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Calculating the CMB power spectrum blah blah

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ABSTRACT

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Key words. cosmic microwave background – large-scale structure of universe

Note to self: "-" is hyphen, "–" is en dash and "—" is em dash

1. Introduction

Remember to include the following in this section:

- GOAL: predict the CMB (and matter) fluctuations through the power spectrum, starting from first principles + learn about the various physical processes that is happening in order to explain the results
- divided into four steps: two concerning background cosmology and two concerning perturbations and ... (?)
- remember to state abbreviations, e.g. cosmic microwave background (CMB)

2. Background cosmology

The geometry of the background is determined by the Friedmann-Robertson-Walker (FRW) metric (Eq. (1)).

As a starting point, we use a flat universe. However, we keep the variables associated with the curvature in our deduced expressions, as they become relevant later.

After creating our cosmological model, we use observational data (Betoule et al. 2014) to find suitable cosmological parameters. In particular, we translate our model to spit out pairs of redshift and luminosity distance for a set of density parameters (today's values), and compare the computed data points with the observed ones. The best-fit model is found by performing a Monte Carlo Markov Chain (MCMC) that goes through the Metropolis algorithm REFERENCE! Remember explainantion in Implementation.

2.1. Theory

The FRW line element in flat space is given by

$$\begin{aligned} \mathrm{d}s^2 &= -c^2\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \qquad \mid \mathrm{d}\eta \equiv c\mathrm{d}t\, a^{-1}(t) \\ &= a^2(t)\left(-\mathrm{d}\eta^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right). \end{aligned} \tag{1}$$

Instead of the scale factor a, we will use its logarithm $x \equiv \ln a$ as our main time variable. In addition, the cosmic time t and Hubble parameter H $(= \mathrm{d}x/\mathrm{d}t)$ will be replaced by the conformal time η $(\mathrm{d}\eta = ce^{-x}\mathrm{d}t)$ and scaled Hubble parameter $\mathcal{H} \equiv aH$ $(= c\mathrm{d}x/\mathrm{d}\eta)$. We write the Friedmann equations in terms of our preferred variables, i.e.

$$\mathcal{H}(x) = H_0\sqrt{\Omega_{\mathrm{M}0}e^{-x} + \Omega_{\mathrm{R}0}e^{-2x} + \Omega_{k0} + \Omega_{\Lambda 0}e^{2x}}, \tag{2}$$

the components of which are to be discussed shortly. The operator

$$\frac{\mathrm{d}}{\mathrm{d}x} = \frac{c}{\mathcal{H}}\frac{\mathrm{d}}{\mathrm{d}\eta} = \frac{1}{H}\frac{\mathrm{d}}{\mathrm{d}t} \tag{3}$$

will prove useful, giving amongst others the expression

$$\eta(x) - \eta(-\infty) = \int_{-\infty}^{x}\mathrm{d}\xi\frac{c}{\mathcal{H}(\xi)}. \tag{4}$$

We assume the constituents of the universe to be cold dark matter (CDM), baryons (b), photons (γ), neutrinos (ν),[1] and a cosmological constant (Λ) dark energy comment?. We leave the curvature (k) as a variable for now. The evolution of the density parameter Ω_s associated with cosmological component $s \in \{\mathrm{CDM, b}, \gamma, \nu, \Lambda, k\}$ can be described in terms of our preferred variables as

$$\Omega_s(x) = \frac{\Omega_{s0}}{e^{(1+3w_s)x}\mathcal{H}^2(x)/H_0^2}, \quad \Omega_{s0} \equiv \Omega_s(x = x_0), \tag{5}$$

where H_0 is the Hubble constant, $x_0 = \ln a_0 = 0$ means *today* and w_s is a constant intrinsic to the species s.[2] As a notational relief, we introduce the parameters associated with total matter (M) and total radiation (R) such that

$$\Omega_{\mathrm{M}} = \Omega_{\mathrm{CDM}} + \Omega_{\mathrm{b}} \quad \text{and} \quad \Omega_{\mathrm{R}} = \Omega_\gamma + \Omega_\nu. \tag{6}$$

The observed CMB temperature today $T_{\mathrm{CMB0}} = 2.755\,\mathrm{K}$ gives today's photon density

$$\Omega_{\gamma 0} = 2\frac{\pi^2}{30}\frac{(k_b T_{\mathrm{CMB0}})^4}{\hbar^3 c^5}\frac{8\pi G}{3H_0^2}, \tag{7}$$

[1] For the sake of consistency — will not consider neutrinos fixxx
[2] $w_{\mathrm{M}} = 0$, $w_{\mathrm{R}} = 1/3$, $w_\Lambda = -1$ and $w_k = -1/3$.

and followingly the neutrino density today $\Omega_{\nu 0} \propto N_{\text{eff}}\Omega_{\gamma 0}$, N_{eff} being the effective number of massless neutrinos. However, this paper neglects neutrinos, setting $N_{\text{eff}} = 0$, and so this is not relevant for us. The total density should add up to one, so we can determine the cosmological constant through $\Omega_{\Lambda 0} = 1 - \sum_s \Omega_{s0}$. Together with current values for the remaining densities, we have the evolution of all the considered constituents' densities as functions of x. This allows us to pinpoint the time where the total matter and radiation densities are equal — the "radiation–matter equality" — as $\Omega_{\mathrm{M}}(x = x_{\mathrm{RM}}) = \Omega_{\mathrm{R}}(x = x_{\mathrm{RM}})$. Further, we find the time at which the universe becomes dominated by the cosmological constant — the "matter–dark energy transition" — as $\Omega_{\Lambda}(x = x_{\mathrm{M}\Lambda}) = \Omega_{\mathrm{M}}(x = x_{\mathrm{M}\Lambda})$.

To study the geometry of the universe, we want to know when the expansion started, i.e. when the universe started accelerating: $\mathrm{d}^2a/\mathrm{d}t^2 = 0$. It is trivial to show that this condition is equivalent to requiring $\mathrm{d}\mathcal{H}/\mathrm{d}x = 0$.[3] In Appendix A we present analytical expressions for the derivatives of $\mathcal{H}(x)$ in x.

2.1.1. Distance measures

2.2. Implementation details

2.3. Results

Here is Tab. 1

Event	**Time**		
	x	z	t
R-M equality	-8.67		
M-DE equality			
$\ddot{a} > 0$			
today			13.86 Gyrs
$\eta(0)/c$			

Table 1. The logarithmic scale factor (x), redshift (z) and cosmic time (t) of various milestones in the history of the universe.

This is Fig. 1 from (Callin 2006)

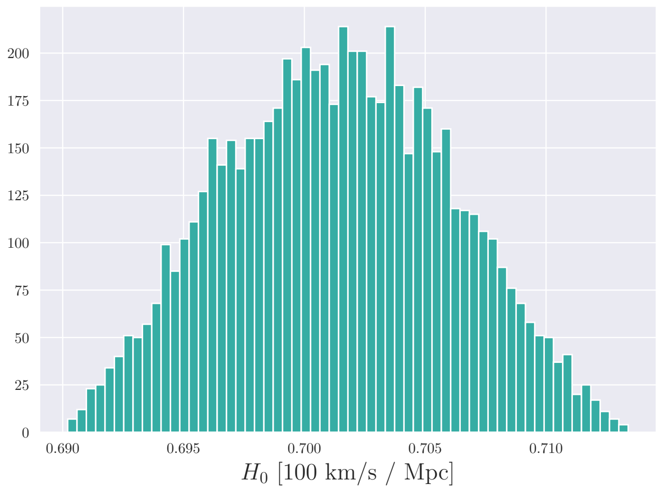

Fig. 1. Duration as

[3] It is implicit that we require $\mathcal{H}(x)$ to increase (and not decrease) at this point.

3. Conclusion

References

Betoule, M., Kessler, R., Guy, J., et al. 2014, A&A, 568, A22
Callin, P. 2006, arXiv e-prints, astro

Appendix A: Derivatives of the scaled Hubble parameter