Домашнее задание

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20.02.2020

1

$$\frac{dx}{\sqrt{x}(x^{\frac{1}{3}}+1)^2} \tag{1}$$

$||x = t^6 \to dx = 6t^5||$
$\int \frac{6t^5dt}{\sqrt{t^6}(t^2+1)^2} = \pm\int \frac{6t^2dt}{(t^2+1)^2} = \pm 6\int \frac{t^2dt}{(t^2+1)^2} = \pm 6\int \frac{t^2+1-1dt}{(t^2+1)^2} = \pm 6\int(\frac{1}{1+t^2} - \frac{1}{(1+t^2)^2})dt = \pm 6(\operatorname{arctg} t + c - \int \frac{1}{(1+t^2)^2}dt)$

$||\int \frac{1}{(1+t^2)^2}dt \to t = \operatorname{tg} v \to dt = \frac{1}{\cos^2 v} \to \int \frac{\cos^4 v}{\cos^2 v}dv = \int \cos^2 vdv = \int(\frac{1}{2}\cos 2v + \frac{1}{2})dv = \frac{1}{2}\int(\frac{1}{2}\cos 2v + \frac{1}{2})d2v = \frac{1}{4}(\sin 2v + 2v)||$

$\pm 6(\operatorname{arctg} t - \frac{1}{4}(\sin 2v + 2v)) + c = \pm 6(\operatorname{arctg} x^{\frac{1}{6}} - \frac{1}{4}(\sin(2\operatorname{arctg} x^{\frac{1}{6}}) + 2\operatorname{arctg} x^{\frac{1}{6}})) + c$

2

$$\int \frac{x^7dx}{\sqrt{x^2-1}} \tag{2}$$

$||x^2 - 1 = t^2 \to x = \sqrt{t^2+1} \to dx = \frac{t}{\sqrt{t^2+1}}||$
$\int \frac{\sqrt{t^2+1}^7 t}{\sqrt{t^2}\sqrt{t^2+1}}dt = \int \frac{(t^2+1)^3\sqrt{t^2+1}t}{|t|\sqrt{t^2+1}}dt = \pm\int(t^2+1)^3dt = \pm\int(t^6+3t^4+3t^2+1)dt = \frac{t^7}{7} + \frac{3t^5}{5} + t^3 + t + c = \frac{\sqrt{x^2-1}^7}{7} + \frac{3\sqrt{x^2-1}^5}{5} + \sqrt{x^2-1}^3 + \sqrt{x^2-1} + c$

3

$$\int \frac{dx}{x^6\sqrt{x^2-1}} \tag{3}$$

$||x = \frac{1}{\cos t} \to dx = \frac{\sin t}{\cos^2 t}||$
$\int \frac{\sin t\cos^6 t}{\sqrt{\frac{1}{\cos^2 t} - 1}\cos^2 t}dt = \int \frac{\sin t\cos^4 t}{\operatorname{tg} t}dt = \int \cos^5 tdt = \int(\cos^2 t)^2\cos tdt = \int(1 - \sin^2 t)^2\cos tdt = \int(1-\sin^2 t)^2 d\sin t = \int(1+\sin^4 t - 2\sin^2 t)d\sin t = sint + \frac{\sin^5 t}{5} - \frac{2t^3}{3} + c = \sin\arccos\frac{1}{x} + \frac{\sin^5\arccos\frac{1}{x}}{5} - \frac{2\sin^3\arccos\frac{1}{x}}{3} + c$

4

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{x}dx \tag{4}$$

$||\frac{1-x}{1+x} = t^2 \rightarrow x = \frac{1-t^2}{1+t^2} \rightarrow dx = -\frac{4t}{(1+t^2)^2}$
$\int \frac{t}{\frac{1-t^2}{1+t^2}}(-\frac{4t}{(1+t^2)^2})dt = -\int \frac{4t^2}{(1-t^2)(1+t^2)}dt = 4\int \frac{t^2}{(t^2-1)(t^2+1)}dt = 4\int \frac{1}{2}(\frac{1}{t^2-1} + \frac{1}{t^2+1})dt = 2\int(\frac{1}{t^2-1}+\frac{1}{t^2+1})dt = 2\int(\frac{1}{t^2-1}+\frac{1}{t^2+1})dt = 2(\frac{1}{2}(\log(1-t)-\log(t+1))+ \operatorname{arctg} t)+c = \log(1-t)-\log(t+1)+2\operatorname{arctg} t+c = \log(1-\sqrt{\frac{1-x}{1+x}})-\log(\sqrt{\frac{1-x}{1+x}}+1)+ 2\operatorname{arctg}\sqrt{\frac{1-x}{1+x}}+c$

5

$$\int \frac{(x^2+1)dx}{\sqrt{-x^2+3x-2}} \tag{5}$$

$\int \frac{(x^2+1)dx}{\sqrt{-x^2+3x-2}} = \int \frac{x^2+1}{\sqrt{-(x-\frac{3}{2})^1+\frac{1}{4}}}dx$
$||x-\frac{3}{2}=t||$
$\int \frac{(t+\frac{3}{2})^2+1}{\sqrt{\frac{1}{4}-t^2}}dt$
$||t = \frac{\sin v}{2} \rightarrow dt = \frac{\cos v}{2}||$
$\int \frac{((\sin v+\frac{3}{2})^2+1)\cos v}{\sqrt{\frac{1}{4}-\frac{\sin^2 v}{4}}}dv = 2\int \frac{((\sin v+\frac{3}{2})^2+1)\cos v}{\sqrt{\cos^2 v}}dv = 2\int((\sin v+\frac{3}{2})^2+1)\cos v dv = 2\int((\sin v+\frac{3}{2})^2+1)d\sin v = 2\int((\sin v+\frac{3}{2})^2+1)d(\sin v+\frac{3}{2}) = 2(\frac{(\sin v+\frac{3}{2})^3}{3} + (\sin v+\frac{3}{2}))+c = 2(\frac{(\sin\arcsin 2t+\frac{3}{2})^3}{3}+(\sin\arcsin 2t+\frac{3}{2}))+c = 2(\frac{(\sin\arcsin 2(x-\frac{3}{2})+\frac{3}{2})^3}{3}+ (\sin\arcsin 2(x-\frac{3}{2})+\frac{3}{2}))+c = 2(\frac{(2(x-\frac{3}{2})+\frac{3}{2})^3}{3}+(2(x-\frac{3}{2})+\frac{3}{2}))+c$