(1) Let $n \in \mathbb{N} \centerdot f(n) = [n]$

(a) f is an injection if $\forall x, y \in \mathbb{N} \centerdot x = y \centerdot f(x) = f(y)$.

Thus, in order for f to be injective, $\forall x, y \centerdot x \neq y \centerdot f(x) \neq f(y)$ must be true.

So, let $x, y \in \mathbb{N} \centerdot x > y$. $f(n) = [n]$, so $f(x) = [x]$ and $f(y) = [y]$.

This is equivalent to $f(x) = \{1, 2, 3, ..., x\}$ and $f(y) = \{1, 2, 3, ..., y\}$

Because $x > y$, there are elements (numbers greater than y and less than x) that are in $f(x)$ and not in $f(y)$. So $f(x) \nsubseteq f(y)$.

Thus, $f(x) \neq f(y)$ when $x \neq y$ so f is injective $\square$.

(b) Using the function $f(n) = [n]$, f is not a surjection if $\exists b \in \mathbb{PN} \centerdot \forall n \in N \centerdot f(n) \neq b$.

By definition, $\{\varnothing\}$ is an element of $\mathbb{P}(\mathbb{N})$.

However, there is no $f(n)$ that can be equal to the empty set because $n \geq 1$, so $[n]$ must have elements (i.e. f(n) = $\{1\}$ when $n = 1$).

So, there is no $n \in \mathbb{N} \centerdot f(n) = \{\varnothing\}$.

Thus, f is not a surjection $\square$.