

A *digraph* is a pair consisting of a set of vertices and a set of edges

$Vertices(digraph) \triangleq digraph[1]$   
 $Edges(digraph) \triangleq digraph[2]$

$IsDigraph(digraph) \triangleq$   
 $\wedge digraph = \langle Vertices(digraph), Edges(digraph) \rangle$   
 $\wedge \forall e \in Edges(digraph) :$   
 $\wedge e = \langle e[1], e[2] \rangle$   
 $\wedge \{e[1], e[2]\} \subseteq Vertices(digraph)$

$Children(digraph, v) \triangleq$   
 $\{c \in Vertices(digraph) : \langle v, c \rangle \in Edges(digraph)\}$

$Descendants(dag, vs)$  is the set of vertices reachable from any vertex in  $vs$

RECURSIVE  $Descendants(-, -)$

$Descendants(dag, vs) \triangleq$  IF  $vs = \{\}$  THEN  $\{\}$  ELSE  
 LET  $children \triangleq \{c \in Vertices(dag) : \exists v \in vs : \langle v, c \rangle \in Edges(dag)\}$  IN  
 $children \cup Descendants(dag, children)$

The sub-*dag* reachable from the set of vertices  $vs$ :

$SubDag(dag, vs) \triangleq$   
 LET  $vs2 \triangleq Descendants(dag, vs) \cup vs$   
 $es2 \triangleq \{e \in Edges(dag) : e[1] \in vs2\}$  implies  $e[2] \in vs2$   
 IN  $\langle vs2, es2 \rangle$