Question 1.5

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0.1 Fundamentals of Macro & Micro Thermodynamics

- Solution by E. Erik Larsen

Consider two equal $1000cm^3$ cubes of copper. Initially separated, one has a temperature of $20°C$ and the other is at $100°C$. They are then brought into contact along one wall, but otherwise isolated from their surroundings. 1. Estimate how long it will take for the two cubes to come into equilibrium.

0.2 Imports

[1]:
```
import numpy as np
from scipy.constants import k, N_A
```

0.3 Physical Constants

[2]:
```
Cu = 63.55
conductivity = 385.0 # W/m K --> J/sm K

print(f'Atomic mass of copper: {Cu} g/mol  |  Thermal conductivity:␣
 ↪{conductivity} W/mK\n')
print('-' * 36 + '|' + '-' * 35, '\n')
print(f'Boltzmann\'s Constant: {k}  |  Avogadro\'s Number: {N_A}\n')
```

```
Atomic mass of copper: 63.55 g/mol  |  Thermal conductivity: 385.0 W/mK

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Boltzmann's Constant: 1.380649e-23  |  Avogadro's Number: 6.02214076e+23
```

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1 Solution

- Find number of atoms in each cube
- Find thermal energy of each cube
- Sum to get total energy
- Find new temperature
- Use thermal conductivity to estimate time

1.1 Atoms & Energies

[3]:
```python
# Calculate mass of each cube
m = 8.96 * 1000
print(f'Mass of each cube: {int(m)} g\n')

# Calculate atoms in each cube
atoms = m * (1 / Cu) * (N_A)
print(f'Atoms in each cube: {atoms}\n')

E1 = 3/2 * k * (20 + 273.15) * atoms
E2 = 3/2 * k * (100 + 273.15) * atoms
E_tot = E1 + E2

print(f'E1: {round(E1, 3)} J   E2: {round(E2, 3)} J\n\nTotal Energy: 
 ↪{round(E_tot, 3)} J')
```

```
Mass of each cube: 8960 g

Atoms in each cube: 8.49069727924469e+25

E1: 515475.226 J   E2: 656147.298 J

Total Energy: 1171622.524 J
```

1.2 Equilibrium Energy & Temperature

[4]:
```python
Eq = E_tot / 2

Eq_temp = E_tot / (3 * k * atoms)
print(f'Energy per cube at equilibrium: {round(Eq, 3)} J\n\n' +
      f'Equilibrium Temperature: {Eq_temp} K  --->  {round(Eq_temp - 273, 2)} 
 ↪C\n')

E_diff = round(Eq - E1, 3)
print(f'Transferred: {E_diff} J')
```

```
Energy per cube at equilibrium: 585811.262 J

Equilibrium Temperature: 333.15 K  --->  60.15 C
```

```
Transferred: 70336.036 J
```

1.3 Estimate Time

[5]:
```
time = round((E_diff ) / (conductivity * Eq_temp * .1), 2)

print(f'Time to equilibrium: {time} s')
```

```
Time to equilibrium: 5.48 s
```

2 Conclusion

- Equilibrium temperature: $60°C$
- It will take approximately 5.5 seconds to come to equilibrium.
