

# Gravitational waves from topological defects

Any short subtitle

### Nanna Bryne

CS: Astrophysics 60 ECTS study points

Institute of Theoretical Astrophysics, Department of Physics Faculty of Mathematics and Natural Sciences



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Supervisor: David Fonseca Mota

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# **Notation**

**Constants and units.** We use ['natural units'], where  $\hbar = c = 1$ , where  $\hbar$  is the reduced Planck constant and c is the speed of light in vacuum. Planck units? Set  $k_B = G_N = 1$ ? The Newtonian constant of gravitation  $G_N$  is referenced explicitly, and we use Planck units such as the Planck mass  $M_{\rm Pl} = (\hbar c/G_N)^{1/2} = G_N^{-1/2} \sim 10^{-8} \text{ kg}$ .

**Tensors.** The metric signature (-,+,+,+) is considered, i.e.  $\det[g_{\mu\nu}] \equiv |g| < 0$ . The Minkowski metric is denoted  $\eta_{\mu\nu}$ , whereas a general metric is denoted  $g_{\mu\nu}$ . A four-vector  $p^{\mu} =$ 

$$[\eta_{\mu\nu}] = \text{diag}(-1, 1, 1, 1)$$

$$f_{,\mu} \equiv \partial_{\mu} f = \frac{\partial f}{\partial x^{\mu}}$$

**Christophel symbols.** The Christophel symbols or "connections" are written

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma} \left( g_{\mu\sigma,\nu} + g_{\mu\sigma,\nu} - g_{\mu\nu,\sigma} \right) \tag{1}$$

"Lambda tensor." We express the *Lambda tensor*—sometimes called the "projection operator"—that projects onto the TT gauge refer to  $\sec^{\textcircled{c}}_{.0}$  as

$$\Lambda_{ij,kl}(\boldsymbol{n}) = P_{ik}(\boldsymbol{n})P_{jl}(\boldsymbol{n}) - \frac{1}{2}P_{ij}(\boldsymbol{n})P_{kl}(\boldsymbol{n}); \quad P_{ij}(\boldsymbol{n}) = \delta_{ij} - n_i n_j$$
 (2)

 $\forall$  **n** of unit length;  $\mathbf{n}^2 = n_1^2 + n_2^2 + n_3^2 = 1$ . We use a dot ('.') instead of the more conventional comma (',') to distinguish from the Minkowskian partial derivative.

**Fourier transforms.** We use the following convention for the Fourier transform of f(x),  $\tilde{f}(k)$ , and its inverse, where x and k are Lorentz four-vectors:

$$f(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot x} \tilde{f}(k)$$

$$\tilde{f}(k) = \int d^4x e^{ik \cdot x} f(x)$$
(3)

Here,  $k \cdot x = k_{\sigma} x^{\sigma} = g_{\rho \sigma} k^{\rho} x^{\sigma}$ .

### **Acronyms**

CDM <u>cold dark matter</u>

CMB cosmic microwave background (radiation)

DW <u>domain wall</u> GR general <u>relativity</u>

GW gravitational wave

 $\Lambda$ CDM Lambda (Greek  $\underline{\Lambda}$ ) cold <u>dark matter model</u>; standard model of cosmology

### **Nomenclature**

In the table below is listed the most frequently used symbols in this paper, for reference.

Table 1: helo

Symbol	Referent	SI-value or definition
Natural consta	ants	
$G_{ m N}$	Newtonian constant of gravitation	1.2 kg
$k_{ m B}$	Boltzmann's constant	1.2 K
Fiducial quant	tities	
$h_0$	Reduced Hubble constant	0.67
Subscripts		
$Q_{ m gw}$	Quantity $Q$ related to gravitational wave	
Functions and	operators	
$\Theta(\xi)$	Heaviside step function	$\begin{cases} 1 & \xi > 0 \\ 0 & \xi < 0 \end{cases}$
$sgn(\xi)$	Signum function	$2\Theta(\xi) - 1$
$\delta^{(n)}(\xi)$	Dirac-Delta function of $\xi \in \mathbb{R}^n$ , $n \in \mathbb{N}$ .	
$\delta^{\mu  u}$	Kronecker delta.	

#### **DRAFT**

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### **Writing Tools**

```
This is a comment.

This needs spelling check.

[Rephrase this.]

[Awkward wording.]

[Needs double-checking.]

This is in need of citation or reference to a section.

[With a comment.]

blah blah [...] (Phantom text.)

PHANTOM PARAGRAPH: THIS IS A PHANTOM PARAGRAPH, MAYBE WITH SOME KEYWORDS.
```

- This is a note.
- This is another note.
  - This is a related note.

### This is very important.

This text is highlighted.

Statement. [←That needs to be shown or proven.]

Notation

# Introduction

#### • GOALS:

- Gather framework about GWs from DWs
- Remove the need for very expensive *N*-body simulations with (semi-)analytical predictions
- Extract as much information as possible from the NANOgrav spectra thingy

#### • WHY RELEVANT:

- NANOgrav data wihoo
- Simulations in this regard are hugely expensive, and will not allow us to constrain the parameters of a model

$$\tilde{h}_{\circledast}^{"} + 2\mathcal{H}\tilde{h}_{\circledast}^{"} + k^{2}\tilde{h}_{\circledast} = 16\pi G_{N}a^{2}\tilde{\sigma}_{\circledast}; \quad \circledast = +, \times$$

$$\tag{1.1}$$

$$\left(\tilde{h}^{\mathrm{TT}}\right)_{ij}(\eta, \mathbf{k}) = \sum_{\circledast = +, \times} e_{ij}^{\circledast}(\hat{\mathbf{k}})\tilde{h}_{\circledast}(\eta, \mathbf{k})$$
(1.2)

$$\tilde{h}^{\text{TT}}_{ij}(\eta, \mathbf{k}) = \sum_{\circledast = +\times} e_{ij}^{\circledast}(\hat{\mathbf{k}}) \tilde{h}_{\circledast}(\eta, \mathbf{k})$$
(1.3)

#### 1.1 Preliminaries

It is assumed that the reader is familiar with variational calculus and linear perturbation theory. [In the following, we briefly (re)capture some concepts that are important starting points for the rest of the thesis.]

- variational calculus/ varying action
- action
- pert. theory?
- line element
- gauge invariance

- FRW cosmology
- classical field theory

$$G_{\mu\nu} = 8\pi G_{\rm N} T_{\mu\nu} \tag{1.4}$$

#### 1.1.1 Field theory

We formulate a theory in four-dimensional spacetime Minkowski in terms of the Lorentz invariant action

$$S = \int d^4x \, \mathcal{L}(\{\phi_i\}, \{\partial_\mu \phi_i\}), \tag{1.5}$$

with  $\mathcal{L}$  being the *Lagrangian density* of the theory, a function of the set of fields  $\{\phi_i\}$  and its first derivatives. We will refer to  $\mathcal{L}$  simply as the Lagrangian, as is customary when working with fields. For a general (i.e. curved) spacetime, blah blah  $[\ldots] \partial_{\mu} \rightarrow \nabla_{\mu}$  blah blah  $[\ldots]$  to construct a Lorentz invariant Lagrangian,

$$S = \int d^4x \underbrace{\mathcal{L}(\{\phi_i\}, \{\nabla_{\mu}\phi_i\})}_{\text{not scalar}} = \int \underbrace{d^4x \sqrt{-|g|}}_{\text{scalar}} \underbrace{\hat{\mathcal{L}}(\{\phi_i\}, \{\nabla_{\mu}\phi_i\})}_{\text{scalar}}, \tag{1.6}$$

<sup>™</sup>Maybe specify that this is only for scalar fields? Or include other fields? ∟

#### 1.1.2 Expanding universe: FRW cosmology

The universe expands with the rate a(t) at cosmic time t.

- expansion rate, cosmic time, conformal time
- why is flat assumption OK?

#### 1.1.3 Method of Green's Functions

A linear ordinary differential equation (ODE)  $L_x f(x) = g(x)$  assumes a linear differential operator L, a [continuous], unknown function f, and a right-hand side g that constitutes the inhomogeneous part of the ODE. The *Green's function G* for the ODE (or L) is manifest as any solution to  $L_x G(x,y) = \delta(x-y)$  [check plagiarism (Bringmann)]. If L is translation invariant (invariant under  $x \mapsto x + a$ )—which is equivalent to L having constant coefficients—we can write G(x,y) = G(x-y) and  $[\leftarrow \text{show}?]$ 

$$f(x) = (G * g)(x) = \int dy G(x - y)g(y)$$
 (1.7)

solves  $L_x f(x) = g(x)$ .

Let  $f_i^{(0)}$ , i = 1, 2, 3, ... be solutions to the homogeneous ODE, i.e.  $L_x f_i^{(0)} = 0$ . Then, by the superposition principle,  $f(x) + \sum_i c_i f_i^{(0)}$  is also a solution of the original, inhomogeneous equation.

Pulse signal. Consider the very common scenario where the source is a temporary pulse;

$$g(x) = \begin{cases} g(x), & x_0 \le x \le x_1, \\ 0, & x \ge x_1. \end{cases}$$
 (1.8)

#### 1.1.4 Special functions (tmp. name, maybe move (appendix)?)

blah blah [...]

For  $\nu = n + 1/2, n \in \mathbb{N}$  we have

$$\mathcal{J}_{n+1/2}(x) = \sqrt{\frac{2}{\pi}} x^{n+\frac{1}{2}} \left( -\frac{1}{x} \frac{d}{dx} \right)^n \frac{\sin x}{x}$$
 (1.9a)

and

$$\mathcal{N}_{n+1/2}(x) = (-1)^{n+1} \mathcal{J}_{-(n+1/2)}(x)$$
(1.9b)

for  $x \in \mathbb{C}xlxl$ 

# Part I Background

# **Classical Field Theory and Gravity**

Alongside quantum mechanics, Einstein's theory of gravity—general relativity (GR)—is widely accepted as the most accurate description of our surroundings. GR can be formulated from a geometrical point of view, or it can be viewed as a classical field theory. In the former approach we meet geometrical tools such as the geodesic equation, whereas the latter allows the application of field-theoretical methods. This chapter lays emphasis on the field interpretation of GR.

PHANTOM PARAGRAPH: Two perspectives insightful; better overall understanding of aspects of concepts in GR

#### 2.1 General Relativity

The Einstein-Hilbert action in vacuum is Check Planck mass def.

$$S_{\rm EH} = 1/2M_{\rm Pl}^2 \int d^4x \sqrt{-|g|} \,\mathcal{R},$$
 (2.1)

where  $\mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}$ . By varying  $S_{EH}$  with respect to  $g_{\mu\nu}$  one obtains the equation of motion

$$\mathcal{G}_{\mu\nu} \equiv \mathcal{R}_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\mathcal{R} = 0. \tag{2.2}$$

Thus, we interpret GR as a *classical* field theory where the tensor field  $g_{\mu\nu}$  is the gravitational field, [with the particle realisation named "graviton"].

- scalar field (ST theories)
- energy momentum tensor
- 2.2 TITLE (Conformal Transformations)
- 2.3 TITLE (Scalar-Tensor Theories (maybe subsec. of prev.))
- 2.4 Perturbation Theory
- 2.5 Classical Solitons

Chapter 2. Classical Field Theory and Gravity

## **Gravitational Waves**

The term "gravitational waves" refers to the tensor perturbations to the background metric  $^{\mathbb{C}}_{0}$ . These "waves" are spacetime distortions whose name comes from the fact that  $\lceil$  they obey the wave equation  $\rfloor_{7}$ .

#### 3.1 Linearised Gravity

CONFORMAL TRAFOS!!!! Carroll ((2019, p. 467))

If we let  $\overline{g}_{\mu\nu}$  be the background metric, the perturbed metric is given by

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + h_{\mu\nu}; \quad \left| h_{\mu\nu} \right| \ll 1, \tag{3.1} \label{eq:3.1}$$

where  $h_{\mu\nu}$  describes a tensor field propagating on said background. Now,  $g^{\mu\nu} = \overline{g}^{\mu\nu} - h^{\mu\nu}$  is the inverse metric, however  $h^{\mu\nu}h_{\nu\lambda} = \overline{g}^{\mu\rho}\overline{g}^{\nu\sigma}h_{\rho\sigma}h_{\nu\lambda} \neq \delta^{\mu}_{\lambda}$ .

PHANTOM PARAGRAPH: FIND THE LINEARISED EINSTEIN EQS — USING COMMA NOTATION — COMMENT ABOUT NO BACKREACTION

The Christophel symbols are now

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\left(g_{\mu\sigma,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma}\right) 
= \overline{\Gamma}^{\rho}_{\mu\nu} + \frac{1}{2}\overline{g}^{\rho\sigma}\left(h_{\mu\sigma,\nu} + h_{\nu\sigma,\mu} - h_{\mu\nu,\sigma}\right) - \frac{1}{2}h^{\rho\sigma}\underbrace{\left(\overline{g}_{\mu\sigma,\nu} + \overline{g}_{\nu\sigma,\mu} - \overline{g}_{\mu\nu,\sigma}\right)}_{=2\overline{g}_{\sigma\lambda}\overline{\Gamma}^{\lambda}_{\mu\nu}} + \mathcal{O}(h^{2}) 
= \overline{\Gamma}^{\rho}_{\mu\nu} + \frac{1}{2}\overline{g}^{\rho\sigma}\left(h_{\mu\sigma,\nu} + h_{\nu\sigma,\mu} - h_{\mu\nu,\sigma}\right) - h^{\rho}_{\lambda}\overline{\Gamma}^{\lambda}_{\mu\nu} + \mathcal{O}(h^{2}) 
= \overline{\Gamma}^{\rho}_{\mu\nu} + \frac{1}{2}\overline{g}^{\rho\sigma}\left(h_{\mu\sigma,\nu} + h_{\nu\sigma,\mu} - h_{\mu\nu,\sigma} - 2h_{\sigma\lambda}\overline{\Gamma}^{\lambda}_{\mu\nu}\right) + \mathcal{O}(h^{2}) 
= \overline{\Gamma}^{\rho}_{\mu\nu} + \frac{1}{2}\overline{g}^{\rho\sigma}\left(h_{\mu\sigma,\nu} + h_{\nu\sigma,\mu} - h_{\mu\nu,\sigma}\right) + \mathcal{O}(h^{2}),$$
(3.2)

where  $h_{\mu\nu;\sigma} = \overline{\nabla}_{\sigma} h_{\mu\nu} + \mathcal{O}(h^2)$ ??... [ $\leftarrow$ Prove this last line.].

「Maybe have such things in an appendix?」

■ The such things in an appendix in the such things in an appendix in the such things in the such that the such that

#### **DRAFT**

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#### 3.1.1 TITLE (Energy-momentum tensor; Eom.; Scalar field)

The energy-momentum tensor is

$$T^{\mu\nu} = -\frac{2}{\sqrt{-q}} \frac{\delta S_{\phi}}{\delta g^{\mu\nu}} = \partial_{\mu}\phi \partial_{\nu}\phi - g_{\mu\nu} \left[ \frac{1}{2} g^{\rho\sigma} \partial_{\rho}\phi \partial_{\sigma}\phi + V(\phi) \right] \tag{3.3}$$

blah blah [...] Assume no cross terms in  $g_{\mu\nu}$ . To retrieve the TT-part of  $T_{\mu\nu}$  we utilise the Lambda tensor $_{\text{(repeat or refer?)}}^{\textcircled{c}}$ 

$$T_{ij}(\eta, \mathbf{k}) = \Lambda_{ij,kl}(\hat{\mathbf{k}}) (T^{\neg \text{TT}})_{ij} (\eta, \mathbf{k})$$

$$\stackrel{!}{=} \Lambda_{ij,kl}(\hat{\mathbf{k}}) \left[ \partial_i \phi \partial_j \phi \right] (\eta, \mathbf{k})$$
(3.4)

$$\left[\partial_{i}\phi\partial_{j}\phi\right](\eta, \mathbf{k}) = \int d^{3}x \, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} \left[\partial_{i}\phi\partial_{j}\phi\right](\eta, \mathbf{x})$$

$$=????$$
(3.5)

Ш

#### 3.2 Generation of Gravitational Waves

• Somehow get to this eq:

$$T_{ij}^{\mathrm{TT}}(\eta, \mathbf{k}) = \Lambda_{ij,kl}(\hat{\mathbf{k}}) \int \frac{d^3p}{(2\pi)^3} p_k p_l \phi(\eta, \mathbf{p}) \phi(\eta, \mathbf{k} - \mathbf{p}). \tag{3.6}$$

• Production instead of generation?

#### 3.2.1 General Formalism

#### DRAFT

<sup>r</sup>Use conformal trafo! ((Carroll, 2019, p. 467)) **→** 

$$ds^{2} = -dt^{2} + a^{2}(t) \left\{ \gamma_{ij} + h_{ij} \right\} dx^{i} dx^{j} = a^{2}(\eta) \left\{ -d\eta^{2} + \left( \gamma_{ij} + h_{ij} \right) dx^{i} dx^{j} \right\}$$
(3.7)

$$h_{ij} = h_{ij}^{\text{TT}}, \gamma_{ij} = \delta_{ij}, (\overline{g}_{\mu\nu} = a^2 \eta_{\mu\nu})$$

 $h_{ij}=h_{ij}^{\rm TT}, \gamma_{ij}=\delta_{ij}, (\overline{g}_{\mu\nu}=a^2\eta_{\mu\nu})$ Linearised Einstein equations: Conformal Newtonian gauge

$$\delta G^{i}_{j} = 8\pi G_{N} T^{i}_{j} = \frac{1}{2a^{2}} \left[ \ddot{h}_{ij} + 2\frac{\dot{a}}{a} \dot{h}_{ij} - \nabla^{2} h_{ij} \right]$$
 (3.8)

where a dot (''') signifies the *conformal* time derivative.  $(T_i^i = a^{-2}T_{ij})$  blah blah [...]

$$\ddot{h}_{ij}(\eta, \mathbf{k}) + 2\frac{\dot{a}}{a}\dot{h}_{ij}(\eta, \mathbf{k}) - k^2 h_{ij}(\eta, \mathbf{k}) = 16\pi G_N T_{ij}(\eta, \mathbf{k})$$
(3.9)

Define  $\mathfrak{h}_{ij} \equiv ah_{ij}$ . By inserting this in Eq. (3.9) and multiplying the equation by a, one finds

$$\ddot{\mathfrak{h}}_{ij}(\eta, \mathbf{k}) + \left[ k^2 - \frac{\ddot{a}(\eta)}{a(\eta)} \right] \mathfrak{h}_{ij}(\eta, \mathbf{k}) = 16\pi G_{\text{N}} a(\eta) T_{ij}(\eta, \mathbf{k}). \tag{3.10}$$

We assume  $a(\eta) \propto \eta^{\alpha}$  and define  $\nu \equiv \alpha - \frac{1}{2}$ . Letting  $\tau = k\eta$ , Eq. (3.10) becomes

$$\left[\frac{\partial^2}{\partial \tau^2} + 1 - \frac{4v^2 - 1}{4\tau^2}\right] \mathfrak{h}_{ij}(\eta, \mathbf{k}) = \frac{16\pi G_{\text{N}} a(\eta)}{k^2} T_{ij}(\eta, \mathbf{k})$$
(3.11)

Now, Eq. (3.11) transforms into a problem of the form  $L_{\tau} f(\tau) = g(\tau)$ ; a problem that can be solved using Green's method (see some section 0 ). Kawasaki and Saikawa ((2011)) propose

$$G(\tau, \tau') = \frac{\pi}{2} \Theta(\tau - \tau') \left[ \mathcal{N}_{\nu}(\tau) \mathcal{J}_{\nu}(\tau') - \mathcal{J}_{\nu}(\tau) \mathcal{N}_{\nu}(\tau') \right]$$
(3.12)

as a solution to  $L_{\tau}G(\tau,\tau')=\delta(\tau-\tau')$ . In some appendix we show that this holds for a matter dominated universe where  $v = 2 - \frac{1}{2} = \frac{3}{2}$ .

Now assume the source is active (emits gravitational radiation) between  $\eta_{ini}$  and  $\eta_{fi}$ , and followingly? initial conditions  $\mathfrak{h}_{ij}(\eta_{\text{ini}}, \mathbf{k}) = \dot{\mathfrak{h}}_{ij}(\eta_{\text{ini}}, \mathbf{k}) = 0$ . Thus,

$$\mathfrak{h}_{ij}(\eta \geq \eta_{\text{ini}}, \mathbf{k}) = \frac{8\pi^2 G_{\text{N}}}{k^2} \int_{k\eta_{\text{ini}}}^{k\eta} d\tau' \sqrt{\tau\tau'} \left[ \mathcal{N}_{\nu}(\tau) \mathcal{J}_{\nu}(\tau') - \mathcal{J}_{\nu}(\tau) \mathcal{N}_{\nu}(\tau') \right] a(\tau') T_{ij}(\tau', \mathbf{k}), \quad (3.13)$$

which reduces to

$$\mathfrak{h}_{ij}(\eta \ge \eta_{\mathrm{fi}}, \mathbf{k}) = A_{ij}(\mathbf{k}) \sqrt{k\eta} \mathcal{J}_{\nu}(k\eta) + B_{ij}(\mathbf{k}) \sqrt{k\eta} \mathcal{N}_{\nu}(k\eta). \tag{3.14}$$

Combining Eq. (3.13) and Eq. (3.14) at  $\eta = \eta_{fi}$  gives the coefficients  $A_{ij}$  and  $B_{ij}$ :

$$A_{ij}(\mathbf{k}) = -\frac{8\pi^2 G_{\rm N}}{k^2} \int_{k\eta_{\rm ini}}^{k\eta_{\rm fi}} d\tau' \sqrt{\tau'} a(\tau') \mathcal{N}_{\nu}(\tau') T_{ij}(\tau', \mathbf{k})$$

$$B_{ij}(\mathbf{k}) = +\frac{8\pi^2 G_{\rm N}}{k^2} \int_{k\eta_{\rm ini}}^{k\eta_{\rm fi}} d\tau' \sqrt{\tau'} a(\tau') \mathcal{J}_{\nu}(\tau') T_{ij}(\tau', \mathbf{k})$$
(3.15)

3.2.2 Scalar Field Source (temp. name)

# Lattice $\frac{?}{}$ and N-body simulations

Chapter 4. Lattice- $^{?}$  and N-body simulations

# Part II Project

# Calculating Gravitational Waves from Domain Walls

TITLE (Possibly different Ch. name: "GWs from perturbed flat DW" or something ) TITLE ("Dynamics of Domain Walls"??)

• BenteBent (thin) → KatjaKaj (thick)

We consider a relatively general four-dimensional spacetime  $(\mathcal{M}, g_{\mu\nu})$  with continuous metric and associated coordinate system  $\{x^{\mu}\}$ . blah blah [...] ((Ishibashi and Ishihara, 1999))

#### 5.1 Dynamics of Domain Walls in the Thin-Wall Limit

Let the (2+1)-dimensional submanifold  $\Sigma$  embedded in  $\mathcal{M}$  represent the thin domain wall we henceforth shall refer to as "BenteBent" (or BB). This hypersurface divides the manifold into two separate regions  $(\mathcal{M}_{\pm})$ , allowing us to write  $\mathcal{M} = \mathcal{M}_{+} \cup \Sigma \cup \mathcal{M}_{-}$ . Allow indices  $a, b, c, \ldots$  run over 0, 1, 2, and assign the coordinate system  $\{y^a\}$  to  $\Sigma$ . Now, the world-volume metric

$$\gamma_{ab} = g_{\mu\nu} \frac{\partial x^{\mu}}{\partial y^{a}} \frac{\partial x^{\nu}}{\partial y^{b}} \tag{5.1}$$

is the induced metric on  $\Sigma$ . Hmmm, maybe something wrong here.  $\square$  ((Carroll, 2019))

We write the covariant action of the domain wall BenteBent as

$$S_{\rm BB} = -\sigma \int_{\Sigma} d^3y \sqrt{-\gamma}, \qquad (5.2)$$

where

Chapter 5. Calculating Gravitational Waves from Domain Walls

# Simulating Gravitational Waves from Domain Walls

Chapter 6. Simulating Gravitational Waves from Domain Walls

# Studying Gravitational Waves from Domain Walls

Chapter 7. Studying Gravitational Waves from Domain Walls

# **Discussion**

# **Conclusion and Outlook**

Chapter 9. Conclusion and Outlook

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# Appendix A I do not have an appendix