
MODULE *Digraph*

A *digraph* is a pair consisting of a set of vertices and a set of edges

$$\begin{aligned} \text{Vertices}(\text{digraph}) &\triangleq \text{digraph}[1] \\ \text{Edges}(\text{digraph}) &\triangleq \text{digraph}[2] \end{aligned}$$

$$\begin{aligned} \text{IsDigraph}(\text{digraph}) &\triangleq \\ &\wedge \text{digraph} = \langle \text{Vertices}(\text{digraph}), \text{Edges}(\text{digraph}) \rangle \\ &\wedge \forall e \in \text{Edges}(\text{digraph}) : \\ &\quad \wedge e = \langle e[1], e[2] \rangle \\ &\quad \wedge \{e[1], e[2]\} \subseteq \text{Vertices}(\text{digraph}) \end{aligned}$$

$$\begin{aligned} \text{Children}(\text{digraph}, v) &\triangleq \\ &\{c \in \text{Vertices}(\text{digraph}) : \langle v, c \rangle \in \text{Edges}(\text{digraph})\} \end{aligned}$$

Descendants(dag, vs) is the set of vertices reachable from any vertex in *vs*

RECURSIVE *Descendants*(-, -)

$$\begin{aligned} \text{Descendants}(\text{dag}, \text{vs}) &\triangleq \text{IF } \text{vs} = \{\} \text{ THEN } \{\} \text{ ELSE} \\ &\text{LET } \text{children} \triangleq \{c \in \text{Vertices}(\text{dag}) : \exists v \in \text{vs} : \langle v, c \rangle \in \text{Edges}(\text{dag})\} \text{ IN} \\ &\quad \text{children} \cup \text{Descendants}(\text{dag}, \text{children}) \end{aligned}$$

The sub-*dag* reachable from the set of vertices *vs*:

$$\begin{aligned} \text{SubDag}(\text{dag}, \text{vs}) &\triangleq \\ &\text{LET } \text{vs2} \triangleq \text{Descendants}(\text{dag}, \text{vs}) \cup \text{vs} \\ &\quad \text{es2} \triangleq \{e \in \text{Edges}(\text{dag}) : e[1] \in \text{vs2}\} \text{ implies } e[2] \in \text{vs2} \\ &\quad \text{IN } \langle \text{vs2}, \text{es2} \rangle \end{aligned}$$
