Домашнее задание

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27.02.2020

1

$$\int \frac{dx}{1+\sin x} \tag{1}$$

$|| \operatorname{tg} \frac{x}{2} = t \to \sin x = \frac{2t}{t^2+1} \to dx = \frac{2}{t^2+1} ||$
$\int \frac{2}{(t^2+1)(1+\frac{2t}{t^2+1})} dt = \int \frac{2(t^2+1)}{(t^2+1)((t^2+1)+2t)} dt = \int \frac{2}{t^2+1+2t} dt = \int \frac{2}{(t+1)^2} dt = 2 \int \frac{1}{(t+1)^2} dt =$
$2 \int (t+1)^{-2} dt = -2(t+1)^{-1} + c = -2(\operatorname{tg} \frac{x}{2} + 1)^{-1} + c$

2

$$\int \frac{1+\operatorname{tg} x}{1-\operatorname{tg} x} dx \tag{2}$$

$\int \frac{\operatorname{tg} \frac{\pi}{4} + \operatorname{tg} x}{\operatorname{tg} \frac{\pi}{4} - \operatorname{tg} x} dx = \int \frac{\frac{\sin \frac{\pi}{4} + x}{\cos \frac{pi}{4} \cos x}}{\frac{\sin \frac{\pi}{4} - x}{\cos \frac{pi}{4} \cos x}} dx = \int \frac{(\sin(\frac{\pi}{4}+x))(\cos \frac{pi}{4} \cos x)}{(\cos \frac{pi}{4} \cos x)(\sin(\frac{\pi}{4}-x))} dx = \int \frac{\sin(\frac{\pi}{4}+x)}{\cos(\frac{\pi}{4}+x)} dx =$
$\int \operatorname{tg}(\frac{\pi}{4} + x) dx$
$|| \frac{\pi}{4} + x = t \to x = t - \frac{\pi}{4} \to dx = 1 ||$
$\int \operatorname{tg} t dt = \int \frac{\sin t}{\cos t} dt$
$|| \cos t = v \to t = \arcsin v \to dv = -\sin x ||$
$\int -\frac{1}{v} dv = -\ln v + c = -\ln \cos t + c = -\ln \cos(\frac{\pi}{4} + x) + c$

3

$$\int \sqrt{\operatorname{tg} x} dx \tag{3}$$

$|| \sqrt{\operatorname{tg} x} = t \to x = \operatorname{arctg} t^2 dx = \frac{2t}{1+t^4} ||$
$\int \frac{2t^2}{1+t^4} dt =$

4

$$\int \sin^3 x \cos^4 x dx \tag{4}$$

$\int \sin^3 x(1-\sin^2 x)^2 dx = \int \sin^3 x - 2\sin^5 x + \sin^7 x dx = \int \sin^3 x dx - \int 2\sin^5 x dx + \int \sin^7 x dx = \int (1-\cos^2 x) d(-\cos x) - 2\int (1-\cos^2 x)^2 d(-\cos x) + \int (1-\cos^2 x)^3 d(-\cos x)$
$|| -\cos x = t||$
$\int (1+(-t)t)dt - 2\int (1+(-t)t)^2 dt) + \int (1+(-t)t)^3 dt = \int (1-t^2))dt - 2\int (1-t^2)^2 dt) + \int (1-t^2)^3 dt = t - \frac{t^3}{3} - 2(\frac{t^5}{5} - \frac{2t^3}{3} + t) - \frac{t^7}{7} + \frac{3t^5}{5} - t^3 + t + c = -\cos x - \frac{(-\cos x)^3}{3} - 2(\frac{(-\cos x)^5}{5} - \frac{2(-\cos x)^3}{3} - \cos x) - \frac{(-\cos x)^7}{7} + \frac{3(-\cos x)^5}{5} - (-\cos x)^3 - \cos x + c$

5

$$\int \frac{dx}{\sqrt{x(a-x)}} \tag{5}$$

$||\sqrt{x} = t \rightarrow x = t^2 \rightarrow dx = 2\sqrt{x}||$
$\int \frac{2}{\sqrt{a-t^2}} dt = \int \frac{2}{\sqrt{a}\sqrt{1-\frac{t^2}{a}}} = \int \frac{2}{\sqrt{1-(\frac{t}{\sqrt{a}})^2}} d(\frac{t}{\sqrt{a}}) = 2\arcsin(\frac{t}{\sqrt{a}}) + c = 2\arcsin(\frac{\sqrt{x}}{\sqrt{a}}) + c$

6

$$\int \ln(\sqrt{1-x} + \sqrt{1+x}) dx \tag{6}$$

$x\ln(\sqrt{1-x} + \sqrt{1+x}) - \frac{1}{2}\int \frac{x(\sqrt{1-x} - \sqrt{1+x})}{\sqrt{1-x^2}(\sqrt{1-x} + \sqrt{1+x})} dx = x\ln(\sqrt{1-x} + \sqrt{1+x}) - \frac{1}{2}\int \frac{x(\sqrt{1-x} - \sqrt{1+x})^2}{\sqrt{1-x^2}(\sqrt{1-x} + \sqrt{1+x})(\sqrt{1-x} - \sqrt{1+x})} dx = x\ln(\sqrt{1-x} + \sqrt{1+x}) - \frac{1}{2}\int \frac{x(\sqrt{1-x} - \sqrt{1+x})^2}{\sqrt{1-x^2}(1-x-1-x)} dx = x\ln(\sqrt{1-x} + \sqrt{1+x}) + \frac{1}{4}\int \frac{x(\sqrt{1-x} - \sqrt{1+x})^2}{\sqrt{1-x^2}x} dx = x\ln(\sqrt{1-x} + \sqrt{1+x}) + \frac{1}{4}\int \frac{(\sqrt{1-x} - \sqrt{1+x})^2}{\sqrt{1-x^2}} dx = x\ln(\sqrt{1-x} + \sqrt{1+x}) + \frac{1}{4}\int (\frac{2}{\sqrt{1-x^2}} - \frac{2\sqrt{1-x}\sqrt{1+x}}{\sqrt{1-x^2}}) dx = x\ln(\sqrt{1-x} + \sqrt{1+x}) + \frac{1}{4}\int (\frac{2}{\sqrt{1-x^2}} - \frac{2\sqrt{1-x^2}}{\sqrt{1-x^2}}) dx = x\ln(\sqrt{1-x} + \sqrt{1+x}) + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} - 1 dx = x\ln(\sqrt{1-x} + \sqrt{1+x}) + \frac{1}{2}(arcsin(x) - x) + c =$