1. Solve the following relation:

a.) $x(n) = x(n-1) + 5$ for $n > 1$ with $x(1) = 0$.

* write down the first two terms to identify the pattern.

$x(1) = 0$

$x(2) = x(1) + 5 = 5$.

$x(3) = x(2) + 5 = 10$.

$x(4) = x(3) + 5 = 15$.

* Identify the pattern (or) the general form.

the first term $x(1) = 0$

the common difference $d = 5$.

* the general formula for the n^{th} term of an AP is

$x(n) = x(1) + (n-1)d$.

substituting the given values

$x(n) = 0 + (n-1) \quad 5 = 5(n-1)$

the solution is $x(n) = 5(n-1)$

* $x(n) = 3x(n-1)$ for $n > 1$ with $x(1) = 4$

write down the first two terms to identify the pattern.

$x(1) = 4$

$x(2) = 3(x)(1) = 3 \times 4 = 12$.

$x(3) = 36$.

$x(4) = 108$.

* Identify the general term.

the first term $x(1) = 4$

the common ratio $r = 3$.

substituting the given values

$x(n) = 4 \cdot 3^{n-1}$

the solution is $x(n) = 4 \cdot 3^{n-1}$.

* $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (solve for $n = 2^k$).

* for $n=2^K$, we can write recurrence in terms of K.

* substitute $n=2^K$ in the recurrance.

$$x(2^K) = x(2^{K-1}) + 2^K.$$

* write down the first two terms to identify the pattern.

$$x(1)=1$$
$$x(2)=3.$$
$$x(4)=7.$$
$$x(8)=15.$$

we observe that :-

$$x(2^K) = x(2^{4-1} + 2^{K-2} + \dots$$

since $x(1)=1$!

$$x(2^K) = 2^K + 2^{K-1} + 2^{K-2} + \dots$$

the geometric series with the terms $a=2$ and the last term 2^K except for the additional $+1$ term.

the sum of a geometric series s with ratio $=2$

$$s = a\,\frac{r^{n-1}}{r-1}$$

Here $a=2$, $r=2$ and $n=K$.

$$S = 2\,\frac{2K-1}{2-1} = 2(2^K-1) = 2^{K+1}.$$

Adding the $+1$ term

$$x(2^K) = 2^{K+1} - 2 + 1 = 2^{K+1} - 1$$

solution is $x(2^K) = 2^{K+1} - 1$

* $x(n) = x(n/3)+1$ for $n>1$ with $x(1)=1$ (solve for $n=3^K$)

for $n=3^K$, we can write in terms of K.

* substitute $n=3^K$ in the recurrence.

$$x(3^K) = x(3^{K-1})+1$$

* write down the first few terms to identify the pattern.

$x(1) = 1$.

$x(3) = x(3^1) = x(1) + 1 = 2$

$x(9) = x(3^2) = x(3) + 1 = 3$.

$x(27) = x(3^3) = x(9) + 1 = 4$.

* Identify the general term

we observe that:-

$$x(3^k) = x(3^{k-1}) + 1$$

summing up the series

$x(3^k) = 1 + 1 + 1 + \quad + 1$

$x(3^k) = k + 1$

the solution is $x(3^k) = k + 1$.

2) evaluate the following recurrences complexity.

* $T(n) = T(n/2) + 1$, where $n = 2^k$ for all $k \geq 0$

the recurrence relation can be solved using iteration method

* substitute $n = 2^k$ in the recurrence.
* Iterate the recurrence

for $k = 0$: $T(2^0) = T(1) - T(1)$

$k = 1$ $T(2^1) = T(1) + 1$

$k = 2$ $T(2^2) = T(8) = T(n) + 1 = (T(1) + 2) + 1 = T(1) + 2$

$k = 3$ $T(2^3) = T(8) = T(n) + 1 = (T(1) + 2) + 1 = T(1) + 3$.

* Generalize the pattern

$T(2^k) = T(1) + k$

since $n = 2^k$, $k = \log_2 n$

$T(n) = T(2^k) = T(1) + \log_2 n$.

* Assume $T(1)$ is a constant c.

$T(n) = 0 + \log_2 n$.

the solution is $T(n) = O(\log n)$.

* the recurrence can be solved by using the master's theorem. for divide-and-conquer theorem.

$T(n) = aT(n/b) + f(n)$

where $a = 2$, $b = 3$ and $f(n) = n$

let's determine the value of $\log_b a$:

$\log_b a = \log_3 2$

using the properties of algorithms

$\log_3 2 = \frac{\log 2}{\log 3}$

Now we compare $f(n) = cn$ with $n^{\log_3 2}$.

$f(n) = O(n)$

$n = n^1$.

since $\log_3 2$ we are in the third case of the master's theorem

$f(n) = O(n^c)$ with $c > \log_b a$

the solution is:

$T(n) = O(f(n)) = O(n) = O(n)$.

* consider the following recurrence algorithm.

```
min [A(0      n-1)]
if n-1 return A[0]
else temp = min (A[0 .    n-2)
 if temp < A(n-1) return temp
else
return A(n-1).
```

* what does this algorithm compute ?

the given algorithm, min [A(0....n-1)] computes the minimum value in the array "a". from the index "0" for 'n-1' it does this by recursively finding the min value in the sub array A(0....n-2) and then comparing it with the last element A(n-1)

* set up a recurrence relation for the algorithm basic operation count and solve it.

the solution is $T(n) = n$

the means the algorithm performs n basic operations for an input array of size n.

4. Analyze the order of growth.

* $f(n) = 2n^2 + 5$ and $g(n) = 7n$ use the $\Omega g(n)$ notation.

To analyze the order of growth and use the Ω notation, we need to compare the given function f(n) and g(n).

Given functions:

$$f(n) = 2n^2 + 5.$$

$$g(n) = 7n.$$

the notation $\Omega g(n)$ describes a lower bound on the growth rate that for sufficiently large n, f(n), grows at least for as g(n).

$$f(n) = c \cdot g(n).$$

let's analyze $f(n) = 2n^2 + 5$ with respect to $g(n) = 7n$.

* Identify dominal terms:
* the dominal terms in f(n) is $2n^2$ since it grows faster than the constant terms as n increases.
* the dominal term in g(n) is 7n.
* establish the inequality :

* we want to find constants c and n_0 such that

$$2n^2+5 \geq c \cdot 7n \quad \text{for all } n \geq n_0$$

* simplify the inequality

$$2n^2 \geq 7cn.$$

* divide both side by n.

$$2n \geq 7c.$$

* solve for n

$$n \geq 7/2$$

* choose constants

let $c=1$

$$n \geq \frac{7}{2} = 3.5$$

$\therefore$ for $n \geq n$, the inequality holds.

$$2n^2+5 \geq 7n \quad \text{for all } n \geq n$$

$$2n^2+5 \geq 7n$$

thus, we can conclude that:-

$$f(n) = 2n^2+5 = \Omega(7n).$$

In Ω notation, the dominant term $2n^2$ in $f(n)$ clearly grows faster than $7n$ hence.

$$f(n) = \Omega(n^2).$$

However for the specific comparasion asked $f(n) = \Omega(7n)$ is also correct.

showing that $f(n)$ grows at least at fast as $7n$.