Домашнее задание

Казаков Никита

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1

$$\int \frac{x^3}{\sqrt{x-1}}dx \tag{1}$$

$||x-1=t \to dx=dt||$
$\int \frac{(t+1)^3}{\sqrt{t}}dt = \int(\sqrt{t^5}+3\sqrt{t^3}+3\sqrt{t}+\frac{1}{\sqrt{t}})dt = \int\sqrt{t^5}dt+\int 3\sqrt{t^3}dt+\int 3\sqrt{t}dt+\int\frac{1}{\sqrt{t}}dt = \frac{2\sqrt{\sqrt{t^7}}}{7}+\frac{6\sqrt{t^5}}{5}+2\sqrt{t^3}+2\sqrt{t}+c = \frac{2\sqrt{\sqrt{(x-1)^7}}}{7}+\frac{6\sqrt{(x-1)^5}}{5}+2\sqrt{(x-1)^3}+2\sqrt{(x-1)}+c$

2

$$\int \frac{\sqrt{x}}{2\sqrt{x}+3}dx \tag{2}$$

$||\sqrt{x}=t \to dx=dt||$
$2\int\frac{t^2}{2t+3}dt = 2\int(\frac{t}{2}+\frac{9}{4(2t+3)}-\frac{3}{4})dt = \int tdt+\frac{9}{2}\int\frac{1}{2t+3}dt-\frac{3}{2}\int 1dt = \frac{t^2}{2}-\frac{3}{2}t+\frac{9}{4}\ln|2t+3|+c = \frac{x}{2}-\frac{3}{2}\sqrt{x}+\frac{9}{4}\ln|2\sqrt{x}+3|+c$

3

$$\int \frac{1}{(1+x^{\frac{1}{3}})\sqrt{x}}dx \tag{3}$$

$\int\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{6}}}dx$
$||x^{\frac{1}{6}}=t \to dx=6t^5dt||$
$\int\frac{6t^5}{t^3+t}dt = 6\int\frac{t^4}{t^2+1}dt = 6\int(t^2+\frac{1}{t^2+1}-1)dt = 6(\int t^2dt+\int\frac{1}{t^2+1}dt-\int 1dt) = \frac{6t^3}{3}-6t+6\operatorname{arctg} t = \frac{6\sqrt{x}}{3}-6x^{\frac{1}{6}}+6\operatorname{arctg} x^{\frac{1}{6}}$

4

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x}}dx \tag{4}$$

$\int \sqrt{x+1}-\sqrt{x}dx = \int \sqrt{x+1}dx - \int \sqrt{x}dx = \frac{2\sqrt{(x+1)^3}}{3} - \frac{2\sqrt{x^3}}{3} + c$

5

$$\int x\sqrt{\frac{x-1}{x+1}}dx \tag{5}$$

$||\sqrt{\frac{x-1}{x+1}} = t \to x = -\frac{t^2+1}{t^2-1} \to dx = \frac{4tdt}{(t^2-1)^2}||$

$-\int \frac{t^2+1}{t^2-1}\frac{4t}{(t^2-1)^2}tdt = -4\int \frac{(t^2+1)t^2}{(t^2-1)^3}dt = 4\int(\frac{1}{8(t+1)} - \frac{3}{8(t+1)^2} + \frac{1}{4(t+1)^3} - \frac{1}{8(t-1)} - \frac{3}{8(t-1)^2} + \frac{1}{4(t-1)^3})dt = \frac{1}{2}\int \frac{1}{t+1}dt - \frac{3}{2}\int \frac{1}{(t+1)^2}dt + \int \frac{1}{(t+1)^3}dt - \frac{1}{2}\int \frac{1}{t-1}dt - \frac{3}{2}\int \frac{1}{(t-1)^2} - \int \frac{1}{(t-1)^3}dt = \frac{1}{2}\ln|\sqrt{\frac{x-1}{x+1}}+1| + \frac{3}{2\sqrt{\frac{x-1}{x+1}}+2} - \frac{1}{2(\sqrt{\frac{x-1}{x+1}}+1)^2} - \frac{1}{2}\ln|\sqrt{\frac{x-1}{x+1}}-1| - \frac{3}{2-2\sqrt{\frac{x-1}{x+1}}} + \frac{1}{2(\sqrt{\frac{x-1}{x+1}}-1)^2} + c =$

6

$$\int \frac{\sqrt{x+1}+2}{(x+1)^2\sqrt{x+1}}dx \tag{6}$$

$||\sqrt{x+1} = t = 2tdt||$

$2\int \frac{t(t+2)}{t^5}dt = 2\int \frac{t+2}{t^4}dt = 2\int \frac{1}{t^3}dt + 4\int \frac{1}{t^4}dt = -\frac{1}{t^2} - \frac{4}{3t^3} + c = -\frac{1}{x+1} - \frac{4}{3(\sqrt{x+1})^3} + c$

7

$$\int \frac{1}{(1+x^2)^3}dx \tag{7}$$

$||x = \operatorname{tg} t \to dx = \frac{dt}{\cos^2 t}||$

$\int \frac{1}{\cos^2 t(1+\operatorname{tg}^2 t)^3}dt = \int \frac{(\cos^2 t)^3}{\cos^2 t}dt = \int \cos^4 t dt = \frac{\int(3+4\cos 2t+\cos 4t)dt}{8} = \frac{3t}{8} + \frac{\sin 2t}{4} + \frac{\sin 4t}{32} + c = \frac{3\operatorname{arctg} x}{8} + \frac{\sin(2\operatorname{arctg} x)}{4} + \frac{\sin(4\operatorname{arctg} x)}{32} + c$