Timing

Part 1 - TOAs

Pulse Times-of-Arrival (TOAs) are the fundamental data used in pulsar timing. The TOA of a pulse is the time at which the pulse was detected at the telescope. In practice, many pulses (commonly 100s or 1000s) are summed together to generate a single TOA. This is done to

- increase the signal-to-noise ratio of the pulse profile, and
- ensure the pulse profile has stabilized (individual pulses are quite variable, but average profiles are remarkably stable).

How TOAs are determined

The TOA of a pulse is the MJD corresponding to the fiducial point (often the peak is chosen to be this point) of a pulsar's profile. The phase of the fiducial point is determined by cross-correlating the observed profile with a standard template. This phase difference is converted to days and added to the MJD marking the start of the observed profile.

Given the observed profile (Fig. 1) what is its phase difference with respect to the standard template (Fig. 2)? (Estimate the phase difference roughly by using the plot.)

Assuming that phase=0.0 of the observed profile is at MJD 56724.251505493712, what is the TOA of the pulse? (Notice that the plot is centered on phase=0.0. Look up the spin period - `P0` - of J1713+0747 in the ATNF pulsar catalogue: http://www.atnf.csiro.au/people/pulsar/psrcat/. If you want to know what the units of MJD are, and how it relates to calendar dates look here: http://en.wikipedia.org/wiki/Julian_day)

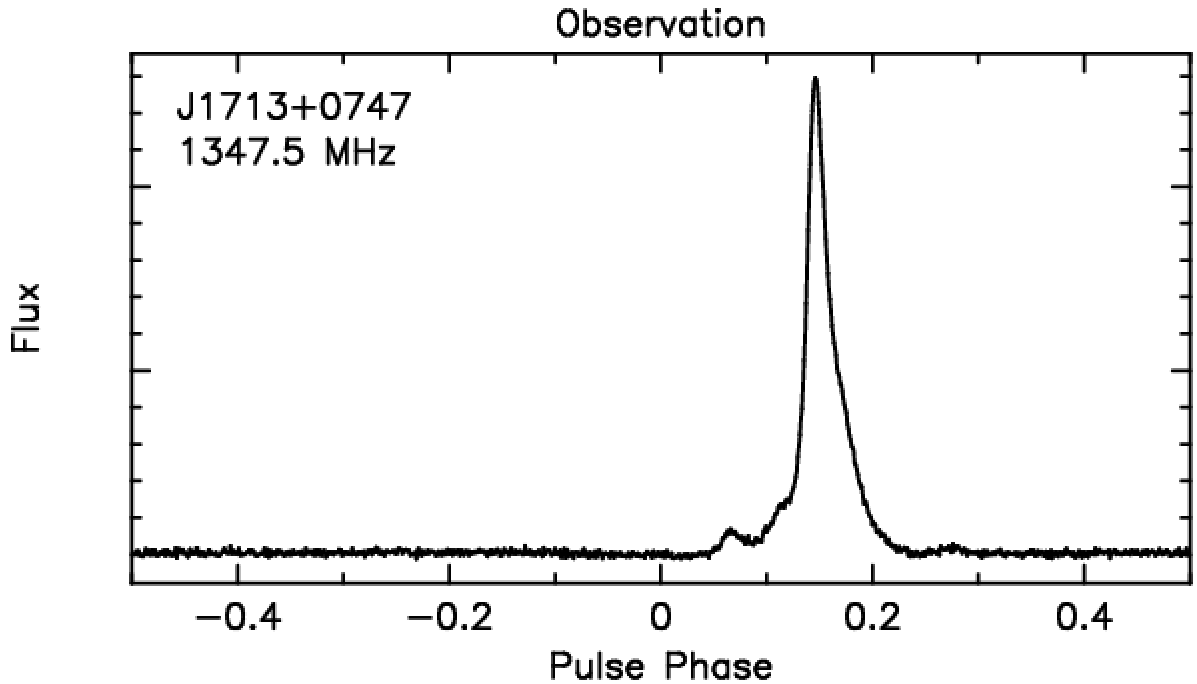


Fig. 1: Observed profile.

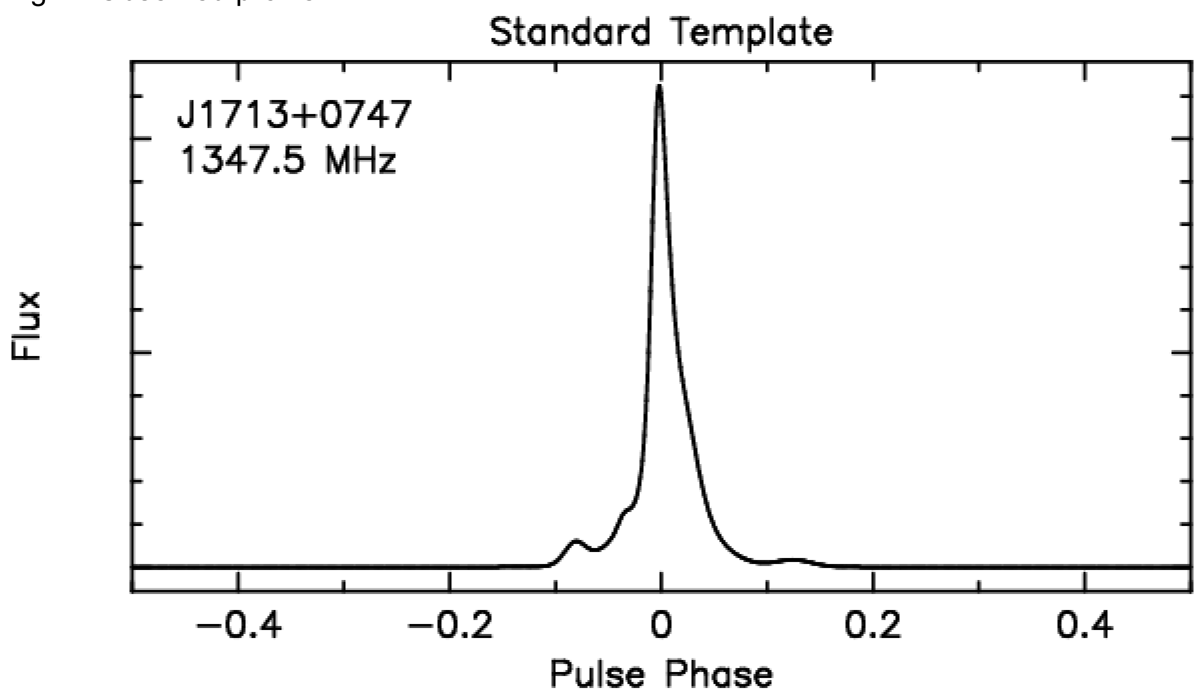


Fig. 2: Standard template.

Anatomy of a TOA

TOAs are specified in a text file, commonly known as a “timfile”. TEMPO2 recognizes several formats. The TEMPO2 format is:

```
<filename> <freq. (MHZ)> <TOA (MJD)> <TOA error (us)> <telescope> [<flags>]
```

Comment lines start with “#” or “C “.

Flags are optional supplementary information describing the TOA. They can have many uses, including

- colourizing plots,
- adding time, phase offsets,
- defining groups of TOAs,
- and much, much more.

Flags consist of a flag-name (starting with a dash), and a value. For example:

```
-rcvr <receiver name>
```

Multiple flags can be specified for a TOA. Flag names and values cannot contain spaces, and must be less than 32 characters.

Some flags trigger predefined behaviour in TEMPO2. For a list of these flags and their meaning see Table 4.1 in
http://www.atnf.csiro.au/research/pulsar/tempo2/index.php?n=Documentation.ObservationFiles.

Part 2 - Ephemerides

Parfile

Parameter files, commonly referred to as “parfiles” contain the ephemeris used to model the rotation of the pulsar. Lines in the parfile contain at least the name of the parameter, and the value, and optionally a flag that tells TEMPO2 to fit (flag=”1”), or not (flag=”0”, or no flag), and an uncertainty. The uncertainty is not used by the general TEMPO2 software.

Here is the a basic ephemeris for B1937+21:
For each of the parameters write the units on the appropriate line

			Units:
PSRJ	J1939+2134		
RAJ	19:39:38.558720	2.000e-06	____________
DECJ	+21:34:59.13745	3.000e-05	____________
DM	71.0398	2.000e-04	____________
PEPOCH	47899.5		____________
F0	641.9282611068082	1.300e-12	____________
F1	-4.331749E-14	9.000e-20	____________
PMRA	-0.46	2.000e-02	____________
PMDEC	-0.66	2.000e-02	____________

POSEPOCH	47500		____________
F2	4.0E-26	2.000e-27	____________
PX	0.12	8.000e-02	____________

For more information see the TEMPO2 documentation:
http://www.atnf.csiro.au/research/pulsar/tempo2/index.php?n=Documentation.Parameters

Part 3 - Using TEMPO2

Running TEMPO2

TEMPO2 requires two input files, specified with the “-f” command-line flag: a parfile, and a timfile. Execute TEMPO2 with the following input:

```
tempo2 -f part3/B1937+21.fit.par part3/B1937+21.tim
```

Looking through the output of `TEMPO2`, which parameters were fit for?

And, what are their uncertainties?

Just for fun

Put the uncertainty on the position of B1937+21 into context:

How far would a soccer ball (diameter ~ 22cm) need to be such that the angle it subtends is equal to the error in RA of B1937+21? If you are in this room in Banff where is the ball?

Just for fun *(cont.)*

The Graphical (PLK) Plugin

It is often helpful to look at the timing residuals, the difference between the measured TOAs, and what is predicted by the ephemeris. TEMPO2 has a graphical plugin called “plk”. The use the graphical plugin, run the following command:

```
tempo2 -gr plk -f part3/B1937+21.fit.par part3/B1937+21.tim
```

The “plk” plugin has many, many features. By pressing the ‘h’ key within the plot window a list of features will be printed in the text terminal.

Take a quick look at the list of options.

How do you get information for a data point (i.e. “identify” a point)?

How do you zoom and unzoom?

How do you use colours to highlight points from TOAs with a specific flag?

How do you delete a data point?

Fitting

It is possible to fit (and re-fit) the timing model to the TOAs within `PLK`. Often, it is more convenient to run `PLK` without having `TEMPO2` automatically fit the data before showing the plot (this is the default behaviour). To turn off `TEMPO2`'s fitting you simply need to provide the "`-nofit`" flag on the command line.

Within the `PLK` window, to select what parameters to fit for press on the boxes at the top of the window. To fit, press "Re-Fit". A few notes:

- Only TOAs that are within the MJD-range current selected in the plot will be used in the fit.
- The fit uses the current parameters values (i.e. the result of the last fit), not the parameters originally read in from the parfile.

The `general2` Plugin

There exist many other `TEMPO2` plugins. (It is even possible to write one yourself - although this is beyond the scope of this worksheet.) One useful plugin is "`general2`". It can print various attributes for each TOAs. This can be used, for example, to output the post-fit timing residuals, either for plotting, or for further analysis with another program.

To print out the pre-fit residuals run:

```
tempo2 -output general2 -f part3/B1937+21.fit.par part3/B1937+21.tim -s
'{bat} {pre} {err}\n'
```

The string provided with the "`-s`" flag specifies the information to be printed for each TOA.

- `{bat}` is the pulse time of arrival at the Solar System barycentre (in MJD)
- `{pre}` is the pre-fit residual (in s)
- `{err}` is the uncertainty on the TOA (in μs)

Use the `general2` plugin to print the **post**-fit residuals.

If you are comfortable with making error-bar plots practice plotting the residuals and compare them with what the `plk` plugin produces.

There are many more attributes that can be printed with general2. See http://www.atnf.csiro.au/research/pulsar/tempo2/index.php?n=Main.T2calculator for a sample.

Part 4 - Pulsar Spin

Basic Timing Model

The most basic model of pulsar rotation is a Taylor Expansion in frequency:

$$N = \nu\ (t - t_0) + \tfrac{1}{2}\dot{\nu}(t - t_0)^2 + \ldots$$

where N is the number of rotations (including the fractional part) since t_0, and $\nu,\ \dot{\nu}$ are the frequency, and frequency derivative, respectively.

Error in Spin Frequency

If the ephemeris used has an error in the spin frequency, that is

$$\nu = \nu_{true} + \delta\nu\,,$$

what would you expect the residuals to look like? Calculate the **mathematical expression** for the residuals as a function of time, and error in the frequency.

Recall that residuals are the difference between the phase of the pulsar predicted by the model (in this case the model is incorrect), and the observed phase of the pulsar (in this case assumed to be perfect).

Sketch the residuals:

Check your result by introducing an error in the value of `F0` in your parfile and looking at the residuals. (You might see some phase-wraps where the residuals go off the top of the plot at phase = 0.5, and continue from the bottom at phase = - 0.5.)

If there is an error of $\delta\nu = 2 \times 10^{-8}$ Hz in the frequency of B1937+21, how much time would it take before a phase wrap occurs?

Confirm your result by introducing this error into your parfile and running `TEMPO2`.

Error in Spin Frequency-Derivative

What would the residuals look like if the ephemeris has an error in the spin frequency,

$$\dot{\nu} = \dot{\nu}_{true} + \delta\dot{\nu} \, ,$$

and all other parameters are exact? Calculate the mathematical expression for the residuals as a function of time, and error in the frequency derivative.

Sketch the residuals:

Again, introduce an error into your parfile (this time in `F1`) and check the residuals to check your intuition.

The Power of Pulsar Timing - Expert

By considering the phase of the pulsar at each observation epoch pulsar timing is far more powerful than doing simple period measurements. To illustrate this you will first fit for **only** `F0` in two separate observations a few years apart.

What are the frequency, its uncertainty, and MJD of this first observing epoch?

```
tempo2 -gr plk -f part4/B1919+21.f1err.par part4/B1919+21.epoch1.tim
```

Repeat the fit for the last group of TOAs. For this second observing epoch, what are the frequency, its uncertainty, and MJD?

```
tempo2 -gr plk -f part4/B1919+21.f1err.par part4/B1919+21.epoch2.tim
```

From these frequency measurements, what do you derive for the error in frequency derivative (in Hz/s) with respect to the value in the parfile? *Hint: The error propagation equation is on the formula sheet provided.*

Now, we will look at all observations. Run `TEMPO2`

```
tempo2 -gr plk -f part4/B1919+21.f1err.par part4/B1919+21.all.tim
```

The residuals should look mostly linear. Try fitting for **only** `F0` over the entire data set. Are the residuals flat?

Now, restart `TEMPO2` and fit all the data for `F1` **only**. Does this flatten the residuals? What do you find for the error in frequency derivative and its uncertainty?

How many orders of magnitude more precise is the pulsar timing method compared to the frequency-difference method (i.e. compare the uncertainty from `TEMPO2` with the uncertainty on the value you computed earlier)?

Part 5 - Pulsar Astrometry

The motion of the Earth around the Sun introduces delays or advances of TOAs of up to ~500 s. The offset depends on the position of the pulsar with respect the ecliptic according to

$$t_c = A\cos(\omega t - \lambda)\cos\beta$$

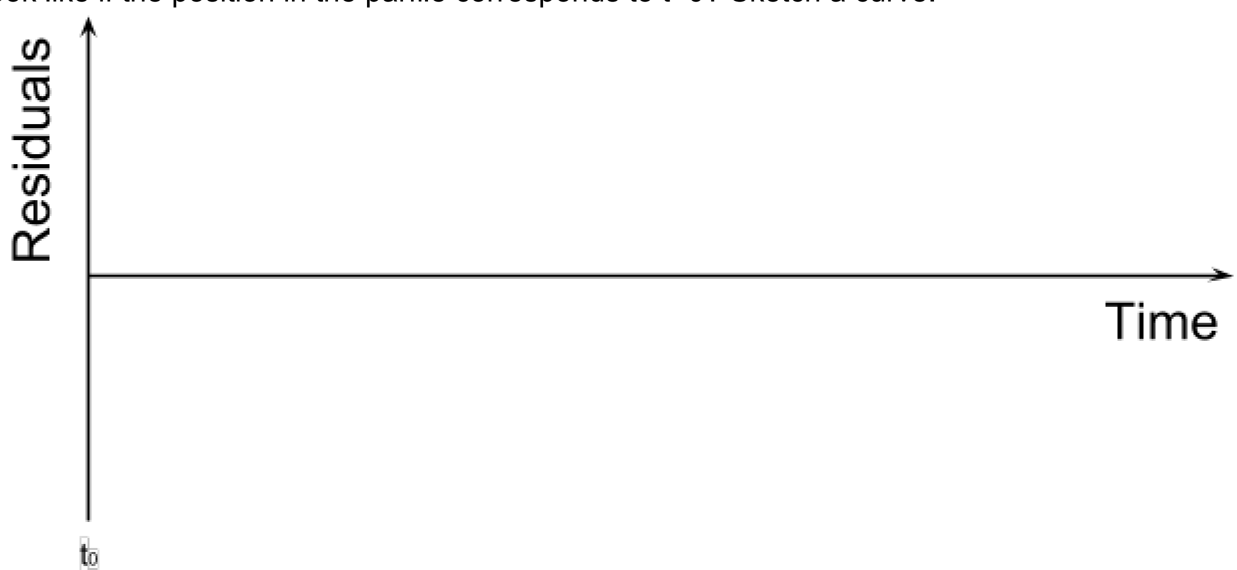
where A is the light travel time from the Earth to the Sun, ω is the angular velocity of the Earth, and λ, and β are the ecliptic longitude, and latitude of the pulsar, respectively. Given this relation derive the mathematical expression describing the residuals for when there is an error in pulsar position?

First consider a small angular error in ecliptic longitude, $\delta\lambda$, and then a small angular error in ecliptic latitude, $\delta\beta$. *Hint: There are some useful trigonometric identities on the formula sheet.*

Check your results using TEMPO2 by introducing a position error. Vary the size of the position error and see how the signature in the residuals changes. (The parfile provided uses the ELONG and ELAT parameters for ecliptic longitude and latitude, respectively.)

```
tempo2 -gr plk -f part5/B1937+21.ecliptic.par part5/B1937+21.tim
```

Based on your above results what does an un-modelled proper motion cause the residuals to look like if the position in the parfile corresponds to t=0? Sketch a curve.



Covariance between Spin-down and Position - Expert

For short time spans a position error can be well fit by F0 and F1. However, after some time absorbing the position error into the incorrect parameters becomes more difficult. To get a feeling for this run

```
tempo2 -gr plk -f part5/B1937+21.posnerr.par part5/B1937+21.tim
```

Zoom in to a short portion of the early residuals. Pick a range where the residuals look linear or slighly quadratic. Fit for F0/F1. The fit should look good. Zoom out and zoom into a slightly larger time span re-fit F0/F1. Does the fit look OK? Repeat the process of fitting with slighly more TOAs until your fit of F0/F1 can no longer absorb the position error. What is the time span? Does this make sense given what you know about the signature of a position error in the residuals, and the shape of the residuals fit out by F0 and F1?

Part 6 - Binary Pulsars

Binary-Specifics

There are quite a few known pulsars in binary systems. An annotated diagram can be found below (modified from Splaver et al., 2005, ApJ, 620, 405)

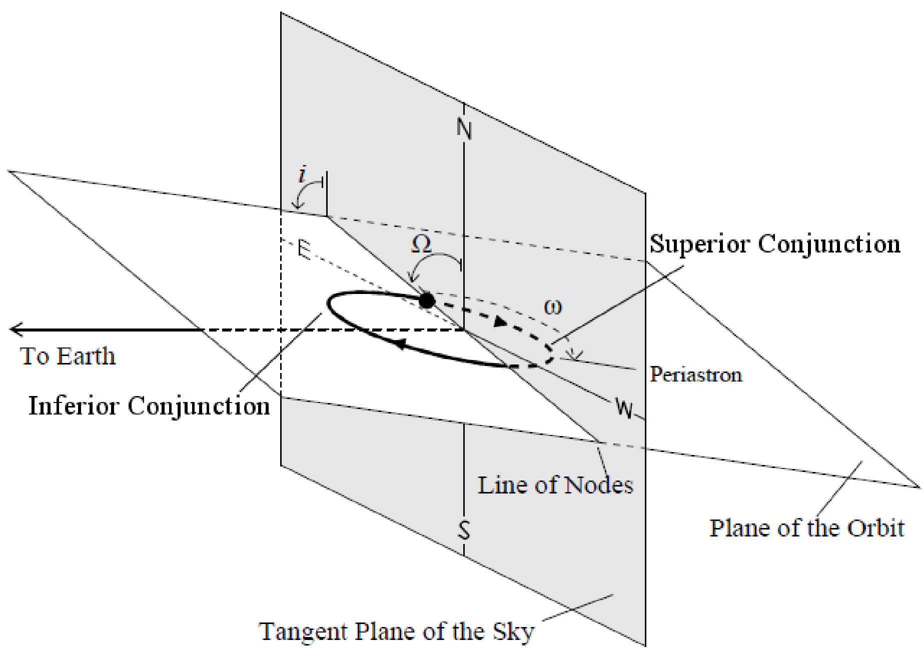


Fig. 3: Annotated schematic of a binary pulsar system (modified from Splaver et al. 2005).

TEMPO2 support many different parameterisations of binary motion of pulsars. Different models are useful in different systems. However, this is beyond the scope of this worksheet.

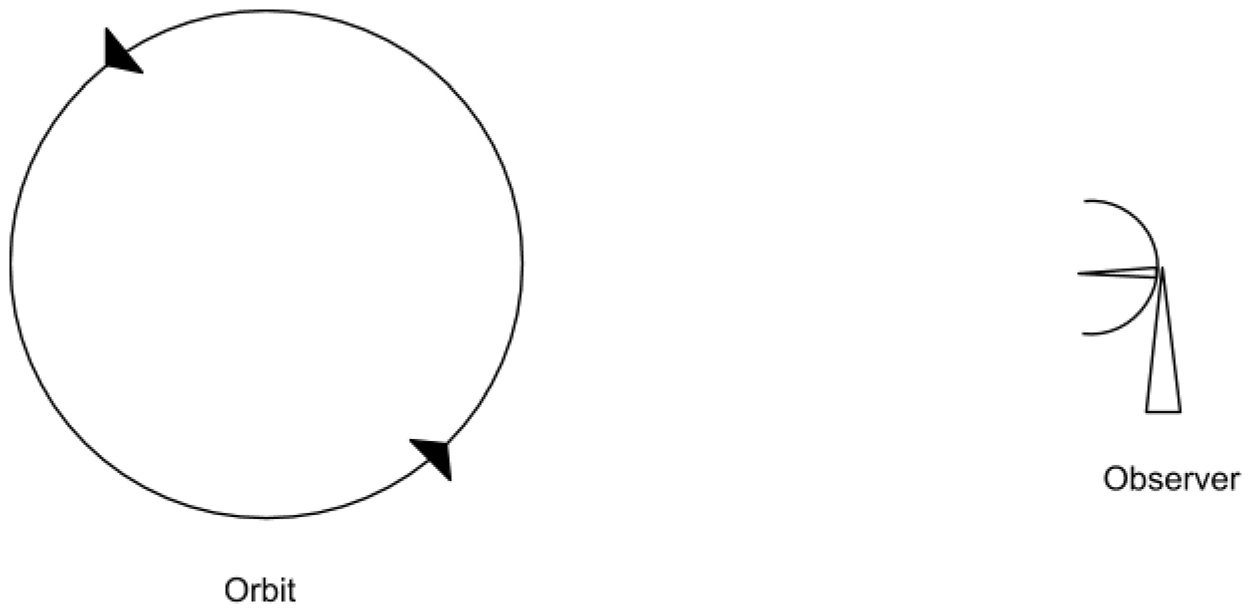
Doppler Shifts

Given a circular orbit viewed edge-on with the following parameters what is the largest observed frequency? *Hint: formula sheet.*

Spin Frequency	220 Hz
Orbital Period	68 days
Radius	24 light-seconds

If you were to observe the pulsar at this point, when the apparent frequency is maximum, but used the true spin frequency to reduce the data (i.e. you ignore the orbital motion), how long would it take for one phase wrap to occur?

Draw the pulsar at the position where the observed frequency is maximum? *(Bonus points for artistic drawings.*)



Mark in the above diagram where the residuals be maximum/minimum?

Part 7 - Phase Connecting (if time permits)

Phase connecting newly discovered pulsars is very rewarding, but can be very a tedious and frustrating process. Don't get discouraged. Rest assured, even seasoned pulsar timers have difficulty. Ask them. It's true.

In this part you will "solve" for the timing model of a pulsar using very rough initial parameters. This is similar to what is required to do after discovering a new pulsar.

Important notes about phase connecting with `TEMPO2`

- Re-fitting with TEMPO2 using PLK only uses TOAs within the time window displayed. This is very useful. Zoom into a subset of the time span where the residuals look structured. Fit parameters that you think model the particular shape of the residuals until they look flat.
- You might need to manually include a phase offset. This is done with "+" and "-" for +1 phase wrap and -1 phase wrap, respectively. The phase wrap is added at the position of the mouse. To remove all phase wraps press "backspace".
- If you include phase offsets you will need to remove them after fitting. (After fitting the phase offsets you manually added are included into the model parameters.)

- If you make a mistake it is usually easier to restart from scratch.

Isolated Pulsars

Start with the “easy” parfile. (It’s parameters are closer to the actual values.)

Open the timfile “`pulsarA.tim`”. On what MJDs was the pulsar observed? This type of cadence is quite common. Why?

Run `TEMPO2` on pulsarA.

```
tempo2 -gr plk -f part7/pulsarA.par part7/pulsarA.tim
```

Before starting look at the scale of the residuals in units of phase (vertical scale on the right-side edge of the plot in `PLK`.) What is the range?

Try to phase connect pulsarA.

Phase connecting with rough starting parameters - Expert

Repeat with “`pulsarB.par`”. The parameter values are quite rough. Phase connecting this pulsar may require you to start over if your parameters values diverge too much from the true values. Don’t be too shy to close `PLK` and re-open it.

*If you haven’t finished the **Expert** questions from previous parts of this worksheet. Go do them before attempting this problem.*

```
tempo2 -gr plk -f part7/pulsarB.par part7/pulsarB.tim
```

Want to learn more?

Check out the following references:

- *Pulsar Astronomy*, by Andrew Lyne , Francis Graham-Smith, Cambridge, UK: Cambridge University Press, 2012
  http://adsabs.harvard.edu/abs/2012puas.book.....L
- *Handbook of pulsar astronomy*, by D.R. Lorimer and M. Kramer. Cambridge observing handbooks for research astronomers, Vol. 4. Cambridge, UK: Cambridge University Press, 2004
  http://adsabs.harvard.edu/abs/2004hpa..book.....L
- *TEMPO2, a new pulsar-timing package - I. An overview*. Monthly Notices of the Royal Astronomical Society, Volume 369, Issue 2, pp. 655-672.
  http://adsabs.harvard.edu/abs/2006MNRAS.369..655H

Formulas

Error propagation

For $y = f(a, b)$ the error on y is

$$\sigma_y = \sqrt{\left(\frac{\partial f}{\partial a}\right)^2 {\sigma_a}^2 + \left(\frac{\partial f}{\partial b}\right)^2 {\sigma_b}^2}$$

Trigonometry

$$cos(\alpha \pm \beta) = cos(\alpha)\, cos(\beta) \mp sin(\alpha)\, sin(\beta)$$

Taylor Expansions

$$cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

$$sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

Uniform Circular Motion

$$v = \frac{2\pi r}{P}$$

Doppler Shift

$$f = \left(1 + \frac{\Delta v}{c}\right) f_0$$

where Δv is the relative velocity between the source and observer (positive for approaching).