MODULE *Digraph*

A *digraph* is a pair consisting of a set of vertices and a set of edges

$Vertices(digraph) \triangleq digraph[1]$

$Edges(digraph) \triangleq digraph[2]$

$IsDigraph(digraph) \triangleq$
- $\land\ digraph = \langle Vertices(digraph),\ Edges(digraph)\rangle$
- $\land\ \forall\, e \in Edges(digraph) :$
  - $\land\ e = \langle e[1],\ e[2]\rangle$
  - $\land\ \{e[1],\ e[2]\} \subseteq Vertices(digraph)$

$Children(v,\ digraph) \triangleq$
$\{c \in Vertices(digraph) : \langle v,\ c\rangle \in Edges(digraph)\}$

RECURSIVE $Reachable(\_,\ \_,\ \_)$

$Reachable(v1,\ v2,\ dag) \triangleq$
- $\lor\ v1 = v2$
- $\lor\ \exists\, c \in Children(v1,\ dag) : Reachable(c,\ v2,\ dag)$

RECURSIVE $Descendants(\_,\ \_)$ union of reachable

$Descendants(vs,\ dag) \triangleq$ IF $vs = \{\}$ THEN $\{\}$ ELSE
- LET $children \triangleq \{c \in Vertices(dag) : \exists\, v \in vs : \langle v,\ c\rangle \in Edges(dag)\}$ IN
  - $children \cup Descendants(children,\ dag)$

$Parents(v,\ digraph) \triangleq$
- LET $incomingEdges \triangleq \{e \in Edges(digraph) : e[2] = v\}$
- IN $\{e[1] : e \in incomingEdges\}$