MODULE *Digraph*

A *digraph* is a pair consisting of a set of vertices and a set of edges

$Vertices(digraph) \triangleq digraph[1]$
$Edges(digraph) \triangleq digraph[2]$

$IsDigraph(digraph) \triangleq$
$\quad \land\ digraph = \langle Vertices(digraph),\ Edges(digraph) \rangle$
$\quad \land\ \forall\, e \in Edges(digraph) :$
$\quad\quad \land\ e = \langle e[1],\ e[2] \rangle$
$\quad\quad \land\ \{e[1],\ e[2]\} \subseteq Vertices(digraph)$

$Children(digraph,\ v) \triangleq$
$\quad \{c \in Vertices(digraph) : \langle v,\ c \rangle \in Edges(digraph)\}$

$Descendants(dag,\ vs)$ is the set of vertices reachable from any vertex in vs

RECURSIVE $Descendants(\_,\ \_)$
$Descendants(dag,\ vs) \triangleq$ IF $vs = \{\}$ THEN $\{\}$ ELSE
$\quad$ LET $children \triangleq \{c \in Vertices(dag) : \exists\, v \in vs : \langle v,\ c \rangle \in Edges(dag)\}$ IN
$\quad\quad children \cup Descendants(dag,\ children)$

The sub-*dag* reachable from the set of vertices vs:

$SubDag(dag,\ vs) \triangleq$
$\quad$ LET $vs2 \triangleq Descendants(dag,\ vs) \cup vs$
$\quad\quad\quad es2 \triangleq \{e \in Edges(dag) : e[1] \in vs2\}$ implies $e[2] \in vs2$
$\quad$ IN $\langle vs2,\ es2 \rangle$