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Spacetime Ripples from Domain-Wall Wiggles

On the analytical prediction of the gravitational-wave signature from perturbed topological defects in expanding spacetime

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Master’s presentation

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Introduction

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1. Introduction
   - Background
   - Overview
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3. Analysis

Coverage

CLASSICAL FIELD THEORY perturbation theory, action principles, topological defects

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DIFFERENTIAL GEOMETRY pseudo-Riemannian manifolds, conformal transformations, hypersurfaces

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MODERN COSMOLOGY concordance model, gravitational waves

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MODERN COSMOLOGY concordance model, gravitational waves

Background

- What is GR?

Background

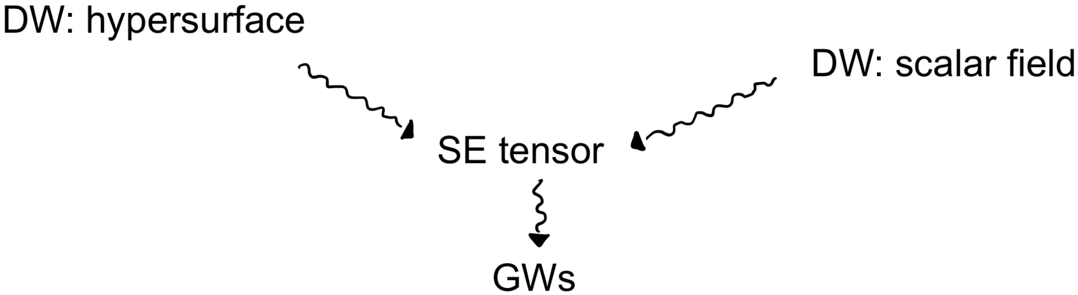
- What is GR?
- What are GWs?
- Symmetron?

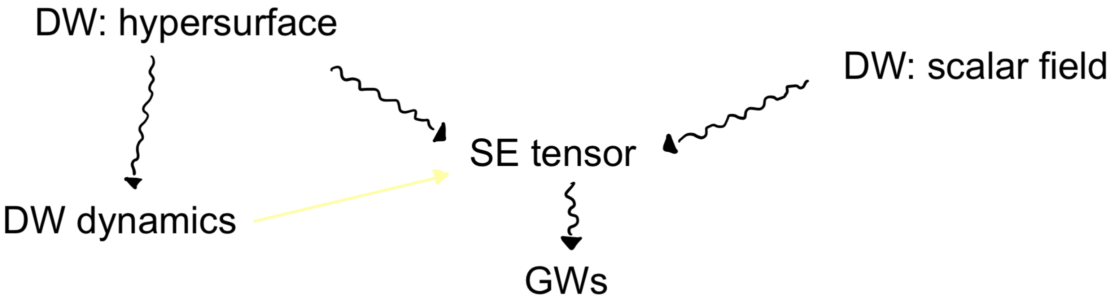
Background

- What is GR?
- What are GWs?
- Symmetron?
- Hubble tension + PTAs

New physics

About GWs as probe for new physics


DW: hypersurface
DW: scalar field
SE tensor
GWs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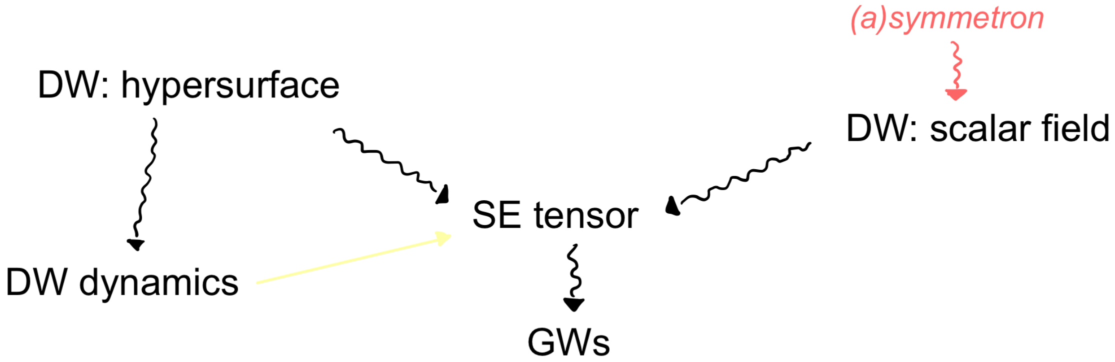

(a)symmetron
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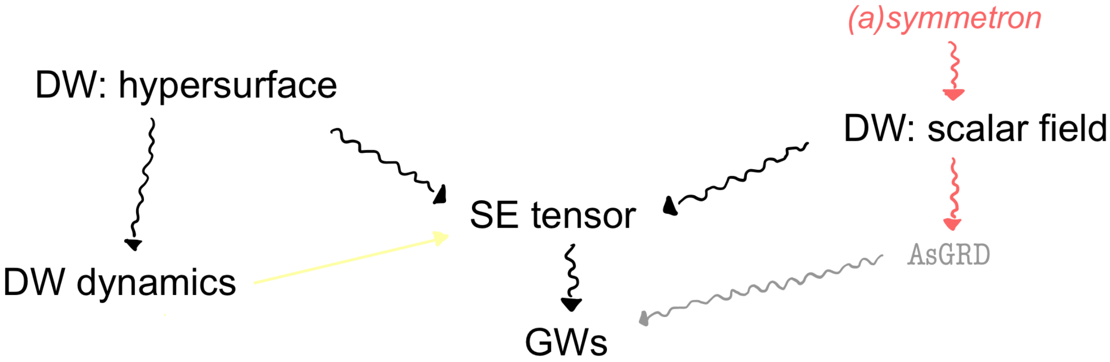
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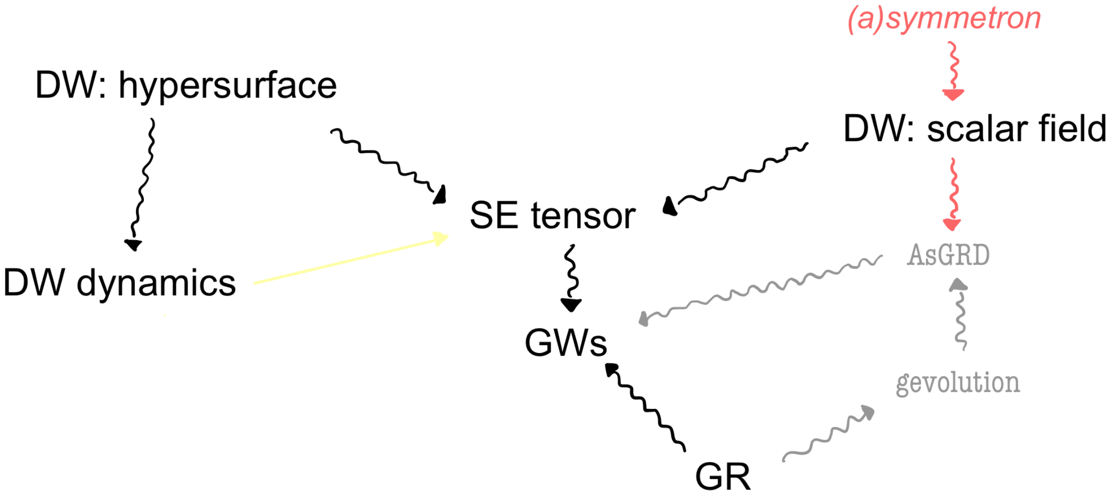
AsGRD
GWs



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AsGRD
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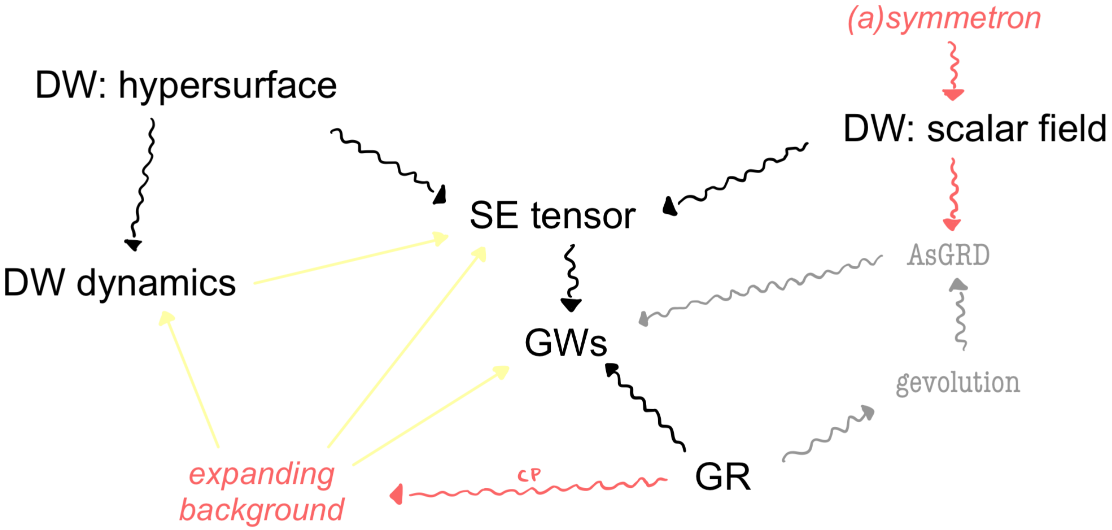
gevolution
GR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expanding background
CP
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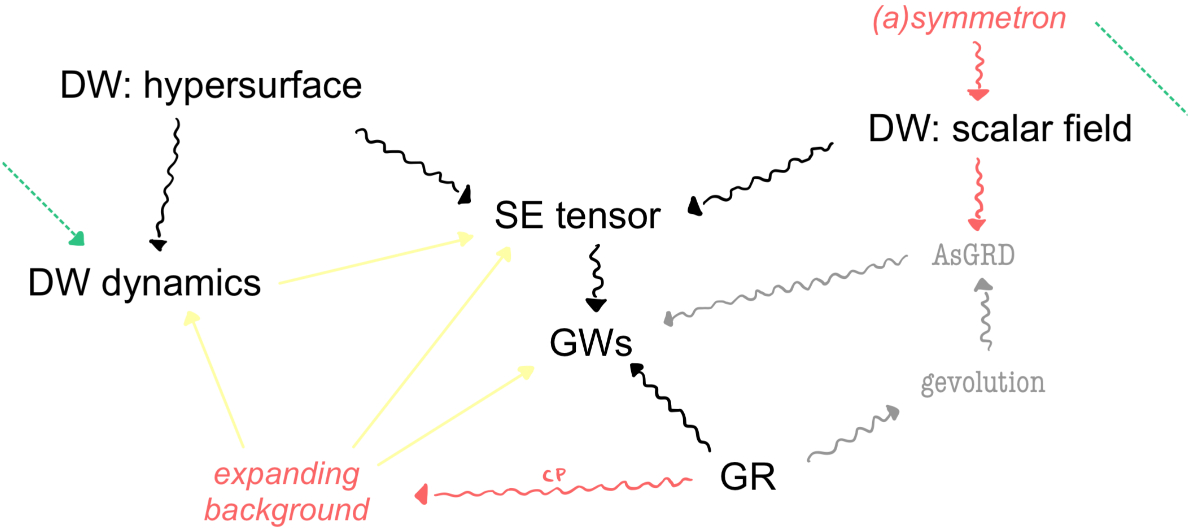

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Project

Contents

1 Introduction

2 Project
- DW $\leftrightarrow$ hypersurface
- DW $\leftrightarrow$ scalar field

3 Analysis

General setup:

- Symmetron
- hypersurface
-

DW $\leftrightarrow$ hypersurface

$$S_{\mathrm{NG}} = -\sigma \int_\Sigma d^3\xi \sqrt{-\gamma} = -\sigma \int_\Sigma d^3\xi \sqrt{-\hat{\gamma}}\, a^3 \tag{1}$$

$$\ddot{\epsilon} + \mathcal{D}(\tau)\dot{\epsilon} - \left[\partial_x^2 + \partial_y^2\right]\epsilon = 0 \tag{2}$$

where $\mathcal{D}(\tau) = 3\dot{a}/a$.

DW ↔ hypersurface

$$S_{\mathrm{NG}} = -\sigma \int_{\Sigma} d^3\xi \sqrt{-\gamma} = -\int_{\Sigma} d^3\xi \sqrt{-\hat{\gamma}}\, a^3 \sigma \tag{1}$$

$$\ddot{\epsilon} + \mathcal{D}(\tau)\dot{\epsilon} - \left[\partial_x^2 + \partial_y^2\right]\epsilon = 0 \tag{2}$$

where $\mathcal{D}(\tau) = 3\dot{a}/a + \dot{\sigma}/\sigma$.

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DW $\leftrightarrow$ hypersurface

Separabel eq etc.

DW ↔ hypersurface

Separabel eq etc.

Symmetron domain walls have $\sigma = \sigma_\infty \sqrt{1 - \upsilon}$ where $\upsilon \equiv (a_*/a)^3$, and this gives

$$\varepsilon'' + \left(\frac{3\alpha}{s} + 2d(s)\right)\varepsilon' + \omega^2 \varepsilon = 0; \quad d(s) \triangleq \frac{9\alpha}{4s(s^{3\alpha} - 1)}. \tag{3}$$

DW $\leftrightarrow$ hypersurface

Separabel eq etc.

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Set $\alpha = 2$.

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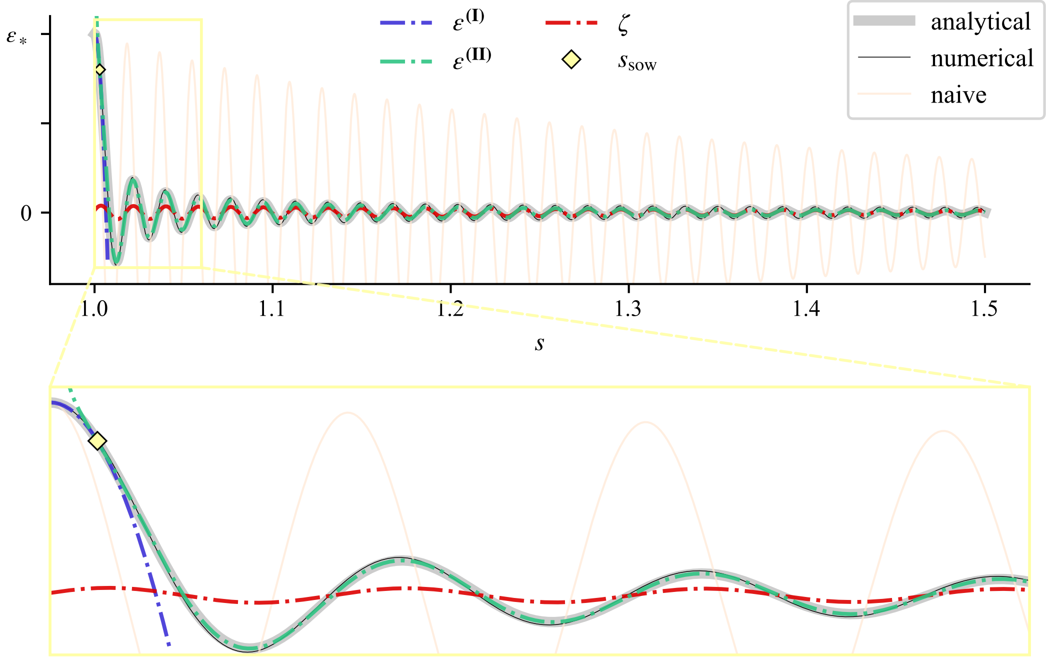

ε_*
0
$\varepsilon^{(\mathbf{I})}$
$\varepsilon^{(\mathbf{II})}$
ζ
s_{sow}
analytical
numerical
naive
1.0
1.1
1.2
1.3
1.4
1.5
s

Stress–energy tensor

$$T_{ij}^{\mathrm{TT}}\big|_{\mathrm{NG}} = \tag{4}$$

Gravitational waves

$$\tilde{h}_{ij}(\tau, \boldsymbol{k}) \sim \int_{\tau_*}^{\tau} \mathrm{d}\hat{\tau}\, \sqrt{k\hat{\tau}} J_\nu(k\hat{\tau}) \tilde{T}_{ij}^{\mathrm{TT}}(\hat{\tau}, \boldsymbol{k}) \tag{5}$$

DW ↔ scalar field

Analysis

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1. Introduction
2. Project
3. Analysis
   - The code
   - Findings

The code

`AsGRD`, based on `gevolution`, computes the full metric perturbations with the asymmetron blahblah

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`AsGRD`, based on `gevolution`, computes the full metric perturbations with the asymmetron blahblah

Periodic BCs $\Rightarrow$ at least 2 walls. In two dim.:

Experiments

Cubic simulation box of side lengths $L_\#$ (fundamental frequency $k_\# = 2\pi/L_\#$) with $N_\#^3$ lattice points, with initial**(explain)** perturbation $\epsilon(\tau_*, x, y) = \varepsilon_* \sin py$. Fiducial symmetron parameters are $a_* = 0.33$, $\xi_* = 3.33 \times 10^{-4}$ and $\beta_* = 1$. Simulation onset is at scale factor $a_\mathrm{i} \gtrsim a_*$, and we finish at $a_\mathrm{f} = 0.50$.

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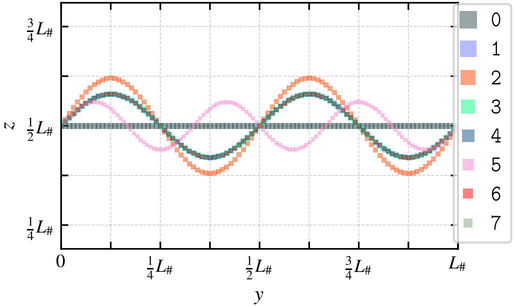
$\varepsilon_* = 0.08 L_{\#}$, $p = 2k_{\#}$, $L_{\#} \approx 1$ Gpc/h, $N_{\#} = 768$, $a_\mathrm{i} \simeq a_* + 0.003$

Experiments

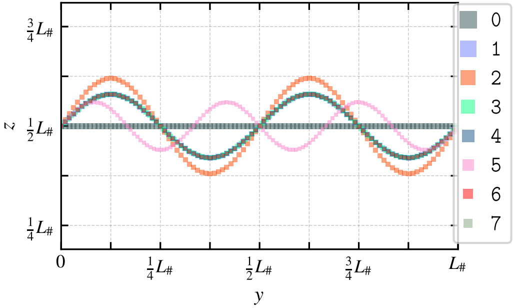
Cubic simulation box of side lengths $L_{\#}$ (fundamental frequency $k_{\#} = 2\pi/L_{\#}$) with $N_{\#}^3$ lattice points, with initial**(explain)** perturbation $\epsilon(\tau_*, x, y) = \varepsilon_* \sin py$. Fiducial symmetron parameters are $a_* = 0.33$, $\xi_* = 3.33 \times 10^{-4}$ and $\beta_* = 1$. Simulation onset is at scale factor $a_\mathrm{i} \gtrsim a_*$, and we finish at $a_\mathrm{f} = 0.50$.

1	$\varepsilon_* = 0.08 L_{\#}$, $p = 2k_{\#}$, $L_{\#} \approx 1$ Gpc/h, $N_{\#} = 768$, $a_\mathrm{i} \simeq a_* + 0.003$
2	$\varepsilon_* \longrightarrow 0.12 L_{\#}$
3	$N_{\#} \longrightarrow 900$
4	$L_{\#} \longrightarrow 1.4$ Gpc/h
5	$p \longrightarrow 3k_{\#}$, $\varepsilon_* \longrightarrow 0.06 L_{\#}$, $N_{\#} \longrightarrow 900$
7	$(a_\mathrm{i} - a_*) \longrightarrow 0.001$

1	—
2	$\varepsilon_* \nearrow$
3	$N_\# \nearrow$
4	$L_\# \nearrow$
5	$p \nearrow \quad \varepsilon_* \searrow \quad N_\# \nearrow$
7	$(a_\mathrm{i} - a_*) \searrow$



0	$\varepsilon_* \searrow$		
1	—		
2	$\varepsilon_* \nearrow$		
3	$N_\# \nearrow$		
4	$L_\# \nearrow$		
5	$p \nearrow$	$\varepsilon_* \searrow$	$N_\# \nearrow$
6	$\xi_* \searrow$		
7	$(a_\mathrm{i} - a_*) \searrow$		



Time measures

SCALE FACTOR $a = (1 + \mathfrak{z})^{-1} = a_* s^\alpha$

COSMIC REDSHIFT $\mathfrak{z} = 1/a - 1$

CONFORMAL TIME $s = \tau/\tau_*$

SCALE-DEPENDENT TIME PARAMETER $t_\omega \equiv \omega(s-1) = p(\tau - \tau_*)$

PHASE-TRANSITION TIME-SCALE $\chi_+ \equiv \sqrt{1-\upsilon} = \sqrt{1-(a_*/a)^3} = \sqrt{1-s^{-3\alpha}}$

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Matter domination $\leftrightarrow \alpha = 2$

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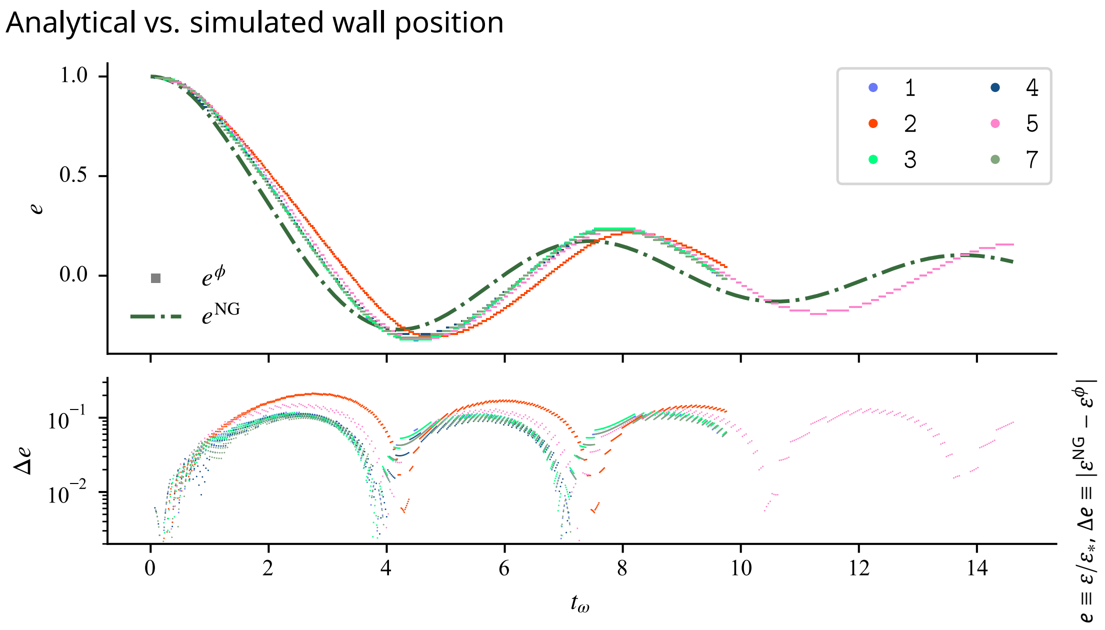

Analytical vs. simulated wall position
1.0
0.5
0.0
e
e^{ϕ}
e^{NG}
1
2
3
4
5
7
10^{-1}
10^{-2}
Δe
0
2
4
6
8
10
12
14
t_{ω}
$e \equiv \varepsilon/\varepsilon_{*}$, $\Delta e \equiv \left|\varepsilon^{\mathrm{NG}} - \varepsilon^{\phi}\right|$

Main results

	Calculation	**Simulation**
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
e independent of ε_*	yes	no
Gravitational waves	$\tilde{h}_+^{\mathrm{NG}}$ $(\tilde{h}_+^{\phi})$	$\tilde{\mathtt{h}}_+$

Main results

	Calculation	**Simulation**
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
e independent of ε_*	yes	no
$e \nsim s^{-5/2} Z_{-5/2}(\omega s)$	yes	yes
Gravitational waves	$\tilde{h}_+^{\mathrm{NG}}$ $(\tilde{h}_+^{\phi})$	$\tilde{\mathtt{h}}_+$

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Main results

	Calculation	**Simulation**
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
e independent of ε_*	yes	no
$e \nsim s^{-5/2}\mathrm{Z}_{-5/2}(\omega s)$	yes	yes
(e vs. t_ω)-plot indep. of parameters	yes	yes*
Gravitational waves	$\tilde{h}_+^{\mathrm{NG}}\ (\tilde{h}_+^{\phi})$	$\tilde{\mathtt{h}}_+$
?	yes	yes*

* To some extent.

$\varepsilon^{\mathrm{NG}}$ vs $\varepsilon^{\mathrm{NG}}$ vs $\varepsilon^{\mathrm{NG}}$

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