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Spacetime Ripples from Domain-Wall Wiggles

On the analytical prediction of the gravitational-wave signature from perturbed topological defects in expanding spacetime

Nanna Bryne

Institute of Theoretical Astrophysics

Master’s presentation

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Contents

Background

Contents

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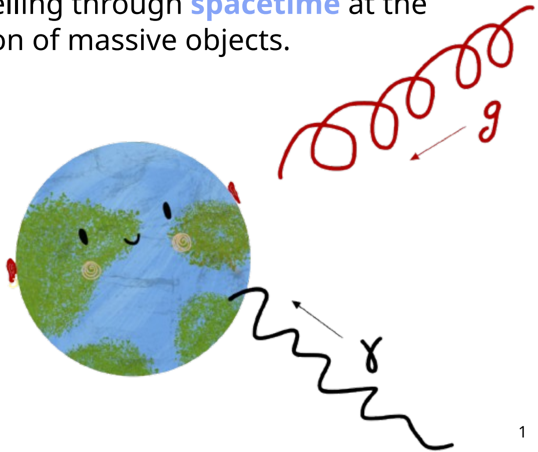

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But why?

Motivation

RELEVANCE Recent observations* provide evidence for a stochastic GW background (GWB) in cosmological frequency ranges. Future GW experiments aim at similar frequency ranges.

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*Agazie et al. ((2023))

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In **general relativity** (GR), gravitation is interpreted as an intrinsic property of spacetime curvature.

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In **general relativity** (GR), gravitation is interpreted as an intrinsic property of spacetime curvature. The existence of GWs is a consequence of GR.

General relativity

Curvature of spacetime manifold $\mathscr{M}$ is described by

$$\overbrace{ds^2}^{\text{line element}} = \overbrace{g_{\mu\nu}}^{\text{metric}} \mathrm{d}x^{\mu} \mathrm{d}x^{\nu}. \tag{1}$$

The components obey the Einstein equations:

$$\mathcal{G}_{\mu\nu} = 8\pi G_{\mathrm{N}} T_{\mu\nu}. \tag{2}$$

Here, $\mathcal{G}_{\mu\nu} = \mathcal{R}_{\mu\nu} - g_{\mu\nu}\mathcal{R}/2$ is the Einstein tensor, $\mathcal{R}_{\mu\nu}$ ($\mathcal{R}$) the Ricci tensor (scalar), and $T_{\mu\nu}$ the stress–energy (SE) tensor.

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SPECIAL RELATIVITY. $g_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$ = Minkowski metric

FLAT, EXPANDING UNIVERSE. $g_{\mu\nu} = a^2(x^0)\eta_{\mu\nu}$ = (flat) FLRW metric

Quick look: Metric perturbations

Under $g_{\mu\nu} \longrightarrow \mathring{g}_{\mu\nu} = g_{\mu\nu} + \delta g_{\mu\nu}$, we get the linearised Einstein equations

$$\mathring{\mathcal{G}}_{\mu\nu} = 8\pi G_{\mathrm{N}} \mathring{T}_{\mu\nu} \,. \tag{3}$$

Minkowski. $\mathring{g}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Flat FLRW. $\mathring{g}_{\mu\nu} = a^2(x^0)\left[\eta_{\mu\nu} + h_{\mu\nu}\right]$

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Background cosmology

COSMOLOGICAL PRINCIPLE. *The universe is homogeneous and isotropic.*

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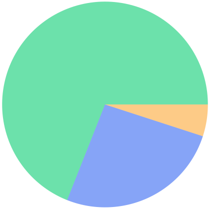
(today)

$$H^2(a) = H_0^2 \sum_s \Omega_{s0} a^{-3(1+w_s)} = H_0^2 \left(\Omega_{m0} a^{-3} + \Omega_{de0} \right). \tag{4}$$

ΛCDM. Dark energy (DE) ↔ a cosmological constant (Λ) + baryonic & cold dark matter (CDM).

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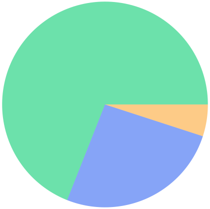

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• **Hubble tension.** There is a discrepancy between measurements of the Hubble constant $H_0 = 100\text{h km s}^{-1}\text{ Mpc}^{-1}$:

$$\text{“Late universe” measurements} \rightsquigarrow \text{h} \sim 0.73 \pm 0.02$$

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•POSSIBLE SOLUTION. A common work-around is to let DE have time-dependent e.o.s. parameter $w_{\text{de}} = w_{\text{de}}(a)$, which can be ensured by a dynamical scalar field ϕ (cf. (a) **symmetron** ,* quintessence, phantom dark energy).

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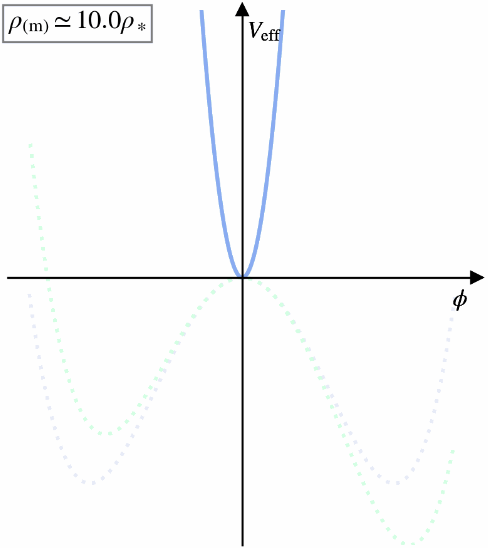
*Hinterbichler et al. ((2011)); Perivolaropoulos & Skara ((2022))

The symmetron effective potential is

$$V_{\text{eff}}(\phi) = \frac{\lambda}{4}\phi^4 + \frac{\mu^2}{2}(\upsilon - 1)\phi^2, \quad (5)$$

where $\upsilon \equiv \rho_{(\text{m})}/\rho_* \simeq (a_*/a)^3$.

Symmetry breaking at critical density ρ_*.



The asymmetron effective potential is

$$V_{\text{eff}}(\phi) = \frac{\lambda}{4}\phi^4 - \frac{\kappa}{3}\phi^3 + \frac{\mu^2}{2}(\upsilon - 1)\phi^2, \quad (5)$$

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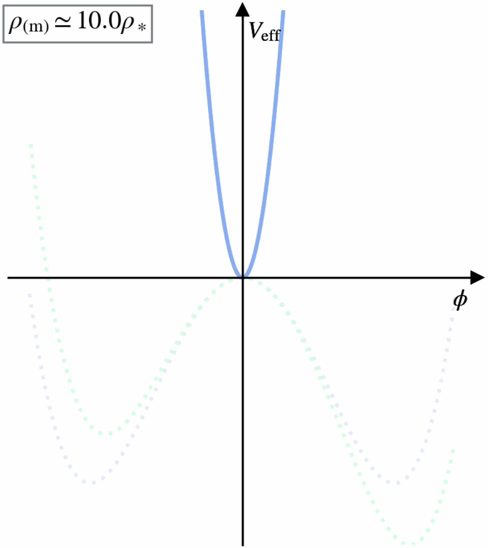
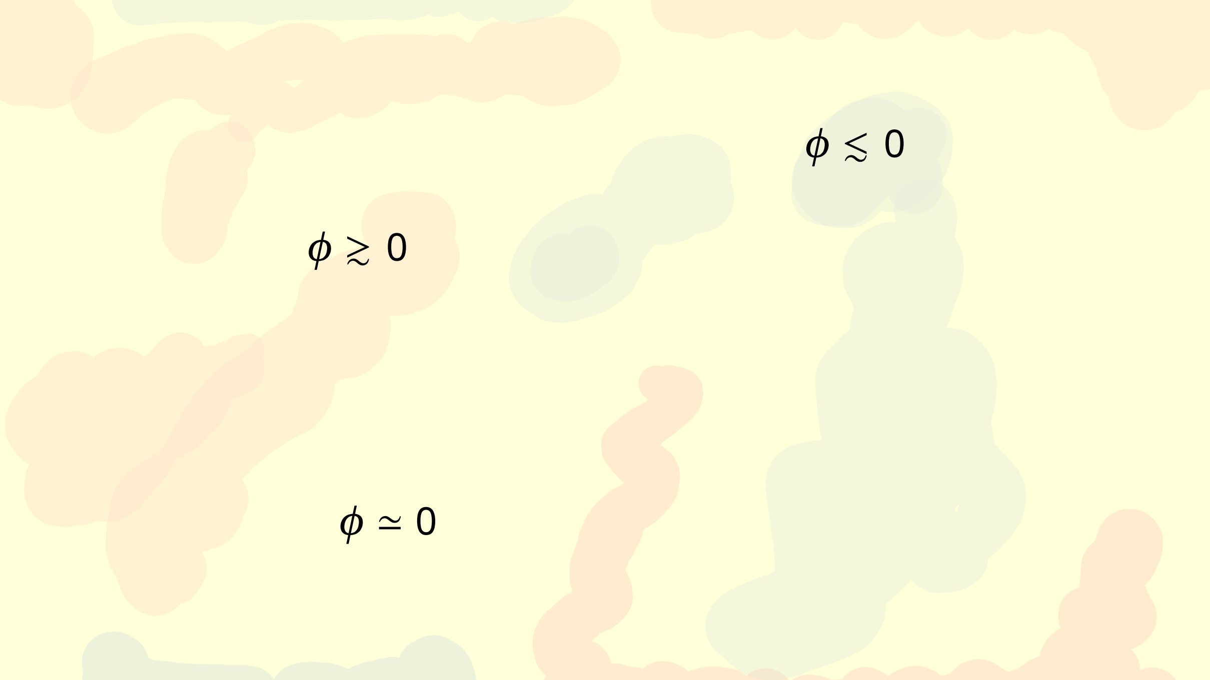
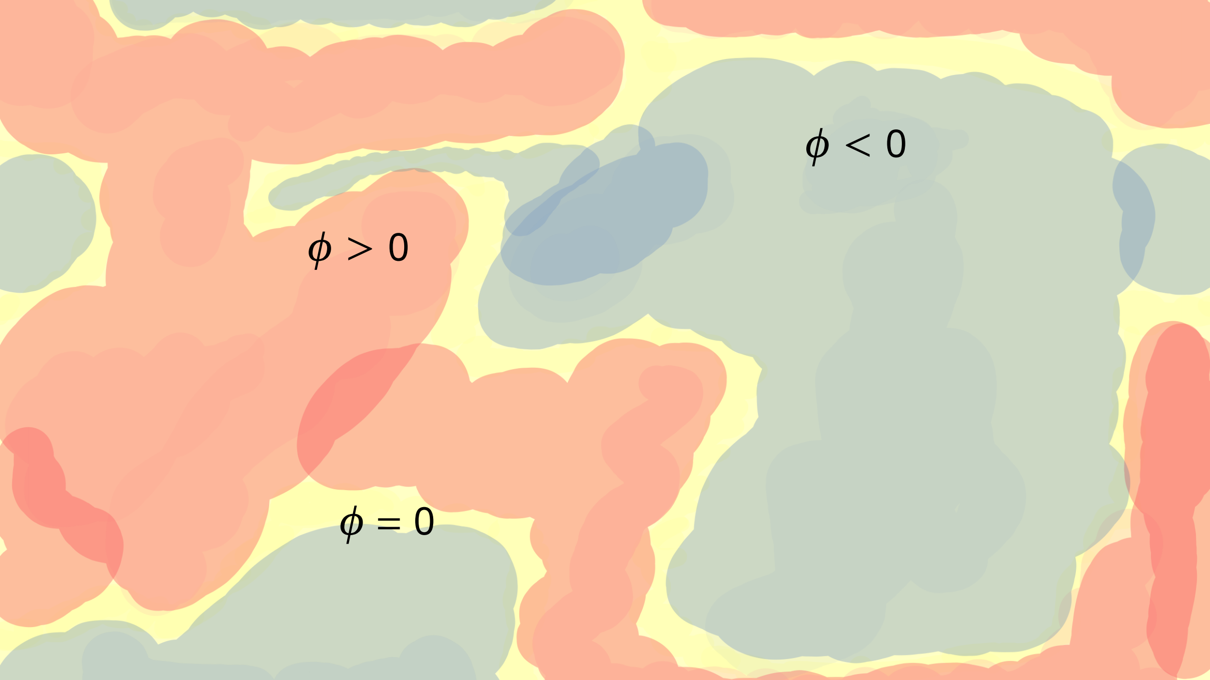
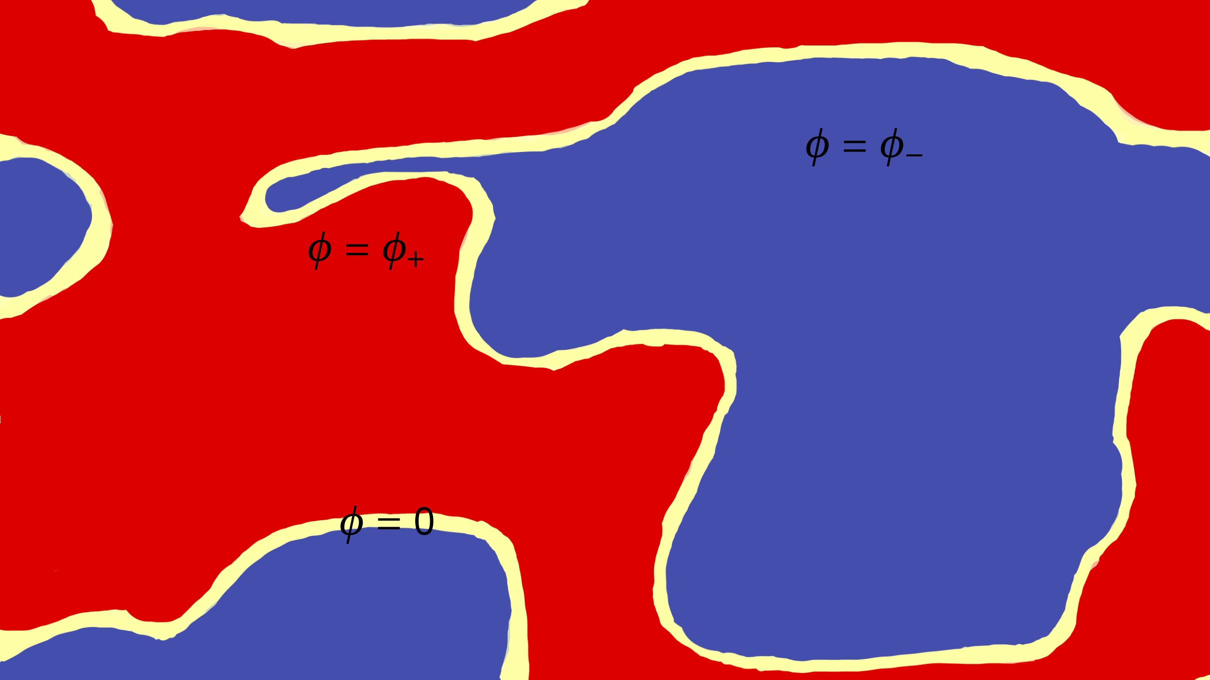



$\phi \lesssim 0$
$\phi \gtrsim 0$
$\phi \simeq 0$



$\phi < 0$
$\phi > 0$
$\phi = 0$



$\phi = \phi_-$
$\phi = \phi_+$
$\phi = 0$

The domain-wall ripple effect

GWs from DWs are expected to be within reach of future experiments.

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$*$**OVERCLOSING PROBLEM.** A DW network modelled as a perfect fluid will have e.o.s.-parameter $w_{\rm dw} = -2/3$. As contributor to the total energy budget of the universe, it would quickly dominate.

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$*$**OVERCLOSING PROBLEM.** A DW network modelled as a perfect fluid will have e.o.s.-parameter $w_{\mathrm{dw}} = -2/3$. As contributor to the total energy budget of the universe, it would quickly dominate.

$*$**PROPOSED SOLUTIONS.** Slightly broken vacuum degeneracy; $V(\phi_+) \neq V(\phi_-)$ (e.g. asymmetron), or let walls melt away.

Project

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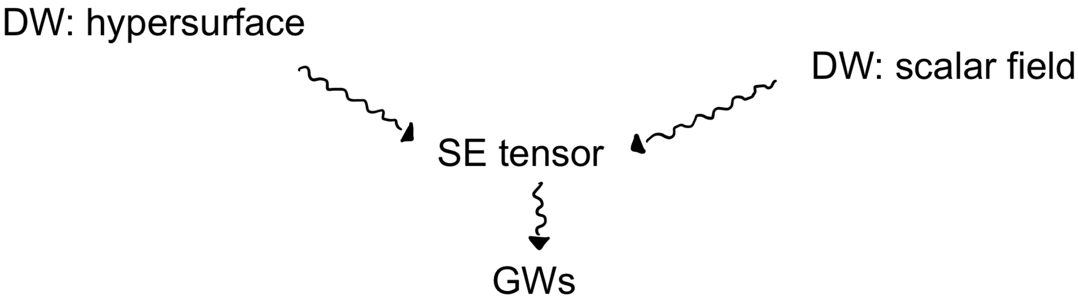
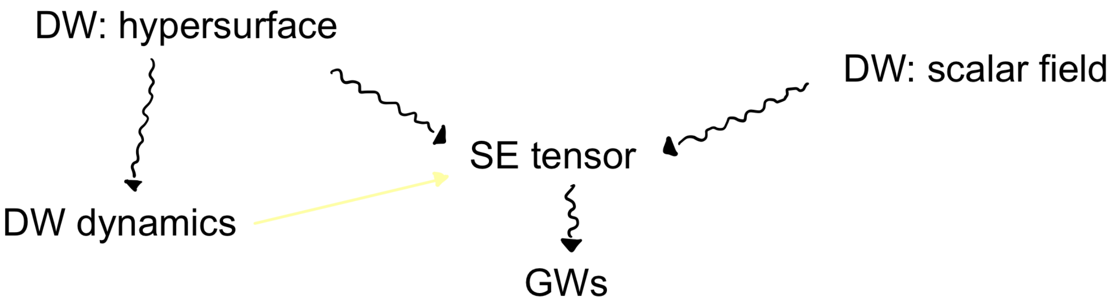

DW: hypersurface
DW: scalar field
SE tensor
GWs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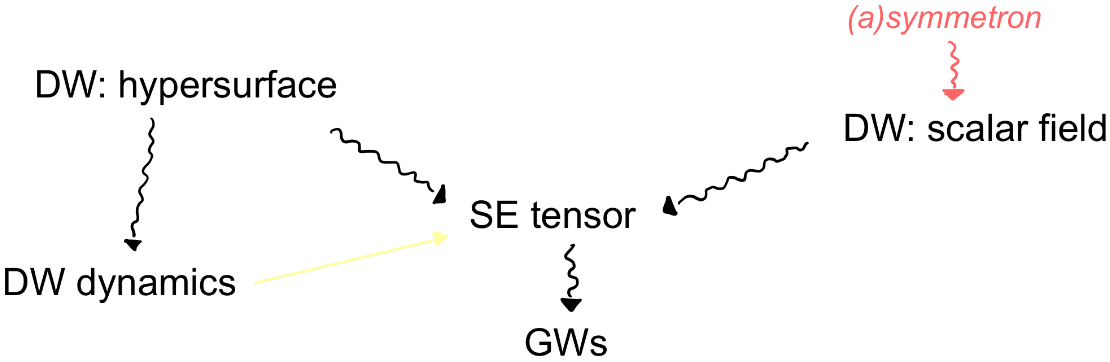
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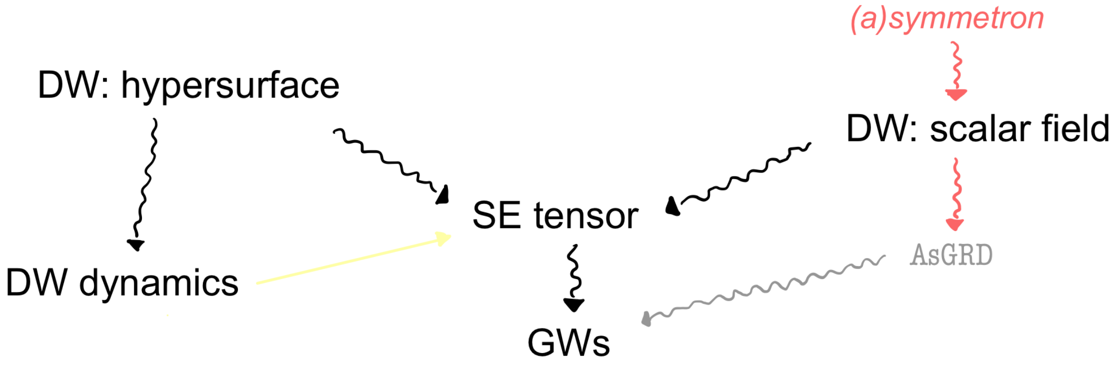
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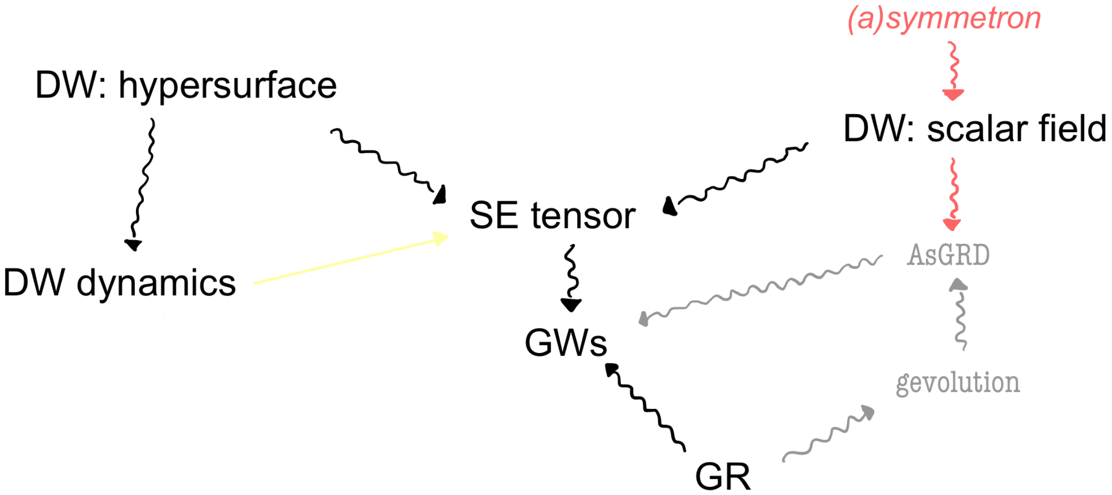

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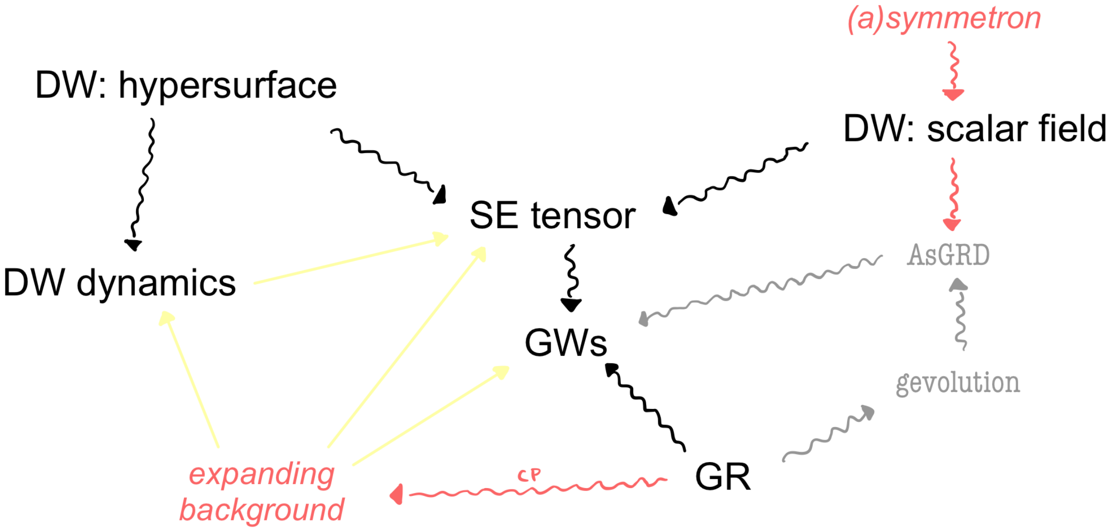

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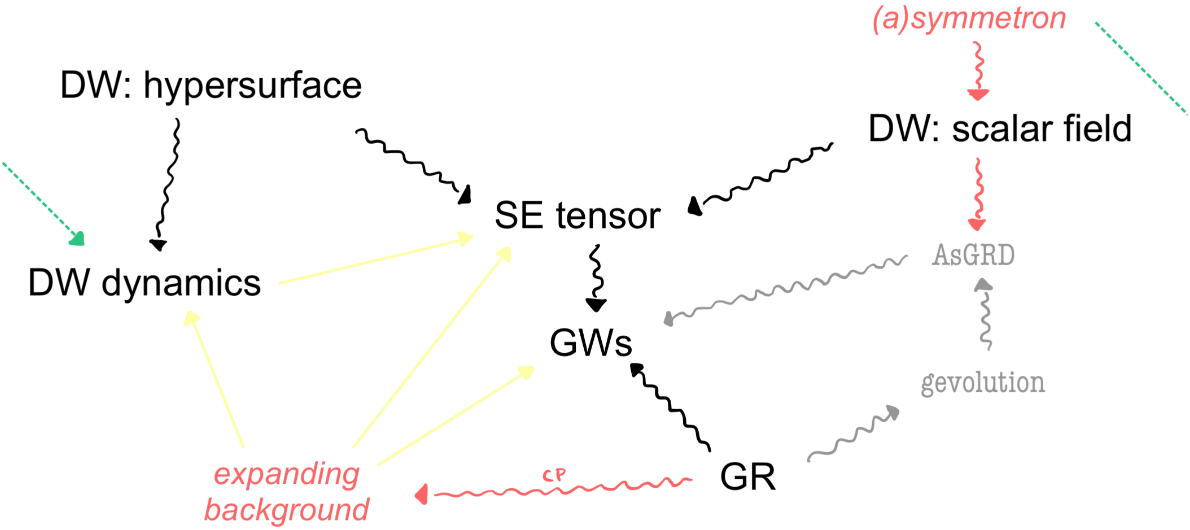
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Toy scenario

The (3+1)-dimensional spacetime $\mathscr{M}$ is described by the metric $g_{\mu\nu} = a^2\eta_{\mu\nu}$ and **Cartesian comoving coordinates** x^μ. We consider a **matter-dominated** ($a \sim \tau^\alpha$, $\alpha = 2$) universe in which a **symmetron** phase transition takes place at scale factor $a = a_*$. This induces the formation of a **domain wall** with (unperturbed) planar structure, oriented perpendicular to the z-axis.

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The defect is represented by the hypersurface Σ that splits the ambient spacetime into vacuum domains $\mathscr{M}_\pm$ with energies $v_\pm = v_+$. We let γ_{ab} be the induced metric on the submanifold and impose the coordinate system ξ^a, where $a = 0, 1, 2$. When $n^\mu = \delta^\mu{}_z$, the hypersurface's energy density—the **surface tension**—is

$$\sigma = -\int_\Sigma \mathrm{d}z\,\sqrt{g_{zz}}T^0{}_0\,. \tag{6}$$

Alternative toy scenario

The $(N+1)$-dimensional spacetime $\mathscr{M}$ is described by the metric $g_{\mu\nu} = a^2\eta_{\mu\nu}$ and arbitrary coordinates x^μ. We consider a [perfect fluid]-dominated ($a \sim \tau^\alpha$, $\alpha \in \mathbb{R}$) universe in which a first-order phase transition takes place at scale factor $a = a_*$. This induces the formation of a topological defect with (unperturbed) simple structure, oriented perpendicular to the unit-normal n^μ.

The defect is represented by the hypersurface Σ that splits the ambient spacetime into vacuum domains $\mathscr{M}_\pm$ with energies $v_\pm$. We let γ_{ab} be the induced metric on the submanifold and impose the coordinate system ξ^a, where $a = 0, 1, \ldots, N-1$. When $n^\mu = \delta^\mu{}_z$, the hypersurface's energy density is

$$\mathcal{E} = -\int_\Sigma \mathrm{d}z\,\sqrt{g_{zz}}T^0{}_0\,. \tag{6}$$

DW $\leftrightarrow$ hypersurface

First approach

DW $\leftrightarrow$ hypersurface

Wall positioned at $X^\mu = \delta^\mu{}_a \xi^a + \delta^\mu{}_z (z_0 + \epsilon) = (\tau, x, y, z_0 + \epsilon)$.

The induced metric is

$$\gamma_{ab} \equiv a^2 \hat{\gamma}_{ab} = a^2 \eta_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$$

and $\epsilon(\xi^a)$ is a first-order displacement to the wall's z-coordinate.

We have $a = a_*(\tau/\tau_*)^\alpha = a_*(\tau/\tau_*)^2$ and let a dot signify conformal time derivative; $\dot{f} \equiv \mathrm{d}f/\mathrm{d}\tau$.

DW ↔ hypersurface

Wall positioned at $X^\mu = \delta^\mu{}_a \xi^a + \delta^\mu{}_z (z_0 + \epsilon) = (\tau, x, y, z_0 + \epsilon)$.

The **Nambu–Goto action**

$$S_{\mathrm{NG}} = -\sigma \int_\Sigma d^3\xi \sqrt{-\gamma} \quad = -\sigma \int_\Sigma d^3\xi \sqrt{-\hat{\gamma}}\, a^3 \tag{7}$$

gives the equation of motion (e.o.m.)

$$\ddot{\epsilon} + \mathcal{D}(\tau)\dot{\epsilon} - \left[\partial_x^2 + \partial_y^2\right]\epsilon = 0 \tag{8}$$

where $\mathcal{D}(\tau) = 3\dot{a}/a$.

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The **Nambu–Goto action** with time-dependent surface tension

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Domain-wall dynamics

Separable equation $\rightsquigarrow$ Solutions $\epsilon(\tau, x, y) \sim \sum_p \varepsilon(\tau) \cdot \mathscr{E}(x, y)$, where

$$\ddot{\varepsilon} + \mathcal{D}\dot{\varepsilon} = -p^2\varepsilon, \tag{9a}$$

$$\left[\partial_x^2 + \partial_y^2\right]\mathscr{E} = -p^2\mathscr{E}, \tag{9b}$$

are the solutions with eigenvalue $-p^2$.

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are the solutions with eigenvalue $-p^2$.

Introduce $s \equiv \tau/\tau_*$ and $\omega \equiv p\tau_*$ such that ($\sigma' = 0$)

$$\varepsilon'' + (6/s)\varepsilon' + \omega^2\varepsilon = 0. \tag{10}$$

General solution: $\varepsilon(s) = s^\nu \mathrm{Z}_\nu(\omega s)$; $\nu = (1 - 3\alpha)/2 = -5/2$.

Domain-wall dynamics

Separable equation $\rightsquigarrow$ Solutions $\epsilon(\tau, x, y) \sim \sum_p \varepsilon(\tau) \cdot \mathscr{E}(x, y)$, where

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are the solutions with eigenvalue $-p^2$.

Introduce $s \equiv \tau/\tau_*$ and $\omega \equiv p\tau_*$ such that $(\sigma' \neq 0)$

$$\varepsilon'' + (\tau_*\mathcal{D})\varepsilon' + \omega^2\varepsilon = 0. \tag{10}$$

General solution: $\varepsilon(s) = \textbf{?}$.

Symmetron solution

Symmetron domain walls have $\sigma = \sigma_\infty \sqrt{1-\upsilon}^3$ where $\upsilon \equiv (a_*/a)^3 = s^{-6}$, and this gives

$$\varepsilon'' + \left(\frac{6}{s} + 2d(s)\right)\varepsilon' + \omega^2\varepsilon = 0; \quad d(s) \triangleq \frac{9}{2s(s^6-1)}. \tag{11}$$

Initial conditions (ICs): $\varepsilon(s=1) = \varepsilon_*$ and $\varepsilon'(s=1) = 0$.

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Initial conditions (ICs): $\varepsilon(s=1) = \varepsilon_*$ and $\varepsilon'(s=1) = 0$. Solve in two regimes and sow together at $s = s_{\text{sow}}$, such that

$$\varepsilon(s) = \begin{cases} \varepsilon^{(\mathrm{I})}(s) & \text{if } s \leq s_{\text{sow}}, \\ \varepsilon^{(\mathrm{II})}(s) & \text{if } s \geq s_{\text{sow}}. \end{cases} \tag{12}$$

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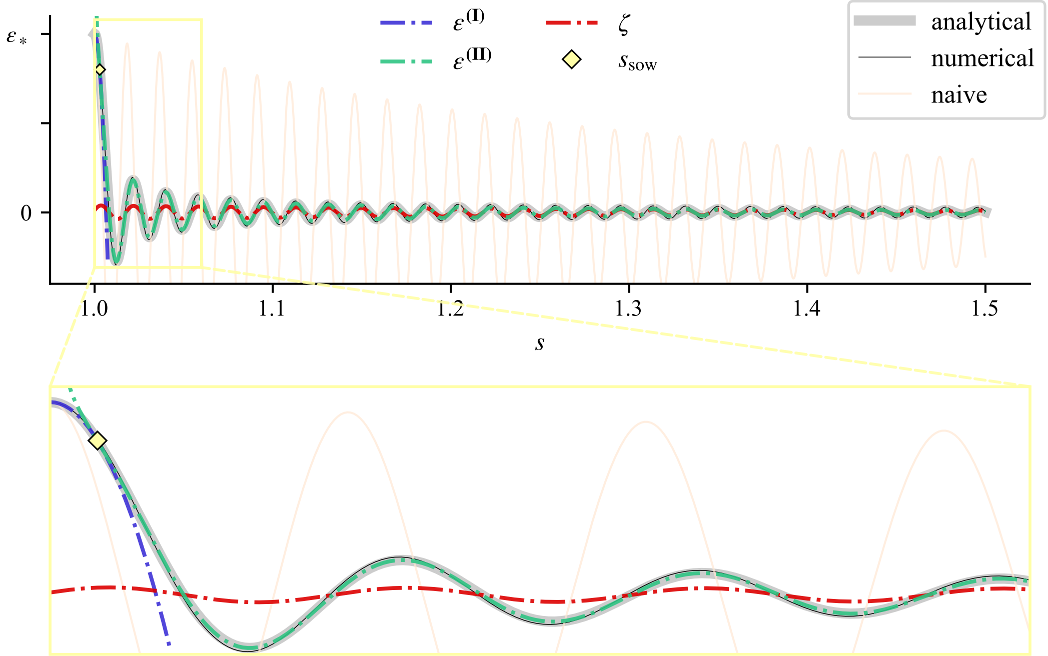
Initial conditions (ICs): $\varepsilon(s=1) = \varepsilon_*$ and $\varepsilon'(s=1) = 0$. Solve in two regimes and sow together at $s = s_{\text{sow}}$, such that $\varepsilon = \Theta(s_{\text{sow}} - s)\varepsilon^{(\mathrm{I})} + \Theta(s - s_{\text{sow}})\varepsilon^{(\mathrm{II})}$;

$$\begin{aligned} \varepsilon^{(\mathrm{I})}/\varepsilon_* &\simeq 1 - \omega^2(s-1)^2/5 + \omega^2(s-1)^3/35, \\ \varepsilon^{(\mathrm{II})}/\varepsilon_* &\simeq \zeta(s) \times s^{9/2}(s^6-1)^{-3/4}; \quad \zeta(s) \simeq \underbrace{s^\nu Z_\nu(\omega s)}_{\text{“naive”}}, \nu = -5/2. \end{aligned} \tag{12}$$



ε_*
0
$\varepsilon^{(\mathrm{I})}$
$\varepsilon^{(\mathrm{II})}$
ζ
s_{sow}
analytical
numerical
naive
1.0
1.1
1.2
1.3
1.4
1.5
s

Gravitational waves

The e.o.m. for GWs, $\Box h_P = -16\pi G_{\mathrm{N}} \pi_P$, is solved in Fourier space through Green's functions:

$$a\tilde{h}_P(\tau, \boldsymbol{k}) \propto k^{-2} \int_{\tau_*}^{\tau} \mathrm{d}\hat{\tau}\, G(k\tau, k\hat{\tau}) a^3(\hat{\tau}) \tilde{\pi}_P(\hat{\tau}, \boldsymbol{k}); \quad P = +, \times, \tag{13}$$

where $G(u, v) = \mathrm{S}_1(u)\mathrm{C}_1(v) - \mathrm{C}_1(u)\mathrm{S}_1(v)$.[¶]

We use ICs $\tilde{h}(\tau_*, \boldsymbol{k}) = \dot{\tilde{h}}(\tau_*, \boldsymbol{k}) = 0$.

[¶] Riccati–Bessel functions: $\mathrm{S}_n(x) = +x\mathrm{j}_n(x)$, $\mathrm{C}_n(x) = -x\mathrm{y}_n(x)$

With $\boxed{\epsilon = \varepsilon(\tau)\sin(py),}$ only *monochromatic plus-polarised GWs* are produced.

For $\ell \equiv k_y/p \in \mathbb{Z}$ and $\vartheta \equiv \arctan\left(k_y/k_z\right) \in [0, 2\pi)$, we have

$$a\tilde{\pi}_+ \propto \sigma\cos^2\vartheta \cdot \mathrm{J}_{-\ell}(\ell\tan\vartheta \cdot p\varepsilon). \tag{14}$$

Finally,

$$a\tilde{h}_+ \propto \int_{\tau_*}^{\tau} \mathrm{d}\hat{\tau}\, G(k\tau, k\hat{\tau})\hat{\tau}^4 \sqrt{1 - (\tau_*/\hat{\tau})^6}^{\,3}\, \mathrm{J}_{\ell}(\ell\tan\vartheta \cdot p\varepsilon(\hat{\tau})). \tag{15}$$

With $\boxed{\epsilon = \varepsilon(\tau)\sin(py),}$ only *monochromatic plus-polarised GWs* are produced.

For $\ell \equiv k_y/p \in \mathbb{Z}$ and $\vartheta \equiv \arctan\left(k_y/k_z\right) \in [0, 2\pi)$, we have

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$$\sim \text{«}\ \mathrm{Z}_{3/2}(k\tau) \times \int \mathrm{d}\hat{\tau}\, \mathrm{Z}_{3/2}(k\hat{\tau}) \times \mathrm{J}_\ell\left[\mathrm{Z}_{-5/2}(p\hat{\tau})\right] \times f(\hat{\tau})\ \text{»}$$

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DW $\leftrightarrow$ scalar field

Second approach

DW ↔ scalar field

The e.o.m. for the symmetron field $\phi = \phi_\infty \chi$ is $\Box\phi = \partial_\phi V_{\text{eff}}$, or

$$\ddot{\chi} + 2(\dot{a}/a)\dot{\chi} - \nabla^2\chi = -\mu^2 \cdot a^2 \left[\chi^2 - \chi_+^2\right]\chi, \tag{16}$$

where $\chi_\pm = \pm\sqrt{1 - \upsilon}$; $\upsilon = (a_*/a)^3 = s^{-6}$.

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where $\chi_\pm = \pm\sqrt{1-\upsilon}$; $\upsilon = (a_*/a)^3 = s^{-6}$.

MINKOWSKI, STATIC CASE. $\chi = \tanh\left(\mu z/\sqrt{2}\right)$ if $a = 1$ and $\chi_+ = 1$ ($a \gg a_*$)

Quasi-static limit

In the quasi-static limit,[††]

$$\boldsymbol{\nabla}^2\chi \simeq \partial_z^2\chi \simeq \mu^2 \cdot a^2\left[\chi^2 - \chi_+^2\right]\chi, \tag{17}$$

where $\chi_\pm = \pm\sqrt{1-\upsilon}$. With boundary conditions (BCs) $\chi|_{z\to\pm\infty} = \chi_\pm$, an approximate solution is

$$\chi_\mathrm{w} = \chi_+ \tanh\left(\frac{\chi_+ a(z - z_\mathrm{w})}{2L_\mathrm{c}}\right), \tag{18}$$

if we can assume that the derivatives of $z_\mathrm{w} = z_0 + \epsilon(\tau, y)$ makes no significant difference.

[††] $\dot{\chi} \ll \nabla^2\chi$

Comoving wall thickness

$$\delta_{\mathrm{w}} = \frac{\delta_\infty}{a\sqrt{1-\upsilon}} \longrightarrow \begin{cases} \infty & \text{for } a \to a_*, \\ \delta_\infty & \text{for } a \gg a_*, \end{cases} \tag{19}$$

and surface tension

$$\sigma_{\mathrm{w}} = \sigma_\infty [1-\upsilon]^{3/2} \longrightarrow \begin{cases} 0 & \text{for } a \to a_*, \\ \sigma_\infty & \text{for } a \gg a_*, \end{cases} \tag{20}$$

where $\delta_\infty = 1/\mu$ and $\sigma_\infty = 2\sqrt{2}\mu^3/(3\lambda)$.

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Asymptotic limit

Far away from the wall,[‡‡] the field takes values $\pm\breve{\chi}$, obeying

$$\ddot{\breve{\chi}} + 2(\dot{a}/a)\dot{\breve{\chi}} = -\mu^2 \cdot a^2 \left[\breve{\chi}^2 - \chi_+^2\right]\breve{\chi}. \tag{21}$$

Generally, $\pm\breve{\chi} \sim \chi_\pm$, depending on ICs.

[‡‡] $\nabla^2\chi \longrightarrow 0$

Asymptotic limit

Far away from the wall,[‡‡] the field takes values $\pm\breve{\chi}$, obeying

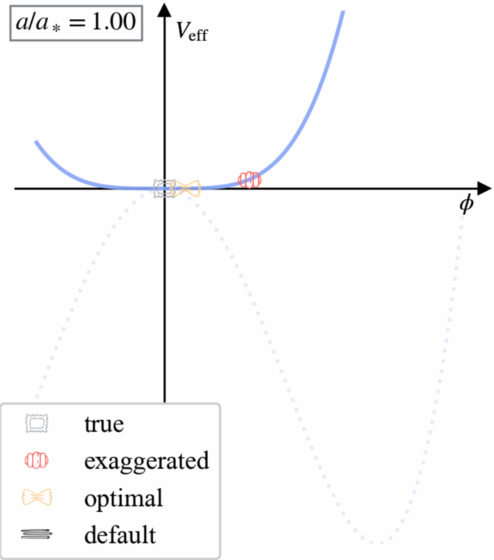
$$\ddot{\breve{\chi}} + 2(\dot{a}/a)\dot{\breve{\chi}} = -\mu^2 \cdot a^2 \left[\breve{\chi}^2 - \chi_+^2\right]\breve{\chi}. \tag{21}$$

Generally, $\pm\breve{\chi} \sim \chi_\pm$, depending on ICs.

STABLE FIELD. We present a protocol for finding the solution with the smallest possible oscillations around χ_+ in Appendix B. This also allows initialisation at PT.

The farfalle is the “optimal” solution (Appendix B), **spaghetti** shows the solution with our default ICs, **gnocchi** shows an “unfortunate” solution and the **ravioli** tracks the potential’s positive minimum, $\chi_+ = \sqrt{1 - \upsilon}$.

Part II of the animation is zoomed from the ravioli’s point of view.



Analysis

Contents

The code

`AsGRD`,[*] based on `gevolution`,[†] computes the full metric perturbations and is compatible with asymmetron scenarios.

[*]Christiansen et al. ((2024a))

[†]Adamek et al. ((2016))

The code

`AsGRD`, based on `gevolution`, computes the full metric perturbations and is compatible with asymmetron scenarios.

Cubic simulation boxes with **periodic boundaries** demand the presence of two or more walls.

Experiments

Cubic simulation box of side lengths $L_\#$ (fundamental frequency $k_\# = 2\pi/L_\#$) with $N_\#^3$ lattice points, with perturbation $\epsilon_* = \varepsilon_* \sin(py)$ to the middle wall. Fiducial symmetron parameters are $a_* = 0.33$, $\xi_* = 3.33 \times 10^{-4}$ and $\beta_* = 1$ (Thesis, Eq. (5.22)). Complete matter domination ($\Omega_{\mathrm{m0}} = 1$) gives $\tau_* \approx 3.4$ Gpc/h. Simulation onset is at scale factor $a_\mathrm{i} \gtrsim a_*$, and we finish at $a_\mathrm{f} = 0.50$.

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Cubic simulation box of side lengths $L_\#$ (fundamental frequency $k_\# = 2\pi/L_\#$) with $N_\#^3$ lattice points, with perturbation $\epsilon_* = \varepsilon_* \sin(py)$ to the middle wall. Fiducial symmetron parameters are $a_* = 0.33$, $\xi_* = 3.33 \times 10^{-4}$ and $\beta_* = 1$ (Thesis, Eq. (5.22)). Complete matter domination ($\Omega_{m0} = 1$) gives $\tau_* \approx 3.4$ Gpc/h. Simulation onset is at scale factor $a_i \gtrsim a_*$, and we finish at $a_f = 0.50$.

1	$\varepsilon_* = 0.08L_\#$, $p = 2k_\#$, $L_\# \approx 1$ Gpc/h, $N_\# = 768$, $a_i \simeq a_* + 0.003$

Experiments

Cubic simulation box of side lengths $L_\#$ (fundamental frequency $k_\# = 2\pi/L_\#$) with $N_\#^3$ lattice points, with perturbation $\epsilon_* = \varepsilon_* \sin(py)$ to the middle wall. Fiducial symmetron parameters are $a_* = 0.33$, $\xi_* = 3.33 \times 10^{-4}$ and $\beta_* = 1$ (Thesis, Eq. (5.22)). Complete matter domination ($\Omega_{m0} = 1$) gives $\tau_* \approx 3.4$ Gpc/h. Simulation onset is at scale factor $a_i \gtrsim a_*$, and we finish at $a_f = 0.50$.

1	$\varepsilon_* = 0.08L_\#$, $p = 2k_\#$, $L_\# \approx 1$ Gpc/h, $N_\# = 768$, $a_i \simeq a_* + 0.003$
2	$\varepsilon_* \longrightarrow 0.12L_\#$
3	$N_\# \longrightarrow 900$
4	$L_\# \longrightarrow 1.4$ Gpc/h
5	$p \longrightarrow 3k_\#$, $\varepsilon_* \longrightarrow 0.06L_\#$, $N_\# \longrightarrow 900$
7	$(a_i - a_*) \longrightarrow 0.001$

1 —

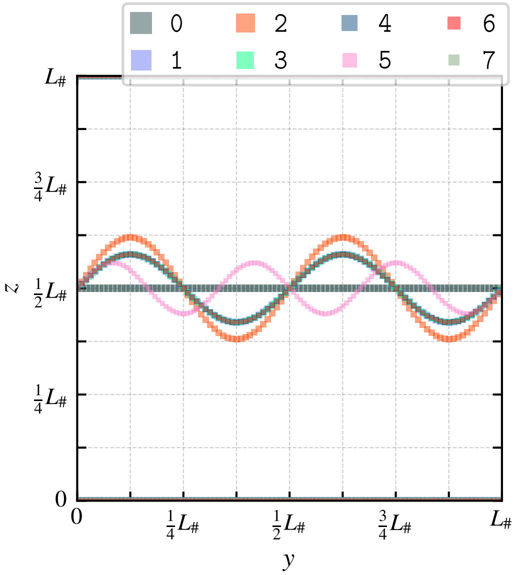
2 ε_* ↗

3 $N_\#$ ↗

4 $L_\#$ ↗

5 p ↗ ε_* ↘ $N_\#$ ↗

7 $(a_\mathrm{i} - a_*)$ ↘



0 $\varepsilon_* \searrow$

1 —

2 $\varepsilon_* \nearrow$

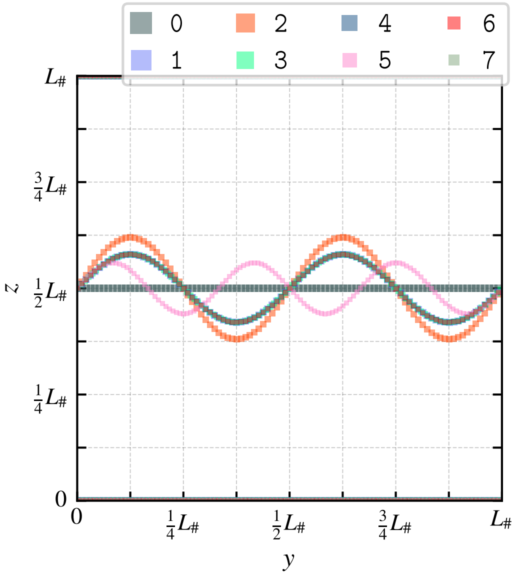
3 $N_\# \nearrow$

4 $L_\# \nearrow$

5 $p \nearrow$ $\varepsilon_* \searrow$ $N_\# \nearrow$

6 $\xi_* \searrow$

7 $(a_\mathrm{i} - a_*) \searrow$



Wall displacement field

Center wall's z-position $z_{\mathrm{w}} = z_0 + \epsilon(\tau, x, y)$ where $z_0 = \mathfrak{D} \equiv L_{\#}/2$ is the unperturbed position and $\epsilon(\tau, x, y) = \varepsilon(\tau)\sin(py)$ the displacement field.

Wall displacement field

Center wall's z-position $z_{\mathrm{w}} = z_0 + \epsilon(\tau, x, y)$ where $z_0 = \mathfrak{D} \equiv L_{\#}/2$ is the unperturbed position and $\epsilon(\tau, x, y) = \varepsilon(\tau) \sin(py)$ the displacement field.

FIRST APPROACH $\rightsquigarrow \varepsilon(\tau) = \varepsilon^{\mathrm{NG}}(\tau)$, analytically determined in the thin-wall limit

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FIRST APPROACH $\rightsquigarrow \varepsilon(\tau) = \varepsilon^{\mathrm{NG}}(\tau)$, analytically determined in the thin-wall limit

SECOND APPROACH $\rightsquigarrow \varepsilon(\tau) = \varepsilon^{\phi}(\tau)$, from the coordinate at which the simulated field $|\chi|$ takes its minimum value along a slice at a y-coordinate that satisfies $\sin(py) = 1$

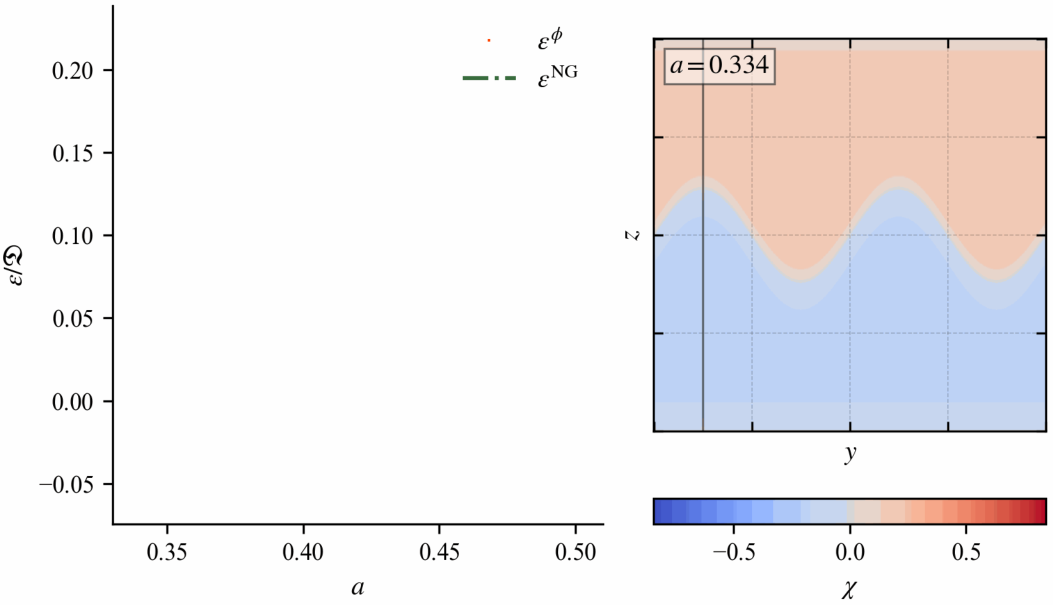
Wall displacement field

Center wall's z-position $z_{\mathrm{w}} = z_0 + \epsilon(\tau, x, y)$ where $z_0 = \mathfrak{D} \equiv L_{\#}/2$ is the unperturbed position and $\epsilon(\tau, x, y) = \varepsilon(\tau)\sin(py)$ the displacement field.

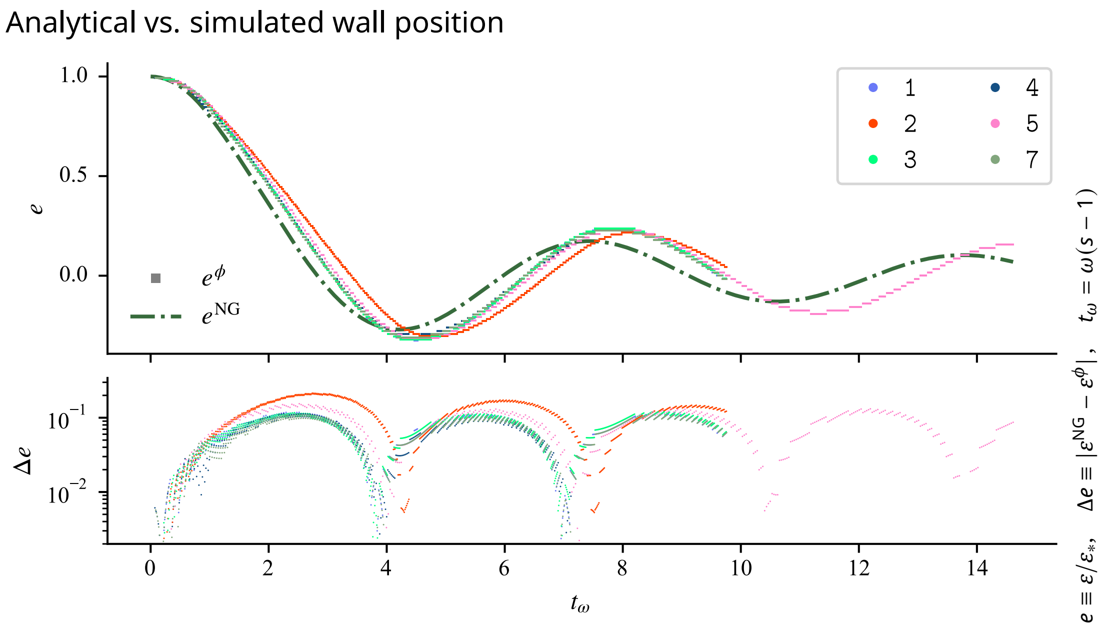
FIRST APPROACH $\rightsquigarrow \varepsilon(\tau) = \varepsilon^{\mathrm{NG}}(\tau)$, analytically determined in the thin-wall limit

SECOND APPROACH $\rightsquigarrow \varepsilon(\tau) = \varepsilon^{\phi}(\tau)$, from the coordinate at which the simulated field $|\chi|$ takes its minimum value along a slice at a y-coordinate that satisfies $\sin(py) = 1$

We do not expect a perfect match.



ε^{ϕ}
$\varepsilon^{\mathrm{NG}}$
0.20
0.15
0.10
0.05
0.00
−0.05
$\varepsilon/\mathfrak{S}$
0.35
0.40
0.45
0.50
a
$a = 0.334$
z
y
−0.5
0.0
0.5
χ



Analytical vs. simulated wall position
1.0
0.5
0.0
e
e^{ϕ}
e^{NG}
1
2
3
4
5
7
10^{-1}
10^{-2}
Δe
0
2
4
6
8
10
12
14
t_ω
$e \equiv \varepsilon/\varepsilon_*$, $\Delta e \equiv |\varepsilon^{\mathrm{NG}} - \varepsilon^{\phi}|$, $t_\omega = \omega(s-1)$

Gravitational waves

We have GWs in Fourier space with $\boldsymbol{k} = (k_x, k_y, k_z) = (\mathtt{u}, \mathtt{v}, \mathtt{w})k_{\#}$.

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Gravitational waves

$a\tilde{h}_+^{\mathrm{NG},\phi} \propto \int \mathrm{d}\hat{\tau}\, [\ldots] \times \mathrm{J}_\ell([\ldots] p \varepsilon^{\mathrm{NG},\phi})$

We have GWs in Fourier space with $\boldsymbol{k} = (k_x, k_y, k_z) = (\mathtt{u}, \mathtt{v}, \mathtt{w})k_{\#}$.

$\tilde{h}_+^{\mathrm{NG}}$ $\leftrightarrow$ semi-analytical formula with $\varepsilon^{\mathrm{NG}}$ as input to the SE tensor
$\tilde{h}_+^{\phi}$ $\leftrightarrow$ semi-analytical formula with ε^{ϕ} as input to the SE tensor

Gravitational waves

$a\tilde{h}_+^{\mathrm{NG},\phi} \propto \int \mathrm{d}\hat{\tau}\,[\ldots] \times \mathrm{J}_\ell([\ldots]p\varepsilon^{\mathrm{NG},\phi})$

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$\tilde{\mathtt{h}}_+ \leftrightarrow$ output from `AsGRD`

Gravitational waves

$a\tilde{h}_+^{\mathrm{NG},\phi} \propto \int \mathrm{d}\hat{\tau}\, [\ldots] \times \mathrm{J}_\ell([\ldots] p \varepsilon^{\mathrm{NG},\phi})$

We have GWs in Fourier space with $\boldsymbol{k} = (k_x, k_y, k_z) = (\mathtt{u}, \mathtt{v}, \mathtt{w}) k_\#$.

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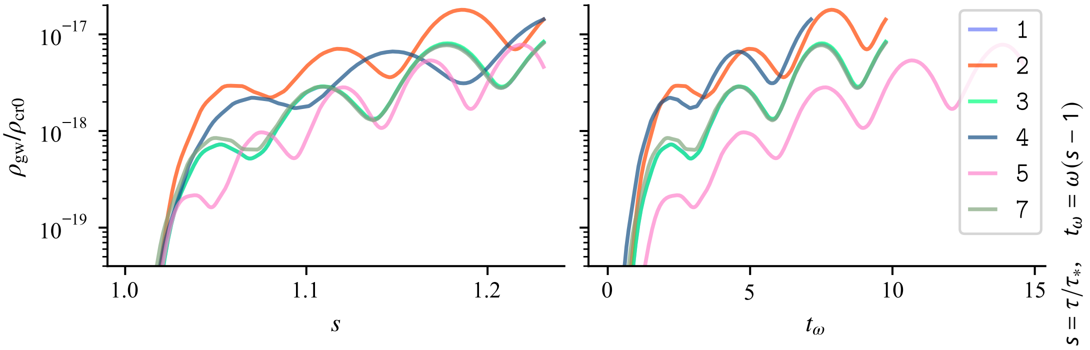
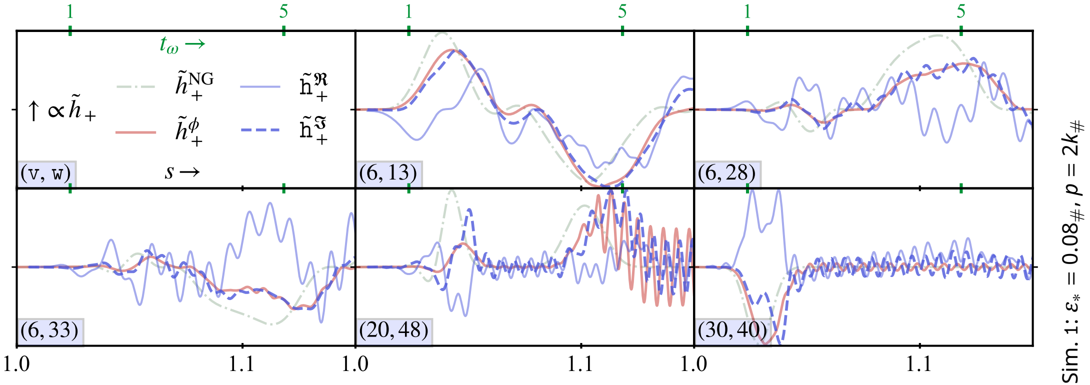
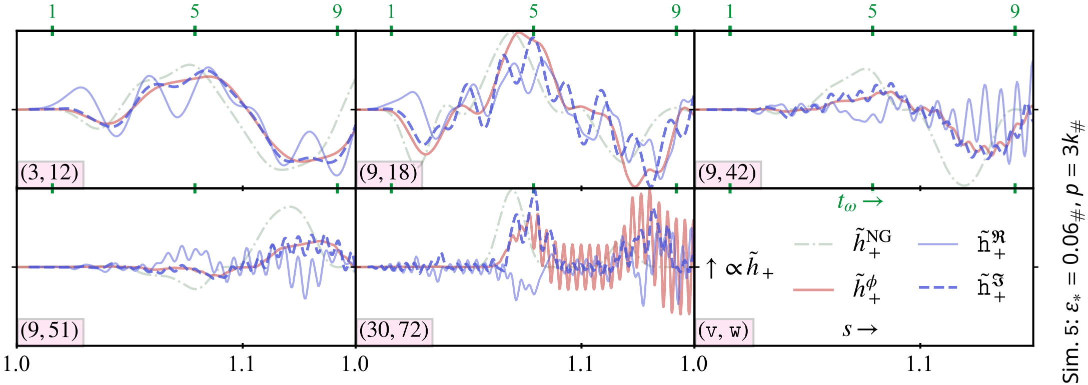
MAGNITUDES. The magnitudes differ unsystematically inbetween these three, and we will not address this issue here.

Gravitational radiation $\rho_{gw} = M_{Pl}^2 a^{-2} \langle \dot{h}^{ij} \dot{h}_{ij} \rangle$





$t_\omega \rightarrow$
$\uparrow \propto \tilde{h}_+$
$\tilde{h}_+^{\mathrm{NG}}$
$\tilde{h}_+^{\phi}$
$\tilde{\mathtt{h}}_+^{\mathfrak{R}}$
$\tilde{\mathtt{h}}_+^{\mathfrak{I}}$
$s \rightarrow$
(v, w)
(6, 13)
(6, 28)
(6, 33)
(20, 48)
(30, 40)
1
5
1.0
1.1
Sim. 1: $\varepsilon_* = 0.08_\#$, $p = 2k_\#$



1
5
9
(3, 12)
(9, 18)
(9, 42)
(9, 51)
(30, 72)
(v, w)
$t_\omega \rightarrow$
$\uparrow \propto \tilde{h}_+$
$\tilde{h}^{\mathrm{NG}}_+$
$\tilde{\mathrm{h}}^{\mathfrak{R}}_+$
$\tilde{h}^{\phi}_+$
$\tilde{\mathrm{h}}^{\mathfrak{I}}_+$
$s \rightarrow$
1.0
1.1
Sim. 5: $\varepsilon_* = 0.06_\#$, $p = 3k_\#$

	Calculation	**Simulation**
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
Gravitational waves	$\tilde{h}_+^{\mathrm{NG},\phi}$	$\tilde{\mathtt{h}}_+$

	Calculation	**Simulation**
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
e independent of ε_*	yes	no
Gravitational waves	$\tilde{h}_+^{\mathrm{NG},\phi}$	$\tilde{\mathtt{h}}_+$

	Calculation	Simulation
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
e independent of ε_*	yes	no
$e \nsim s^{-5/2} Z_{-5/2}(\omega s)$	yes	yes
Gravitational waves	$\tilde{h}_+^{\mathrm{NG},\phi}$	$\tilde{\mathtt{h}}_+$

* To some extent.

	Calculation	Simulation
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
e independent of ε_*	yes	no
$e \nsim s^{-5/2} Z_{-5/2}(\omega s)$	yes	yes
(e vs. t_ω)-plot indep. of parameters	yes	yes*
Gravitational waves	$\tilde{h}_+^{\mathrm{NG},\phi}$	$\tilde{\mathtt{h}}_+$

* To some extent.

	Calculation	Simulation
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
e independent of ε_*	yes	no
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(e vs. t_ω)-plot indep. of parameters	yes	yes*
e unaffected by large fifth-force oscillations	yes*	yes*
Gravitational waves	$\tilde{h}_+^{\mathrm{NG},\phi}$	$\tilde{\mathtt{h}}_+$

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	Calculation	Simulation
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
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$e \nsim s^{-5/2} Z_{-5/2}(\omega s)$	yes	yes
(e vs. t_ω)-plot indep. of parameters	yes	yes*
e unaffected by large fifth-force oscillations	yes*	yes*
Gravitational waves	$\tilde{h}_+^{\mathrm{NG},\phi}$	$\tilde{\mathtt{h}}_+$
$\tilde{h}_\times = 0$	yes	yes

* To some extent.

	Calculation	Simulation
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
e independent of ε_*	yes	no
$e \nsim s^{-5/2}\mathrm{Z}_{-5/2}(\omega s)$	yes	yes
(e vs. t_ω)-plot indep. of parameters	yes	yes*
e unaffected by large fifth-force oscillations	yes*	yes*
Gravitational waves	$\tilde{h}_+^{\mathrm{NG},\phi}$	$\tilde{\mathtt{h}}_+$
$\tilde{h}_\times = 0$	yes	yes
$\tilde{h}_+ \in \mathbb{R}$	yes	no

* To some extent.

	Calculation	**Simulation**
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
e independent of ε_*	yes	no
$e \nsim s^{-5/2} Z_{-5/2}(\omega s)$	yes	yes
(e vs. t_ω)-plot indep. of parameters	yes	yes*
e unaffected by large fifth-force oscillations	yes*	yes*
Gravitational waves	$\tilde{h}_+^{\mathrm{NG},\phi}$	$\tilde{\mathtt{h}}_+$
$\tilde{h}_\times = 0$	yes	yes
$\tilde{h}_+ \in \mathbb{R}$	yes	no
$\tilde{h}_+ \simeq 0$ for $k_x \neq 0$, and for $k_y \neq np$	yes	yes

* To some extent.

	Calculation	Simulation
Wall position, $e = \varepsilon/\varepsilon_*$	e^{NG}	e^{ϕ}
e independent of ε_*	yes	no
$e \propto s^{-5/2} \mathrm{Z}_{-5/2}(\omega s)$	yes	yes
(e vs. t_ω)-plot indep. of parameters	yes	yes*
e unaffected by large fifth-force oscillations	yes*	yes*
Gravitational waves	$\tilde{h}_+^{\mathrm{NG},\phi}$	$\tilde{\mathtt{h}}_+$
$\tilde{h}_\times = 0$	yes	yes
$\tilde{h}_+ \in \mathbb{R}$	yes	no
$\tilde{h}_+ \simeq 0$ for $k_x \neq 0$, and for $k_y \neq np$	yes	yes
$\tilde{h}_+ = 0$ for $k_y = 0$	yes	no

* To some extent.

Foreground

Contents

Possible ways forward

Testing the limits of the framework $\rightsquigarrow$
Comparing thin-wall description to full-theory simulations...

Possible ways forward

- Perform higher-resolution experiments.
  - Dissociate discrepancies due to numerical limitations.
  - Investigate the effect of changing the wall thickness.

Possible ways forward

- Perform higher-resolution experiments.
- Investigate the effect of a Yukawa-like force term,

$$\ddot{\epsilon} \supset A \exp(-\mu \mathfrak{d}), \quad \mathfrak{d} \triangleq |\mathfrak{D} - \epsilon(\tau, y)|.$$

Possible ways forward

- Perform higher-resolution experiments.
- Investigate the effect of a Yukawa-like force term, $\ddot{\epsilon} \supset A \exp(-\mu\mathfrak{d})$.
- Find suitable statistical measures for the GWs.
  - Perhaps asymptotic analyses are the ways to go.

Possible ways forward

- Perform higher-resolution experiments.
- Investigate the effect of a Yukawa-like force term, $\ddot{\epsilon} \supset A \exp(-\mu \mathfrak{d})$.
- Find suitable statistical measures for the GWs.

Further development of the framework $\rightsquigarrow$
Assume limitations are well understood. . .

Possible ways forward

- Perform higher-resolution experiments.
- Investigate the effect of a Yukawa-like force term, $\ddot{\epsilon} \supset A\exp(-\mu\mathfrak{d})$.
- Find suitable statistical measures for the GWs.

- Vary parameters of the theory.
  - Establish relationship between symmetron parameters μ, λ and M, and GW signature.

Possible ways forward

- Perform higher-resolution experiments.
- Investigate the effect of a Yukawa-like force term, $\ddot{\epsilon} \supset A \exp(-\mu \mathfrak{d})$.
- Find suitable statistical measures for the GWs.

- Vary parameters of the theory.
- Apply more complicated displacement ϵ.
  - $\epsilon = \varepsilon(\tau) \sin(py)$ is the simplest non-trivial solution, and a very unlikely configuration.

Possible ways forward

- Perform higher-resolution experiments.
- Investigate the effect of a Yukawa-like force term, $\ddot{\epsilon} \supset A \exp(-\mu \mathfrak{d})$.
- Find suitable statistical measures for the GWs.

- Vary parameters of the theory.
- Apply more complicated displacement ϵ.
- Introduce energy bias $v = v_+ - v_- \neq 0$.
  - Does it change the motion significantly?
  - Is there analytical solutions for this?

Possible ways forward

- Perform higher-resolution experiments.
- Investigate the effect of a Yukawa-like force term, $\ddot{\epsilon} \supset A \exp(-\mu\mathfrak{d})$.
- Find suitable statistical measures for the GWs.

- Vary parameters of the theory.
- Apply more complicated displacement ϵ.
- Introduce energy bias $v = v_+ - v_- \neq 0$.

Concluding remarks

- Analytical solution ϵ^{NG} is not found in other literature (to the best of our knowledge).
- Formula for gravitational waves needs further validation.
- Solid foundation is laid for further similar analyses with `AsGRD`.

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((All figures are created by the author.))

Nanna Bryne

Spacetime Ripples from Domain-Wall Wiggles

On the analytical prediction of the gravitational-wave signature from perturbed topological defects in expanding spacetime