Домашнее задание

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1 9.3.4

$$y = \operatorname{tg}^2 x,\ x = \frac{\pi}{4},\ y = 0$$

$\int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x dx = \int_0^{\frac{\pi}{4}} \frac{1-\cos^2 x}{cos^2 x} dx = \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x} dx - \int_0^{\frac{\pi}{4}} 1 dx = \operatorname{tg} x|_0^{\frac{\pi}{4}} - x|_0^{\frac{\pi}{4}} = 1 - \frac{\pi}{4}$

2 9.3.12

$$y = x^2 - 2x + 3,\ y = 3x - 1$$

$x^2 - 2x + 3 = 3x - 1$
$x^2 - 5x + 4 = 0$
$\begin{cases} x = 1 \\ x = 4 \end{cases}$
$\int_1^4 |x^2 - 5x + 4| dx = |\frac{x^3}{3} - \frac{5x^2}{2} + 4x||_1^4 = |\frac{64}{3} - 40 + 16 - \frac{1}{3} + \frac{5}{2} - 4| = \frac{9}{2}$

3 9.3.21

$$\begin{cases} x = 3t^2 \\ y = 3t - t^3 \end{cases} \quad S = \int_{t_1}^{t_2} y(t)x'(t)dt$$

$$\begin{cases} t = -\sqrt{3} \\ t = \sqrt{3} \end{cases} \quad x' = 6tS = \int_{-\sqrt{3}}^{\sqrt{3}} (3t - t^3)6tdt = \int_{-\sqrt{3}}^{\sqrt{3}} 18t^2 - 6t^4 dt = (6t^3 - \frac{6t^5}{5})|_{-\sqrt{3}}^{\sqrt{3}} = \frac{36\sqrt{3}}{5} + \frac{36\sqrt{3}}{5} = \frac{72\sqrt{3}}{5}$$

4 9.3.29

$$r = 3(1 + \sin\varphi)$$
$$\alpha = 0,\ \beta = 2\pi$$
$$S = \frac{1}{2}\int_\alpha^\beta r^2 \varphi d\varphi$$

$S = \frac{1}{2}\int_0^{2\pi} (3 + 3\sin\varphi)^2 d\varphi = \frac{1}{2}\int_0^{2\pi} 9 + 18\sin\varphi + 9\sin^2\varphi d\varphi = (\frac{1}{2}(9\varphi - 18\cos\varphi + \frac{9\varphi}{2} - \frac{9\sin 2\varphi}{4}))|_0^{2\pi} = \frac{27\pi}{2} + 9 - 9 = \frac{27\pi}{2}$

5 9.3.89

$$y = \frac{1}{2}x^2 - 4x + \frac{15}{2},\ y = 0$$

$\frac{1}{2}x^2 - 4x + \frac{15}{2}$
$x^2 - 8x + 15 = 0$
$x^2 - 8x + 15 = 0$
$\begin{cases} x = 5 \\ x = 3 \end{cases}$
$L = \int_3^5 \sqrt{1 + (x-4)^2}dx$
$y' = x - 4$
$||x - 4 = t \to dx = dt||$
$\int \sqrt{1 = t^2}dt$
$||t = \operatorname{tg} v \to v = \operatorname{arctg} t \to dt = \frac{1}{\cos^2 v}dv$
$\int \frac{\sqrt{1+\operatorname{tg}^2 v}}{\cos^2 v}dv = \int \frac{1}{\cos^3 v}dv = \frac{\operatorname{tg} v}{2\cos v} + \frac{1}{2}\int \frac{1}{\cos v}dv = \frac{\operatorname{tg} v}{2\cos v} + \frac{\ln(\operatorname{tg} v + \frac{1}{\cos v})}{2} + c = \frac{v\sqrt{v^2+1}}{2} + \frac{\ln(v+\sqrt{1+v^2})}{2}|_3^6 = \frac{\sqrt{2}+\ln(1+\sqrt{2})}{2} + \frac{\sqrt{2}-\ln(\sqrt{2}-1)}{2}||$

6 9.3.145

Найти объём конуса с радиусом R и высотой H

$$V = \int_a^b S dx = \pi \int_a^b f^2(x)dx$$

$f(x) = kx = \operatorname{tg}\alpha x = \frac{R}{H}x$
$V = \pi \int_0^H (\frac{R}{H}x)^2 dx = \frac{\pi R^2}{H^2}\int_0^H x^2 dx = \frac{\pi R^2}{3H^2}x^3|_9^H = \frac{\pi R^2 H}{3}$

$$V = \frac{\pi R^2 H}{3}$$

7 9.4.150

$$x^2 + y^2 + z^2 = 25,\ y = 1,\ y = 4$$

$$V = \int_a^b S(y)dy$$

$S(y) : x^2 + z^2 = 25 - y^2 \Rightarrow r = \sqrt{25 - y^2} \Rightarrow \pi(25 - y^2)$
$V = \pi \int_1^4 (25 - y^2)dy = 2S\pi \int_1^4 dy - \pi \int_1^4 y^2 dy = 25\pi y|_1^4 - \frac{\pi}{3}y^3|_1^4 = 54\pi$

$$V = 54\pi$$

8 9.3.179

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ вокруг оси OY}$$

$$\begin{cases} x = a\cos t \\ y = b\sin t \end{cases}$$

$\frac{1}{4}V = \pi \int_0^{\frac{\pi}{2}} a^2 \cos^2 t b \cos t dt = \pi a^2 b \int_0^{\frac{\pi}{2}} \cos^3 t dt = \pi a^2 b \int_0^{\frac{\pi}{2}} (1 - \sin^2 t) d\sin t = \pi a^2 b \sin t|_0^{\frac{\pi}{2}} - \pi a^2 b \frac{\sin^3 t}{3}|_0^{\frac{\pi}{2}} = \pi a^2 b - \frac{\pi a^2 b}{3} = \frac{2\pi a^2 b}{3}$

$$V = \frac{2\pi a^2 b}{3}$$