Practicum 5: The Hellings-Downs (HD) curve and the JHLM statistic

In this exercise you will compute the JHLM (see Ref. [1]) statistic for three pulsar timing residual data sets. Ref. [1] is included in you materials.

With your materials you also have a file called SkyPositions.txt. This file contains the right ascension (0-2π) and declination (0-π) in radians for 20 pulsars. You also have three sets of 20 files with pulsar timing residuals (dataset1-1.txt to dataset1-20.txt, dataset2-1.txt to dataset2-20.txt, dataset3-1.txt to dataset3-20.txt). Each file contains 500 data points of equally spaced timing residuals that span 10 years. These files potentially contain a gravitational wave signal (which is flat when expressed in terms of Ω).

The idea here is to use Eq. (1), Eq. (3), and Eq. (4) of Ref. [1] to make an estimate of the significance, which is $\rho\sqrt{N_p}$ [where ρ is given in Eq. (4)]. If M is the number of pulsars, $N_p = M(M-1)/2$ is the number of pulsar pairs.

Here's a suggestion on how to proceed:

- The practicum materials include a file (Practicum5.py) that loads in a data file with the sky locations of the pulsars. You'll first need to add some code that computes the and stores the value of the Hellings-Downs curve (using Eq. (3)), along with the angular separation for each pulsar pair. You'll need two loops over the number of pulsars to determine the angular separation and expected value of the HD curve for all pulsar pairs.
- For now just focus on the first data set (you can copy and paste your code for data sets 2 and 3 later once everything is working). The Practicum5.py code also reads in the dataset1 files for each of the pulsars. Add some code to Practicum5.py that computes and stores the correlation for each pulsar pair, i.e. Eq. (1) of [1]. Again, you'll just need add a couple of for loops over the number of pulsars.
- At this point you can make a plot of the correlation vs. angular separation for your pulsar pairs (this plot is included in your materials, dataset1HD.pdf, which shows the correlation versus angular separation in radians). Do you think there are gravitational waves in this data?
- You can now calculate the JHLM statistic Eq. (4) (which is also called the correlation or coherence, and you have code for this that you wrote for Practicum4) between the HD values you expected to get for each of your pairs and the ones that you actually found. You'll need to first compute the mean of the HD values $\bar{r}$, the mean of the correlations $\bar{\zeta}$, and their standard deviations (σ_r, σ_ζ).
- To calculate the significance you need to multiply your value of ρ by $\sqrt{N_p}$
- Once your code is working, copy and paste it and edit it to read data sets 2 and 3 and compute the significances. The sky positions of the pulsars are the same. You're done coding now.
- You should now have 3 values of the significance. Using Figure 1 below estimate the value of Ω in each of the three data sets (note that your estimates of the significance are for a particular instance of timing residuals, so you shouldn't expect perfect agreement with the expected significance curve). Examples of typical values of Ω (with flat spectra) are 0-10^{-6} for cosmic string scenarios, or around 10^{-14} for inflation.
- Why do you think the expected significance curve flattens out at large values of Ω?
- If you have extra time, here's something you could do. As you've learned earlier this week, timing residuals are produced by subtrating out a timing model from the TOAs. The largest effect is the subtraction of a quadratic, because it takes out the low frequencies where much of our signal is coming from. Try subtracting out quadratics from the data sets and see how that changes the significance.

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[1] Jenet et al.; ApJ. Letters 625, L123-L126 (2005).

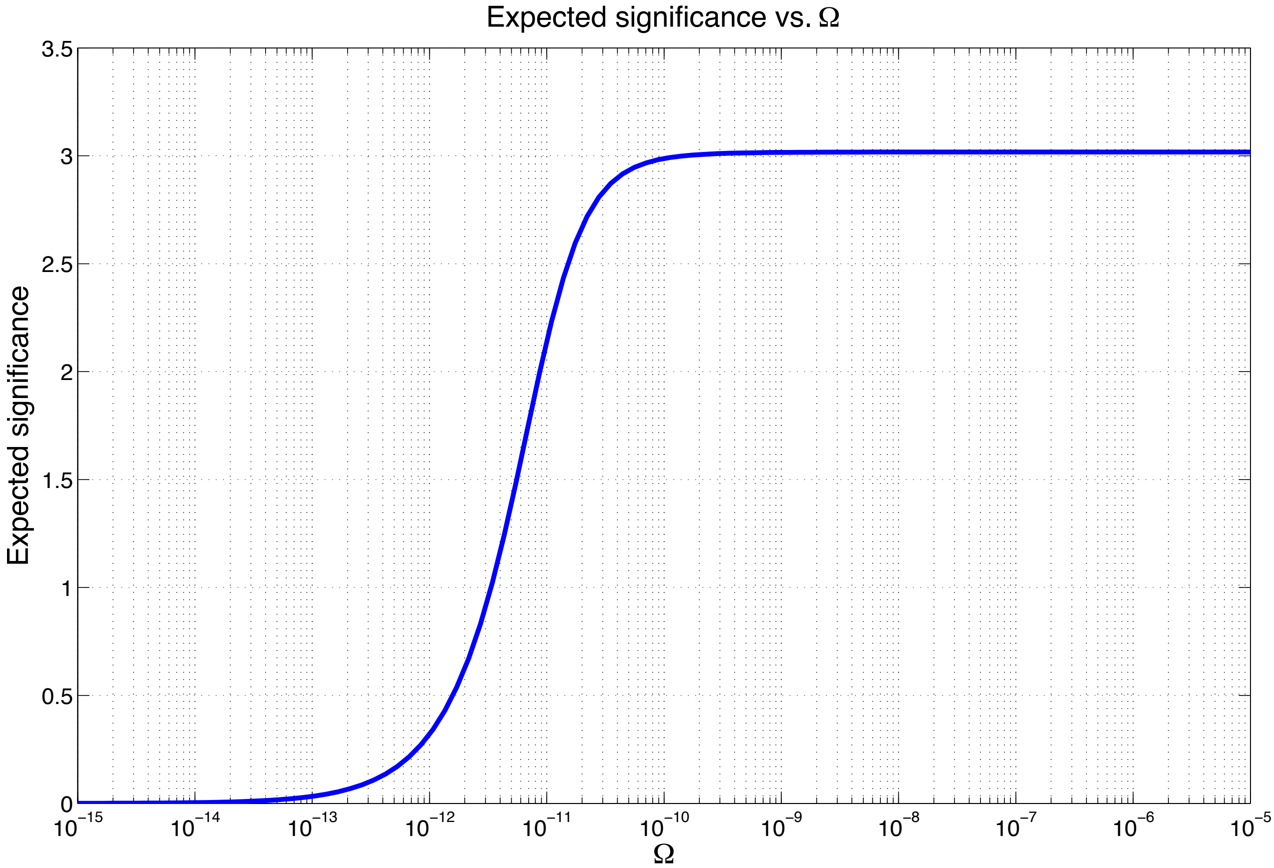

FIG. 1: Expected (average) significance versus Ω (flat spectrum).