Домашнее задание

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1

$$\int \frac{x^3}{\sqrt{x-1}}dx \tag{1}$$

$||\ x-1=t \rightarrow x=t+1 \rightarrow dx=1||$
$\int \frac{(t+1)^3}{\sqrt{t}}dt = \int t^{\frac{5}{3}} + 3t^{\frac{3}{2}} + 3t^{\frac{1}{2}} + t^{-\frac{1}{2}} = \frac{5}{8}t^{\frac{8}{5}} + \frac{3*2}{5}t^{\frac{5}{2}} + \frac{3*2}{3}t^{\frac{3}{2}} + 2t^{\frac{1}{2}} = \frac{5}{8}(x-1)^{\frac{8}{5}} + \frac{6}{5}(x-1)^{\frac{5}{2}} + 2(x-1)^{\frac{3}{2}} + 2(x-1)^{\frac{1}{2}}$

2

$$\int \frac{\sqrt{x}}{2\sqrt{x}+3}dx \tag{2}$$

$||2\sqrt{x}+3=t \rightarrow \sqrt{x}=\frac{t-3}{2} \rightarrow x=(\frac{t-3}{2})^2 \rightarrow dx=\frac{t-3}{2}||$
$\int \frac{(\frac{t-3}{2})^2}{t}dt = \int \frac{t^2-6t+9}{4t}dt = \int \frac{t}{4} - \frac{6}{4} + \frac{9}{4t}dt = \frac{t^2}{8} - \frac{6}{4}t + 9logt + c = \frac{(2\sqrt{x}+3)^2}{8} - \frac{6}{4}(2\sqrt{x}+3) + 9log(2\sqrt{x}+3) + c$

3

$$\int \frac{dx}{(1+x^{\frac{1}{3}})\sqrt{x}} \tag{3}$$

$||t=x^{\frac{1}{6}} \rightarrow t^6=x \rightarrow dx=6t^5||$
$\int \frac{6t^5}{(1+t^2)t^3}dt = \int \frac{6t^2}{1+t^2}dt = \int \frac{6(1+t^2-1)}{1+t^2}dt = 6\int 1 - \frac{1}{1+t^2}dt = 6(t - \operatorname{arctg} t) + c = 6(x^{\frac{1}{6}} - \operatorname{arctg} x^{\frac{1}{6}}) + c$

4

$$\int \frac{dx}{\sqrt{x+1}+\sqrt{x}} \tag{4}$$

$\int \frac{\sqrt{x+1}-\sqrt{x}}{(\sqrt{x+1}+\sqrt{x})(\sqrt{x+1}-\sqrt{x})}dx = \int \frac{\sqrt{x+1}-\sqrt{x}}{x+1-x}dx = \int \sqrt{x+1} - \sqrt{x}dx = \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2x^{\frac{3}{2}}}{3} + c$

5

$$\int x\sqrt{\frac{x-1}{x+1}}dx \tag{5}$$

$||t = \frac{x-1}{x+1} \rightarrow x = \frac{1+t}{1-t} \rightarrow dx = \frac{2}{(1-t)^2}||$
$\int \frac{1+t}{1-t}\sqrt{t}\frac{2}{(1-t)^2}dt = \int \frac{2(\sqrt{t}(1+t))}{(1-t)^3}dt = 2\int \frac{\sqrt{t}(1+t)}{(1-t)^3}dt = -2\int \sqrt{t}\frac{(t-1+2)}{(t-1)^3}dt = -2\int(\frac{\sqrt{t}}{(t-1)^2} + \frac{2\sqrt{t}}{(t-1)^3}dt$
$||\sqrt{t} = y \rightarrow t = y^2 \rightarrow dt = ||$

6

$$\int \frac{\sqrt{x+1}+2}{(x+1)^2\sqrt{x+1}}dx \tag{6}$$

$||t = x+1 \rightarrow x = t-1 \rightarrow dx = 1||$
$\int \frac{\sqrt{t}+2}{t^2\sqrt{t}}dt = \int t^{-2} + 2t^{-\frac{5}{2}}dt = -\frac{1}{t} - \frac{4}{3}t^{\frac{3}{2}} + c = -\frac{1}{(x+1)} - \frac{4}{3}(x+1)^{\frac{3}{2}} + c$

7

$$\int \frac{dx}{(1+x^2)^3} \tag{7}$$

$||x = \operatorname{tg} t \rightarrow dx = \frac{1}{\cos^2 t}||$
$\int \frac{1}{(\frac{1}{\cos^2 t})^3}\frac{1}{\cos^2 t}dt = \int \frac{1}{(\frac{1}{\cos^2 t})^2}dt = \int \cos^4 t dt = \int(\frac{1+\cos 2t}{2})^2 dt = \int \frac{1+2\cos 2t+\cos^2 2t}{4}dt = \int \frac{1+2\cos 2t+\frac{1+\cos 4t}{2}}{4}dt = \int \frac{2+4\cos 2t+1+\cos 4t}{8}dt = \frac{1}{8}\int 2+4\cos 2t+1+\cos 4t dt = \frac{2t+2\sin 2t+t+\frac{1}{4}\sin 4t}{8}+c = \frac{3t+2\sin 2t+\frac{1}{4}\sin 4t}{8}+c = \frac{3\operatorname{arctg}(x)+2\sin(2\operatorname{arctg}(x))+\frac{1}{4}\sin(4\operatorname{arctg}(x))}{8}+c$