

EXTENDS *Integers, FiniteSets*

CONSTANTS

V the set of values to decide on
 , P the set of processes (typically $3f + 1$ nodes)
 , $Quorum$ the set of quorums (typically sets of $2f + 1$ nodes out of $3f + 1$)
 , $Blocking$ the set of blocking sets (typically sets $f + 1$ nodes out of $3f + 1$)
 , B the set of malicious nodes (typically f nodes)
 , $Leader(-)$ assigns a leader to each round

$Round \triangleq Nat$ the set of rounds

Each round consists of 3 voting phases:

$Phase \triangleq 1 \dots 3$

A votes is cast in a phase of a round and for a value:

$Vote \triangleq [round : Round, phase : Phase, value : V]$

A proposal is for a round and a value:

$Proposal \triangleq [round : Round, value : V]$

--algorithm *FastConsensus*{

variables

$proposals = \{\}$;
 $votes = [p \in P \mapsto \{\}];$
 $round = [p \in P \mapsto 0];$

define {

$Decided \triangleq \{v \in V : \exists Q \in Quorum, r \in Round : \forall p \in Q \setminus B : \\ [round \mapsto r, phase \mapsto 3, value \mapsto v] \in votes[p]\}$

$Safety \triangleq Cardinality(Decided) \leq 1$

$Proposable(v, r, HO) \triangleq HO$ is intended to be the set of processes the leader hears of

$\vee r = 0$ no constraint if it's round 0
 $\vee \wedge \forall p \in HO \setminus B : round[p] \geq r$
 $\wedge \exists Q \in Quorum : Q \subseteq HO$
 $\wedge \exists r2 \in 0 \dots (r - 1) : \forall p \in HO \setminus B : \forall vt \in votes[p] :$
 $vt.round < r \wedge vt.phase = 3 \Rightarrow$
 $\wedge vt.round \leq r2$
 $\wedge vt.round = r2 \Rightarrow vt.value = v$
 $\wedge \exists Bl \in Blocking :$
 $\wedge Bl \subseteq HO$
 $\wedge \forall p2 \in Bl \setminus B : \exists vt2 \in votes[p2] :$
 $\wedge vt2.round = r2$
 $\wedge vt2.phase = 2$
 $\wedge vt2.value = v$

$$\begin{aligned}
& Safe(v, r) \triangleq \\
& \quad \vee \quad r = 0 \\
& \quad \vee \quad \exists r2 \in 0 \dots (r - 1) : \exists Q \in Quorum : \\
& \quad \quad \forall p \in Q \setminus B : \forall vt \in votes[p] : \\
& \quad \quad \quad vt.round < r \wedge vt.phase = 3 \Rightarrow \\
& \quad \quad \quad \wedge \quad vt.round \leq r2 \\
& \quad \quad \quad \wedge \quad vt.round = r2 \Rightarrow vt.value = v \\
& \quad \quad \wedge \quad \exists Bl \in Blocking : \forall p2 \in Bl \setminus B : \exists vt2 \in votes[p2] : \\
& \quad \quad \quad \wedge \quad vt2.round = r2 \\
& \quad \quad \quad \wedge \quad vt2.phase = 1 \\
& \quad \quad \quad \wedge \quad vt2.value = v \\
& \quad \} \\
& \text{macro } vote(p, v, r, ph) \{ \\
& \quad votes[p] := votes[p] \cup \{ [round \mapsto r, phase \mapsto ph, value \mapsto v] \}; \\
& \quad \} \\
& \text{process } (proc \in (P \setminus B)) \{ \\
l0: & \quad \text{while } (TRUE) \\
& \quad \text{either } \{ \\
& \quad \quad \text{when } Leader(round[self]) = self ; \\
& \quad \quad \text{with } (v \in V) \{ \\
& \quad \quad \quad \text{if } (self \notin B) \\
& \quad \quad \quad \quad \text{with } (HO \in SUBSET P) \\
& \quad \quad \quad \quad \quad \text{when } Proposable(v, round[self], HO); \\
& \quad \quad \quad \quad \quad proposals := proposals \cup \{ [round \mapsto round[self], value \mapsto v] \} \\
& \quad \quad \quad \} \\
& \quad \quad \} \\
& \quad \text{or with } (v \in V, Q \in Quorum) \{ \\
& \quad \quad \text{when } Leader(round[self]) \notin B \Rightarrow [round \mapsto round[self], value \mapsto v] \in proposals ; \\
& \quad \quad \text{when } Safe(v, round[self]); \\
& \quad \quad \text{when } \forall vt \in votes[self] : \neg(vt.round = round[self]); \\
& \quad \quad \quad vote(self, v, round[self], 1); \\
& \quad \quad \} \\
& \quad \text{or with } (v \in V, Q \in Quorum) \{ \\
& \quad \quad \text{when } \forall p \in Q \setminus B : \\
& \quad \quad \quad [round \mapsto round[self], phase \mapsto 1, value \mapsto v] \in votes[p]; \\
& \quad \quad \text{when } \forall vt \in votes[self] : \neg(vt.round = round[self] \wedge vt.phase \geq 2); \\
& \quad \quad \quad vote(self, v, round[self], 2); \\
& \quad \quad \} \\
& \quad \text{or with } (v \in V, Q \in Quorum, Bl \in Blocking) \{ \\
& \quad \quad \text{when } \forall vt \in votes[self] : \neg(vt.round = round[self] \wedge vt.phase = 3); \\
& \quad \quad \text{when} \\
& \quad \quad \quad \vee \quad \forall p \in Q \setminus B : \\
& \quad \quad \quad \quad [round \mapsto round[self], phase \mapsto 2, value \mapsto v] \in votes[p] \\
& \quad \quad \quad \vee \quad \forall p \in Bl \setminus B :
\end{aligned}$$

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        [round  $\mapsto$  round[self], phase  $\mapsto$  3, value  $\mapsto$  v]  $\in$  votes[p];
        vote(self, v, round[self], 3);
    }
    or {
        round[self] := round[self] + 1;
    }
}
}

```
