~-> Based on ch(XP) are independent / (XP)~~

L ⊥ B / S
D ⊥ X / E
D ⊥ A / T
D ⊥ L, X, T, A / S, E
D ⊥ A / S, T
D ⊥ A / X, T
D ⊥ X / E, A
D ⊥ S / L B
D ⊥ X, T, A / L, E
D ⊥ A / B, T
D ⊥ X, A / E, T
D ⊥ L, T, A / S, X, E
D ⊥ A / S, X, T
D ⊥ L, X, T / S, E, A
D ⊥ S / L, B, T, A
D ⊥ X / L, E, T, A
A ⊥ L, S, B
A ⊥ L, B / S
A ⊥ X / E
A ⊥ S, B / L
A ⊥ L, S / B
A ⊥ D, S, X, L, B, E / T

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$A \perp B / S, X$

$A \perp X, B, D / S, E$

$A \perp B / L, S$

$A \perp L / S, B$

$A \perp D, X, L, B, E / S, T$

$A \perp S, B / L, X$

$A \perp D, S, L, B, E / X, T$

$A \perp S, X, B, D / L, E$

$A \perp X, D / B, E$

$A \perp X / D, E$

$A \perp S, X, L, B, D / E, T$

$A \perp S / L, B.$

$L \perp T, A$

$L \perp B, T, A / S$

$L \perp T / A$

$L \perp X / E$

$L \perp T, A / B$

$L \perp A / T$

$L \perp B / S, X$

$L \perp B, T / S, A$

$L \perp X, B, D / S, E$

$L \perp T, A / S, B$

$L \perp B, A / S, T$

$L \perp A / X, T$

$L \perp X / E, A$

$L \perp T / B, A$

$L \perp X, D / B, E$

$L \perp X / D, E$

$L \perp X, A / E, T$

$L \perp A / B, T.$

$L \perp A / D, T$

$L \perp B / S, X, A$

$S \perp T, A$

$S \perp X / E$

$S \perp T / A$

$S \perp X, E, T, A / L$