Домашнее задание

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1

$$\int_0^2 \frac{\sqrt{e^x}}{\sqrt{e^x + e^{-x}}} dx \tag{1}$$

$||e^x = t \to x = \ln t \to dx = \frac{1}{t} dt$
$\int \frac{\sqrt{t}}{\sqrt{t+\frac{1}{t}}t} dt = \int \frac{\sqrt{t}}{\sqrt{(t^2+1)\frac{1}{t}}t} dt = \int \frac{\sqrt{t}}{\sqrt{t^2+1}\sqrt{\frac{t^2}{t}}} dt = \int \frac{1}{\sqrt{t^2+1}} dt$
$||t = \operatorname{tg} v \to dt = \frac{1}{\cos^2 v} \to v = \operatorname{arctg} t||$
$\int \frac{1}{\sqrt{\operatorname{tg}^2 v+1} \cos^2 v} dv = \int \frac{1}{\sqrt{\frac{\sin^2 v}{\cos^2 v}+1} \cos^2 v} dv = \int \frac{1}{\sqrt{\frac{1}{\cos^2 v}} \cos^2 v} dv = \int \frac{1}{\cos v} dv$
$||\frac{1}{\cos v} = u \to du = \frac{\sin v}{\cos^2 v} \to du = \frac{\sqrt{1-\cos^2 v}}{\cos^2 v}||$
$\int u \frac{u^{-2}}{\sqrt{1-\cos^2 x}} du = \int \frac{u^{-1}}{\sqrt{1-u^{-2}}} du = \int \frac{u^{-1}}{\sqrt{u^2-1}u^{-1}} du = \int \frac{1}{\sqrt{u^2-1}} du = \ln \sqrt{u^2-1} + u + c = \ln \sqrt{(\frac{1}{\cos \operatorname{arctg} e^x})^2 - 1} + \operatorname{arctg} e^x + c = \ln \sqrt{(\frac{1}{\cos \operatorname{arctg} e^2})^2 - 1} + \operatorname{arctg} e^2 - \ln \sqrt{(\frac{1}{\cos \operatorname{arctg} 1})^2 - 1} - \operatorname{arctg} 1 = \operatorname{arctg} e^2 - \frac{\pi}{4}$

2

$$\int_0^1 \sqrt{\frac{x}{2-x}} dx \tag{2}$$

$\int_0^1 \frac{\sqrt{x}}{\sqrt{2-x}} dx$
$||\sqrt{x} = t \to dt = \frac{dx}{2\sqrt{x}}||$
$2\int \frac{t^2}{\sqrt{2-t^2}} dt$
$||t = \sqrt{2} \sin v \to v = \arcsin \frac{t}{\sqrt{2}} \to dt = \sqrt{2} \cos v dv||$
$2\int \frac{2\sin^2 v \sqrt{2} \cos v}{\sqrt{2-2\sin^2 v}} dv = 2\int sin^2 v dv = 2\int \frac{2-2\cos 2v}{2} dv = 2(\int 1 dv - \int \cos 2v) dv = 2v - \sin 2v + = 2\arcsin \frac{t}{\sqrt{2}} - t\sqrt{2-t^2} + c$
$2\arcsin \sqrt{\frac{x}{2}} - \sqrt{x}\sqrt{2-x}|_0^1 = \frac{\pi}{2} - 1$

3

$$\int_{-3}^{-2} \frac{dx}{x\sqrt{x^2-1}} \tag{3}$$

$||\sqrt{x^2-1} = t \to dt = \frac{\sqrt{x^2-1}}{x}dx||$
$\int \frac{dt}{t^2+1} = \operatorname{arctg} t + c = \operatorname{arctg}\sqrt{x^2-1} + c$
$\operatorname{arctg}\sqrt{x^2-1}|_{-3}^{-2} = \frac{\pi}{3} - \operatorname{arctg} 2\sqrt{2}$

4

$$\int_{0}^{\frac{\pi}{3}} \frac{dx}{3+2\cos 3x} \tag{4}$$

$||3x = t \to dx = \frac{dt}{3}||$
$\frac{1}{3}\int \frac{dt}{3+2\cos t}$
$||\operatorname{tg}\frac{t}{2} = v \to dt = \frac{2}{1+v^2}dv \to \cos t = \frac{1-v^2}{1+v^2}||$
$\frac{1}{3}\int \frac{\frac{1}{1+v^2}}{\frac{2-2u^2}{1+u^2}+3}dv = \frac{1}{3}\int \frac{\frac{2}{1+v^2}}{\frac{2+v^2+3}{1+v^2}}dv = \frac{1}{3}\int \frac{2}{5+v^2}dv = \frac{2}{3}\frac{1}{\sqrt{5}}\operatorname{arctg} v + c = \frac{2}{3}\frac{1}{\sqrt{5}}\operatorname{arctg}\operatorname{tg}\frac{t}{2} + c = \frac{2}{3}\frac{1}{\sqrt{5}}\frac{t}{2} + c = \frac{x}{\sqrt{5}}|_{0}^{\frac{\pi}{3}} = \frac{\pi}{3\sqrt{5}}$

5

$$\int_{0}^{\frac{1}{2}} \arccos 2x dx \tag{5}$$

$||2x = t \to dx = \frac{1}{2}dt||$
$\frac{1}{2}\int \arccos t dt = \frac{1}{2}(t\arccos t + \int \frac{t}{\sqrt{1-t^2}}dt) = \frac{t\arccos t}{2} - \frac{\sqrt{1-t^2}}{2} + c = x\arccos 2x - \frac{\sqrt{1-4x^2}}{2}|_{0}^{\frac{1}{2}} = \frac{\sqrt{1-\frac{1}{4}}}{2} - \frac{\sqrt{1}}{2} = \frac{1}{2}$