Summer 2019 Project 1: Martingale

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1. In Experiment 1, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning.

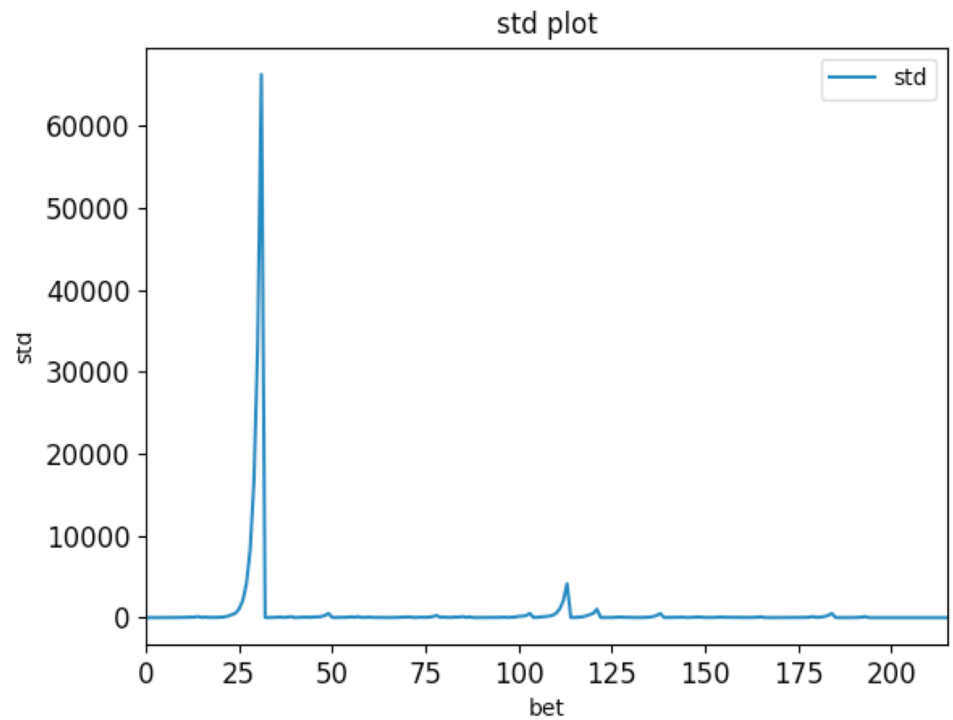
	1	2	3	4	5	6	7	8	9	10
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	1.0	1.0	-1.0	1.0	-1.0	1.0	1.0	-1.0	-1.0	1.0
2	0.0	2.0	1.0	2.0	1.0	0.0	2.0	-3.0	1.0	0.0
3	2.0	1.0	2.0	3.0	2.0	2.0	3.0	1.0	2.0	2.0
4	1.0	3.0	3.0	4.0	3.0	1.0	2.0	2.0	3.0	1.0
5	3.0	4.0	4.0	5.0	4.0	-1.0	0.0	1.0	4.0	3.0
..	...	...	...	...	...	...	...	...	...	...
142	80.0	80.0	80.0	80.0	80.0	80.0	80.0	76.0	80.0	72.0
143	80.0	80.0	80.0	80.0	80.0	80.0	80.0	78.0	80.0	80.0
144	80.0	80.0	80.0	80.0	80.0	80.0	80.0	77.0	80.0	80.0
145	80.0	80.0	80.0	80.0	80.0	80.0	80.0	79.0	80.0	80.0
146	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0

Based on my 10 times experiments, all of winnings reach cap value $80 before about 150 sequential bets. My conclusion is the probability of winning $80 within 1000 sequential bets are **100%**. This is because the strategy used in gambling. When the players lose, they keep doubling their bet amount until their next win, which will make the players gain more than what they lost before. And the chance of the next win is 48.6%, which is the probability of winning by betting on black. And without the limitation of bank roll, players are confident to keep betting until they win $80.

2. In Experiment 1, what is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning.

Based on the same results from my simple simulator shown in question 1, after 1000 sequential bets, the winnings will be $80 100%. So the expected value, which equals to the average of winnings after 1000 sequential bets are **$80**. The first reason is the strategy used in gambling. Players doubled their bet amount after lose. So when they spin win, they will get back what they lost in previous. Another reason for this guaranteed win is because the players have unlimited bank roll to bet. They don't need worry about how much they lost. So they can keep doubling their bet amount until they spin the next win.

3. In Experiment 1, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not).



Above plot shows the Standard Deviation change, it does not reach a maximum value and stabilize as the number of sequential bets increases. Standard deviation fluctuates and downs to zero until the winnings reach its max value $80. Based on the probability of 48.6% by winning the bet on black, win or lose share the similar chance for each spin. In most of case, win spin and lose spin occur one after another, so the standard deviation is close to 0. In rare case, continuing win spin or lose spin will increase standard deviation dramatically. But it will be back to zero due to the general probability of win spin and lose spin.

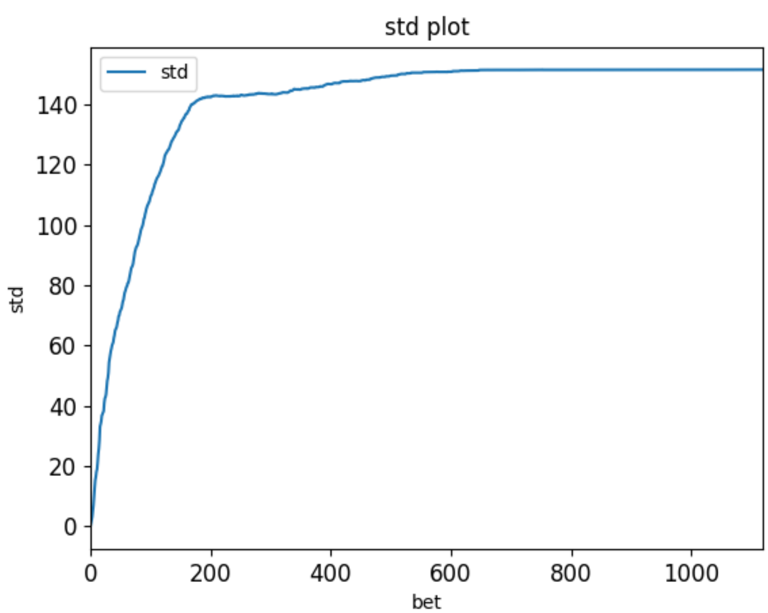
4. In Experiment 2, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning using the experiment. (not based on plots)

For estimating the probability of winning $80 after 1000 sequential bets, I run my realistic simulator 1000 times, and slide the row 1001 to calculate the percentage of winnings reaching cap value $80. I got **72.1%** of these 1000 runs reaching the cap value $80. And the rest lost all the bank roll $256. This is because the limited bank roll and the same strategy used in the first experiment. In this experiment, if the players keep losing, their balance will be not enough to allow them to double their bet amount to guarantee them get back what they lost before. When players are lucky, they spin win and increase their winnings to $80. When players are not lucky, they spin lose and their bank roll will decrease. Especially when they keep spin lose and double their bets, their bank roll will decrease very fast and they will lose all their money and get out of the game.

5. In Experiment 2, what is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning. (not based on plots)

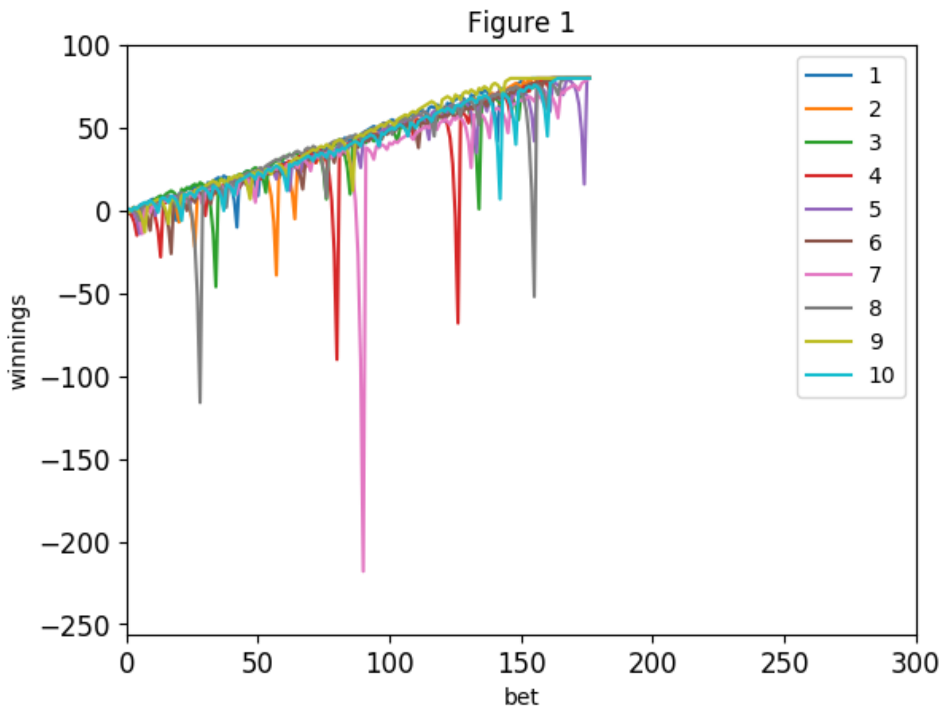
   For estimating the expected value of winning after 1000 sequential bets, I run my realistic simulator 1000 times, and slide the row 1001 to calculate the mean of winnings of this row. I got **-$13.74** from my calculation. This means we will expect a loss in this experiment in general. This is because we use the same strategy as experiment 1 but with limited bank roll. The limited bank roll limits the strategy used in gambling to get back what lost before. Also, the unbalance between cap value $80 and bank roll limitation $256 leads the expected winnings to a loss. Even though there is 70% chance the players will hit the cap value $80, 30% of total loss of $256 weighted the mean value to a negative number.

6. In Experiment 2, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not).



As shown above, the standard deviation reaches a maximum value around 180 bets and stabilized after that point. That is because in most cases, the players either win $80 or lose all their $256 bank roll after about 180 sequential bets. Double betting strategy with limited bank roll will make unlucky players lose their money very quick. After the maximum point, most of the player either keep what their max gain or lose all their money. So the data filling forward will keep the standard deviation stable as a high level.

7. Include figures 1 through 5.



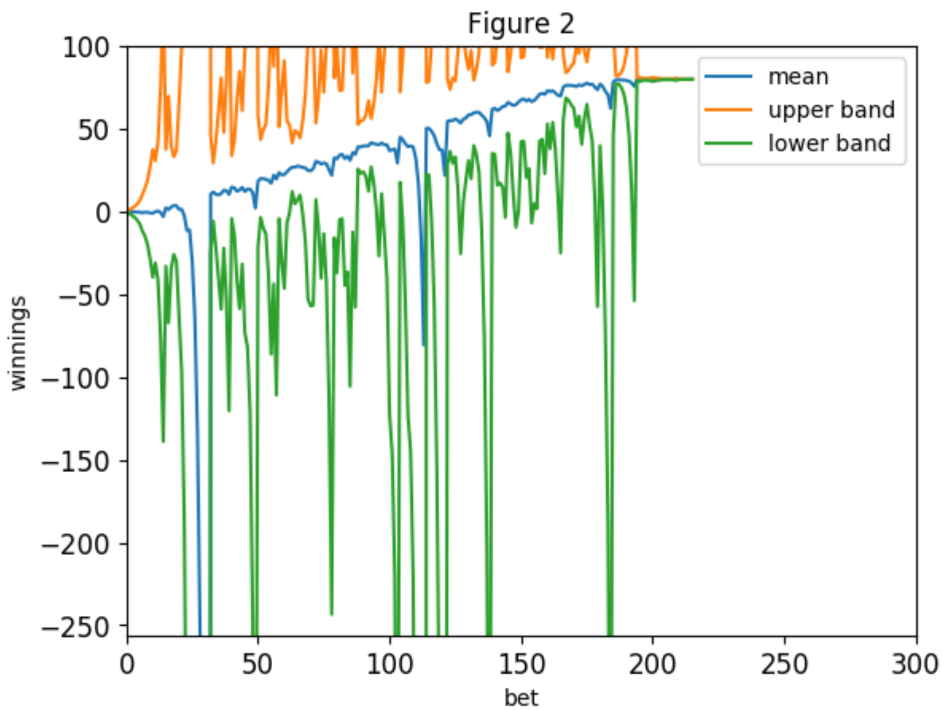


Figure 2
mean
upper band
lower band
winnings
bet
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300

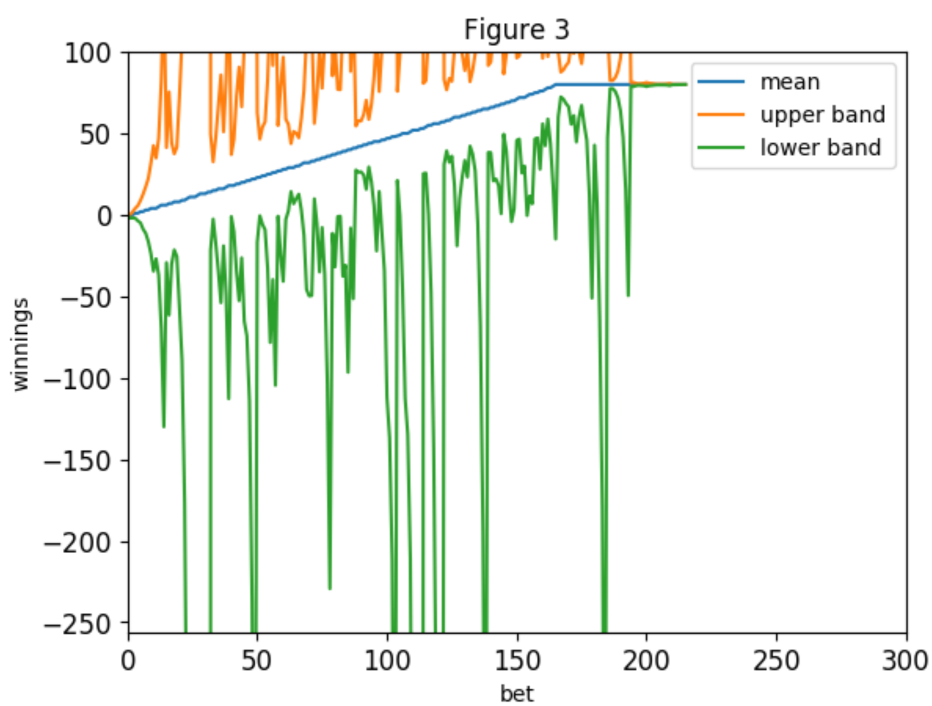


Figure 3
mean
upper band
lower band
winnings
bet
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300

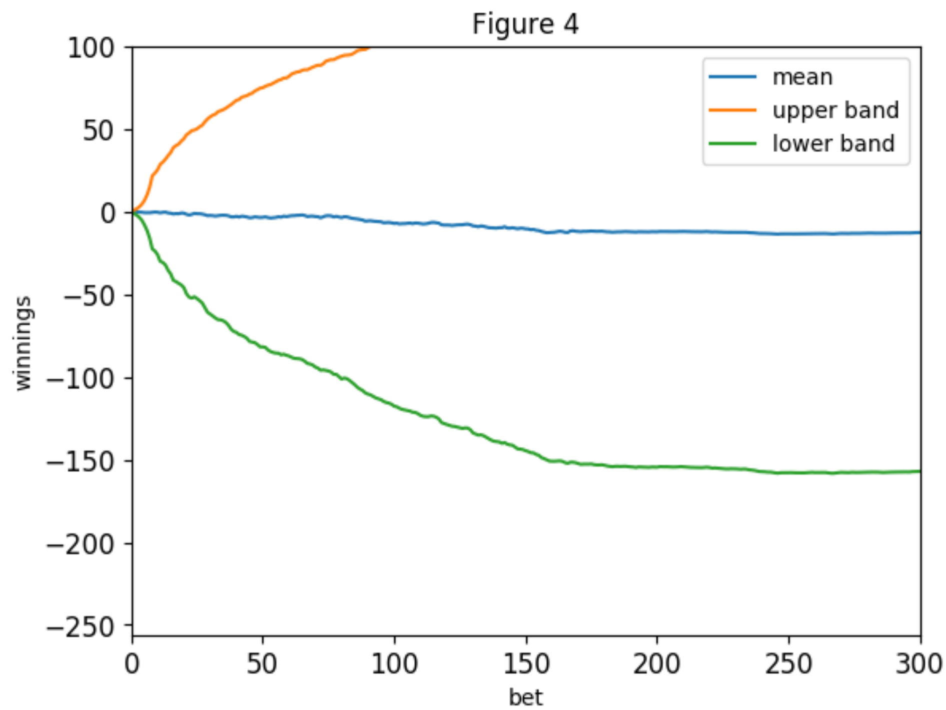


Figure 4
100
50
0
−50
−100
−150
−200
−250
winnings
0
50
100
150
200
250
300
bet
mean
upper band
lower band

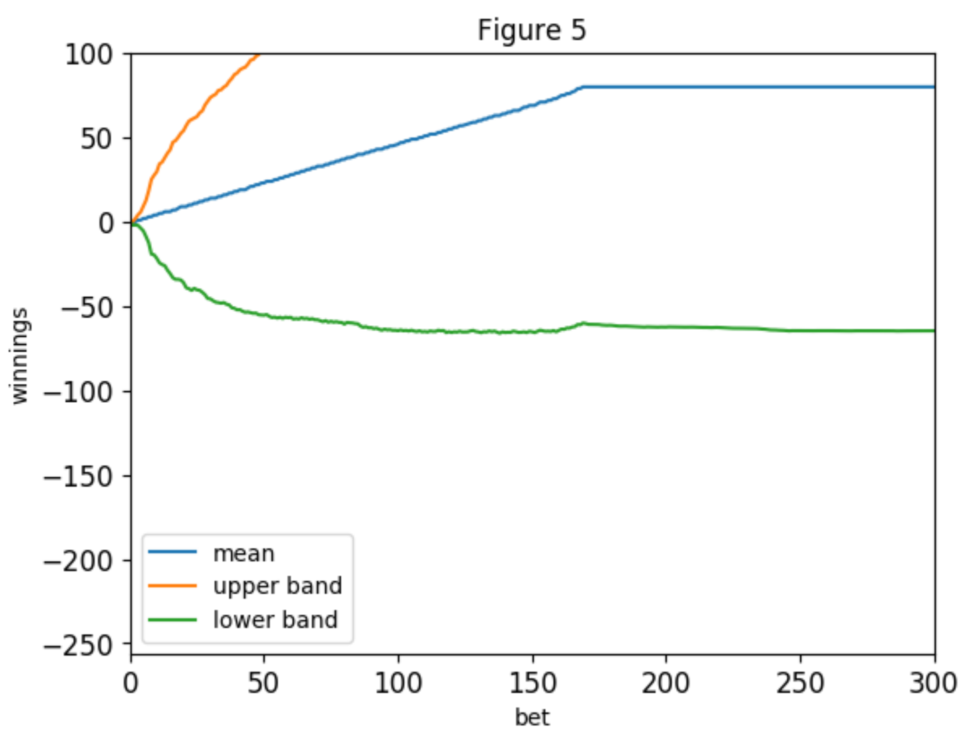


Figure 5
100
50
0
−50
−100
−150
−200
−250
winnings
0
50
100
150
200
250
300
bet
mean
upper band
lower band